# Functor Calculus and Operads
# March 13–March 18, 2011

## MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

## MEETING ROOMS

**All lectures will be held in Max Bell 159 (Max Bell Building accessible by walkway on 2nd floor of Corbett Hall). LCD projector, overhead projectors and blackboards are available for presentations.** Note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155–159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.

## SCHEDULE

### Sunday

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| **16:00** | Check-in begins (Front Desk - Professional Development Centre - open 24 hours) |
| | Lecture rooms available after 16:00 (if desired) |
| **17:30–19:30** | Buffet Dinner, Sally Borden Building |
| **20:00** | Informal gathering in 2nd floor lounge, Corbett Hall (if desired) |
| | Beverages and a small assortment of snacks are available on a cash honor system. |

### Monday

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| **7:00–8:45** | Breakfast |
| **8:45–9:00** | Introduction and Welcome by BIRS Station Manager, Max Bell 159 |
| **9:00–10:00** | Ryan Budney (Victoria) *Talk 1: A primer on concordance and pseudoisotopy* |
| **10:00–10:30** | Coffee Break, 2nd floor lounge, Corbett Hall |
| **10:30–11:30** | Kathryn Hess (EPFL) *Talk 1: What is André–Quillen (co)homology and why is it important?* |
| **11:30–13:00** | Lunch |
| **13:00–14:00** | Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall |
| **14:00–15:00** | Greg Arone (Virginia) *Talk 1: Operads, modules, and the chain rule* |
| **15:00–15:30** | Coffee Break, 2nd floor lounge, Corbett Hall |
| **15:30** | Group Photo; meet on the front steps of Corbett Hall |
| **16:00–17:00** | Mark Behrens (MIT) *Talk 1: Survey of the Goodwillie tower of the identity* |
| **17:30–19:30** | Dinner |

**Tuesday**

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| **7:00–8:45** | Breakfast |
| **9:00–10:00** | Ryan Budney (Victoria) *Talk 2: Manifold embeddings and operads* |
| **10:00–10:30** | Coffee Break, 2nd floor lounge, Corbett Hall |
| **10:30–11:30** | Kathryn Hess (EPFL) *Talk 2: André–Quillen homology and towers* |
| **11:30–13:00** | Lunch |
| **14:00–15:00** | Greg Arone (Virginia) *Talk 2: Beyond the module structure* |
| **15:00–15:30** | Coffee Break, 2nd floor lounge, Corbett Hall |
| **15:30–16:30** | Mark Behrens (MIT) *Talk 2: Survey of the Goodwillie tower of the identity* |
| **17:30–19:30** | Dinner |

**Wednesday**

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| **7:00–8:45** | Breakfast |
| **9:00–9:50** | Mike Mandell (Indiana) *Quillen cohomology of operadic algebras and obstruction theory* |
| **9:50–10:10** | Coffee Break, 2nd floor lounge, Corbett Hall |
| **10:10–11:00** | Benoit Fresse (Lille) *Homotopy automorphisms of $E_2$–operads and Grothendieck-Teichmüller groups* |
| **11:10–12:00** | Eric Finster (EPFL) *The Goodwillie tower for homotopy limits* |
| **11:30–13:30** | Lunch |
| | Free Afternoon |
| **17:30–19:30** | Dinner |

**Thursday**

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| **7:00–8:45** | Breakfast |
| **9:00–10:00** | Michael Weiss (Aberdeen)) *Smooth maps to the plane and Pontryagin classes* |
| **10:00–10:30** | Coffee Break, 2nd floor lounge, Corbett Hall |
| **10:30–11:30** | Bill Dwyer (Notre Dame) *Symmetric powers versus the calculus filtration* |
| **11:30–13:00** | Lunch |
| **14:00–14:50** | Brenda Johnson (Union College) *Cross-effects, cotriples and calculus: an update* |
| **14:50–15:10** | Coffee Break, 2nd floor lounge, Corbett Hall |
| **15:10–16:00** | Tom Goodwillie (Brown) *Tangent bundle analogy* |
| **16:10–17:00** | Pascal Lambrechts (Louvain-la-Neuve) *Formality of the little $N$–disks operads* |
| **17:30–19:30** | Dinner |

