How Helpful is Algebraic Structure in Network Information Theory?

Sae-Young Chung

Dept. Electrical Engineering, KAIST

August 16, 2011
Algebraic Structure in Network Information Theory
Overview
Overview

- Relay networks
Overview

- Relay networks
- Compute-and-forward
Overview

- Relay networks
- Compute-and-forward
- Noisy network coding
Network model

- $N$ nodes consisting of sources, relays, and destinations.
- Node $k$ transmits $X_k \in \mathcal{X}_k$ and receives $Y_k \in \mathcal{Y}_k$.
- Channel: $p(y_1, \ldots, y_N | x_1, \ldots, x_N)$
Encoding, relaying, decoding

- $W_{i,j}$: message from node $i$ to node $j$, uniformly distributed in $[1, 2^{nR_{i,j}}] = \{1, \ldots, 2^{nR_{i,j}}\}$. $R_{i,i} = 0$, $\forall i$.
- $X_k^n \triangleq \{X_{k,1}, \ldots, X_{k,n}\}$: transmitted vector at node $k$ over $n$ channel uses
- $Y_k^n$: received vector at node $k$
- $X_{k,i} = X_{k,i}(\{W_{k,j}|j \in [1,N]\}, Y_k^{i-1})$
- $\{\hat{W}_{j,k}(Y_k^n, \{W_{k,i}|i \in [1,N]\})|j \in [1,N]\}$: message estimates at node $k$
- Discrete memoryless channel

$$p(y_1^n, \ldots, y_N^n|x_1^n, \ldots, x_N^n) = \prod_{i=1}^{n} p(y_{1,i}, \ldots, y_{N,i}|x_{1,i}, \ldots, x_{N,i})$$
A set of rates \( \{R_{i,j}| i, j \in [1, N]\} \) is said to be \textit{achievable} if there exists a sequence of encoding (relaying) and decoding functions such that

\[
P_e \triangleq \Pr(W_{i,j} \neq \hat{W}_{i,j} \text{ for some } i, j \in [1, N]) \to 0 \quad \text{as } n \to \infty
\]

Capacity region: closure of the set of achievable rates
Two-way relay channel

- $P_1, P_2$: Uplink SNR’s of users 1 and 2
- $Q_1, Q_2$: Downlink SNR’s of users 1 and 2
Two-way relay channel

- $P_1, P_2$: Uplink SNR’s of users 1 and 2
- $Q_1, Q_2$: Downlink SNR’s of users 1 and 2
- Compress-and-forward (CF), decode-and-forward (DF) [Rankov, Wittneben ’06]
Two-way relay channel

- $P_1, P_2$: Uplink SNR's of users 1 and 2
- $Q_1, Q_2$: Downlink SNR's of users 1 and 2
- Compress-and-forward (CF), decode-and-forward (DF) [Rankov, Wittneben ’06]
  - Arbitrarily large gap from the cut-set bound
Two-way relay channel

- $P_1, P_2$: Uplink SNR’s of users 1 and 2
- $Q_1, Q_2$: Downlink SNR’s of users 1 and 2
- Compress-and-forward (CF), decode-and-forward (DF) [Rankov, Wittneben ’06]
  - Arbitrarily large gap from the cut-set bound
- Nested lattice code (equal power case): Narayanan, Wilson, Sprintson, Allerton ’07
Two-way relay channel

- $P_1, P_2$: Uplink SNR’s of users 1 and 2
- $Q_1, Q_2$: Downlink SNR’s of users 1 and 2
- Compress-and-forward (CF), decode-and-forward (DF) [Rankov, Wittneben ’06]
  - Arbitrarily large gap from the cut-set bound
- Nested lattice code (equal power case): Narayanan, Wilson, Sprintson, Allerton ’07
- Unequal power case: Nam, C, Lee, IZS ’08, IT ’10
Nested lattice code

\( \Lambda_C \quad \circ \quad \mathcal{C}_2 \quad \odot \quad \mathcal{C}_1 \quad (\Lambda_1 \subseteq \Lambda_2 \subseteq \Lambda_C) \)
Two-way relay channel

Theorem (Nam, C, Lee, IZS ’08, IT ’10)

