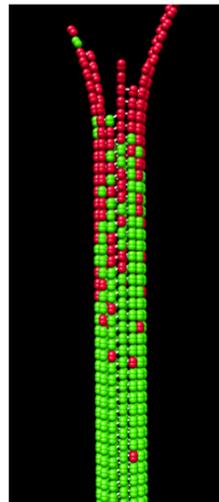
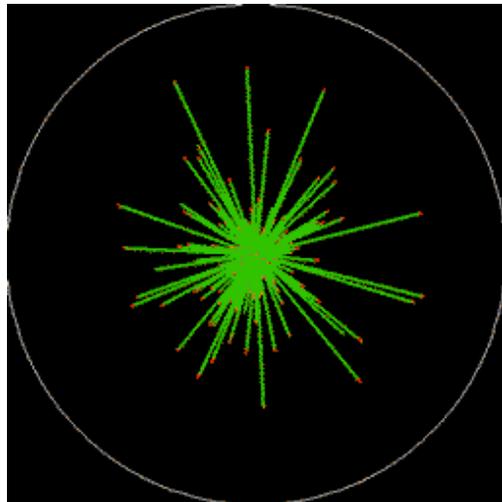


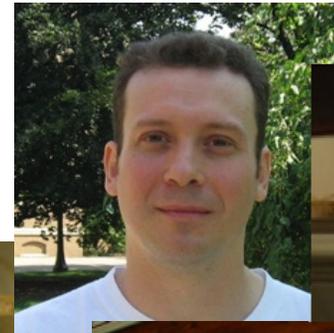
# Biological Insights from Computational Modeling of Microtubule Dynamics



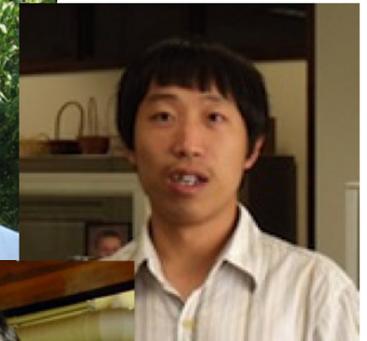
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University of Notre Dame

Thanks to:  
NSF 0951264  
University of Notre Dame  
David Odde, Sid Shaw

# Long-term interest: MT cytoskeleton in cell function

GFP-CLIP-170  
In COS-7 cells

## MT binding proteins

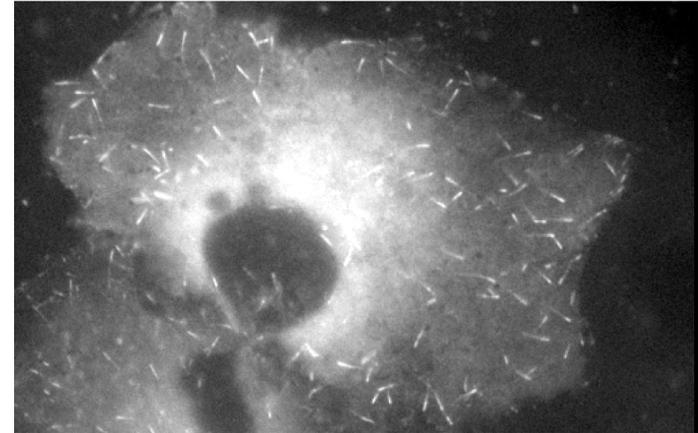
- effect on MT dynamics
- mechanism of effect on MT dynamics

## Problem:

- How does one make **quantitative predictions** about effect of a MT binding protein on MT dynamics?
- How can you make *any* quantitative predictions about MT dynamics?

## Basic Questions:

- What happens if a given amount of tubulin is polymerized?  
How much polymer? MT lengths? Transition frequencies?
- How are answers influenced by:  
# nuclei? spatial constraints of a cell? MAPs?
- Mechanism of dynamic instability? Effect of MAPs on DI?



**Goal:** use computational modeling to **address these questions**, gain **intuitive feel** for MT systems

# Computational models of MT dynamics

## Requirements:

- Based on known structural, biochemical attributes
- Minimal number assumptions

**Explicit** and **intuitively understandable**

- Fast enough to allow simulation of a system of many MT over a biologically relevant span of time

## Approach:

Borrow enthusiastically from existing work!  
Adjust existing models to answer our questions

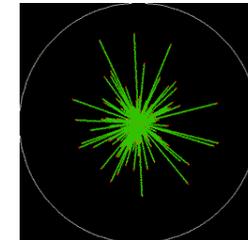
{  
Chen and Hill PNAS 1985  
Flyvbjerg et al., PhysRev 1994  
Van Buren et al., PNAS 2002, BJ 2005  
OTHERS!  
}

## Two models, two scales, different applications

### 1) Mesoscopic or “microscope-scale” model

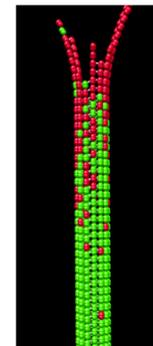
*Influence of physical constraints on behavior  
of a system of MTs*

Gregoretto et al. J Cell Sci. 2006



### 2) Microscopic or “molecular-scale” model

*Mechanisms of catastrophe, rescue, MAPs  
Effect of MAPs on systems of MTs*



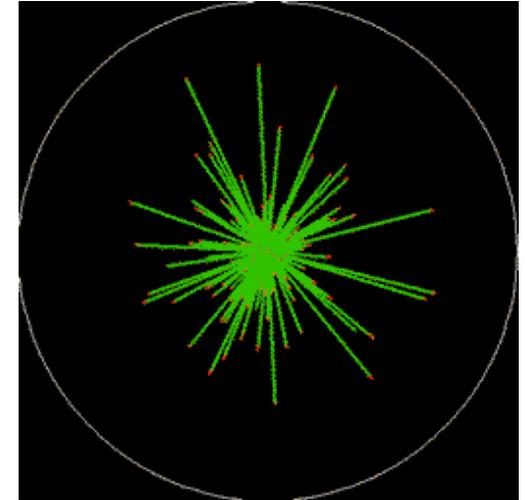
**Both models:** *relationship\_between behavior of **bulk polymer** and **single MTs***

# Outline:

## Biological Insights from Computational Modeling of Microtubule Dynamics

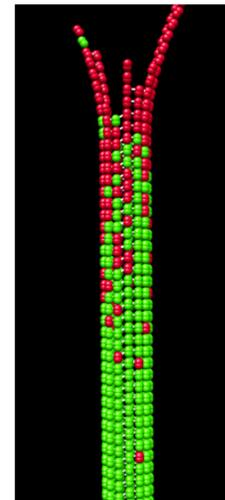
### I. Brief summary of what learned from study of mesoscopic model

Gregoretto et al. J Cell Sci. 2006



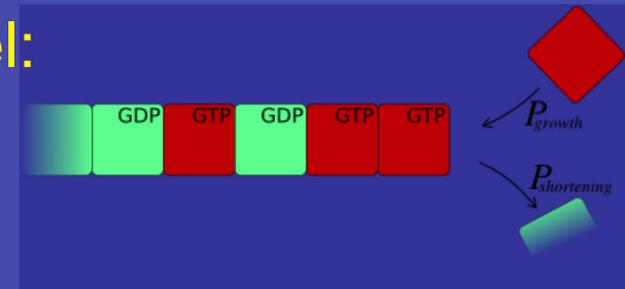
### II. Discuss findings with molecular scale model

- Mechanism of MT dynamics
- Revisiting concept of “critical concentration”



## Key elements of “microscope-scale” model:

- “Monte Carlo” (stochastic) model
- MTs are simplified linear polymers  
*obvious oversimplification...*



- Polymerized tubulin has two states: “GTP” or “GDP”
  - states could represent other conformations
- Addition and loss of tubulin subunits, GTP hydrolysis occur according to defined probabilities

- subunit addition depends on  $[\text{tubulin}_{\text{free}}]$ ; loss does not
- probabilities depend on the identity of the terminal subunit
- GTP hydrolysis is stochastic, not vectorial
  - ↳ Find parameters that give DI like that seen at interphase
  - Explore parameter space for robustness

