

How Helpful is Algebraic Structure in Network Information Theory?

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Algebraic Structure in Network Information Theory

Overview

Overview

- ▶ Relay networks

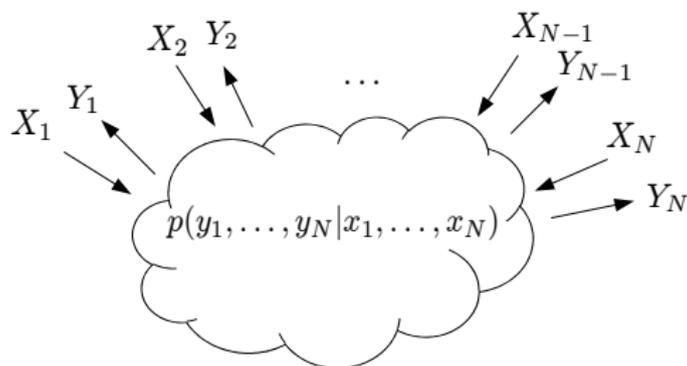
Overview

- ▶ Relay networks
- ▶ Compute-and-forward

Overview

- ▶ Relay networks
- ▶ Compute-and-forward
- ▶ Noisy network coding

Network model



- ▶ N nodes consisting of sources, relays, and destinations.
- ▶ Node k transmits $X_k \in \mathcal{X}_k$ and receives $Y_k \in \mathcal{Y}_k$.
- ▶ Channel: $p(y_1, \dots, y_N | x_1, \dots, x_N)$

Encoding, relaying, decoding

- ▶ $W_{i,j}$: message from node i to node j , uniformly distributed in $[1, 2^{nR_{i,j}}] \triangleq \{1, \dots, 2^{nR_{i,j}}\}$. $R_{i,i} = 0, \forall i$.
- ▶ $X_k^n \triangleq \{X_{k,1}, \dots, X_{k,n}\}$: transmitted vector at node k over n channel uses
- ▶ Y_k^n : received vector at node k
- ▶ $X_{k,i} = X_{k,i}(\{W_{k,j}|j \in [1, M]\}, Y_k^{i-1})$
- ▶ $\{\hat{W}_{j,k}(Y_k^n, \{W_{k,i}|i \in [1, M]\})|j \in [1, M]\}$: message estimates at node k
- ▶ Discrete memoryless channel

$$p(y_1^n, \dots, y_N^n | x_1^n, \dots, x_N^n) = \prod_{i=1}^n p(y_{1,i}, \dots, y_{N,i} | x_{1,i}, \dots, x_{N,i})$$

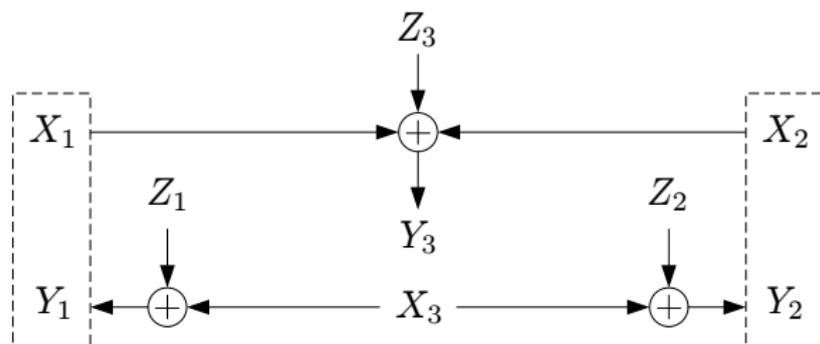
Capacity

- ▶ A set of rates $\{R_{i,j} | i, j \in [1, N]\}$ is said to be *achievable* if there exists a sequence of encoding (relaying) and decoding functions such that

$$P_e \triangleq \Pr(W_{i,j} \neq \hat{W}_{i,j} \text{ for some } i, j \in [1, N]) \rightarrow 0 \text{ as } n \rightarrow \infty$$

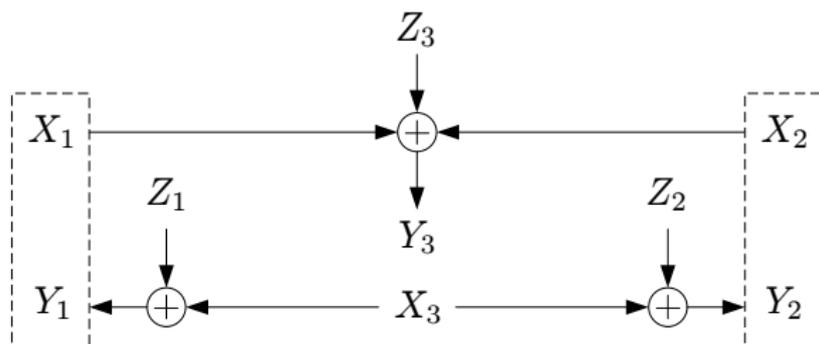
- ▶ Capacity region: closure of the set of achievable rates