For the Gaussian two-way relay channel, the following rate pair is achievable

\[
R_1 < \min \left\{ \left[ \frac{1}{2} \log \left( \frac{P_1}{P_1 + P_2} + P_1 \right) \right]^+, C(Q_2) \right\}
\]

\[
R_2 < \min \left\{ \left[ \frac{1}{2} \log \left( \frac{P_2}{P_1 + P_2} + P_2 \right) \right]^+, C(Q_1) \right\}
\]
Two-way relay channel

Theorem (Nam, C, Lee, IZS '08, IT '10)

For the Gaussian two-way relay channel, the following rate pair is achievable

\[
R_1 < \min \left\{ \left[ \frac{1}{2} \log \left( \frac{P_1}{P_1 + P_2} + P_1 \right) \right]^+ , C(Q_2) \right\}
\]

\[
R_2 < \min \left\{ \left[ \frac{1}{2} \log \left( \frac{P_2}{P_1 + P_2} + P_2 \right) \right]^+ , C(Q_1) \right\}
\]

- Cut-set bound

\[
R_1 \leq \min \{ C(P_1), C(Q_2) \}
\]

\[
R_2 \leq \min \{ C(P_2), C(Q_1) \}
\]
Two-way relay channel

Theorem (Nam, C, Lee, IZS ’08, IT ’10)

For the Gaussian two-way relay channel, the following rate pair is achievable

\[
R_1 < \min \left\{ \left[ \frac{1}{2} \log \left( \frac{P_1}{P_1 + P_2} + P_1 \right) \right]^+, C(Q_2) \right\}
\]

\[
R_2 < \min \left\{ \left[ \frac{1}{2} \log \left( \frac{P_2}{P_1 + P_2} + P_2 \right) \right]^+, C(Q_1) \right\}
\]

- Cut-set bound

\[
R_1 \leq \min\{C(P_1), C(Q_2)\}
\]

\[
R_2 \leq \min\{C(P_2), C(Q_1)\}
\]

- Gap to capacity: \( \frac{1}{2} \) bit per user, \( \log \frac{3}{2} \sim 0.58 \) bits for the sum rate
Compress-and-forward

Compress-and-forward for 2-hop noisy RN can achieve

\[
\max_{p(x_1)p(x_2)p(\hat{y}_2|y_2)} I(X_1; \hat{Y}_2) \geq I(Y_2; \hat{Y}_2)
\]
Compress-and-forward

Compress-and-forward for 2-hop noisy RN can achieve

\[
\max_{p(x_1)p(x_2)p(\hat{y}_2|y_2)} I(X_1; \hat{Y}_2) \geq I(Y_2; \hat{Y}_2)
\]

\[
I(X_2; Y_3) > I(Y_2; \hat{Y}_2)
\]

needed to be able to send the compression index \( l \) over the second hop
Compress-and-forward for 2-hop noisy RN can achieve

$$\max_{p(x_1)p(x_2)p(\hat{y}_2|y_2)} I(X_1; \hat{Y}_2) \geq I(Y_2; \hat{Y}_2)$$

- $I(X_2; Y_3) > I(Y_2; \hat{Y}_2)$ needed to be able to send the compression index $l$ over the second hop
- Destination decodes $l$ and (roughly speaking) gets an effective channel $X_1 \rightarrow \hat{Y}_2$ for $X_1$ that supports rates up to $I(X_1; \hat{Y}_2)$. 
Compress-and-forward for 2-hop noisy RN can achieve

\[
\max \quad I(X_1; \hat{Y}_2) \quad \text{subject to} \quad p(x_1)p(x_2)p(\hat{y}_2|x_2): I(X_2; Y_3) \geq I(Y_2; \hat{Y}_2)
\]
Compress-and-forward

- CF for 3-node relay channel (Cover & El Gamal '79)

\[
\max \quad I(X_1; \hat{Y}_2, Y_3|X_2) \\
P(x_1)p(x_2)p(\hat{y}_2|y_2,x_2): I(X_2; Y_3) \geq I(Y_2; \hat{Y}_2|X_2, Y_3)
\]
Compress-and-forward

\\[ Y_2 \quad X_2 \]

\\[ X_1 \rightarrow p(y_2, y_3|x_1, x_2) \rightarrow Y_3 \]