### User-defined parameters:

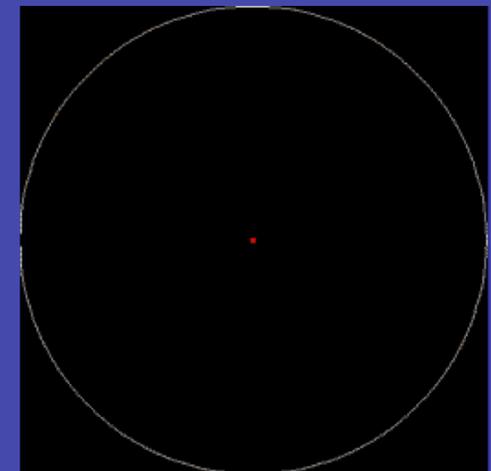
- [total tubulin]
- cell size
- # nucleation sites
- GTPase,  $k_{\text{on}}$ ,  $k_{\text{off}}$

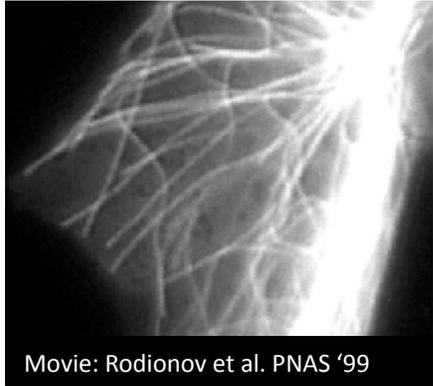
### Emergent parameters:

- growth, depol rates
- transition frequencies
- [polymer], [free dimer]

### Differences:

- **Model SYSTEM of competing MTs**
  - MTs grow from defined numbers of nucleation sites
  - Growing MTs **compete** for unpolymerized dimers
  - DI parameters are **emergent, not assigned!**
- **Addition of tubulin subunits can be limited by the cell "edge"**  
(probability of tubulin addition is less likely)





## Questions for “microscope scale” model:

### 1) What causes persistent growth of MTs inside cells?

- growth in interior of cells is persistent
- classic dynamic instability out near cell edge

Komarova et al., JCS 2002

↪ Asymmetry in MT dynamics induced by interaction of MT with cell edge

Komarova et al., JCS 2002, Janson et al., JCB 2003

But: What causes persistent growth in first place?

MAPs? Gradient of MAPs??

### 2) What causes dramatic changes in MT dynamics through cell cycle?

interphase: MTs are long, persistent growth in interior

mitosis: MTs are short, dynamic

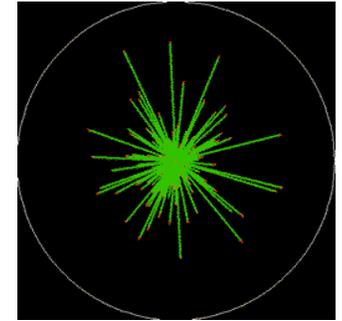
e.g., Rusan et al., MBOC 2001

↪ Regulation of MAPs induces changes

↪ Only answer??

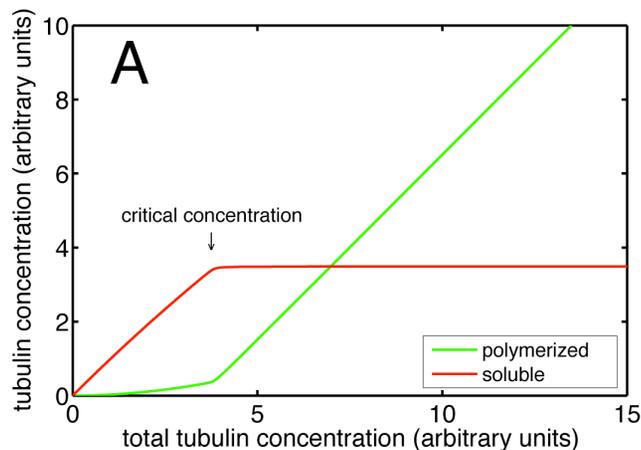
# Biological conclusions from studying behavior of “microscope-scale” computational model

Gregoretta et al. J Cell Sci. 2006

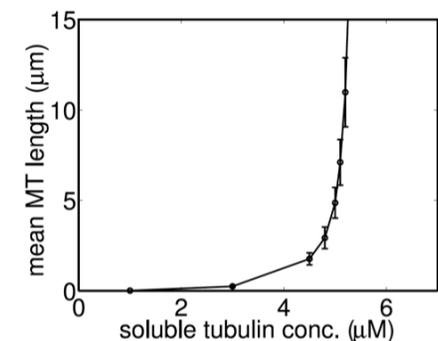
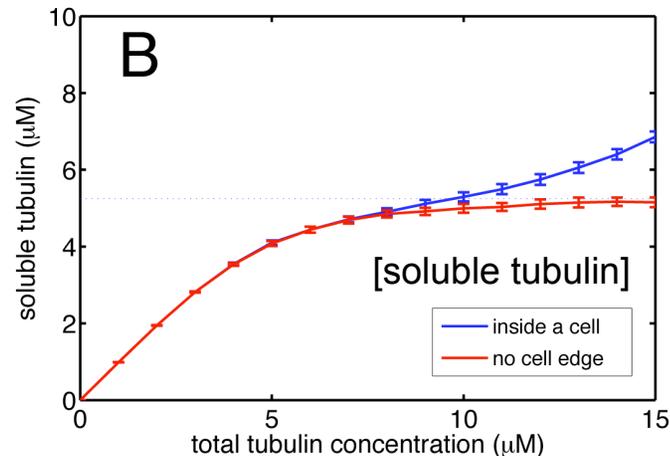


## 1. Persistent MT growth seen *in vivo* is an unavoidable outcome of putting sufficient tubulin in a confined space

Classical “critical concentration”



The model



## Spatial limitation causes [free tubulin] to rise above the $C_c$

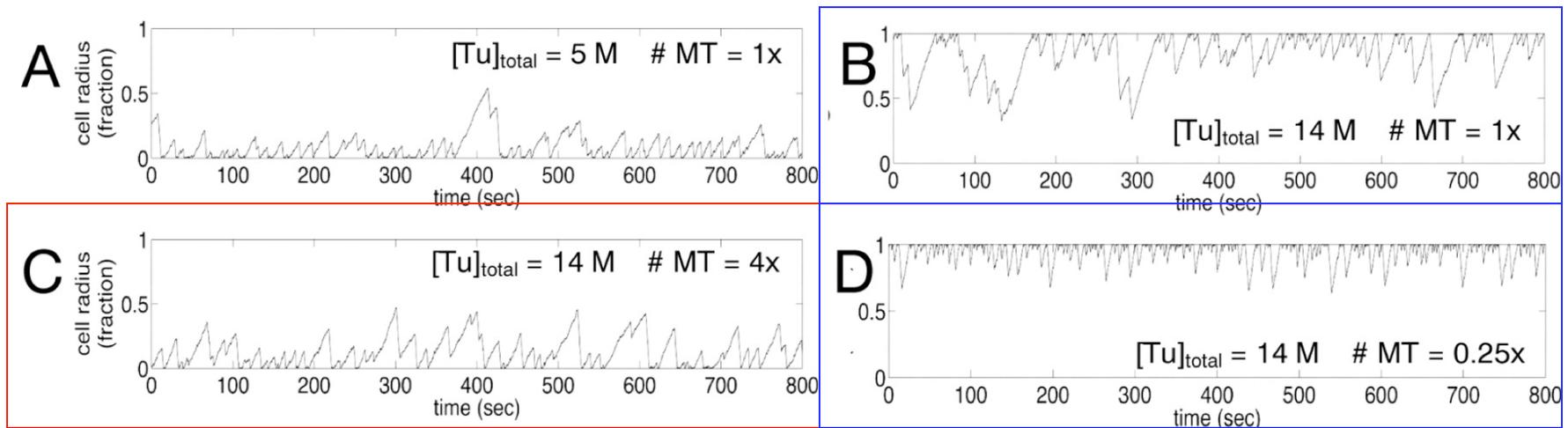
- more total tubulin → longer MTs
- long MTs reach cell edge
- cell edge induces catastrophe
- induction of catastrophe increases [free tubulin] above natural  $C_c$

↳ Maps involved  
Not required

**Increased [free tubulin] ↑ rescue, ↓ catastrophe: persistent growth**

## 2. Physical environment (cell size, # nuclei) influences MT dynamics

- Effects of physical constraint are **global** -- not just at cell edge
- Mutations that alter nucleation are *expected* to cause changes at + end
- Increase in MT dynamics seen at mitosis could be a simple outcome of the mitotic increase in MT nucleation



### MAPs “tune” dynamics, but do not dictate them

- Even with MAPs, transition frequencies depend on [free tubulin]  
[free tubulin] depends on cell size, total tubulin, # nuclei...

