Banff International Research Station

for Mathematical Innovation and Discovery

Harmonic Analysis in Convex Geometry
May 15 - May 20, 2011

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in Max Bell 159 (Max Bell Building accessible by walkway on 2nd floor of Corbett Hall). LCD projector, overhead projectors and blackboards are available for presentations. Note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155–159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.

SCHEDULE

Sunday

16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
17:30–19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall Beverages and a small assortment of snacks are available on a cash honor system.

Monday

7:00 - 8:45	Breakfast
8:45 - 9:00	Introduction and Welcome by BIRS Station Manager, Max Bell 159
9:00 - 9:30	Wolfgang Weil, *Flag measures for convex bodies*
9:35 - 10:05	Rolf Schneider, *Zonoids with isotropic generating measures*
10:05 - 10:40	Coffee Break, 2nd floor lounge, Corbett Hall
10:40 - 11:10	Paul Goodey, *Harmonic analysis aspects of local and equatorial determination*
11:15 - 11:45	Gabriel Maresch, *The sine transform of isotropic measures*
11:45 - 13:00	Lunch
13:00 - 14:00	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:00	Group Photo; meet on the front steps of Corbett Hall
14:15 - 14:45	Franz Schuster, *Harmonic analysis of translation invariant valuations*
14:45 - 15:20	Coffee Break, 2nd floor lounge, Corbett Hall
15:20 - 15:50	Judit Abardia, *Projection bodies in complex vector spaces*
15:55 - 16:25	Thomas Wannerer, *Even Minkowski valuations*
17:30 - 19:30	Dinner

Tuesday

7:00 - 9:00	Breakfast
9:00 - 9:30	Eric Grinberg, *Tomography in Affine and Projective Geometries over finite fields*
9:35 - 10:05	Carla Peri, *Characterization of* $\{-1, 0, +1\}$ *valued functions in discrete tomography under sets of four directions*
10:05 - 10:40	Coffee Break, 2nd floor lounge, Corbett Hall
10:40 - 11:10	Hermann König, *On the maximal measure of sections of the n-cube*
11:15 - 11:45	Gautier Berck, *Regularized Integral Geometry*
11:45 - 13:30	Lunch
13:30 - 14:00	Igor Rivin, *Asymptotic geometry of convex sets.*
14:05 - 14:35	Alex Iosevich, *Distribution of lattice points near families of convex surfaces*
14:35 - 15:10	Coffee Break, 2nd floor lounge, Corbett Hall
15:10 - 15:40	Ivan Soprunov, *Lattice points in polytopes*
15:45 - 16:15	Viktor Vigh, *How to sew in practice?*
17:30 - 19:30	Dinner

Wednesday

7:00 - 9:00	Breakfast
9:00 - 9:30	Mark Rudelson, *Row products of random matrices*
9:35 - 10:05	Rafal Latala, *Tail inequalities for order statistics of log-concave vectors and applications*
10:05 - 10:40	Coffee Break, 2nd floor lounge, Corbett Hall
10:40 - 11:10	Peter Pivovarov, *Rearrangements and Isoperimetric Inequalities*
11:30 - 13:30	Lunch
	Free Afternoon
17:30 - 19:30	Dinner

Thursday

7:00 - 9:00	Breakfast
9:00 - 9:30	Elisabeth Werner, *How often is a random quantum state k–entangled?*
9:35 - 10:05	Deping Ye, *Threshold for separability of random induced states*
10:05 - 10:40	Coffee Break, 2nd floor lounge, Corbett Hall
10:40 - 11:10	Mathieu Meyer, *The convex intersection body*
11:15 - 11:45	Jaegil Kim, *Busemann's theorem and Convexity*
11:45 - 13:30	Lunch
13:30 - 14:00	Carsten Schütt, *Mahler's conjecture and curvature*
14:05 - 14:35	Shlomo Reisner, *On the volume product of polygons*
14:35 - 15:10	Coffee Break, 2nd floor lounge, Corbett Hall
15:10 - 15:40	Yehoram Gordon, *Some Geometric functional inequalities*
15:45 - 16:15	David Alonso-Gutiérrez, *On the factorization of Sobolev inequalities through classes of functions*
17:30 - 19:30	Dinner

Friday

7:00 - 9:00	Breakfast
9:00 - 11:30	Informal Discussions
10:00 - 11:00	Coffee Break, 2nd floor lounge, Corbett Hall
11:30 - 13:30	Lunch
	Checkout by 12 noon.