Two-way relay channel



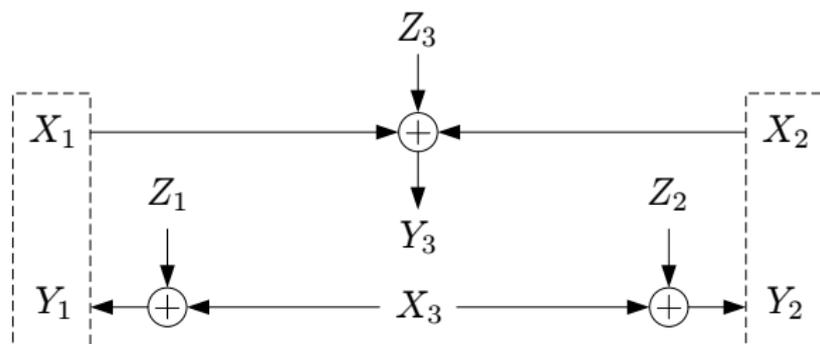
- ▶ P_1, P_2 : Uplink SNR's of users 1 and 2
- ▶ Q_1, Q_2 : Downlink SNR's of users 1 and 2

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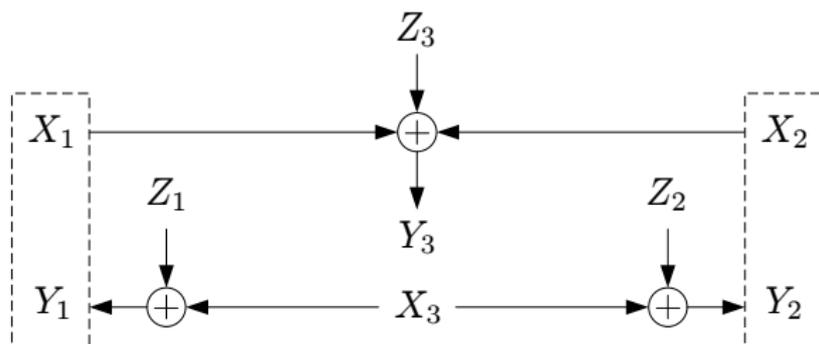
- ▶ P_1, P_2 : Uplink SNR's of users 1 and 2
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- ▶ Compress-and-forward (CF), decode-and-forward (DF) [Rankov, Wittneben '06]

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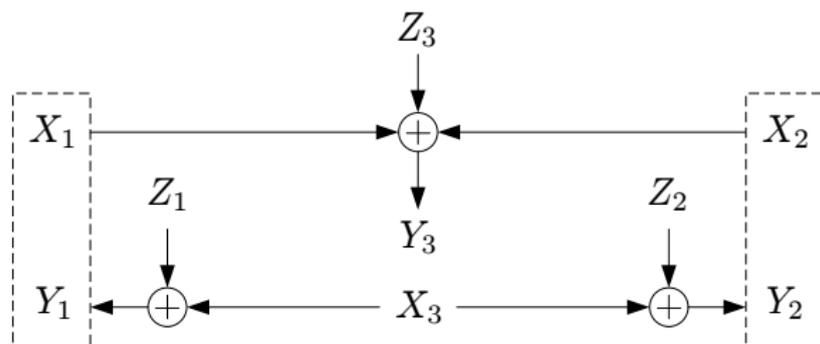
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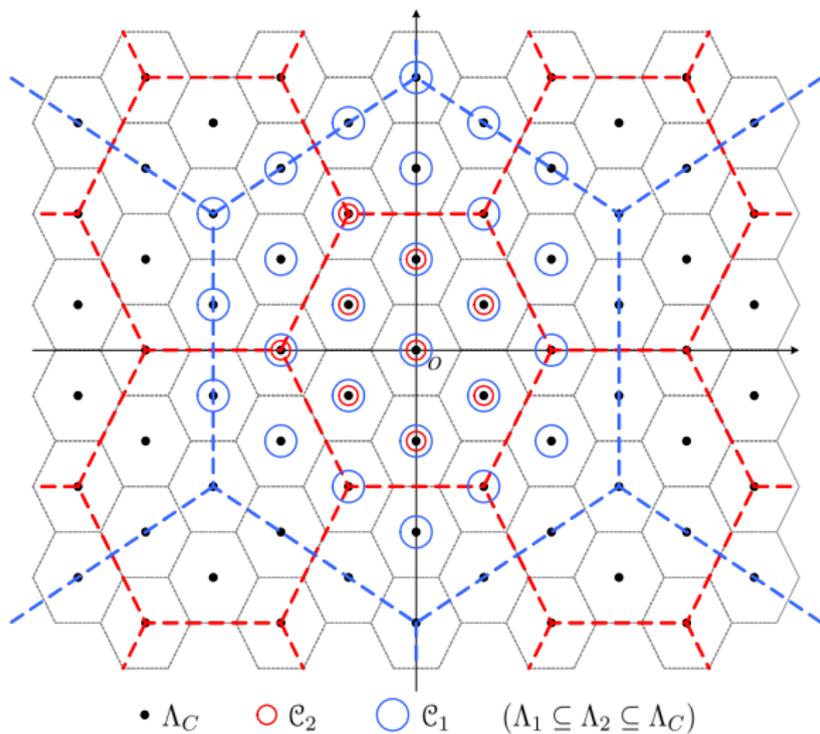
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- ▶ Unequal power case: Nam, C, Lee, IZS '08, IT '10