**Friday**

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| **7:00–9:00** | Breakfast |
| **9:00** | Informal Discussions |
| **11:30–13:30** | Lunch |
| **Checkout by 12 noon.** | |

** 5-day workshops are welcome to use BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

## ABSTRACTS

Speaker: **Gregory Arone** (Virginia)
Title: *Part 1: operads, modules and the chain rule*
Abstract: Let $F$ be a homotopy functor between the categories of pointed topological spaces or spectra. By the work of Goodwillie, the derivatives of $F$ form a symmetric sequence of spectra $\partial_* F$. This symmetric sequence determines the homogeneous layers in the Taylor tower of $F$, but not the extensions in the tower. In these two talks we will explore the following question: what natural structure does $\partial_* F$ possess, beyond being a symmetric sequence? Our ultimate goal is to describe a structure that is sufficient to recover the Taylor tower of $F$ from the derivatives. Such a description could be considered an extension of Goodwillie's classification of homogeneous functors to a classification of Taylor towers.

By a theorem of Ching, the derivatives of the identity functor form an operad. In the first talk we will see that the derivatives of a general functor form a bimodule (or a right/left module, depending on the source and target categories of the functor) over this operad. Koszul duality for operads plays an interesting role in the proof. As an application we will show that the module structure on derivatives is exactly what one needs to write down a chain rule for the calculus of functors.

Title: *Part 2: beyond the module structure*
Abstract: The (bi)module structure on derivatives is not sufficient to recover the Taylor tower of a functor. In the second talk we will refine the structure as follows. We will see that there is a naturally defined comonad on the category of (bi)-modules over the derivatives of the identity functor, and that the derivatives of a functor are a coalgebra over this comonad. From this coalgebra structure one can, in principle, reconstruct the Taylor tower of a functor. Thus this coalgebra structure seems to give a complete description of the structure possessed by the derivatives of a functor.

We give an explicit description (as explicit as permitted by our current understanding) of the structure possessed by the derivatives of functors from Spectra to Spectra, from Spaces to Spectra and from Spaces to Spaces. An interesting example is the functor $X \mapsto \Sigma^\infty \Omega^\infty (E \wedge X)$. Here $E$ is a fixed spectrum, and the functor can be thought as a functor from either the category of Spaces or Spectra to the category of Spectra. The derivatives of this functor are given by the sequence $E, E^{\wedge 2}, \ldots, E^{\wedge n}, \ldots$. The fact that this sequence is the sequence of derivatives of a functor seems to tell us something about the structure possessed by spectra in general. In particular it tells us that spectra possess a natural structure of a restricted algebra over the Lie operad (an observation first made by Bill Dwyer).

Speaker: **Mark Behrens** (MIT)
Title: *Survey of the Goodwillie tower of the identity I*
Abstract: The Goodwillie tower of the identity functor from spaces to spaces is a powerful tool for understanding unstable homotopy from the stable point of view. I will describe the derivatives of this functor, which were studied by Johnson and Arone-Mahowald. I will also explain the Arone-Mahowald computation of the homology of the layers of the tower evaluated on spheres. These layers were shown by Arone-Dwyer to be equivalent to stunted versions of the $L(k)$ spectra studied by Kuhn, Mitchell, Priddy, and others.

Associated to the Goodwillie tower is a spectral sequence which computes the unstable homotopy groups of a space from the stable homotopy groups of the layers. I will explain consequences for unstable $v_k$-periodic homotopy discovered by Arone and Mahowald. I will also discuss relations with the EHP sequence at the prime 2. Specifically, differentials in the Goodwillie spectral sequence can often be computed in terms of Hopf invariants, and differentials in the EHP sequence can often be computed in terms of attaching maps in the $L(k)$-spectra.