- CF for 3-node relay channel (Cover & El Gamal '79)

\[
\max \quad I(X_1; \hat{Y}_2, Y_3|X_2) \\
\text{s.t.} \quad p(x_1)p(x_2)p(\hat{y}_2|y_2, x_2): I(X_2; Y_3) \geq I(Y_2; \hat{y}_2|X_2, Y_3)
\]

- Equivalent min-cut-like form (El Gamal, Mohseni, Zahedi '06)

\[
\max \min \{ I(X_1; \hat{Y}_2, Y_3|X_2), I(X_1, X_2; Y_3) - I(Y_2; \hat{y}_2|X_1, X_2, Y_3) \}
\]

where the maximization is over \( p(x_1)p(x_2)p(\hat{y}_2|y_2, x_2) \).
CF for TWRC

- Achievable rate of CF for DM-TWRC, Rankov, Wittneben ’06

\[ R_1 < I(X_1; Y_2, \hat{Y}_3|X_2, X_3) \]
\[ R_2 < I(X_2; Y_1, \hat{Y}_3|X_1, X_3) \]

for some \( p(x_1)p(x_2)p(x_3)p(\hat{y}_3|x_3, y_3) \) such that
\[ \max_{k=1,2} I(\hat{Y}_3; Y_3|X_3, X_k, Y_k) < \min_{k=1,2} I(X_3; Y_k|X_k). \]
CF for TWRC

- Achievable rate of CF for DM-TWRC, Rankov, Wittneben '06

\[ R_1 < I(X_1; Y_2, \hat{Y}_3|X_2, X_3) \]
\[ R_2 < I(X_2; Y_1, \hat{Y}_3|X_1, X_3) \]

for some \( p(x_1)p(x_2)p(x_3)p(\hat{y}_3|x_3, y_3) \) such that
\[ \max_{k=1,2} I(\hat{Y}_3; Y_3|X_3, X_k, Y_k) < \min_{k=1,2} I(X_3; Y_k|X_k). \]

- Gaussian TWRC w/o direct links

\[ R_1 < C \left( \frac{P_1}{1 + \sigma^2} \right) \]
\[ R_2 < C \left( \frac{P_2}{1 + \sigma^2} \right) \]

where \( \sigma^2 = \frac{1+\max\{P_1, P_2\}}{\min\{Q_1, Q_2\}}. \)
CF for TWRC

- Achievable rate of CF for DM-TWRC, Rankov, Wittneben '06

\[
R_1 < I(X_1; Y_2, \hat{Y}_3|X_2, X_3) \\
R_2 < I(X_2; Y_1, \hat{Y}_3|X_1, X_3)
\]

for some \(p(x_1)p(x_2)p(x_3)p(\hat{y}_3|x_3, y_3)\) such that
\[
\max_{k=1,2} I(\hat{Y}_3; Y_3|X_3, X_k, Y_k) < \min_{k=1,2} I(X_3; Y_k|X_k).
\]

- Gaussian TWRC w/o direct links

\[
R_1 < C \left( \frac{P_1}{1 + \sigma^2} \right) \\
R_2 < C \left( \frac{P_2}{1 + \sigma^2} \right)
\]

where \(\sigma^2 = \frac{1+\max\{P_1, P_2\}}{\min\{Q_1, Q_2\}}\).

- \(R_1, R_2 \to 0\) if \(Q_1 \to 0\). Arbitrarily large gap to capacity.
Achievable rate of CF for DM-TWRC, Rankov, Wittneben '06

\[ R_1 < I(X_1; Y_2, \hat{Y}_3 | X_2, X_3) \]

\[ R_2 < I(X_2; Y_1, \hat{Y}_3 | X_1, X_3) \]

for some \( p(x_1)p(x_2)p(x_3)p(\hat{y}_3|x_3, y_3) \) such that
\[
\max_{k=1,2} I(\hat{Y}_3; Y_3 | X_3, X_k, Y_k) < \min_{k=1,2} I(X_3; Y_k | X_k).
\]

Gaussian TWRC w/o direct links

\[ R_1 < C \left( \frac{P_1}{1 + \sigma^2} \right) \]

\[ R_2 < C \left( \frac{P_2}{1 + \sigma^2} \right) \]

where \( \sigma^2 = \frac{1 + \max\{P_1, P_2\}}{\min\{Q_1, Q_2\}} \).

