Prospects of Incoherent Control by Continuous Measurements

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Motivation

- coherent control inefficient in systems with many dof (e.g. polyatomic molecules)
- incoherent dynamics: contractive evolution & steady states

→ robustness

- ways to induce controlled incoherent dynamics:
  1. environment engineering (Prezhdo, PRL 85, 4413 (2000))
  2. optical pumping (Wang, Schirmer, PRA 81, 062306 (2010))
  3. measurements (Roa et al., PRA 73, 012322 (2006))
Generalized position measurement

- measurement operators: $M_i(x)$, with $\sum_i \int M_i^\dagger(x) M_i(x) dx = 1$
- detection probability: $P_i(\psi) = \int |M_i(x)\psi(x)|^2 dx$
- post measurement state: $\psi(x) \xrightarrow{\text{outcome } i} \frac{M_i(x)\psi(x)}{\sqrt{P_i(\psi)}}$
- left-right measurement: $M_l(x) = \Theta(-x), M_r(x) = \Theta(x)$

Dynamics under continuous non-selective measurement

$$\mathcal{L}\rho = \dot{\rho} = -\frac{i}{\hbar} [H, \rho] + \gamma \left( \sum_{i \in \{l,r\}} M_i \rho M_i^\dagger - \frac{1}{2} \{M_i^\dagger M_i, \rho\} \right)$$

$$= -\frac{i}{\hbar} (H_{\text{eff}} \rho - \rho H_{\text{eff}}^\dagger) + \gamma \sum_i M_i \rho M_i^\dagger = (\mathcal{L}_0 + \mathcal{J})\rho$$

Deterministic evolution \hspace{2cm} Jumps

where $H_{\text{eff}} = H - \frac{i\hbar \gamma}{2} \sum_i M_i^\dagger M_i$
Generalized position measurement

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- detection probability: \( P_i(\psi) = \int |M_i(x)\psi(x)|^2 dx \)
- post measurement state: \( \psi(x) \xrightarrow{\text{outcome } i} M_i(x)\psi(x)/\sqrt{P_i(\psi)} \)
- left-right measurement: \( M_l(x) = \Theta(-x) \quad M_r(x) = \Theta(x) \)

Dynamics under continuous non-selective measurement

\[
\mathcal{L}\rho = \dot{\rho} = -\frac{i}{\hbar} [H, \rho] + \gamma \left( \sum_{i \in \{l, r\}} M_i \rho M_i^\dagger - \frac{1}{2} \{ M_i^\dagger M_i, \rho \} \right)
\]

\[
= -\frac{i}{\hbar} (H_{\text{eff}} \rho - \rho H_{\text{eff}}^\dagger) + \gamma \sum_i M_i \rho M_i^\dagger = (\mathcal{L}_0 + \mathcal{J})\rho
\]

\[
= -\frac{i}{\hbar} [H, \rho] + 2\gamma (M_r \rho M_r^\dagger - \frac{1}{2} \{ M_r^\dagger M_r, \rho \}) = (\mathcal{L}_r + \mathcal{J}_r)\rho
\]

\[
= -\frac{i}{\hbar} [H, \rho] + 2\gamma (M_l \rho M_l^\dagger - \frac{1}{2} \{ M_l^\dagger M_l, \rho \}) = (\mathcal{L}_l + \mathcal{J}_l)\rho
\]
Analytical treatment in terms of jump expansion and reordering

- expand time evolution in Dyson series
  \[ e^{(\mathcal{L}_0 + \mathcal{J})t} = e^{\mathcal{L}_0 t} + \sum_i \int dt_i \ldots dt_1 e^{\mathcal{L}_0 (t_i - t_{i-1})} \mathcal{J} e^{\mathcal{L}_0 (t_{i-1} - t_{i-2})} \mathcal{J} \ldots \mathcal{J} e^{\mathcal{L}_0 t_1} \]

- reordering of jump expansion:
  \[ e^{\mathcal{L}_t} = e^{\mathcal{L}_r t} + \int dt_1 e^{\mathcal{L}_i (t-t_1)} \mathcal{J}_r e^{\mathcal{L}_r t_1} \]
  \[ + \int dt_1 dt_2 e^{\mathcal{L}_r (t-t_2)} \mathcal{J}_l e^{\mathcal{L}_i (t_2 - t_1)} \mathcal{J}_r e^{\mathcal{L}_r t_1} + \ldots \]

  ➔ few jumps
  ➔ most relevant terms in front
Steering molecular wave packets: reflection by measurement & Zeno effect

- reflection coefficient of imaginary potential step (0th order)

\[
R^{(0)}(k) = \left| \frac{1 - \sqrt{1 + i\gamma/k^2}}{1 - \sqrt{1 + i\gamma/k^2}} \right|^2 \quad \gamma \to \infty \to 1 \quad \text{(Zeno effect)}
\]

- next contribution (2nd order): approximate \( \psi_{t_1}(k) \) after first jump & integrate negative velocity components

\[
R^{(2)}(k) \approx \int_{-\infty}^{0} (1-R^{(0)}(k)) |\psi_{t_1}(k)|^2
\]
Control of branching ratio between coupled Born-Oppenheimer surfaces

- Zeno control ($\gamma \to \infty$): trap wave packet inside strong coupling region

Free evolution & transitions measurement switch off measurement
Control of branching ratio between coupled Born-Oppenheimer surfaces

• Zeno control ($\gamma \to \infty$): trap wave packet inside strong coupling region

• for finite measurement rate $\gamma$, use Stückelberg interference
Summary and outlook

- incoherent scattering formalism
- optimal incoherent control