Title: *Survey of the Goodwillie tower of the identity II*
Abstract: I will recall Ching's operad structure on the derivatives of the identity. Coupled with operadic structures recently discovered by Arone-Ching, this gives an action of a certain algebra of Dyer-Lashof-like

operations on the layers of the Goodwillie tower of any functor from spaces to spaces. Specializing to the case of functors concentrated in degrees n and 2n, we will derive a formula for the homological behavior of the k-invariant of the associated Goodwillie tower.

The 2-primary Goodwillie tower of $S^1$ is closely related to the Whitehead conjecture (a.k.a. Kuhn's theorem). Work of Arone-Dwyer-Lesh shows that the k-invariants in the tower for $S^1$ deloop to give candidates for a contracting homotopy to Kuhn's Kahn-Priddy sequence, and conjectured this was indeed the case. The homological formulas for k-invariants will be used to compute these delooped k-invariants, verifying the conjecture (Arone-Dwyer-Lesh have simultaneously developed a different proof of their conjecture.)

Speaker: **Ryan Budney** (Victoria)
Title: *A primer on concordance or pseudoisotopy*
Abstract: I'll describe the context for concordance and pseudoisotopy, the Morlet Disjunction lemma, the pre-80's work on the subject. Tom's dissertation, and his work with Weiss and Klein on The Embedding Calculus, up to around Dev Sinha's early work on the embeddings.

Title: *Manifold embeddings and operads*
Abstract: This talk will be about "where we are and where we're going", describing recent work of Sinha, Salvatore, Turchin, Arone, Lambrechts, Dwyer and Hess on rational homotopy-type and iterated loop-space structures on embedding spaces. I'll also talk about other operads that act on embedding spaces and how they might fit into what is known.

Speaker: **Bill Dwyer** (Notre Dame)
Title: *Symmetric powers vs. the calculus filtration*
Abstract: The talk will discuss ideas which shed light on the relationship between the (increasing) filtration of the Eilenberg-MacLane spectrum $H\mathbb{Z} = \mathrm{SP}^\infty(S^0)$ by the symmetric power spectra $\mathrm{SP}^n(S^0)$, and the (decreasing) filtration of the circle $S^1$ by the Goodwillie tower of the identity functor. Figuring in the picture is a peculiar object which is both simplicial and cosimplicial, and, in either guise, is both trivial and is not. Many Bredon (co)homology spectral sequences collapse, and the circle does double duty as the sphere $S^1$ and as the Eilenberg-MacLane space $K(\mathbb{Z}, 1)$. Kuhn's Theorem (the Whitehead Conjecture) is in the background.

Speaker: **Eric Finster** (EPFL)
Title: *The Goodwillie tower for homotopy limits*
Abstract: In view of the many fruitful applications of the Goodwillie Calculus to the study of mapping spaces of various types, a natural question is the extent to which similar techniques can be applied to the study of their twisted cousins, that is, to spaces of sections. Homotopy theorists have two closely related convenient ways of modeling a space of this type: as the totalization of a cosimplicial space or as the homotopy limit of a diagram. These two models are essentially equivalent to each other in the sense that every homotopy limit can be realized as the total space of its cosimplicial replacement, and conversely, the total space of a fibrant cosimplicial space is equivalent to its homotopy limit when viewed as a diagram over $\Delta$.

One advantage of a cosimplicial model, however, is that it comes with a canonical filtration, the totalization tower. Applying a generalized homology theory to the totalization tower yields the Bousfield-Kan spectral sequence, which under favorable conditions allows one to compute homological invariants of the associated total space. Viewed in another light, the Bousfield-Kan spectral sequence can be seen as a canonical filtration of the spectrum $\Sigma^\infty \operatorname{Tot} \mathcal{X}$ and we can discuss convergence in these terms: we say the Bousfield-Kan spectral sequence is convergent when the natural family of maps

$$\{\Sigma^\infty \operatorname{Tot}_n \mathcal{X}\}_{n\in\mathbb{N}} \longrightarrow \{\operatorname{Tot}_n \Sigma^\infty \mathcal{X}\}_{n\in\mathbb{N}}$$

constitutes a pro-equivalence of spectra from the constant tower at $\Sigma^\infty \operatorname{Tot} \mathcal{X}$ to the totalization tower of the cosimplicial spectrum $\Sigma^\infty \mathcal{X}$.

