Mathematics: Muse, Maker, and Measure of the Arts (11w5070)

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1 Overview of the Field

Mathematics and arts have a long historical relationship. Tile mosaics since the early civilizations combine both artistic beauty and mathematical complexity. The ancient Egyptians and ancient Greeks knew about the golden ratio, regarded as an aesthetically pleasing ratio, and incorporated it into the design of monuments including the Great Pyramid, the Parthenon, the Colosseum. There are many examples of artists who have been inspired by mathematics and studied mathematics as a means of complementing their works. The Greek sculptor Polykleitos prescribed a series of mathematical proportions for carving the ideal male nude. Renaissance painters developed the theory of perspective, and many, including Piero della Francesca, became accomplished mathematicians themselves. The interplay of mathematics and art has continued to flourish throughout our history. Mathematics had greatly inspired artists as M. C. Escher, Picasso, Salvador Dali and artistic movements such as the minimalist and abstract art. Conversely, the art of tiling had contributed to the discovery of Penrose tiles and the study of aperiodic structures such quasicrystals, one of the most important areas in mathematics.

The study of geometry and advent of digital age have sown the seeds for a revolution in the arts. The processing power of modern computers allows mathematicians and non-mathematicians to visualize complex mathematical objects such as the Mandelbrot set and other fractal sets. The artistic beauty of such sets had attracted many mathematicians to discover fundamental properties they play in dynamical systems and chaos. In the modern industry of computer animation, fractals play a key role in modeling mountains, fire, trees and other natural objects. Fractals are an example of the growing field of generative art, which refers to ways to systematically and autonomously generating artwork in an algorithmic way using a computer. The workings of systems in generative art often rely on various fundamental scientific theories such as Complexity theory and Information theory. Generative art is not limited to abstract art. By combining it with Learning Theory it is even possible to artificially generate paintings and music that mimic known masters. While generative art refers to an autonomous system for generating artwork, artists today are increasingly relying on mathematics and computers to aide their creative work. Besides painting and music, the intricacy of origami highlights the fusion of mathematics and art. Modern computing has allowed us to make complex geometric designs that have led to the design and creation of origami figures whose complexity and delicacy cannot even be imagined in the past.

Mathematics is increasingly making its impact in the study of stylometry in art. The term stylometry was coined in 1897 by the historian of philosophy, Wincenty Lutaslowski, as a catch-all for a collection of statistical techniques applied to questions of authorship and evolution of style in the literary arts. The advent of high definition digitization for works of art may have opened up a whole new venue for art authentication. With scans of paintings capturing minute details that even human experts can easily miss it is hoped that more sophisticated mathematical and statistical tools can be developed to shed light on the question of provenance. It is also hoped that these new mathematical and statistical techniques will become valuable tools of research for art historians. Indeed, increasingly, huge treasuries of visual works are being digitized for the purposes of art historical education and research as well as restoration and conservation. Several research projects in this direction, such as the study of Jackson Pollock by Richard Taylor et al, the restoration of digital imagery of medieval draperies by the Chudnovsky brothers and the recent Van Gogh Project, had received headline media coverage.

2 Recent Developments and Challenges

The intended purpose of the workshop is to bring together leading mathematical researchers whose work pertain to arts. In doing so we hope to promote the application of mathematics to the study of arts, and to develop some coherent frameworks for moving this area of research further ahead. Given the diversity of the research activities this area spans it is impossible to cover all topics comprehensively. The workshop focused on two growing areas of research: visual stylometry analysis and mathematical techniques for art making.These selected topics will highlight how mathematical and statistical techniques can be valuable tools for the study of arts.

The evaluation of a work of art for attribution has historically relied on the techniques of connoisseurship, a process by which a questioned work is subjected to the evaluation by a few experts who are steeped in the work and life of the artist in question. The analyses are usually based on their extensive visual experience and encyclopedic knowledge of the career of the would-be artist, as well as other kinds of art historical data. The advent of high definition digitization for works of art has opened up a whole new venue for stylometry analysis. Powerful mathematical and statistical techniques are now available for the study of art, literature and music in terms of authentication and style analysis, motion capturing and more.