** 5-day workshops are welcome to use BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

Harmonic Analysis in Convex Geometry
May 15 - May 20, 2011

ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **Judit Abardia** (Frankfurt University)
Title: *Projection bodies in complex vector spaces*
Abstract: The projection body of a convex body in the Euclidean space was characterized by Monika Ludwig as the unique Minkowski valuation which is continuous, translation invariant and contravariant under the (real) special linear group. In a joint work with Andreas Bernig, we study the complex analog of this characterization result, i.e. we describe the space of Minkowski valuations on an n-dimensional complex vector space which are continuous, translation invariant and contravariant under the complex special linear group. A family of valuations satisfying these properties appear. We show that each of these valuations satisfy geometric inequalities of Brunn-Minkowski, Alexandrov-Fenchel and Minkowski type.

Speaker: **David Alonso-Gutiérrez** (University of Alberta)
Title: *On the factorization of Sobolev inequalities through classes of functions*
Abstract: We recall two approaches to recent improvements of the classical Sobolev inequality and give a relation between them. (Joint work with J. Bastero and J. Bernués)

Speaker: **Gautier Berck** (Polytechnic Institute of NYU)
Title: *Regularized Integral Geometry*
Abstract: Divergent integrals may appear in classical integral geometry due to the lack of compacity of certain isotropy subgroups. We will show how to get round this difficulty in specific situations replacing the invariant measures by invariant distributions.

Speaker: **Paul Goodey** (University of Oklahoma)
Title: *Harmonic analysis aspects of local and equatorial determination*
Abstract: We will discuss the harmonic analysis that underlies a number of recent results concerning the local or equatorial determination of various classes of bodies, both symmetric and non-symmetric. We will also explain how these results are related to certain support properties of functional operators associated with these geometric objects.

Speaker: **Yehoram Gordon** (Technion)
Title: *Some Geometric functional inequalities*

Speaker: **Eric Grinberg** (University of Massachusetts, Boston)
Title: *Tomography in Affine and Projective Geometries over finite fields*
Abstract: In the standard mathematical model of tomography, an unknown function in euclidean space is to be recovered from data regarding its integrals over certain families of lines, planes, etc. The treatment of this problem involves both the geometry of the collection of lines, planes etc., and the analysis of function spaces that model the data. Here we replace euclidean space by an affine or projective space over a finite field, so as to focus the recovery and inversion problem on the collection lines involved. We give a series

of properties of the Radon transform in this context culminating in a Gelfand-style admissibility theorem, which characterizes minimal sets of lines whose x-rays determine a function.This is joint work with David Feldman.

Speaker: **Alex Iosevich** (University of Rochester)
Title: *Distribution of lattice points near families of convex surfaces*
Abstract: We shall use operator bounds for generalized Radon transforms to obtain lattice point bounds previously approached using hands on number theoretic methods.

Speaker: **Jaegil Kim** (Kent State University)
Title: *Busemann's theorem and Convexity*
Abstract: Busemann's theorem states that the intersection body of an origin-symmetric convex body is also convex. In this talk we discuss how much of convexity is preserved under the intersection body operator. We provide a version of Busemann's theorem for each class of quasi-convex bodies and uniformly convex bodies. Indeed, it is proved that the intersection body of a p-convex body is q-convex for certain q, and also shown that every uniformly convex body with modulus of convexity of power type p has the intersection body of the same power type. Furthermore, we discuss the sharpness of the above results and a generalization to some general measure spaces with log-concave or s-concave measures. Part of this talk is based on joint work with Vladyslav Yaskin and Artem Zvavitch.

Speaker: **Hermann König** (University of Kiel)
Title: *On the maximal measure of sections of the n-cube*

Speaker: **Rafal Latala** (University of Warsaw)
Title: *Tail inequalities for order statistics of log-concave vectors and applications*
Abstract: We will present new tail estimates for order statistics of isotropic log-concave vectors and show how they may be applied to derive deviation inequalities for l_r norms and norms of projections of such vectors. Part of the talk is based on the joint work with Radoslaw Adamczak, Alexander Litvak, Alain Pajor and Nicole Tomczak-Jaegermann.

Speaker: **Wenbo Li** (University of Delaware)
Title: *Minimum volume for intersection of symmetric convex body and its rotation*
Abstract: We mainly focus on two special bodies, the cube (L_∞-ball) and the cross-polytope (L_1-ball). Connections with Gaussian correlation conjecture will be discussed.

Speaker: **Gabriel Maresch** (Vienna University of Technology)
Title: *The Sine Transform of Isotropic Measures*
Abstract: The cosine transform plays a fundamental role in modern geometric analysis. While not as well known as the cosine transform, its natural dual - the sine transform - appears in different guises in geometric tomography. In this talk we present sharp isoperimetric inequalities for the sine transform of even isotropic measures. The corresponding reverse inequalities are obtained in an asymptotically optimal form.