Homotopy limits, on the other hand, might be said to have a slight conceptual advantage in that they correspond nicely with our geometric intuition. Taking $K$ to be a simplicial set, a diagram $F : \Delta K \to \mathcal{S}$ indexed by the simplex category of $K$ can be thought of as a presheaf of spaces on $K$, and its homotopy limit as the space of global sections, which we will here denote by $\operatorname{holim}_K F$.

In this lecture, we present a stable filtration

$$\Sigma^\infty \operatorname{holim}_K F \to \operatorname{holim}_{P_n K} \Sigma^\infty F_n$$

which plays the role of the Goodwillie tower for homotopy limits, generalizing Arone's model for $\Sigma^\infty \operatorname{Map}(K, X)$ when X is the common value of a constant functor F. After outlining the basic construction, we discuss convergence issues, examine its relationship to the Bousfield-Kan spectral sequence, and provide examples.

Speaker: **Benoit Fresse** (Lille)
Title: *Homotopy automorphisms of* $\mathbf{E_2}$*-operads and Grothendieck-Teichmüller groups*
Abstract: The notion of $E_2$-operad usually refers to a topological operad which is weakly equivalent to Boardman-Vogt' operad of little 2-cubes $\mathsf{C}_2$. This structure is used to model operations acting on 2-fold loop spaces. The structure of an $E_2$-operad is also used in simplicial and in differential graded algebra in order to model a first level of homotopy commutative structures.

In the algebraic setting, one considers the singular chain complex of the operad of little 2-cubes $\mathit{Sing}_\bullet(\mathsf{C}_2)$, which defines an operad in simplicial cocommutative coalgebras. For our purpose, we form a dual structure $\mathit{Sing}^\bullet(\mathsf{C}_2)$, which is a cooperad in cosimplicial commutative algebras. In what follows, we prefer to adopt the terminology of cosimplicial commutative Hopf cooperad (also commonly used in the literature) for the category of cooperads in cosimplicial commutative algebras and we use the notation $c\,\mathcal{H}opf\,\mathcal{O}p_1^c$ to refer to this category. The first objective of this talk is to explain that the category $c\,\mathcal{H}opf\,\mathcal{O}p_1^c$ gives a faithful algebraic model for the rational homotopy category of operads (assuming that $\mathbb{Q}$ is our ground ring). In the case of little 2-cubes, we will prove that the application of a space-like functor to a fibrant replacement of $\mathit{Sing}^\bullet(\mathsf{C}_2)$ yields a topological operad $(\mathsf{C}_2)^\wedge_\mathbb{Q}$ such that $\pi_*(\mathsf{C}_2)^\wedge_\mathbb{Q} = 0$ for $* > 1$ and $\pi_1(\mathsf{C}_2)^\wedge_\mathbb{Q}$ is the rational prounipotent completion (the Malcev completion) of the fundamental group of $\mathsf{C}_2$.

Our main goal is to determine the homotopy of the space of homotopy automorphisms $\operatorname{haut}_{\mathcal{T}op\mathcal{O}p}(\mathsf{C}_2)^\wedge_\mathbb{Q}$ associated to this topological operad $(\mathsf{C}_2)^\wedge_\mathbb{Q}$. Formally, we have a simplicial function space $\operatorname{Map}_{\mathcal{T}op\mathcal{O}p}(\mathsf{P}, \mathsf{P})$ associated to any topological operad $\mathsf{P}$. The space of homotopy automorphisms $\operatorname{haut}_{\mathcal{T}op\mathcal{O}p}(\mathsf{P})$ consists of the connected components of $\operatorname{Map}_{\mathcal{T}op\mathcal{O}p}(\mathsf{P}, \mathsf{P})$ which are attached to homotopy equivalences of operads $\phi : \mathsf{P} \xrightarrow{\sim} \mathsf{P}$. Basically, we can also identify $\pi_0(\operatorname{haut}_{\mathcal{T}op\mathcal{O}p}(\mathsf{P}))$ with the group of homotopy classes of these operad homotopy equivalences $\phi : \mathsf{P} \xrightarrow{\sim} \mathsf{P}$. Actually, the construction of $\operatorname{haut}_{\mathcal{T}op\mathcal{O}p}(\mathsf{P})$ makes sense if $\mathsf{P}$ is cofibrant as an operad only. Therefore, we tacitly apply the definition to a cofibrant replacement of $\mathsf{P}$ when the operad $\mathsf{P}$ is not itself cofibrant.