Stylometry analysis of literary style has its origins dating back to the mid 1800s. It is Augustus de Morgan, an English logician who first suggested to his friend in a letter in 1851 that questions of authorship might be settled by determining the length of words "*if one text does not deal in longer words than another.*" This technique and later more sophisticated ones have been used on stylometric analysis of works attributed to Shakespeare. Today, stylometric analysis of literature is already a field that has truly come into its own. Although stylometric analysis of art has lagged and is undoubtedly more challenging, time is ripe for it to take off. More importantly, stylometric analysis of art calls for a more diverse and (perhaps more sophisticated) repertoire of mathematical and statistical techniques. The stylometric study by Taylor et al (1999) of Jackson Pollock is based on the fractal property of Pollock's drip paintings, which shows that Pollock paintings have rather unique fractal signatures. Another pioneer work on stylometry of art was the analysis of drawings that had been attributed to the great Flemish artist Pieter Bruegel the Elder by Lyu et al (2004). In it the use of multiscale wavelet analysis was proposed as a tool for visual stylometry. Beside the Bruegel paintings the technique was also used to analyze a large altarpiece generally attributed to the workshop of the Renaissance master Perugino. The results of these experiments provided evidence indicating that the wavelet coefficients could be used as a source of information identifying the artist. More recently, in a comprehensive effort to study stylometry of art, several research teams had been put together to study the paintings of Vincent Van Gogh using high resolution digital scans in the "Van Gogh Project (VGP)." The research by these teams focused on the brush stroke analysis of these paintings using a variety of techniques such as wavelets, hidden Markov trees, sparse coding, c.f. a survey in Johnson et al (2008). A study by Hughes et al (2010) focuses on the use of Empirical Mode Decomposition for stylometry analysis. Wendt, Roux, Abry and Jaffard (2009) have and analyzed provenance and authorship of paintings by the analysis of multifractals. The results of these studies provide evidence that the mathematical study of stylometry of art is a fertile field.

Nevertheless challenges remains in the field of visual stylometry. Aside from the obvious challenge of obtaining high quality digital imageries for stylometric analysis, there are a number of mathematical challenges

as well. For example, the style of a particular artist is no static, and there are a great deal of subject-dependent variations in style even from an artist whose style remained rather constant over the years. The mathematical techniques used for stylomery analysis mostly focus on brush strokes, by doing so some important defining qualities of an artist are not taken into account. Furthermore, the mathematical techniques used so far are based mostly on multiscale decomposition techniques such as wavelets and Empirical Mode Decomposition. More techniques must be developed for the field to truly take off.

Another important topic of this workshop is to bring experts in these areas to discuss the latest mathematical techniques for art making. The advances in the study of geometry, dynamical systems, information theory, learning theory, and others have led to rapid advances in many areas of art making. And conversely, the pursue of new ideas and trends in art such as tiling, origami, computer graphics and abstract art has contributed greatly to the advances in some areas of mathematics. The visually stunning Mandelbrot set and many other fractal sets come from nonlinear dynamical systems, from which the study of chaos was born. Today fractals have been used not only as a generative tool for making beautiful pictures but also a tool for modeling natural objects and physical phenomena such as rough surfaces in material science. The discovery of Penrose tiles has led to the study of aperiodic orders and the study of quasicrystals, and has inspired the Escheresque artistic aspirations of many mathematicians, artists and students.

Today generative art is an active and broad area of research and practice. Generative art aims for the creation of artwork using algorithms (both deterministic and random). While Mandelbrot set and fractals represent a typical generative art, the field has gone far beyond it. Sophisticated mathematical and statistical techniques are now available to create a wide array of intricate artworks. For example, the texture model by Gousseau, Morel and others have led to exquisite abstract paintings that could often mimic those of the masters. The work by Robert Lang and John Montroll have developed geometrical and computational techniques that have taken origami to a state that was unimaginable just a few years ago. Some of these techniques in origami are now found applications in robotics. The study of ancient mosaics has led to new insights on how geometry was used in the making of patterns and tilings in the ancient time. Realistic modelling and rendering of textures and complex systems such as water waves has taken computer graphics to a new level that it becomes a vital part of film making. The advent of digital photography has opened up a whole new frontier of interplay between mathematics and arts. With the explosive growth of digital photography comes many challenging mathematical problems. Today "computational photography" is an extremely active research area in mathematics and computer science, which has also led to an attempt by mathematicians and computer scientists to generate artificial "photographs" that are realistic and artistic in its rendering. Mathematical models have also been used to evaluate the "artistry" of photographs with some degree of success. Mathematical techniques have also been used to enhance and to restore artwork. The work by Chudnovsky brothers on the digital archiving of medieval draperies and the digital restoration and enhancement of old films are also such examples. There are enormous challenges ahead in these areas. For example, although there is a model to evaluate artistry for digital photos, it is still relatively primitive at this stage. While we have been able to build more and more sophisticated mathematical models to generate arts, artistry rarely figures into the generating process. How to integrate it into the process remain one of the great challenges in mathematical arts.