→ *Cell simulations that fix DI parameters are missing part of the picture....*

## Lots of remaining questions....

- **How do MAPs alter MT dynamics?**

How would a lateral X-linker differ from a GTPase inhibitor?

- **How do MAPs alter systems of dynamic MTs?**

*Important to compare:*

- Confined systems (*in vivo*-like) - Non-confined systems (*in vitro*-like)

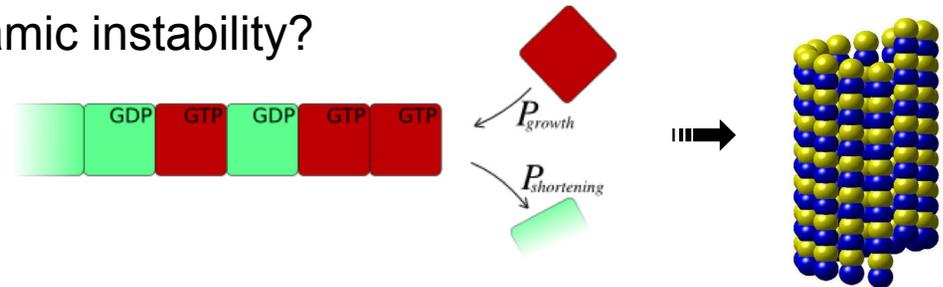
↳ *MAP effects won't always be the same!*

→ Need for comparing *in vitro* expts to each other, *in vivo* work

- **How do MTs work?**

What is the mechanism of dynamic instability?

Need more detailed model:



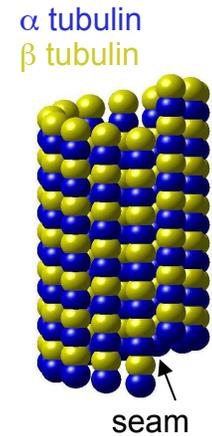
## Additional Requirements for Molecular Scale Model:

- **Detailed** enough to incorporate MAP binding, release, varied activities
  - Account separately lateral and longitudinal bonds
  - Consistent with knowledge of **MT structure, biochemistry**
- **Fast** enough to allow simulation of individual MTs, systems of *many* MTs over tens of minutes (allow comparison to DI experiments)

# Molecular Scale Model:

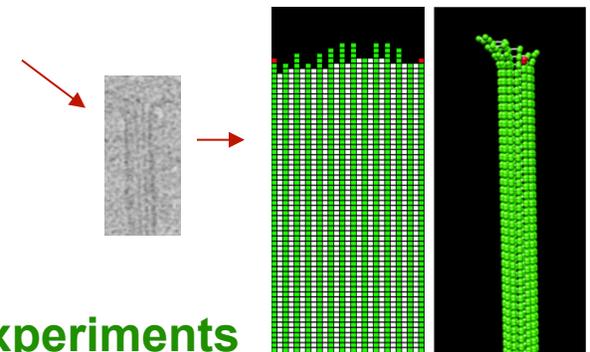
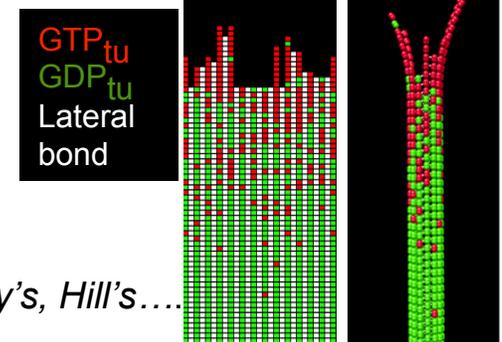
## Approach:

- utilize existing work (VanBuren 2005)
- modify to optimize combination of structural detail and speed  
*no need to reinvent wheel!*
- **Similar to ‘microscope-scale’ model, but higher resolution**
  - cylindrical lattice of 13 protofilaments (13\_3 lattice with seam)
    - ↳ *projected as flat structure* seam protofilament duplicated in visualization
  - subunits: individual tubulin dimers
    - user defined values: rate constants, [tubulin], cell size...
    - emergent values: all else



## Differences from previous models:

- **Interactions are consistent with structure** *not helical like Bayley's, Hill's....*
- **Lateral bonds are modeled explicitly**
  - Bonds form, break according to user-defined rate constants
  - ↳ **laterally unbound GDP protofilaments**  $\approx$  “ram’s horns”
    - ↳ *visualized straight, act kinetically as curved*
      - exploration of the mechanisms of rescue, catastrophe
      - regulation by MAPs



*All like VanBuren2005, but:*

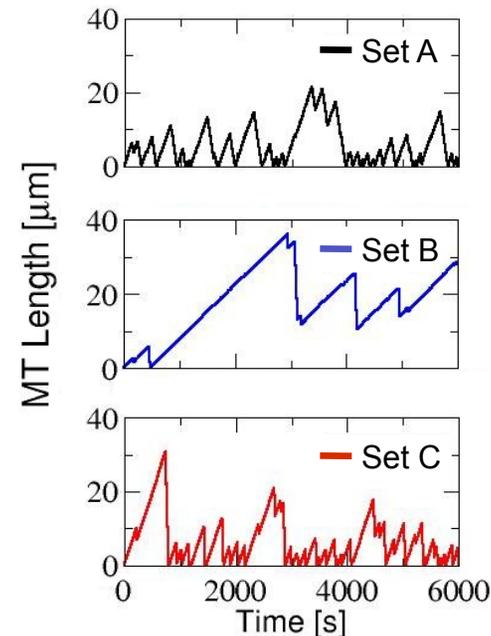
- **Simulate > tens of minutes: full dynamic instability experiments**
  - Mechanical influences on subunits approximated kinetically
    - presence of laterally bound neighbors influences lateral  $k_{\text{bond}}$ ,  $k_{\text{break}}$

# First: Model recapitulates experimental MTs...

## 3 parameter sets: A,B,C

- Differ significantly  
e.g.,  $k_{h-A} = .2\text{sec}^{-1}$ ;  $k_{h-B} = .7\text{sec}^{-1}$
- Focus on C: *most tuned to bovine-brain tubulin*  
Examine all three → parameter specific?

## 1. Displays dynamic instability (catastrophes AND rescues)



## 2. DI similar to BB tubulin

Parameter Set	[tubulin] ( $\mu\text{M}$ )	$K_h$ ( $\text{aut}^{-1}$ )	$V_g$ (dimer lengths/sec)	$V_s$ (dimer lengths/sec)	$F_c$	$F_r$	cap (dimer lengths)
A	14	0.2	$5.31 \pm 0.07$	$11.7 \pm 0.1$	$0.00793 \pm 0.0005$	$0.00243 \pm 0.0007$	$25.3 \pm 3.1$
B	10	0.25	$1.84 \pm 0.01$	$46.1 \pm 4$	$0.00181 \pm 0.0001$	$0.0405 \pm 0.01$	$5.7 \pm 1.2$
C	10	0.7	$5.64 \pm 0.04$	$60.8 \pm 3$	$0.00962 \pm 0.002$	$0.0185 \pm 0.007$	$8.8 \pm 1.5$

Walker 1988  
+ others

Experiment:

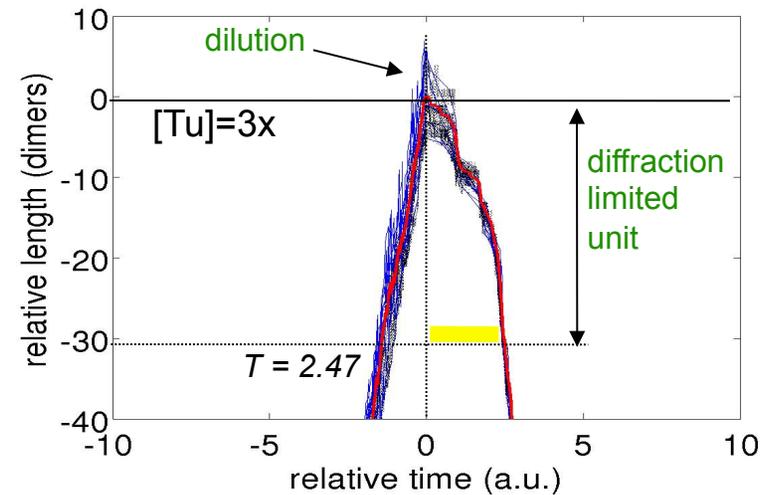
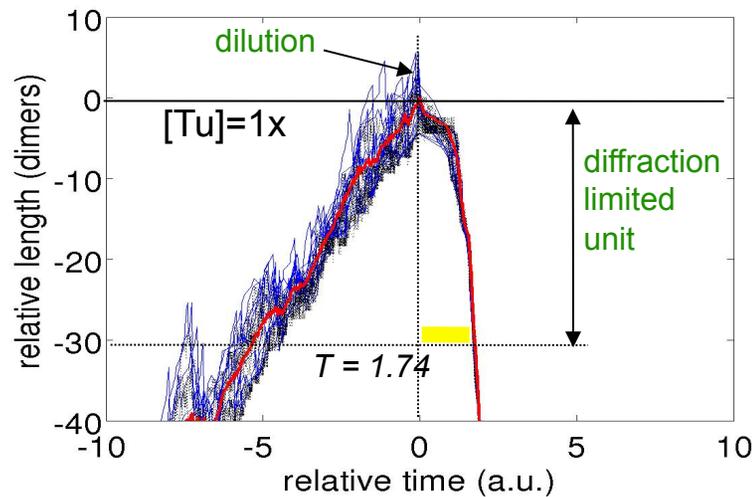
10    0.2-1    ~3    ~60    0.008    0.024    1-20

### 3. Reproduces experimental “sudden dilution” experiments:

➤ pre-dilution growth velocity has little effect on “time to catastrophe”

Parameter Set	Time to depolymerization (s)	
	10 $\mu\text{M}$ [Tu]	30 $\mu\text{M}$ [Tu]
A	4.44 +/- .57	4.29 +/- .78
B	2.23 +/- .44	3.39 +/- .39
C	1.78 +/- .47	2.5 +/- .30

↪ has been used to argue against existence of extended GTP cap



**Summary: all as expected**

In addition:

4. **GMPCPP**-bound MTs are **stable to dilution** over extended time (set C)
5. **Dependence of DI parameters on [tubulin]** does show some **deviation...**

## Inferences/Predictions from molecular-scale model:

### Shape of tip during growth:

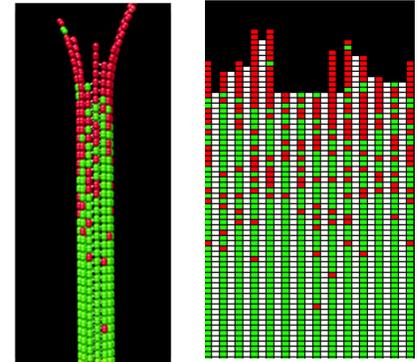
#### 1. Closed tube with multi-protofilament extensions

*regardless of parameter set*

➔ **Not able to observe expected open tube**

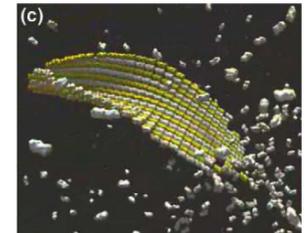
Considerable tuning: *no open sheets*

weaker seams: more frequent catastrophes



#### ↪ Failure to observe sheets due to failure of model?

### Suggest: Reconsideration of idea that MTs grow as sheets



Nogales COCB  
2006

a) CryoEM work showing “sheets” seems equally consistent with extensions

Chretien, 1995



b) Growth of an extended open sheet implies

lateral bonds > longitudinal bonds

- Otherwise subunits at sheet edge should fall off
- resolve sheet to tube

- Blunt sheets particularly unlikely

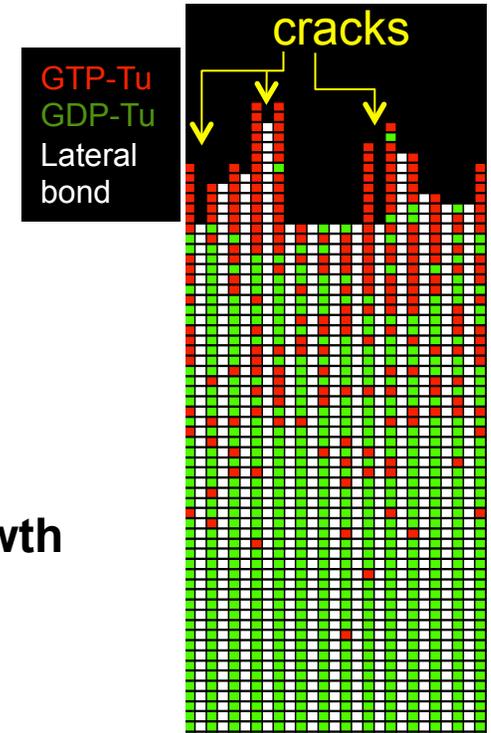


## 2. Cracks between protofilaments exist even in growing tips

*Expected from:*

- Entropic considerations  
    unlikely that all bonds form simultaneously
- Longitudinal bonds stronger than lateral  
(VanBuren 2002, Sept 2003)

**∴ laterally unbonded regions at tip**



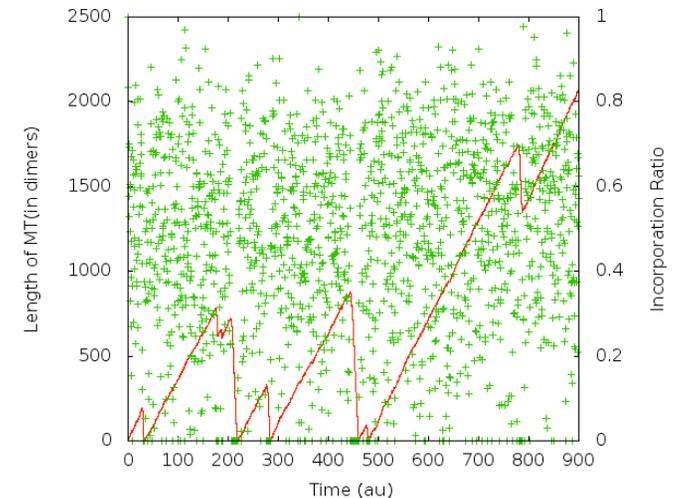
### Consequences:

- **Large fraction** of attached subunits **detach during growth**

(Walker et al., 1991, Scheck et al., 2008, Odde this meeting)

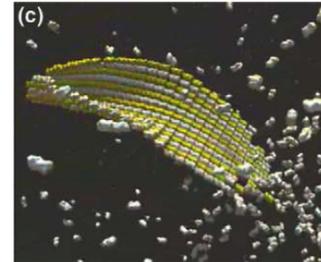
- **Any subunit likely to detach before laterally bonded**  
*not just those in “unfavorable” environments*

Fraction of subunits  
incorporated  $\text{sec}^{-1}$   
(Set C)



### 3. Simple 1st order GTP hydrolysis on non-terminal subunits is sufficient to account for dynamic instability

- no need for vectorial hydrolysis
- no need for sheet-closure → to explain catastrophe



Nogales COCB 2006

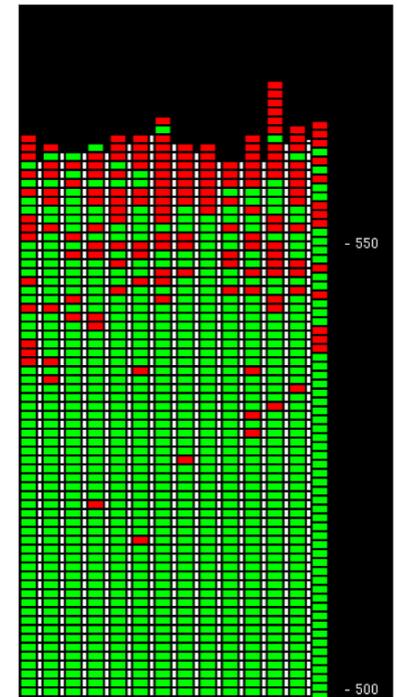
### 4. GTP cap is:

- **not a well-defined structure with discrete edges**
- a heterogeneous, dynamic, functionally defined entity
  - region rich in laterally bonded GTP dimers?
  - *effective cap* < total # GTP-Tu
- too short, short-lived to detect by most methods
  - why so hard to detect cap

Predicted by other models (VanBuren 2005)

Consistent with experimental data

But: idea of solid cap, vectorial hydrolysis persist

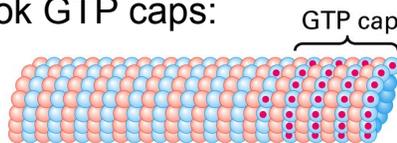


Dilution simulation

Set C

~real time

Textbook GTP caps:



Lodish MBOC 2004



Howard NRMCB 2009

## 5. GTP hydrolysis reduces strength of longitudinal bonds

Generally assumed: GDP weakens only lateral bonds

*Evidence:*

Required to match behaviors of GTP and GMPCPP MTs

GMPCPP: slowly hydrolyzable GTP analog

GMPCPP MTs: extremely stable

→ GMPCPP MTs: strong constraint for GTP-Tu state!