For basic reasons, we are mostly interested in a subspace $\operatorname{haut}^1_{\mathcal{T}op\mathcal{O}p}(\mathsf{C}_2)^\wedge_\mathbb{Q} \subset \operatorname{haut}_{\mathcal{T}op\mathcal{O}p}(\mathsf{C}_2)^\wedge_\mathbb{Q}$, formed by restricting ourselves to the connected components of operad homotopy equivalences inducing the identity in homology. The operad $(\mathsf{C}_2)^\wedge_\mathbb{Q}$, which has no homotopy in degree $* \neq 1$, has a simple categorical model yielded by the Malcev completion of the operad of colored braid groupoids. The prounipotent version of the Grothendieck-Teichmüller group $GT(\mathbb{Q})$, defined by Drinfeld, can be identified with the group of automorphisms of this operad in groupoids. Accordingly, we can form a group morphism

$$\eta : GT(\mathbb{Q}) \to \pi_0(\operatorname{haut}_{\mathcal{T}op\mathcal{O}p}(\mathsf{C}_2)^\wedge_\mathbb{Q}),$$

which is obviously injective (use fundamental groupoids). For our purpose, we consider a group $GT^1(\mathbb{Q}) \subset GT(\mathbb{Q})$, formed by removing a central factor $\mathbb{Q}^\times$ in $GT(\mathbb{Q})$. Our main result asserts:

**Theorem.** *We have*

$$\pi_*(\operatorname{haut}_{\mathcal{T}op\mathcal{O}p}((\mathsf{C}_2)^\wedge_\mathbb{Q}) = \begin{cases} GT^1(\mathbb{Q}), & \text{if } * = 0, \\ 0, & \text{otherwise.} \end{cases}$$

Most of the talk will be devoted to the formulation of this theorem. The remaining time will be used to give an outline of the proof. In brief, our arguments rely on a study of the deformation complex of $\mathit{Sing}^{\bullet}(\mathsf{C}_2)$ in the category of cosimplicial Hopf cooperads.

Speaker: **Tom Goodwillie** (Brown)
Title: *Tangent Bundle Analogy*
Abstract: Some of the ideas and much of the language of functor calculus reflect an analogy between functors and functions. Differentiation is the approximation of a function by a linear function; the analogue for functors is a kind of stabilization. This talk will take the analogy further, attempting a bit of differential geometry, both coordinate-free and otherwise, on categories like Top. The starting point for this is to view stable homotopical categories as vector spaces. The category of spectra is viewed as the tangent space of the category of spaces (at the one-point space).

Speaker: **Kathryn Hess** (EPFL)
Title: *What is André-Quillen (co)homology and why is it important?* Abstract: In this talk I will first describe classical André-Quillen homology, which was invented, essentially simultaneously, by Michel André and by Dan Quillen in the late 1960's. Quillen defined a very general homology theory, calculated as derived functors of abelienization, for objects in any category with all finite limits and enough projectives; special cases include both singular homology of spaces and (reduced) homology of groups. The Quillen homology of commutative rings is called *André-Quillen homology.*

I'll present Quillen's homology theory in the general setting and sketch proofs of its important elementary properties: the transitvity long exact sequence, invariance under base change and a sort of additivity. I'll then describe briefly the particular cases of commutative rings and of associative algebras.