\( R_1, R_2 \to 0 \) if \( Q_1 \to 0 \). Arbitrarily large gap to capacity.

Can we do better?
Noisy Network Coding

- Multi-source multicast relay networks
- $R_k$: rate of node $k$
- $D_k$: set of destination nodes receiving message from $k$

**Theorem (Noisy network coding (Lim, Kim, El Gamal, C ’10))**

*For multi-source multicast RN with $D = D_1 = \ldots = D_N$, the following is achievable*

$$\sum_{k \in T} R_k < \min_{d \in T^c \cap D} I(X_T; \hat{Y}_{T^c}, Y_d|X_{T^c}, Q) - I(Y_T; \hat{Y}_T|X^N, \hat{Y}_{T^c}, Y_d, Q)$$

*for all cuts $T$ s.t. $T^c \cap D \neq \emptyset$ for some $p(q) \prod_{k=1}^N p(x_k|q)p(\hat{y}_k|y_k, x_k, q)$.*
Key ingredients
Key ingredients

- Simultaneous decoding of the message and compression indices
Key ingredients

- Simultaneous decoding of the message and compression indices
  - A must for general noisy networks, e.g., TWRC
Key ingredients

- Simultaneous decoding of the message and compression indices
  - A must for general noisy networks, e.g., TWRC
  - Explicit Wyner-Ziv binning not needed anymore if relay’s transmitted codeword, compression index, and message are all simultaneously decoded
Key ingredients

- Simultaneous decoding of the message and compression indices
  - A must for general noisy networks, e.g., TWRC
  - Explicit Wyner-Ziv binning not needed anymore if relay’s transmitted codeword, compression index, and message are all simultaneously decoded
- A single big message instead of multiple independent messages
Key ingredients

- Simultaneous decoding of the message and compression indices
  - A must for general noisy networks, e.g., TWRC
  - Explicit Wyner-Ziv binning not needed anymore if relay’s transmitted codeword, compression index, and message are all simultaneously decoded
- A single big message instead of multiple independent messages
  - Similar to time expansion in network coding by Ahlswede, Cai, Li, Yeung ’00 and unfolding of graph by Avestimehr, Diggavi, Tse ’07
Key ingredients

- Simultaneous decoding of the message and compression indices
  - A must for general noisy networks, e.g., TWRC
  - Explicit Wyner-Ziv binning not needed anymore if relay’s transmitted codeword, compression index, and message are all simultaneously decoded

- A single big message instead of multiple independent messages
  - Similar to time expansion in network coding by Ahlswede, Cai, Li, Yeung ’00 and unfolding of graph by Avestimehr, Diggavi, Tse ’07

- Non-unique decoding of compression indices and some unwanted messages
Noisy network coding

Message $W \in [1, 2^{nbR}]$

$X_{1,1}^n$ → $Y_{1,1}^n$ → $\hat{Y}_{1,1}^n$ → $X_{1,2}^n$ → $\cdots$

$X_{2,1}^n$ → $Y_{2,1}^n$ → $\hat{Y}_{2,1}^n$ → $X_{2,2}^n$ → $\cdots$

$X_{N,1}^n$ → $Y_{N,1}^n$ → $\hat{Y}_{N,1}^n$ → $X_{N,2}^n$ → $\cdots$

$\hat{W}$

$\cdots$

$Y_{1,b}^n$ → $\cdots$ → $Y_{1,b}^n$ → $X_{1,b}^n$ → $\cdots$

$Y_{2,b}^n$ → $\cdots$ → $Y_{2,b}^n$ → $X_{2,b}^n$ → $\cdots$

$Y_{N,b}^n$ → $\cdots$ → $Y_{N,b}^n$ → $X_{N,b}^n$ → $\cdots$
Theorem (Noisy network coding (Lim, Kim, El Gamal, C ’10))

For multi-source multicast RN with $D = D_1 = \ldots = D_N$, the following is achievable

$$\sum_{k \in T} R_k < \min_{d \in T^c \cap D} I(X_T; \hat{Y}_T^c, Y_d | X_{T^c}, Q) - I(Y_T; \hat{Y}_T | X^N, \hat{Y}_{T^c}, Y_d, Q)$$

for all cuts $T$ s.t. $T^c \cap D \neq \emptyset$ for some $p(q) \prod_{k=1}^N p(x_k | q)p(\hat{y}_k | y_k, x_k, q)$. 