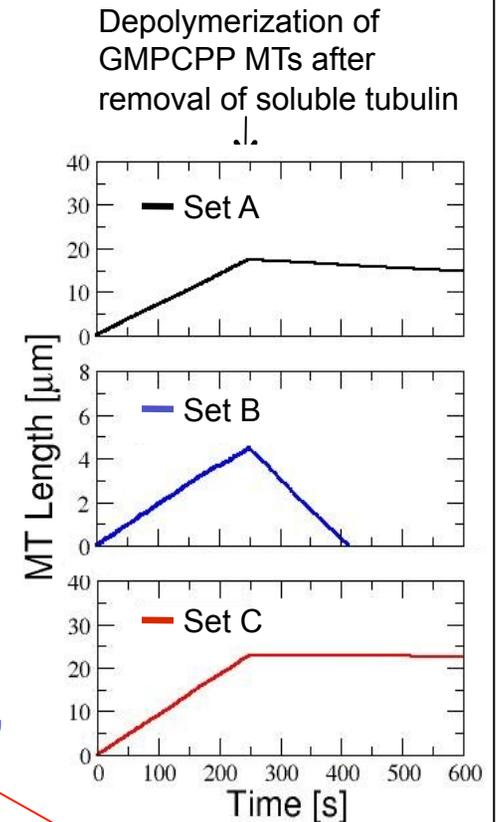
**Sets A, B: life-like DI with GTP-Tu**

➤ GMPCPP-Tu: Sets A, B depolymerized *too quickly*

- Could not identify set that matched GTP and GMPCPP *unless* changed model
- GTP/GDP affects lateral *and* longitudinal bonds - **Set C**

**Consistent with nearness of nucleotide to dimer interface, structural changes at interface:**

Rice PNAS 2008; Nogales, COCB 2006



Unique (?) among computational models of MT dynamics

- importance of tuning parameters to many types of experimental data

Nogales  
COCB  
2006



# Mechanisms of rescue and catastrophe??

Observation:

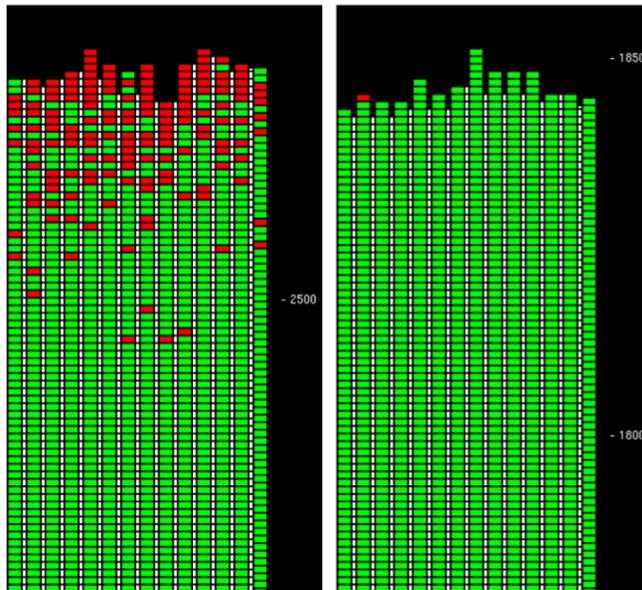
- Growing MT likely to keep growing
- Depolymerizing MT likely keep depolymerizing

↳ **What “tips the tip” to transition?**

**Expect:**

Some obvious attribute of tip structure will predict transition:

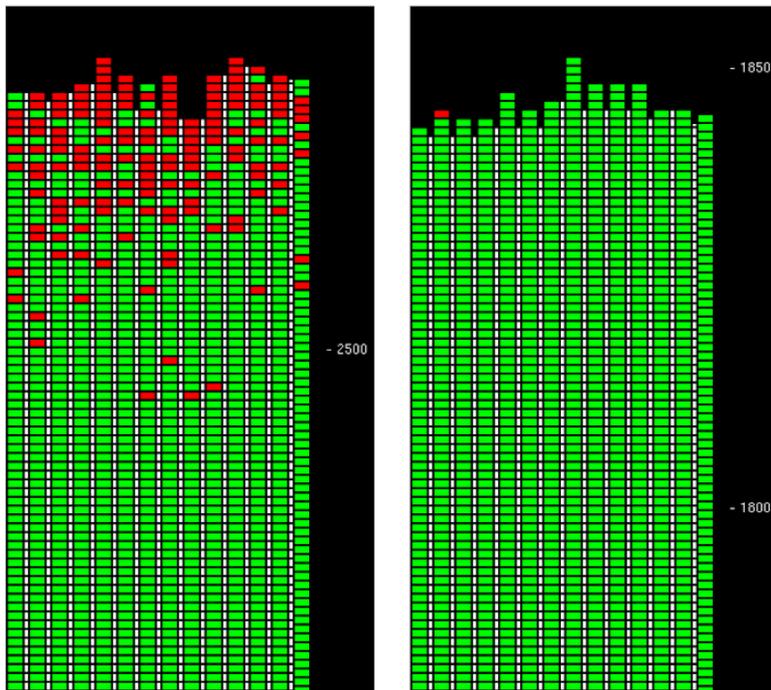
- number of GTPs in cap
- tip “raggedness”
- depth of inter-protofilament cracks
- length of laterally bonded GTP cap



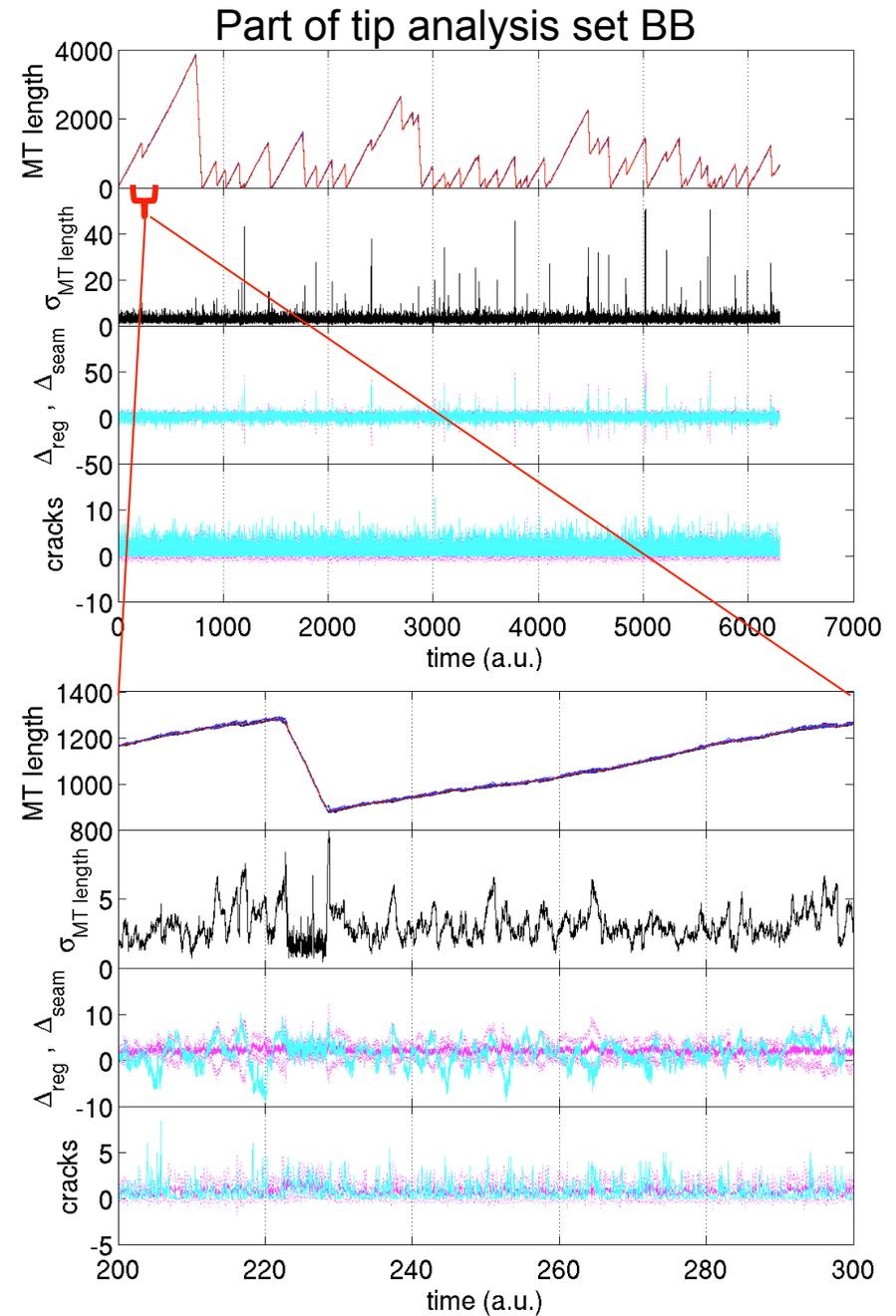
## Observation:

- No one attribute “predicts” catastrophe or rescue

↳ Tip fluctuates too quickly!

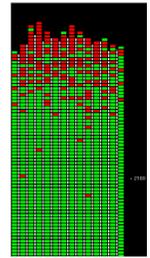


6 frame per sec ~real time



## Another Approach

- Identify, characterize true “tipping point” MT tip structures
  - take “snapshots” of tip configurations during transitions
  - use these configurations as starting point for 10 new simulations
  - find, study subset of structures that can “go either way”



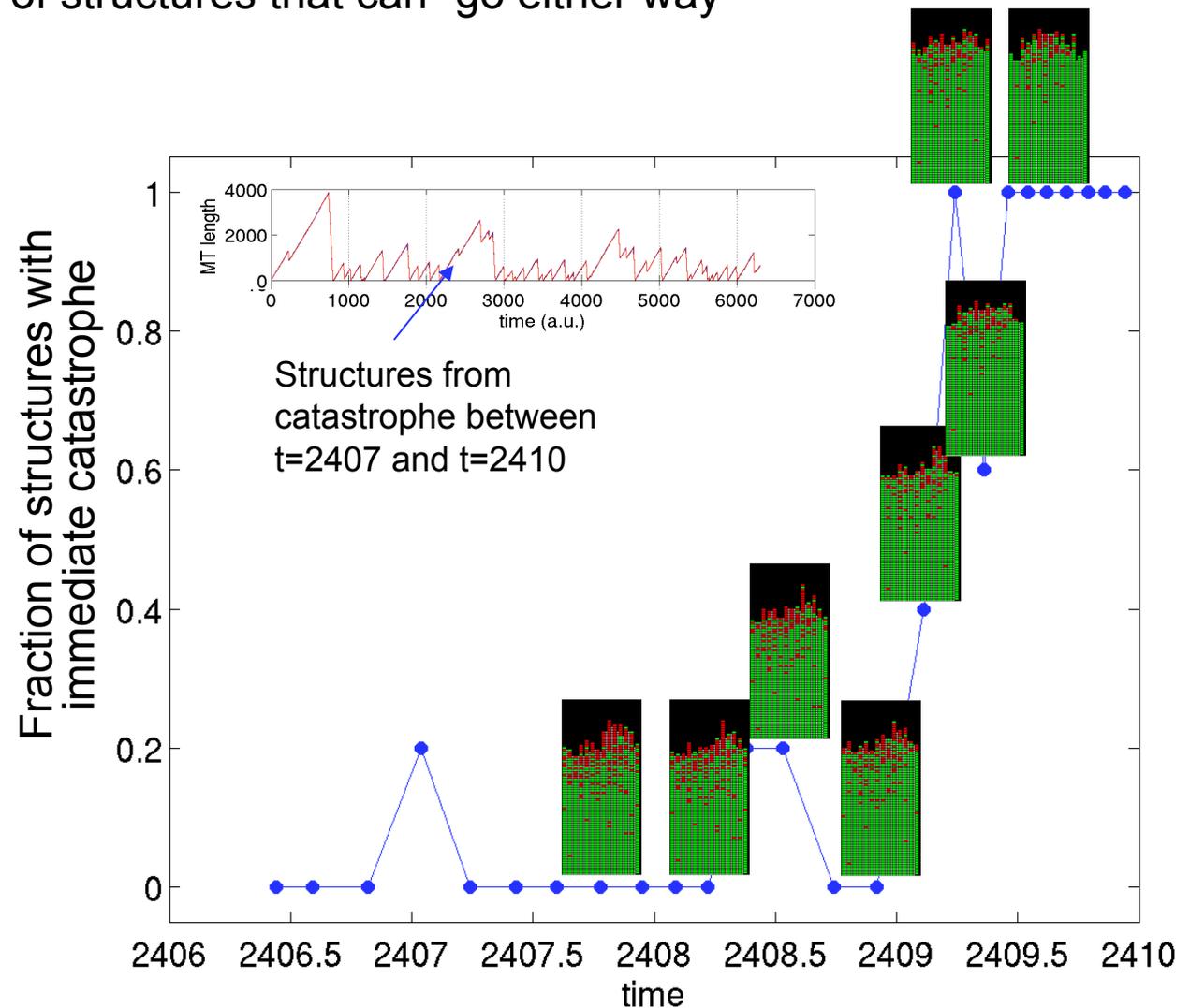
## “Tip fate analysis”

Transition likelihood  
for naturally occurring  
tip structures

shown as images  
5 simulations each structure

### Conclusions:

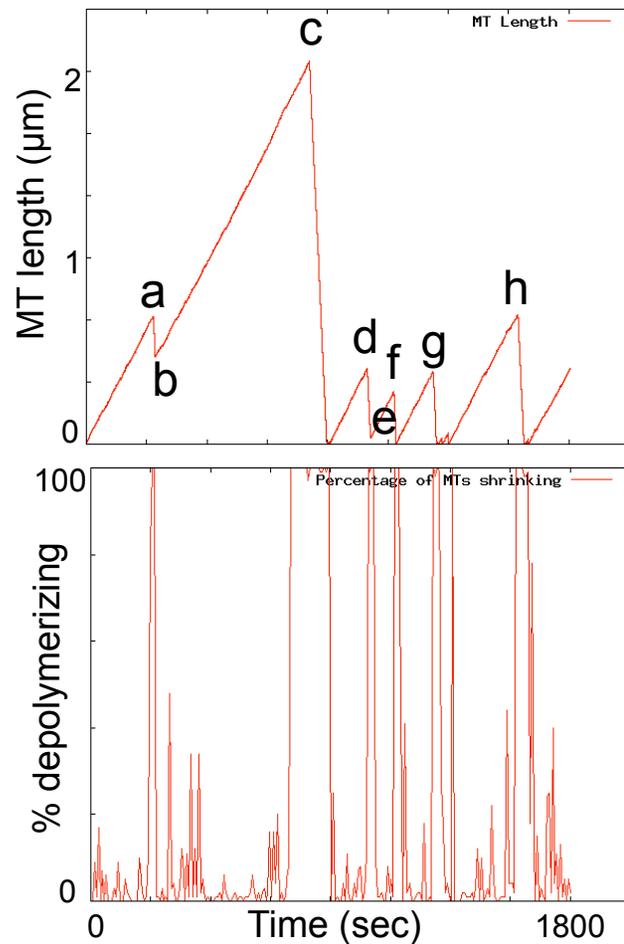
- True transition-prone structures are rare
- Transition is *fast* (1-2 sec)