In the late 1990's, Maria Basterra defined *topological André-Quillen cohomology (TAQ)* of commutative ring spectra and proved that homotopical analogues of the transitvity long exact sequence and invariance under base change held for this new theory. I will describe the construction of TAQ and explain why it is of interest to stable homotopy theorists. In particular, any connective, commutative ring spectrum admits a Postnikov tower decomposition, where the $k$-invariants correspond to classes in TAQ. Also, as shown by Basterra and Mike Mandell, TAQ is universal, in the sense that every cohomology theory of $E_\infty$-ring spectra is TAQ with appropriate coefficients. Finally, as an aide to computation, I will state Mandell's theorem relating André-Quillen homology of $E_\infty$-simplicial algebras and of commutative ring spectra to that of $E_\infty$-differential graded algebras.

In the second part of this talk, I will briefly survey applications of André-Quillen (co)homology. In particular I will describe the role of André-Quillen homology in Haynes Miller's proof of the Sullivan conjecture. I will then sketch Bousfield's proof that André-Quillen homology is the natural home of obstructions to realizing a morphism $H_*(K;\mathbb{F}_p) \to H_*(L;\mathbb{F}_p)$ of cocommutative coalgebras with compatible Steenrod algebra structure as a map of spaces, as well as its generalization by Paul Goerss and Mike Hopkins, where $H_*(-;\mathbb{F}_p)$ is replaced by $E_*$ for any commutative ring spectrum satisfying the Adams conditions and the commutative operad by any operad in spectra. If time permits, I will also talk about work of Natàlia Castellana, Juan Crespo and Jérôme Scherer on finite generation of mod $p$ cohomology over the Steenrod algebra, in which André-Quille homology played an important role.

Title: *André-Quillen homology and towers*
Abstract: This second talk will be more closely related to the theme of this workshop, as I will describe various towers related to (André-)Quillen homology that are, at the very least, highly analogous to the towers of Goodwillie calculus.

I will begin by presenting work of Nick Kuhn, who, for any commutative ring spectrum $R$, constructed a filtration of $\mathrm{TAQ}(R)$ such that the filtration quotients look just like the fibers of stages of a Goodwillie tower. The filtration gives rise, for any field $\mathbb{F}$, to a spectral sequence converging to $H^*\big(\mathrm{TAQ}(R);\mathbb{F}\big)$, which Kuhn applied to computing $\mathrm{TAQ}(R)$ for $R = \Sigma^\infty S^1_+, \Sigma^\infty \mathbb{Z}/2_+$, and $D(S^1_+)$, where $D$ denotes the Spanier-Whitehead dual.

Let $X$ be spectrum, and let $\mathbb{P}X$ denote the free commutative ring spectrum generated by $X$. To elucidate the relationship between $K(n)_*(\mathbb{P}X)$ and $K(n)_*(\Omega^\infty X)$, Kuhn constructed and studied the *André-Quillen tower* of an augmented, commutative $R$-algebra (in spectra) $A$, which is the Goodwillie tower of the identity functor on the category of $R$-algebras, evaluated on $A$. Its limit is essentially an $I$-adic completion of $A$, where $I$ is the augmentation ideal of $A$. I will present Kuhn's results on properties of the André-Quillen tower, such as when it converges and its relation with localization.

I will conclude by talking about joint work with John Harper on a *homotopy completion tower* interpolating between Quillen homology and homotopy completion of algebras in symmetric spectra over a general operad $\mathcal{O}$, which generalizes Kuhn's André-Quillen tower. Our understanding of this homotopy completion tower enabled us to prove that, under reasonable conditions on the operad $\mathcal{O}$, if an $\mathcal{O}$-algebra map induces a weak equivalence on Quillen homology, then it also induces a weak equivalence on homotopy completions. Moreover, under mild connectivity conditions, an $\mathcal{O}$-algebra $X$ is weakly equivalent to its homotopy completion, i.e., the homotopy completion tower for $X$ converges to $X$. As almost immediate corollaries, we obtained Whitehead and Hurewicz theorems for Quillen homology. We have also established a Serre-type finiteness theorem for Quillen homology: under reasonable cofibrancy and connectivity conditions on the operad $\mathcal{O}$, if the homotopy groups of each level of the operad are finitely generated, then finiteness (respectively, finite generation) of the Quillen homology groups of an $\mathcal{O}$-algebra implies finiteness (respectively, finite generation) of its homotopy groups.