Noisy Network Coding
Noisy Network Coding

Theorem (Noisy network coding (Lim, Kim, El Gamal, C ’10))

For multi-source multicast RN with $D = D_1 = \ldots = D_N$, the following is achievable

$$\sum_{k \in T} R_k < \min_{d \in T^c \cap D} I(X_T; \hat{Y}^c_T, Y_d | X^c_T, Q) - I(Y_T; \hat{Y}_T | X^N, \hat{Y}^c_T, Y_d, Q)$$

for all cuts $T$ s.t. $T^c \cap D \neq \emptyset$ for some $p(q) \prod_{k=1}^N p(x_k | q)p(\hat{y}_k | y_k, x_k, q)$.

Includes the following as special cases:

- Max-flow min-cut theorem (Ford, Fulkerson ’56)
- CF for 3-node relay channel (Cover, El Gamal ’79)
- Network coding (Ahlswede, Cai, Li, Yeung ’00)
- Wireless erasure networks (Dana et al ’06)
- Deterministic relay networks (Avestimehr, Diggavi, Tse ’07)
Noisy Network Coding

Theorem (Nonunique decoding of unwanted messages)

For multi-source multicast RN, the following is achievable

\[
\sum_{k \in T} R_k \leq \min_{d \in T^c \cap D_T} I(X_T; \hat{Y}_{T^c}, Y_d|X_{T^c}, Q) - I(Y_T; \hat{Y}_T|X^N, \hat{Y}_{T^c}, Y_d, Q)
\]

for all cuts \( T \) s.t. \( T^c \cap D_T \neq \emptyset \) for some

\[
p(q) \prod_{k=1}^{N} p(x_k|q)p(\hat{y}_k|y_k, x_k, q), \text{ where } D_T = \cup_{k \in T} D_k.
\]
Noisy Network Coding

Theorem (Nonunique decoding of unwanted messages)
For multi-source multicast RN, the following is achievable

\[
\sum_{k \in T} R_k < \min_{d \in T^c \cap D_T} I(X_T; \hat{Y}_{T^c}, Y_d|X_{T^c}, Q) - I(Y_T; \hat{Y}_T|X^N, \hat{Y}_{T^c}, Y_d, Q)
\]

for all cuts \(T\) s.t. \(T^c \cap D_T \neq \emptyset\) for some
\[p(q) \prod_{k=1}^N p(x_k|q)p(\hat{y}_k|y_k, x_k, q),\] where \(D_T = \bigcup_{k \in T} D_k\).

Theorem (Treating interference as noise)
For multi-source multicast RN, the following is achievable

\[
\sum_{k \in T} R_k < I(X_T, U_S; \hat{Y}_{S^c}, Y_d|X_{T^c}, U_{S^c}, Q) - I(Y_S; \hat{Y}_S|X_{S_d}, U^N, \hat{Y}_{S^c}, Y_d, Q)
\]

for all cuts \(S, T\) and \(d \in D_S\) such that \(S \cap S_d \subseteq T \subseteq S_d\) and \(S^c \cap D_S \neq \emptyset\) for some
\[p(q) \prod_{k=1}^N p(u_k, x_k|q)p(\hat{y}_k|y_k, u_k, q),\] where \(T^c = S_d \setminus T\).
Network coding vs. NNC

Network coding

1

$\text{nC}_{1,3}$ bits

2

$\text{nC}_{2,3}$ bits

3

$\text{nC}_{3,4}$ bits

4

Noisy network coding

1

$X_1^n$

2

$X_2^n$

3

$Y_3^n$

4

$nI(Y_3; \hat{Y}_3 | X_3)$
Network coding vs. NNC

Network coding: Compression can happen due to bottleneck, i.e., if $C_{1,3} + C_{2,3} > C_{3,4}$
Network coding vs. NNC