Nested lattice code



Two-way relay channel

Theorem (Nam, C, Lee, IZS '08, IT '10)

For the Gaussian two-way relay channel, the following rate pair is achievable

$$R_1 < \min \left\{ \left[\frac{1}{2} \log \left(\frac{P_1}{P_1 + P_2} + P_1 \right) \right]^+, C(Q_2) \right\}$$

$$R_2 < \min \left\{ \left[\frac{1}{2} \log \left(\frac{P_2}{P_1 + P_2} + P_2 \right) \right]^+, C(Q_1) \right\}$$

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► Cut-set bound

$$R_1 \leq \min\{C(P_1), C(Q_2)\}$$

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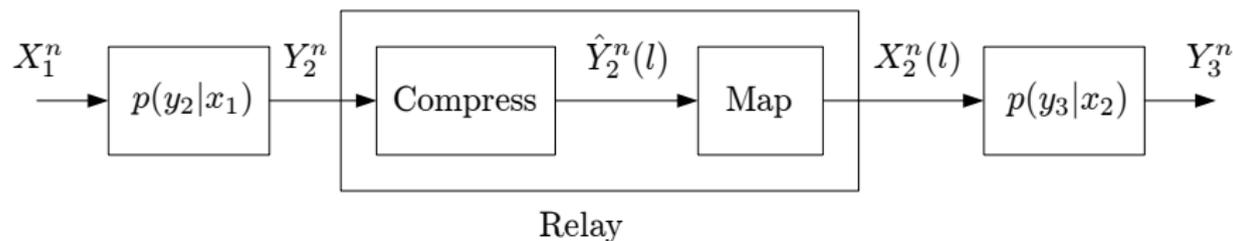
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- ▶ Gap to capacity: $\frac{1}{2}$ bit per user, $\log \frac{3}{2} \sim 0.58$ bits for the sum rate

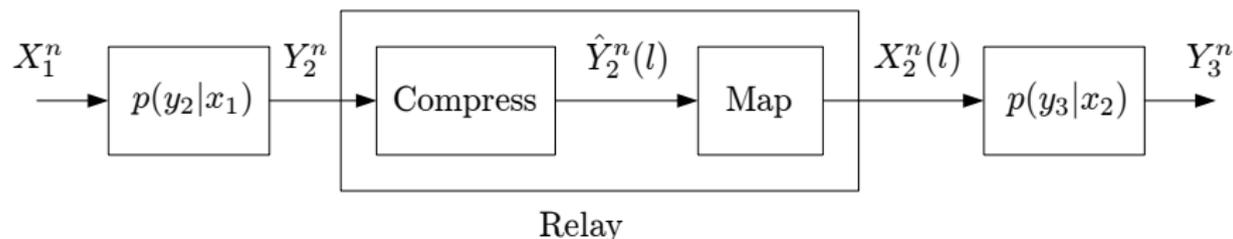
Compress-and-forward



- Compress-and-forward for 2-hop noisy RN can achieve

$$\max_{p(x_1)p(x_2)p(\hat{y}_2|y_2): I(X_2; Y_3) \geq I(Y_2; \hat{Y}_2)} I(X_1; \hat{Y}_2)$$

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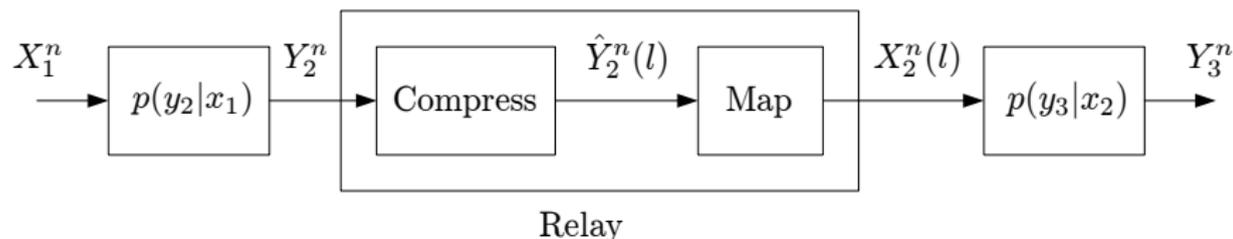