Speaker: **Brenda Johnson** (Union)
Title: *Cross-effects, cotriples and calculus: an update*
Abstract: In a series of papers published between 1998 and 2004, Randy McCarthy and I developed an analogue of Goodwillie's calculus of homotopy functors in an abelian setting using cross effects and cotriples. For functors $F : \mathcal{A} \to \mathcal{B}$ where $\mathcal{A}$ is a pointed category with finite coproducts and $\mathcal{B}$ is an abelian category, the $(n+1)$st cross effect $cr_{n+1}F$ is a symmetric functor of $n+1$ variables first used by Eilenberg and Mac Lane. The functor $cr_{n+1}$ and the diagonal functor $\Delta^*$ form an adjoint pair between the categories of functors from $\mathcal{A}$ to $\mathcal{B}$ and functors from $\mathcal{A}^{\times n+1}$ to $\mathcal{B}$ which in turn yields a cotriple $\perp_{n+1} = \Delta^* \circ cr_{n+1}$ on the category of functors from $\mathcal{A}$ to $\mathcal{B}$. For $F : \mathcal{A} \to \mathcal{B}$ the cotriple produces an augmented simplicial object $\perp_{n+1}^{*+1} F \to F$ whose homotopy cofiber gives us a functor $\Gamma_n F := \text{cofiber}(\perp_{n+1}^{*+1} F \to F)$. These $\Gamma_n F$ can be assembled into a tower of functors $\cdots \to \Gamma_{n+1}F \to \Gamma_n F \to \ldots \Gamma_1 F \to \Gamma_0 F$. The $n$th term in this tower, $\Gamma_n F$, is a degree $n$ functor, i.e., its $(n+1)$st cross effect is quasi-isomorphic to 0. This is a weaker condition than the $n$-excisive condition satisfied by the terms in Goodwillie's Taylor tower of a homotopy functor. In his thesis, Andrew Mauer-Oats extended this cotriple tower construction to endofunctors of the category of based spaces. He also proved that when $F$ is a functor that commutes with geometric realization, $\Gamma_n F \simeq P_n F$, the $n$th term in Goodwillie's Taylor tower of the functor $F$.

In this talk I will discuss recent work with K. Bauer, R. Eldred, and R. McCarthy to extend these ideas to a more general context. We work with functors $F : \mathcal{C}_f \to \mathcal{D}$ where $\mathcal{C}$ and $\mathcal{D}$ are simplicial model categories, $f : A \to B$ is a morphism in $\mathcal{C}$, $\mathcal{C}_f$ is the full subcategory of objects $A \to X \to B$ that factor the morphism $f$, and $\mathcal{D}$ is stable. In this setting, the functors $cr_{n+1}$ and $\Delta^*$ only form an adjoint pair up to homotopy. Mauer-Oats handled this problem for spaces by using combinatorial techniques to show directly that $\perp_{n+1}$ is a cotriple. In the current work, we identify two adjoint pairs whose composition yields $\perp_{n+1}$ as its associated cotriple. With this, the construction of the tower

$$\ldots \Gamma^f_{n+1}F \to \Gamma^f_n F \to \ldots \Gamma^f_1 F \to \Gamma^f_0 F$$

proceeds much as it did in the abelian case. The $n$th term is now a functor that is $n$-excisive relative to $f$, that is, it takes strongly cocartesian $(n+1)$-cubes of objects built by using the morphism $f$ to cartesian diagrams. When $F$ commutes with realizations, we generalize Mauer-Oats' results, showing that $\Gamma^f_n F$ is $n$-excisive and $P_n F$ and $\Gamma^f_n F$ are weakly equivalent functors. More generally, for an arbitrary $F : \mathcal{C}_f \to \mathcal{D}$ (that does not necessarily commute with realizations) evaluation at the initial object $A$ in $\mathcal{C}_f$ yields a weak equivalence $\Gamma^f_n F(A) \simeq P_n F(A)$.