3 Presentation Highlights and Outcomes

The workshop is characterized by many excellent presentations. They include very detailed overviews of key related areas of research and cutting edge researches. The workshop kicked off with a beautiful presentation by Ingrid Daubechies, who is a leading researcher in the area of stylometry analysis for visual arts. She gave a very in-depth overview of the field and highlighted many challenges in the area. In particular, she has gone into details about how the ongoing Van Gogh Project has shaped the field and has lead to many novel new techniques. Her talk had set the tone for many subsequent talks. David Mumford gave a beautiful overview on random models for image synthesis, drawing from both his earlier work in the 90's on image statistics and latest work by him and many others such as the Paris Group. Jim Coddington from MoMA spoke about the latest technological advances in analyzing arts that allow us to detect a painting underneath another one, and the mathematical challenges to restore such a painting. Craig Kaplan illustrated the use of computer graphics techniques in the generation of arts.

Several speakers have presented their findings in stylometry analysis, which are among the latest advances in this new field. Shannon Hughes presented the Hidden Markov model for authentication. James Wang developed a brush stroke model that is significantly different from the conventional wavelet based approach, and it can be applied effectively to Chinese watercolor paintings and calligraphy. James Hughes (joint with Dan Rockmore and Yang Wang) showed that the latest advances in Empirical Mode Decomposition (EMD) present a powerful tool for stylometric analysis. An exciting new development is presented by Patrice Abry and Stephan Jaffard, who showed that multifractal analysis, commonly used in the study of statistical mechanics and nonlinear dynamics, can be an effective tool for the study of brushstrokes and hence for visual stylometry. In one of the highlighted presentations, Jason Brown showed how he used Fourier analysis to unravel the mystery of the Beetle's *A Hard Day's Night*, a feat that has been featured in a number of media outlets.

beside the presentations in stylometry there are a number of engaging talks in other areas related to mathematics and arts. Reza Sarhangi spoke about the polyhedral modularity for the creation of artistic geometric patterns, and show how the idea was used for making patterns in the Near East. George Hart presented various beautiful geometric constructions for the making of artistic and intriguing geometric objects. Robert Moody gave a beautiful presentation on aperiodic tiling and Penrose tiles, and how the research blends in with physics and arts. Both Robert Schneider and Luke Wolcott illustrated the interplay of mathematics and music. Alice Major showed how mathematics is an integral part of her beautiful poems. The presentations by James Wang and James Hughes also included a very intriguing but relatively undeveloped concept: the measurement of artistic beauty. In particular, James Wang developed a system that ranks digital photos. His presentation has sparked a lively discussion on how and whether mathematics can or should be used to evaluate the *artistry* of an art.

This workshop is the first major meeting on stylomtery, and it will not be the last as the field gains more and more attention. Our hope is that from this experience we will be able to share our ideas and latest findings to move the research further ahead. The fact that there are experts in arts and mathematics (including accomplished artists such as Robert Moody, David Mumford, Jim Coddington, George Hart, Jason Brown, Robert Schneider, Alice Major) in attendance will provide a great boost for the field.

The workshop has been memorable for many participants because of the interactions between mathematicians and those who have strongly established record in art making (e.g. those aforementioned participants). To many of the participants whose primary specialty is mathematical research, it has open their eyes on many intriguing possibilities. The workshop has already spurred several ongoing and potential collaborations, which would not have happened without it. Sarhangi and Wang have already submitted an NSF proposal for the upcoming BRIDGE conference, which will for the first time include a session on stylometry. Daubechies, Hughes, Kaplan and Wang are contemplating a proposal for the next Banff meeting. S. Hughes have discussed collaborating with P. Abry and S. Jaffard. Coddington has generously offered to help researchers with digital images in the MoMA collection. Brown and Y. Wang are looking into a deeper study of music and stylometry using the latest mathematical and machine learning techniques.

References

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