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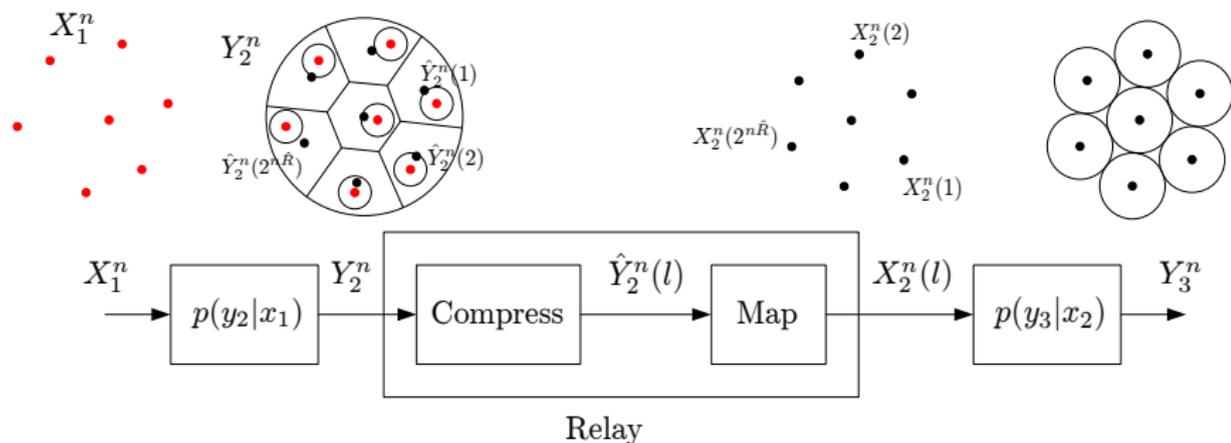


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- ▶ $I(X_2; Y_3) > I(Y_2; \hat{Y}_2)$ needed to be able to send the compression index l over the second hop
- ▶ Destination decodes l and (roughly speaking) gets an effective channel $X_1 \rightarrow \hat{Y}_2$ for X_1 that supports rates up to $I(X_1; \hat{Y}_2)$.

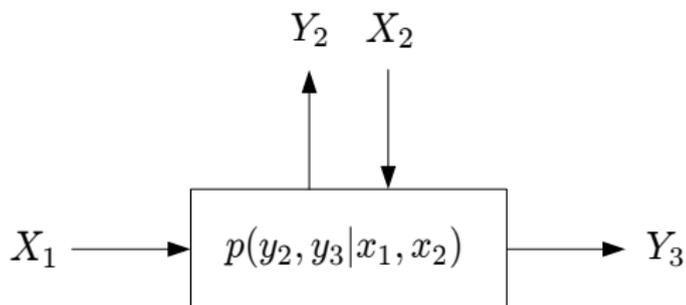
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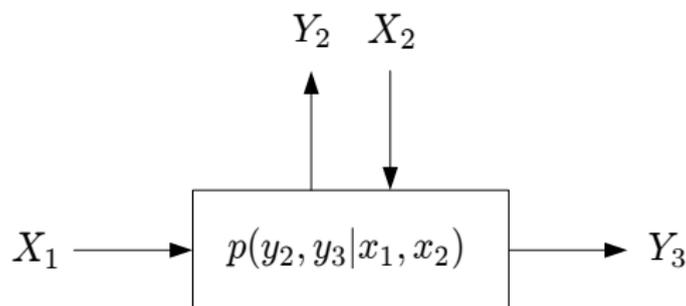
Compress-and-forward



- ▶ CF for 3-node relay channel (Cover & El Gamal '79)

$$\max_{p(x_1)p(x_2)p(\hat{y}_2|y_2,x_2): I(X_2;Y_3) \geq I(Y_2;\hat{Y}_2|X_2,Y_3)} I(X_1; \hat{Y}_2, Y_3 | X_2)$$

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- ▶ Equivalent min-cut-like form (El Gamal, Mohseni, Zahedi '06)

$$\max \min \{ I(X_1; \hat{Y}_2, Y_3 | X_2), I(X_1, X_2; Y_3) - I(Y_2; \hat{Y}_2 | X_1, X_2, Y_3) \}$$

where the maximization is over $p(x_1)p(x_2)p(\hat{y}_2|y_2,x_2)$.

CF for TWRC

- ▶ Achievable rate of CF for DM-TWRC, Rankov, Wittneben '06

$$R_1 < I(X_1; Y_2, \hat{Y}_3 | X_2, X_3)$$

$$R_2 < I(X_2; Y_1, \hat{Y}_3 | X_1, X_3)$$

for some $p(x_1)p(x_2)p(x_3)p(\hat{y}_3|x_3, y_3)$ such that
 $\max_{k=1,2} I(\hat{Y}_3; Y_3 | X_3, X_k, Y_k) < \min_{k=1,2} I(X_3; Y_k | X_k)$.

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- ▶ Gaussian TWRC w/o direct links

$$R_1 < C \left(\frac{P_1}{1 + \sigma^2} \right)$$

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- ▶ $R_1, R_2 \rightarrow 0$ if $Q_1 \rightarrow 0$. Arbitrarily large gap to capacity.
- ▶ Can we do better?

Noisy Network Coding

- ▶ Multi-source multicast relay networks
- ▶ R_k : rate of node k
- ▶ D_k : set of destination nodes receiving message from k

Theorem (Noisy network coding (Lim, Kim, El Gamal, C '10))

For multi-source multicast RN with $D = D_1 = \dots = D_N$, the following is achievable

$$\sum_{k \in T} R_k < \min_{d \in T^c \cap D} I(X_T; \hat{Y}_{T^c}, Y_d | X_{T^c}, Q) - I(Y_T; \hat{Y}_T | X^N, \hat{Y}_{T^c}, Y_d, Q)$$

for all cuts T s.t. $T^c \cap D \neq \emptyset$ for some $p(q) \prod_{k=1}^N p(x_k | q) p(\hat{y}_k | y_k, x_k, q)$.