Network coding:
- Compression can happen due to bottleneck, i.e., if \( C_{1,3} + C_{2,3} > C_{3,4} \)

Noisy network coding (quantization + network coding):
- \( nI(Y_3; \hat{Y}_3 | X_3) \)
Network coding vs. NNC

- **Network coding**: Compression can happen due to bottleneck, i.e., if \( C_{1,3} + C_{2,3} > C_{3,4} \)
- **Noisy network coding** (quantization + network coding)
  - **Quantization**: Explicit compression
Network coding vs. NNC

Network coding

Noisy network coding

Network coding: Compression can happen due to bottleneck, i.e., if $C_{1,3} + C_{2,3} > C_{3,4}$

Noisy network coding (quantization + network coding)

- Quantization: Explicit compression
- Network coding: Additional implicit compression can happen due to bottleneck, i.e., if $I(Y_3; \hat{Y}_3|X_3) > H(X_3)$
Noisy Network Coding for TWRC

- Noisy Network Coding for DM-TWRC, Lim, Kim, El Gamal, C ’10

\[ R_1 < \min \{ I(X_1; Y_2, \hat{Y}_3|X_2, X_3), I(X_1, X_3; Y_2|X_2) - I(Y_3; \hat{Y}_3|X_1, X_2, X_3, Y_2) \} \]

\[ R_2 < \min \{ I(X_2; Y_1, \hat{Y}_3|X_1, X_3), I(X_2, X_3; Y_1|X_1) - I(Y_3; \hat{Y}_3|X_1, X_2, X_3, Y_1) \} \]

for some \( p(x_1)p(x_2)p(x_3)p(\hat{y}_3|x_3, y_3) \).
Noisy Network Coding for TWRC

- Noisy Network Coding for DM-TWRC, Lim, Kim, ElGamal, C ’10

\[ R_1 < \min \{ I(X_1; Y_2, \hat{Y}_3|X_2, X_3), I(X_1, X_3; Y_2|X_2) - I(Y_3; \hat{Y}_3|X_1, X_2, X_3, Y_2) \} \]

\[ R_2 < \min \{ I(X_2; Y_1, \hat{Y}_3|X_1, X_3), I(X_2, X_3; Y_1|X_1) - I(Y_3; \hat{Y}_3|X_1, X_2, X_3, Y_1) \} \]

for some \( p(x_1)p(x_2)p(x_3)p(\hat{y}_3|x_3, y_3) \).

- NNC for Gaussian TWRC w/o direct links

\[ R_1 < \min \left\{ C \left( \frac{P_1}{1 + \sigma^2} \right), C(Q_2) - C \left( \frac{1}{\sigma^2} \right) \right\} \]

\[ R_2 < \min \left\{ C \left( \frac{P_2}{1 + \sigma^2} \right), C(Q_1) - C \left( \frac{1}{\sigma^2} \right) \right\} \]
Noisy Network Coding for TWRC

- Noisy Network Coding for DM-TWRC, Lim, Kim, El Gamal, C ’10

\[ R_1 < \min \{ I(X_1; Y_2, \hat{Y}_3|X_2, X_3), I(X_1, X_3; Y_2|X_2) - I(Y_3; \hat{Y}_3|X_1, X_2, X_3, Y_2) \} \]

\[ R_2 < \min \{ I(X_2; Y_1, \hat{Y}_3|X_1, X_3), I(X_2, X_3; Y_1|X_1) - I(Y_3; \hat{Y}_3|X_1, X_2, X_3, Y_1) \} \]

for some \( p(x_1)p(x_2)p(x_3)p(\hat{y}_3|x_3, y_3) \).