## Test predictions:

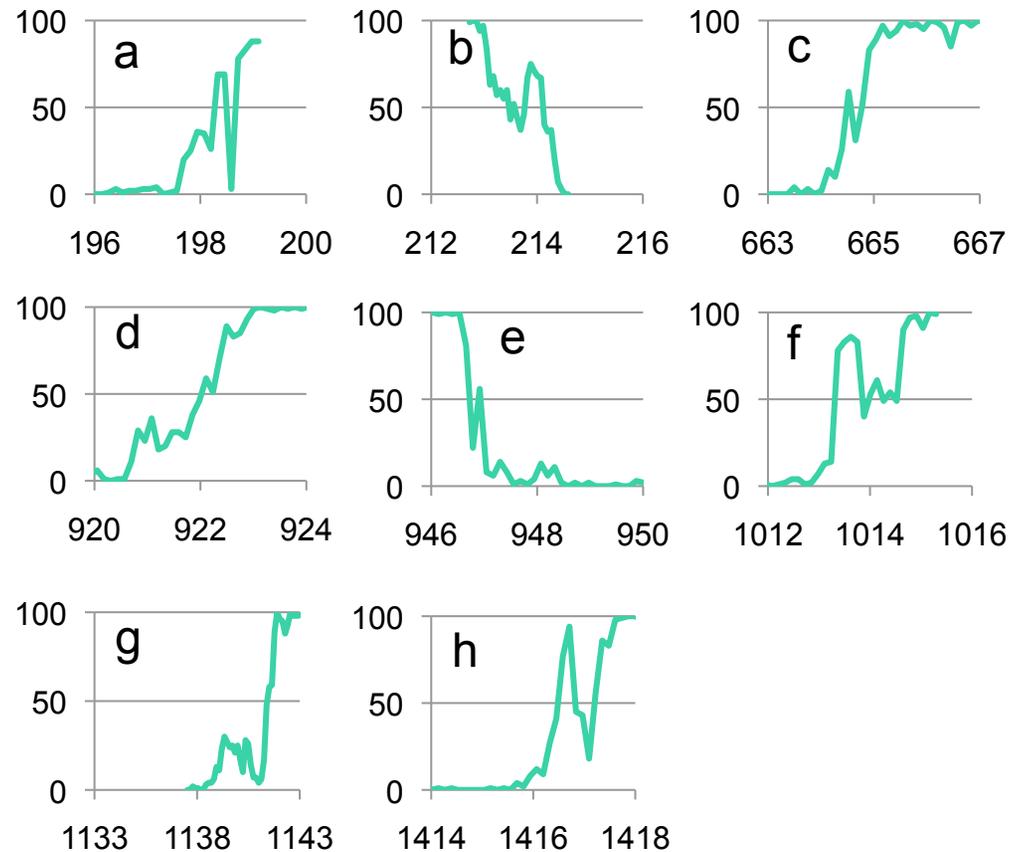
→ Sample structures from full life-history plot

- 1 sample/sec
- 100 new simulations/tip structure



→ Examine transitions more systematically

- 10 samples/sec
- 100 new simulations/tip structure



True transition-prone structures *are* rare  
Transitions happen in 1-2 seconds

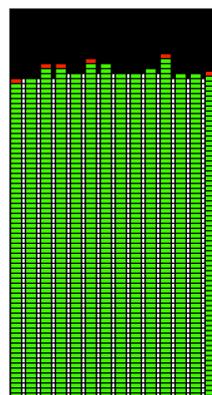
## Features of transitional structures?

→ No universal attribute  
*Not simply more/less GTP*

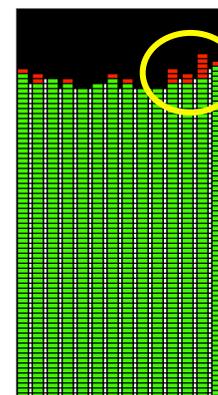
### Rescue:

Becomes likely when have a few  
*laterally bonded GTPs...*

Depol ~100%



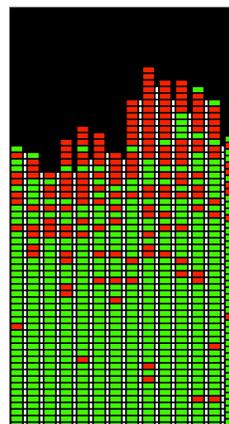
Rescue ~50%



### Catastrophe:

Correlates with cracks extending into  
GDP-rich region...

Grow ~100%



Catastrophe ~50%



### Propose:

- Fluctuations in depth, distribution of **cracks** play **pivotal role in DI**
  - ↳ provides mechanism for action of MT binding proteins

### Conclude

- MT dynamics best explained by refinement of “fluctuating cap model” originally proposed by Chen and Hill (1984)

#### Stochastic cap model:

- MTs hydrolyze GTP according to 1<sup>st</sup> order rate constant
- Catastrophe and rescue result from stochastic fluctuations in shape, depth, and lateral bonding of the cap

*In addition:* MTs grow as closed tubes with extensions

### Why support this conceptual model?

*not simply that it produces life-like dynamic instability*

- Lots of models, multiple real systems produce dynamic instability

### Question to ponder:

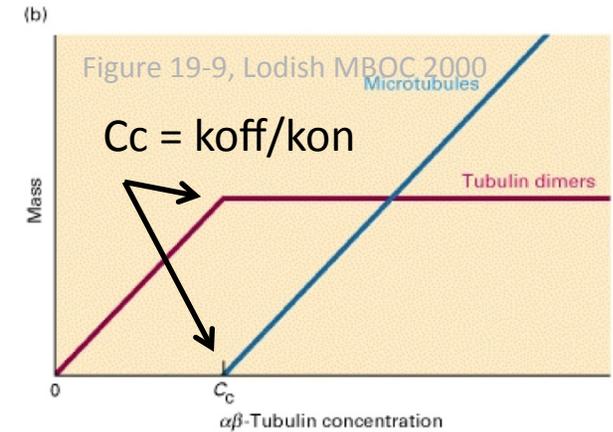
- What characteristics of a system are necessary to produce DI?

# Re-evaluation of “Critical concentration”

↳ Why??

## Commonly used concept:

- ~150 papers on MTs have “critical concentration” in *abstract*
- Discussed in cell biology textbooks: Pollard, Lodish, Alberts...
- Cell biologists/biochemists use  $C_c$  concept to design and interpret experiments



## What IS the critical concentration?

### 3 textbook definitions:

- 1) Concentration of subunits needed to get polymer assembly
- 2) Concentration of subunits in solution once steady-state is reached
- 3) Equilibrium constant for binding of monomer to polymer

**Problem:** relationships above were derived for *equilibrium polymers*

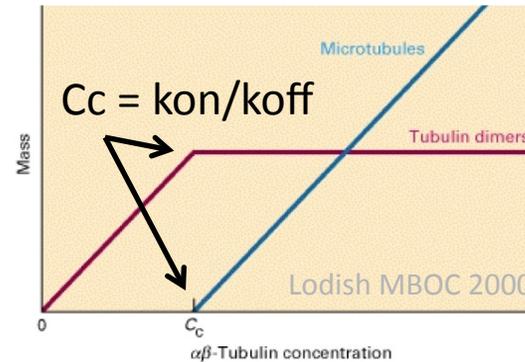
MTs and actin are *steady-state polymers*: energy is used in polymerization process

➤ How should understanding of critical concentration be modified for steady-state polymers?

**To begin, look back to textbook definitions:**

- 1) Concentration of subunits needed to get polymer assembly " $C_{c1}$ "
- 2) Concentration of subunits in solution once steady-state is reached " $C_{c2}$ "
- 3) Equilibrium constant for binding of monomer to polymer  $C_c = k_{on}/k_{off}$  " $C_{c3}$ "

All equivalent:  
 $C_{c1} = C_{c2} = C_{c3}$   
"THE critical concentration"



### Questions:

- Are these definitions still **valid** when considering **steady-state polymers**?
- Are valid definitions still **equivalent**?
  - ↳ **One** critical concentration, or **more**?
- How do these critical concentrations **relate to dynamic instability behavior**?
- How do they relate to **biochemical rate constants**?

**Goal:** Develop a more complete and **intuitive understanding** of

- behavior of populations of biologically relevant polymers
- How this population-level behavior relates to behavior of individual filaments.

**Focus:** Microtubules

**Approach:** computational models of MT dynamics

## 1. Are these definitions still valid when considering steady-state polymers?

Start with the easy part:

3) Equilibrium constant for binding of monomer to polymer:  $C_c = k_{off}/k_{on}$



*Well-established that definition #3 is not valid:*

- GTP and GDP forms of tubulin have different  $k_{on}$  and  $k_{off}$  values  
Different  $C_c$  values for GTP and GDP tubulin

-> **“Real”  $C_c$  will be in between  $C_c_{GTP}$  and  $C_c_{GDP}$**

$C_c_{GMPCPP} < 1\mu M$

$C_c_{GDP} \dots > 20\mu M??$

$1\mu M < C_c < 20\mu M$

*Sophisticated textbooks (Alberts) note this  
others (Lodish) don't*

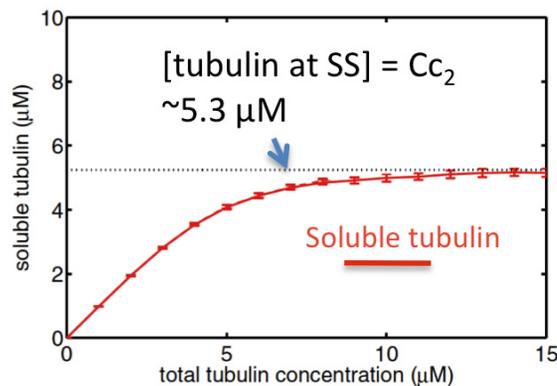
## 2. Are remaining definitions of $C_c$ equivalent in steady-state polymer systems??