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- ▶ Non-unique decoding of compression indices and some unwanted messages

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for all cuts T s.t. $T^c \cap D \neq \emptyset$ for some $p(q) \prod_{k=1}^N p(x_k | q) p(\hat{y}_k | y_k, x_k, q)$.

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Includes the following as special cases:

- ▶ Max-flow min-cut theorem (Ford, Fulkerson '56)
- ▶ CF for 3-node relay channel (Cover, El Gamal '79)
- ▶ Network coding (Ahlswede, Cai, Li, Yeung '00)
- ▶ Wireless erasure networks (Dana et al '06)
- ▶ Deterministic relay networks (Avestimehr, Diggavi, Tse '07)

Noisy Network Coding

Theorem (Nonunique decoding of unwanted messages)

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$p(q) \prod_{k=1}^N p(x_k | q) p(\hat{y}_k | y_k, x_k, q)$, where $D_T = \cup_{k \in T} D_k$.

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for all cuts T s.t. $T^c \cap D_T \neq \emptyset$ for some $p(q) \prod_{k=1}^N p(x_k | q) p(\hat{y}_k | y_k, x_k, q)$, where $D_T = \cup_{k \in T} D_k$.

Theorem (Treating interference as noise)

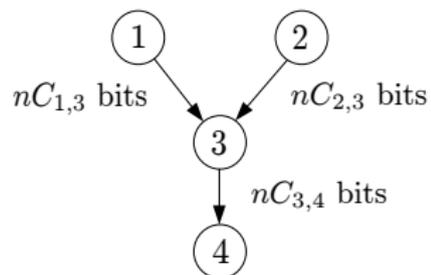
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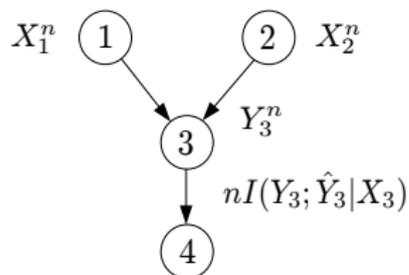
for all cuts S, T and $d \in D_S$ such that $S \cap S_d \subseteq T \subseteq S_d$ and $S^c \cap D_S \neq \emptyset$ for some $p(q) \prod_{k=1}^N p(u_k, x_k | q) p(\hat{y}_k | y_k, u_k, q)$, where $T^c = S_d \setminus T$.

Network coding vs. NNC

Network coding

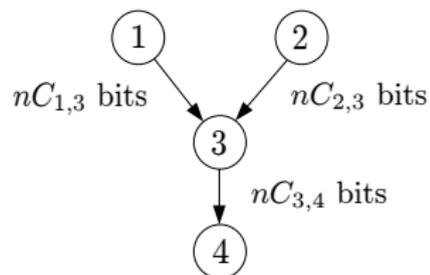


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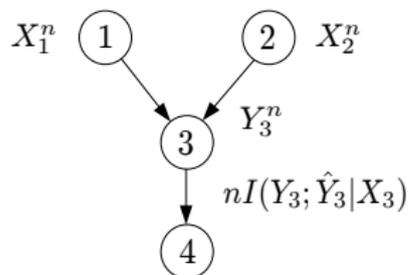


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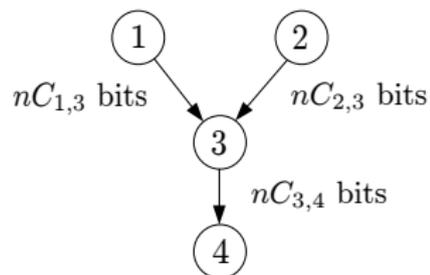
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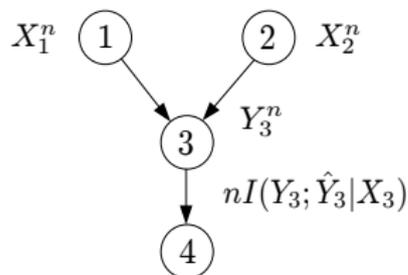
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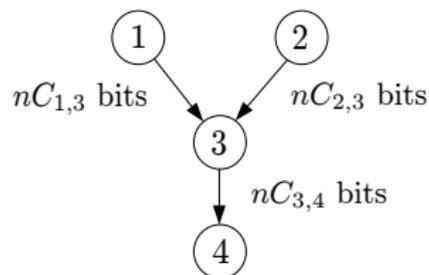
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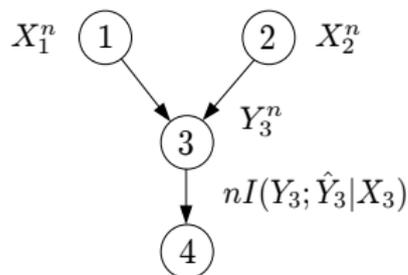
- ▶ Network coding: Compression can happen due to bottleneck, i.e., if $C_{1,3} + C_{2,3} > C_{3,4}$
- ▶ Noisy network coding (quantization + network coding)