Speaker: **Mathieu Meyer** (Université Paris-Est Marne-la-Vallée)
Title: *The convex intersection body*
Abstract: We define the convex intersection body of a convex body, prove its convexity and ask some open questions about it (common work with Shlomo Reisner).

Speaker: **Carla Peri** (Università Cattolica)
Title: *Characterization of $\{-1, 0, +1\}$ valued functions in discrete tomography under sets of four directions*
Abstract: Recently, Hajdu and Tijdman introduced an algebraic approach for discrete tomographical

problems which is based on generating functions and divisibility properties of polynomials. We follow this approach to provide a characterization of $\{-1, 0, +1\}$ valued functions having zero line sums in four lattice directions. We then deduce uniqueness results for the reconstruction problem. Joint work with Sara Brunetti and Paolo Dulio.

Speaker: **Peter Pivovarov** (Texas A&M University)
Title: *Rearrangements and Isoperimetric Inequalities*
Abstract: I will discuss rearrangement inequalities and their use in isoperimetric problems for convex bodies and classes of measures. The main example will be from joint work with G. Paouris on the expected volume of random convex sets.

Speaker: **Shlomo Reisner** (University of Haifa)
Title: *On the volume product of polygons*
Abstract: We present a method that allows us to prove that the volume product of polygons in $\mathbb{R}^2$ with at most n vertices is bounded from above by the volume product of regular polygons with n vertices. The same method shows that the volume product of polygons is bounded from below by the volume product of triangles (or parallelograms in the centrally symmetric case). These last results give a new proof of theorems of K. Mahler. The cases of equality are completely described. Joint with Mathieu Meyer.

Speaker: **Igor Rivin** (Institute for Advanced Study)
Title: *Asymptotic geometry of convex sets.*
Abstract: I will talk about the "limit sets" convex sets of finite volume in hyperbolic space (and a little in euclidean space). I will indicate results on the dimension (Minkowski and Hausdorff) of such sets, and give some geometric corollaries.

Speaker: **Mark Rudelson** (University of Michigan)
Title: *Row products of random matrices*
Abstract: We define the row product of K matrices of size d by n as a d^K by n matrix, whose rows are entry-wise products of rows of these matrices. This construction arises in certain computer science problems. We study the question, to which extent the spectral and geometric properties of the row product of independent random matrices resemble those properties for a matrix with independent random entries. In particular, we show that while the general volume ratio property doesn't hold for these matrices, it still holds in case of a cross-polytope.

Speaker: **Rolf Schneider** (University of Freiburg)
Title: *Zonoids with isotropic generating measures*
Abstract: For the mixed volume of zonoids $Z_1, \dots, Z_k$ with isotropic generating measures in Euclidean space $\mathbb{R}^n$ and of the unit ball B^n, we show the inequality

$$V(Z_1, \dots, Z_k, B^n, \dots, B^n) \geq 2^k \kappa_{n-k}$$

for $k = 1, \dots, n$. For $k = 1$, this is an equality, and for $k \geq 2$, equality holds if and only if $Z_1 = \cdots = Z_k$ is a cube of side length 2. As a corollary, we obtain, for zonoids, reverse inequalities (in the sense of K. Ball's 'reverse isoperimetric inequality') to the Urysohn inequality and its generalizations. These inequalities were motivated, and have applications to, extremal problems for intersection densities of certain random hyperplane systems.

Speaker: **Franz Schuster** (Vienna University of Technology)
Title: *Harmonic Analysis of Translation Invariant Valuations*
Abstract: As a generalization of the notion of measure, valuations on convex bodies have long played a central role in geometry. The starting point for many important new results in valuation theory is Hadwiger's remarkable characterization of the continuous rigid motion invariant real valued valuations as

linear combinations of the intrinsic volumes. Among many applications, this result allows an effortless proof of the famous Principal Kinematic Formula from integral geometry.

In this talk, the decomposition of the space of continuous and translation invariant valuations into a sum of $\mathrm{SO}(n)$ irreducible subspaces is presented. It will be explained how this result can be reformulated in terms of a Hadwiger type theorem for translation invariant and $\mathrm{SO}(n)$ equivariant valuations with values in an arbitrary (finite dimensional) $\mathrm{SO}(n)$ module. From this perspective the classical theorem of Hadwiger becomes the special case when the $\mathrm{SO}(n)$ module is the trivial 1-dimensional one. (joint work with Semyon Alesker and Andreas Bernig)

Speaker: **Carsten Schütt** (University of Kiel)
Title: *Mahler's conjecture and curvature*
Abstract: It is based on a joint paper by S. Reisner, C. Schütt, E. Werner. Let K be a convex body in $\mathbb{R}^n$ with Santaló point at 0. We show that if K has a point on the boundary with positive generalized Gauß curvature, then the volume product $|K||K^\circ|$ is not minimal. This means that a body with minimal volume product has Gauß curvature equal to 0 almost everywhere and thus suggests strongly that a minimal body is a polytope.