Speaker: **Pascal Lambrechts** (Louvain-la-Neuve)
Title: *Formality of the little N-disks operad*
Abstract: This is joint work with Ismar Volic. The little N-disk operad is an important object in functor calculus, notably in the manifold calculus of embeddings of a manifold into a vector space. Since the work of Fred Cohen in the 1970's, the homology of this operad is well known to be a (generalized) Poisson operad. In the paper "Operads, motives and deformation quantization", M. Kontsevich stated and sketched the proof that this operad is formal. This means that there exists a zigzag of quasi-isomorphims of operads connecting the chains on the little disks operad with its homology (with coefficients in the field of real numbers). The special case of operads of little two-dimensional disks this has been proved by Tamarkin.

In this talk we will explain the proof of the formality of this operad. The main idea of the proof is to build a certain graph-complex and to show 1) that it is quasi-isomorphic to the chains of the little disks operad, thanks to a construction which is called the Kontsevich configuration integral; 2) that it is also quasi-isomorphic to its homology, which is proved by a simple algebraic argument. In order to properly interpret the integral we will replace the little disks operad by an equivalent one, namelythe operad of configuration spaces compactified  la Fulton-MacPherson. We will recall some interesting geometry of this operad.

Actually the formality result is more precise than initially stated by Kontsevich: (i) formality holds in the category of CDGA's, which encode rational homotopy type; (ii) we consider that the little disk operad has an operation in arity 0; (iii) some relative version of the operad holds.

An important technical point in the proof is that we need to use a variant of deRham theory for semi-algebraic sets instead of smooth manifolds. We probably will not have the time to explain this subtelity. There is also a cyclic version of formality for the framed little 2-disks operad proved by J. Giansiracusa and P. Salvatore

Speaker: **Mike Mandell** (Indiana)
Title: *Quillen Cohomology of Operadic Algebras and Obstruction Theory*
Abstract: Quillen defined homology in terms of abelianization. For operadic algebras Quillen homology is the derived functor of indecomposibles, and the bar duality (or "derived Koszul duality") construction provides a model for the Quillen homology. The $k$-invariants of Postnikov towers for algebras over an operad $\mathcal{O}$ lie in the Quillen cohomology groups, and this leads to an obstruction theory for $\mathcal{O}$-algebra structures and $\mathcal{O}$-algebra maps. I will discuss my recent paper with Maria Basterra where we apply this obstruction theory to show that BP is an $E_4$ ring spectrum.

Speaker: **Michael Weiss** (Aberdeen)
Title: *Smooth maps to the plane and Pontryagin classes*
Abstract: This is joint work with Rui Reis.

The long-term goal is to prove that the Pontryagin classes in the $4i$-dimensional rational cohomology of $BTOP(n)$, due to Novikov and Thom, satisfy the standard relations that we expect from Pontryagin classes of vector bundles, for example: vanishing when $i > 2n$. Here $TOP(n)$ is the topological group of homeomorphisms from $\mathbb{R}^n$ to itself. It is not difficult to recast this as a problem in orthogonal calculus, for the functor $V \mapsto BTOP(V)$ from real vector spaces with inner product to based spaces. Throwing in smoothing theory as well, one can recast it as a problem about spaces of regular (nonsingular) smooth maps from $D^n \times D^2 \to D^2$, extending the standard projection on the boundary. Imitating the parametrized Morse theory of Cerf and K Igusa, we investigate these spaces of regular maps by thinking of them as being contained in spaces of smooth maps to the plane with only moderate singularities. Then we need homotopy formulae for spaces of smooth maps with only moderate singularities. This leads us to Vassiliev's discriminant method. We reformulate that as a method for proving theorems in manifold calculus (formerly "embedding calculus"). The reformulation gives us some generalizations, allowing us to impose good behavior conditions not just on singularities in the source, $D^n \times D^2$, but also on singularity sets in the target, $D^2$. That gives us a lot to play with. We hope it is enough.