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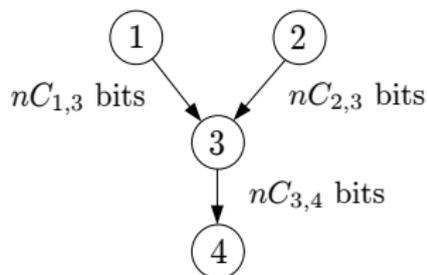
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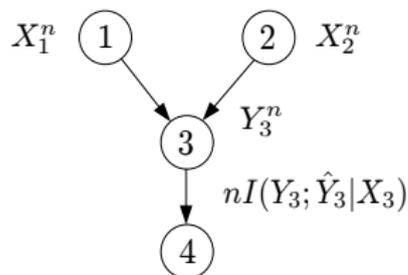
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- ▶ Noisy network coding (quantization + network coding)
 - ▶ Quantization: Explicit compression

Network coding vs. NNC

Network coding



Noisy network coding



- ▶ Network coding: Compression can happen due to bottleneck, i.e., if $C_{1,3} + C_{2,3} > C_{3,4}$
- ▶ Noisy network coding (quantization + network coding)
 - ▶ Quantization: Explicit compression
 - ▶ Network coding: Additional implicit compression can happen due to bottleneck, i.e., if $I(Y_3; \hat{Y}_3 | X_3) > H(X_3)$

Noisy Network Coding for TWRC

- ▶ Noisy Network Coding for DM-TWRC, Lim, Kim, El Gamal, C '10

$$R_1 < \min\{I(X_1; Y_2, \hat{Y}_3 | X_2, X_3), I(X_1, X_3; Y_2 | X_2) - I(Y_3; \hat{Y}_3 | X_1, X_2, X_3, Y_2)\}$$

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for some $p(x_1)p(x_2)p(x_3)p(\hat{y}_3 | x_3, y_3)$.

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- ▶ Gap to capacity: $\frac{1}{2}$ bit per user, 1 bit for the sum rate

Constant gap for Gaussian RN

Theorem (Gaussian RN)

For multi-source multicast Gaussian RN with a single destination set, if (R_1, \dots, R_N) is in the cut-set bound, then $(R_1 - 0.63N, \dots, R_N - 0.63N)$ is achievable by NNC.

- ▶ Generalization of constant gap result by ADT

Lessons

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 - ▶ A big MAC problem

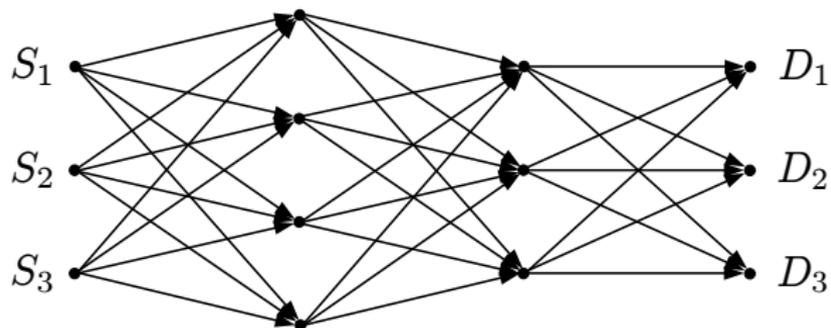
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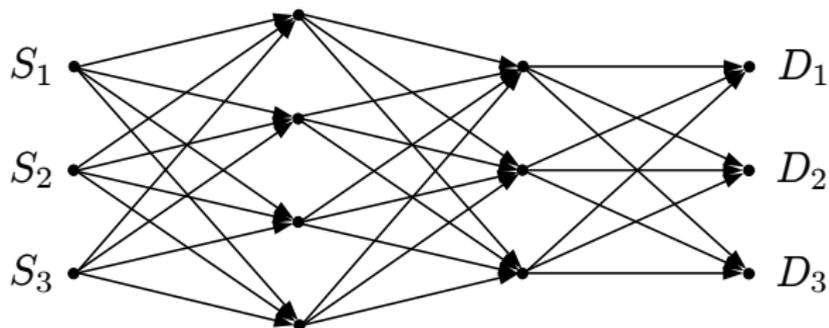
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Multiple unicast