Speaker: **Ivan Soprunov** (Cleveland State University)
Title: *Lattice points in polytopes*
Abstract: Let P be a lattice polytope (i.e. its vertices have integer coordinates). It is intuitively clear that if P has many lattice points (or interior lattice points) then the volume of P must be large. The opposite is, however, not true: there are lattice polytopes of arbitrary large volume with no interior lattice points. Therefore, in general there is no non-trivial lower bound on the number of interior lattice points of P in terms of the volume of P. Nevertheless, if P is a large enough multiple of a lattice polytope or, more generally, a Minkowski sum of sufficiently many lattice polytopes then such a lower bound exists. We will see how this bound is related to zeroes of polynomial systems, and will prove it in particular cases using properties of the Ehrhart polynomial.

Speaker: **Viktor Vigh** (University of Calgary)
Title: *How to sew in practice?*
Abstract: Neighbourly polytopes play an important role in the theory of convex polytopes. The most widely known neighbourly polytopes are the cyclic polytopes, which are quite simple to construct. In 1982, I. Shemer introduced the sewing construction, which allows one to take a given neighbourly polytope, and create a new (sewn) neighbourly polytope that has the same vertices as the original polytope plus one additional vertex. This allows one to construct a wide variety of neighbourly polytopes that are not necessarily cyclic. In this talk, we present some new results on the sewing construction and, as a corollary, a fast algorithm for sewing in practice. This is joint work with Ryan Trelford.

Speaker: **Thomas Wannerer** (Vienna University of Technology)
Title: *Even Minkowski valuations*
Abstract: Using harmonic analysis of translation invariant valuations and properties of cosine and Radon transforms on Grassmannian manifolds, we establish a new representation of even $SO(n)$-equivariant Minkowski valuations involving surface area measures. Furthermore, explicit formulas relating a previously obtained description of such valuations with the new more accessible one are also derived.

Speaker: **Wolfgang Weil** (Universitaet Karlsruhe)
Title: *Flag measures for convex bodies*
Abstract: Curvature measures and area measures are classical local analogs of intrinsic volumes (quermassintegrals), they arise naturally from a local version of the Steiner formula. Using a generalized local Steiner formula, measures on flag manifolds can be introduced for convex bodies. They are useful to obtain integral representations of mixed volumes and projection functions and also gives insight on properties of

valuations. The talk will present some results of this work in progress (jointly with P. Goodey, W. Hinderer, D. Hug and J. Rataj).

Speaker: **Elisabeth Werner** (Case Western Reserve University)
Title: *How often is a random quantum state k–entangled?*
Abstract: The set of trace preserving, positive maps acting on density matrices of size d forms a convex body. We investigate its nested subsets consisting of k–positive maps, where $k = 2, \ldots, d$. Working with the measure induced by the Hilbert-Schmidt distance we derive asymptotically tight bounds for the volumes of these sets. Joint with S. Szarek and K. Zyczkowski

Speaker: **Deping Ye** (Carleton University, University of Ottawa, and the Fields Institute)
Title: *Threshold for separability of random induced states*
Abstract: Quantum information theory is now one of the most active fields in science since the prospect of building quantum computers becomes more and more concrete. First discovered by Einstein-Podolsky-Rosen in 1935, quantum entanglement serves as fundamental and key ingredients for many objects in quantum information, such as, quantum algorithms, quantum key distributions, and quantum teleportation. Thus, detecting quantum entanglement is a central problem in quantum information theory.

A quantum state ρ on the N dimensional system $\mathcal{H}_N$ may be identified as a density matrix, i.e., an $N \times N$ positive semi-definite matrix with trace 1. It can be obtained by partial tracing over the K dimensional environmental system $\mathcal{H}_K$; namely, $\rho = MM^\dagger$ where M is a $N \times K$ (complex) matrix and $M^\dagger$ denotes its complex conjugate.

In this talk, I will present recent progress on estimating the threshold K, such that a random induced quantum state being separable and/or entangled. Our proofs rely on Random Matrices and Geometric Functional Analysis. This talk is based on a recent joint work with G. Aubrun and S. Szarek.