- 1) Concentration of subunits needed to get polymer assembly " $C_{c1}$ "
- 2) Concentration of subunits in solution once steady-state is reached " $C_{c2}$ "

↳ Hint that they are *not* equivalent form work with "microscope-scale" models

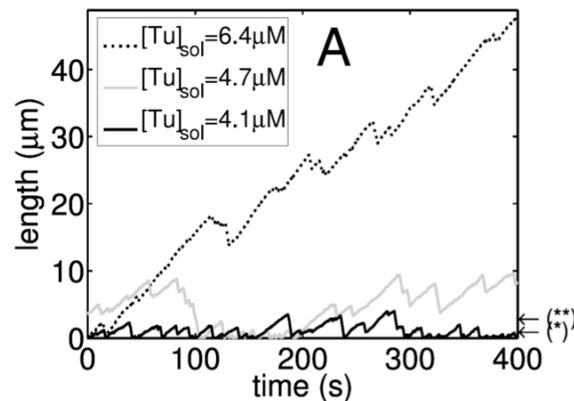
Gregoretti JCS 2006

Examine behavior of population of dynamic MTs



Prediction: No MTs below  $C_{c2}$

Examine individual MTs

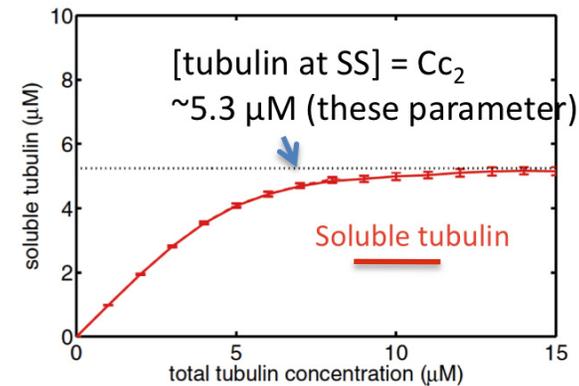


Observation: MTs are growing at [tubulin] *lower* than  $C_{c2}$ ! (same parameters)

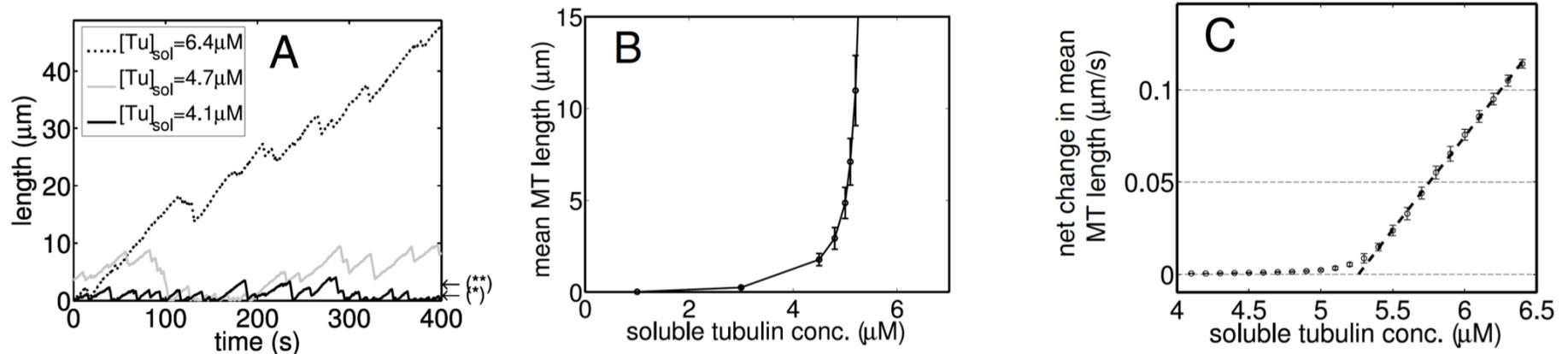
→ Suggests two definitions of  $C_c$  are *not* equivalent:  $C_{c1} \neq C_{c2}$

➤ If  $Cc_2 = [\text{tubulin}_{ss}]$  is *not* “the concentration of subunits needed for MT growth”

➔ What IS  $Cc_2$  ?



Examine behavior of MTs in steady-state systems in more detail



➤  $[\text{tubulin}_{ss}] = Cc_2$  is the concentration of tubulin needed for *persistent growth*

Also called “unbounded” growth (e.g. Verde et al. 1992)

**New (?) definition for [free tubulin] in pool of dynamic MTs at steady state:**

$[\text{tubulin}_{ss}] = Cc$  for persistent growth  $\rightarrow$  “ $Cc_p$ ”

*Note:*  $Cc_p$  is the *asymptote* that is approached:  $[\text{tubulin}_{ss}]$  is not actually a constant

# Conclusions of CC work *in progress*

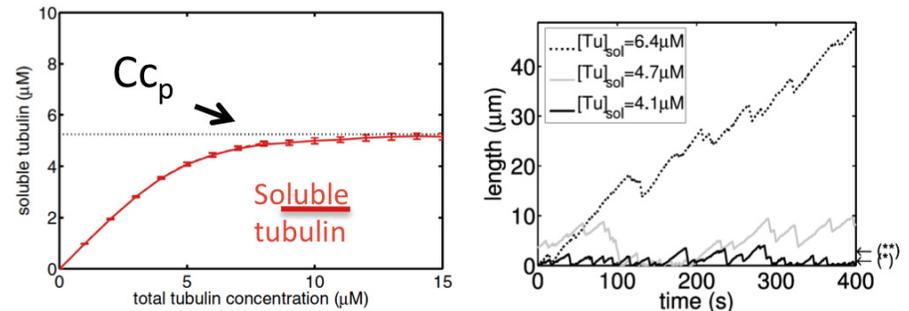
## At least two different “critical concentrations”

$Cc_2 = Cc_p = [\text{tubulin}_{ss}] = [\text{tubulin}]$  needed for persistent growth

persistent growth = unbounded growth

~10 $\mu\text{M}$  experimentally in vitro

~12  $\mu\text{M}$  detailed model standard parameters

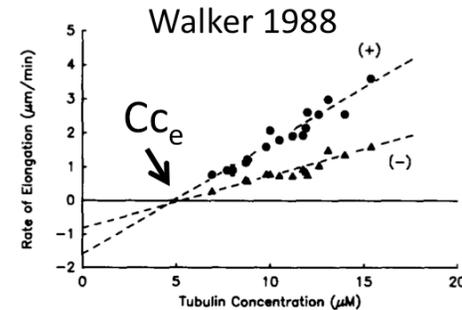


$Cc_1 = Cc_e = [\text{tubulin}]$  needed for MT elongation

$k_{\text{on}}[\text{tubulin}] > k_{\text{off}}$

~3-5 $\mu\text{M}$  experimentally

~12  $\mu\text{M}$  detailed model standard parameters

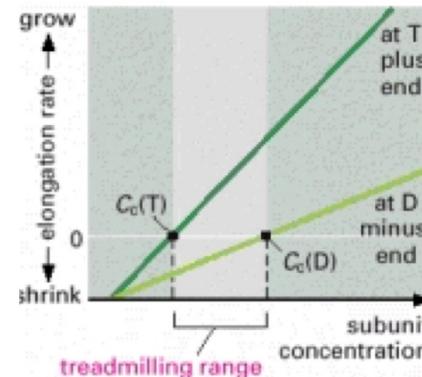


## “Regular” MT dynamic instability occurs *between these two concentrations*

Treadmilling??

Should occur when  $[\text{tubulin}] > Cc_p$  for +end?

$> Cc_e$  is not sufficient!



Alberts textbook: (also Howard) **-> Not correct!**