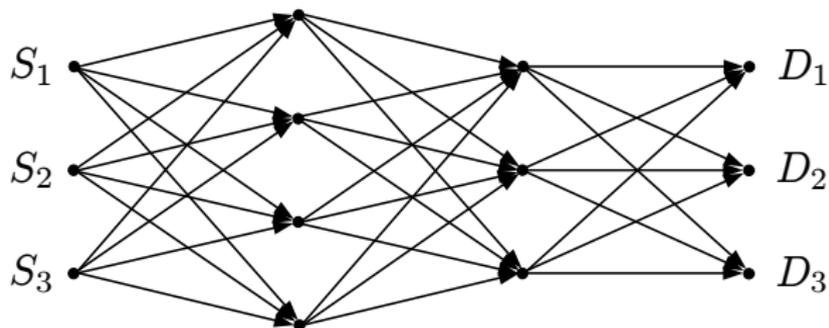


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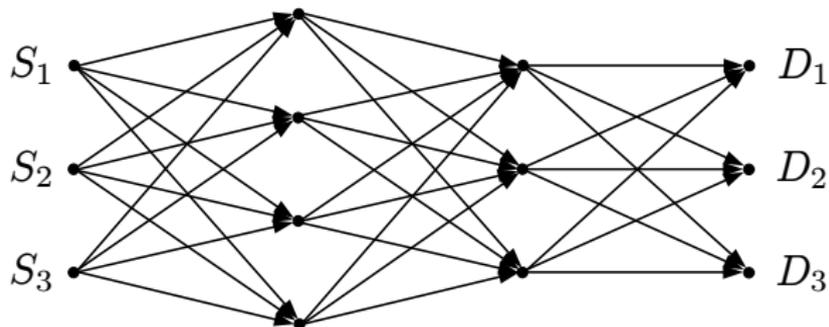
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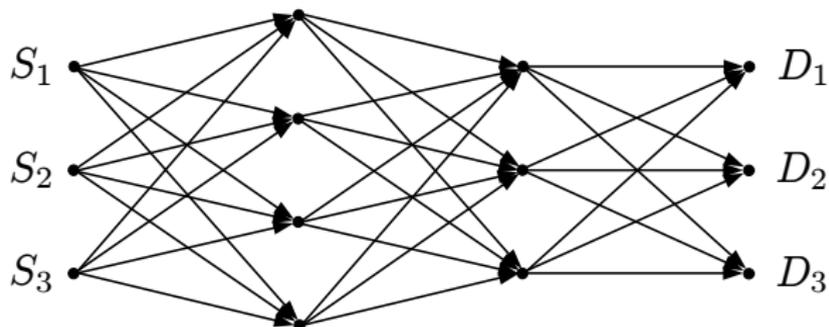
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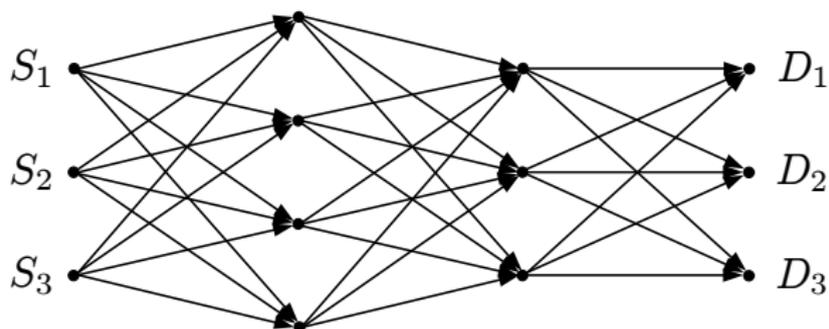
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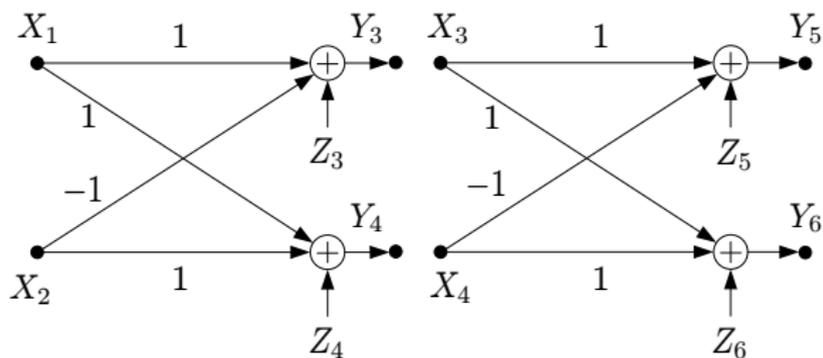
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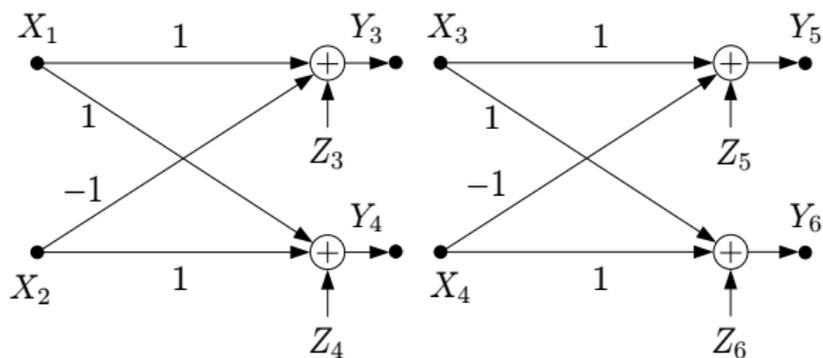


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- ▶ What do we need?
- ▶ Careful control of interference

Interference Neutralization

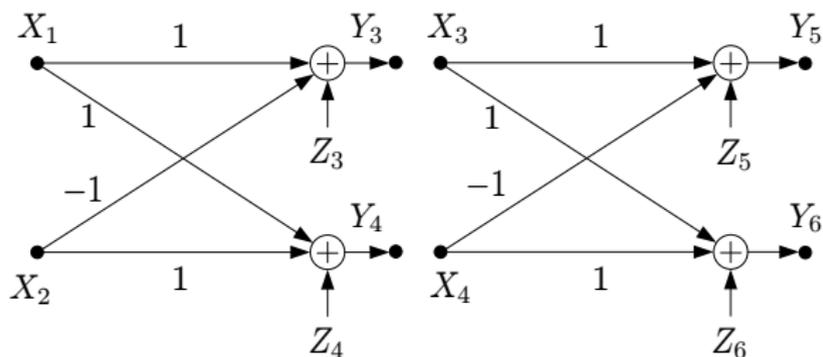


Interference Neutralization



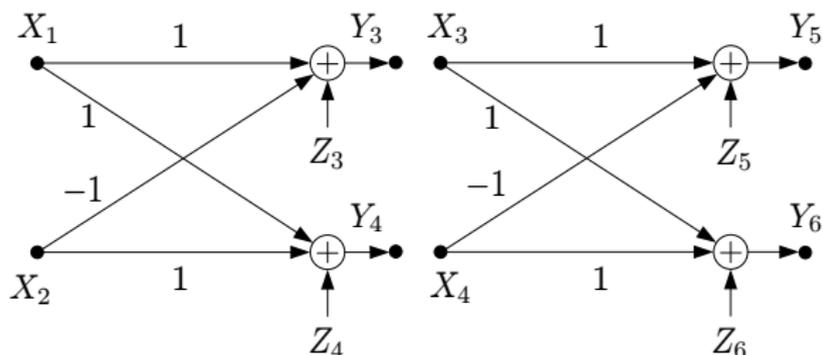
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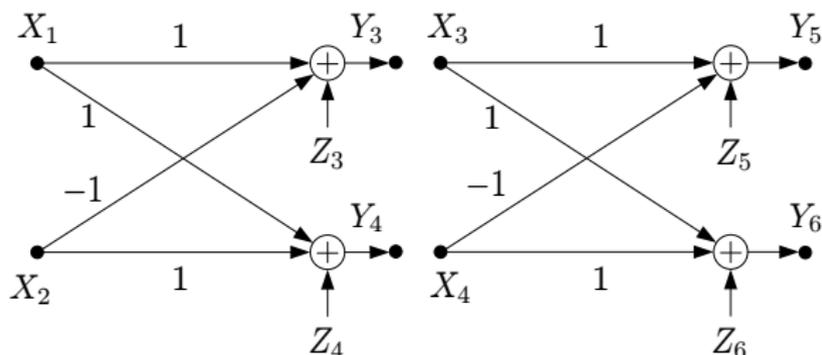
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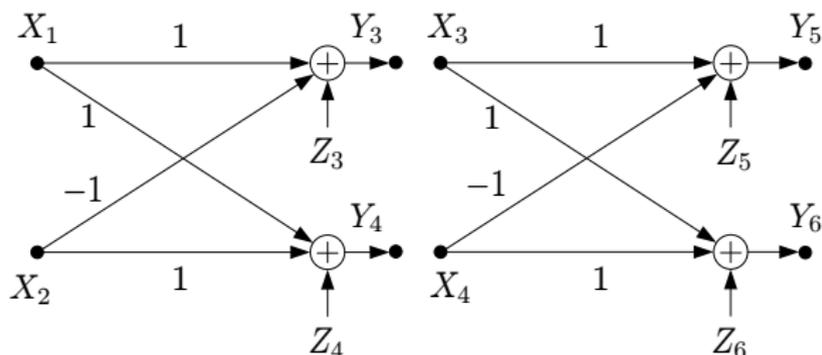
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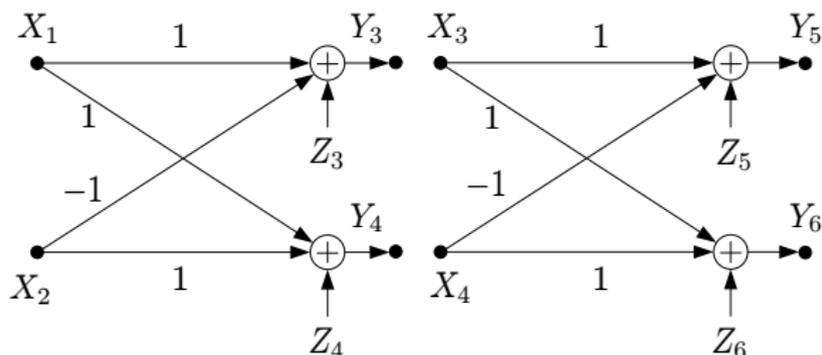
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Conclusions

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