- NNC for Gaussian TWRC w/o direct links

\[ R_1 < \min \left\{ C \left( \frac{P_1}{1 + \sigma^2} \right), C(Q_2) - C \left( \frac{1}{\sigma^2} \right) \right\} \]

\[ R_2 < \min \left\{ C \left( \frac{P_2}{1 + \sigma^2} \right), C(Q_1) - C \left( \frac{1}{\sigma^2} \right) \right\} \]

- Cut-set bound

\[ R_1 \leq \min \{ C(P_1), C(Q_2) \} \]

\[ R_2 \leq \min \{ C(P_2), C(Q_1) \} \]
Noisy Network Coding for TWRC

- Noisy Network Coding for DM-TWRC, Lim, Kim, El Gamal, C ’10
  \[ R_1 < \min \left\{ I(X_1; Y_2, \hat{Y}_3|X_2, X_3), I(X_1, X_3; Y_2|X_2) - I(Y_3; \hat{Y}_3|X_1, X_2, X_3, Y_2) \right\} \]
  \[ R_2 < \min \left\{ I(X_2; Y_1, \hat{Y}_3|X_1, X_3), I(X_2, X_3; Y_1|X_1) - I(Y_3; \hat{Y}_3|X_1, X_2, X_3, Y_1) \right\} \]

  for some \( p(x_1)p(x_2)p(x_3)p(\hat{y}_3|x_3, y_3). \)

- NNC for Gaussian TWRC w/o direct links
  \[ R_1 < \min \left\{ C \left( \frac{P_1}{1 + \sigma^2} \right), C(Q_2) - C \left( \frac{1}{\sigma^2} \right) \right\} \]
  \[ R_2 < \min \left\{ C \left( \frac{P_2}{1 + \sigma^2} \right), C(Q_1) - C \left( \frac{1}{\sigma^2} \right) \right\} \]

- Cut-set bound
  \[ R_1 \leq \min \{ C(P_1), C(Q_2) \} \]
  \[ R_2 \leq \min \{ C(P_2), C(Q_1) \} \]

- Gap to capacity: \( \frac{1}{2} \) bit per user, 1 bit for the sum rate
Theorem (Gaussian RN)

For multi-source multicast Gaussian RN with a single destination set, if \((R_1, \ldots, R_N)\) is in the cut-set bound, then \((R_1 - 0.63N, \ldots, R_N - 0.63N)\) is achievable by NNC.

- Generalization of constant gap result by ADT
Lessons
Lessons

- Multisource multicast with a single destination set
Lessons

- Multisource multicast with a single destination set
  - A big MAC problem
Lessons

- Multisource multicast with a single destination set
  - A big MAC problem
  - If a constant gap is good enough, then
    - Structured codes are not essential
Lessons

- Multisource multicast with a single destination set
  - A big MAC problem
  - If a constant gap is good enough, then
    - Structured codes are not essential
    - Use noisy network coding
Multiple unicast
Multiple unicast

New challenges: How to manage inter-user interference?
New challenges: How to manage inter-user interference?

Why was this not a problem in multi-source multicast with a single destination set?
Multiple unicast

- New challenges: How to manage inter-user interference?
- Why was this not a problem in multi-source multicast with a single destination set?
  - It was safe to mix (network coding) different signals since each destination is required to decode messages from all sources.
Multiple unicast

- New challenges: How to manage inter-user interference?
- Why was this not a problem in multi-source multicast with a single destination set?
  - It was safe to mix (network coding) different signals since each destination is required to decode messages from all sources.
- What do we need?
New challenges: How to manage inter-user interference?
Why was this not a problem in multi-source multicast with a single destination set?
> It was safe to mix (network coding) different signals since each destination is required to decode messages from all sources.

What do we need?
> Careful control of interference
Interference Neutralization

\[
\begin{align*}
X_1 & \quad 1 \quad Y_3 \\
1 & \quad Z_3 & \quad Y_4 \\
-1 & \quad &
\end{align*}
\]

\[
\begin{align*}
X_2 & \quad 1 \\
1 & \quad Z_4 & \quad
\end{align*}
\]

\[
\begin{align*}
X_3 & \quad 1 \quad Y_5 \\
1 & \quad Z_5 & \quad Y_6 \\
-1 & \quad &
\end{align*}
\]

\[
\begin{align*}
X_4 & \quad 1 \\
1 & \quad Z_6 & \quad
\end{align*}
\]
Interference Neutralization

AF can do better than CF and DF.
AF can do better than CF and DF.

Set $X_3 = Y_3$ and $X_4 = -Y_4$. 
AF can do better than CF and DF.

Set $X_3 = Y_3$ and $X_4 = -Y_4$.

Can get two interference free channels.
AF can do better than CF and DF.
Set $X_3 = Y_3$ and $X_4 = -Y_4$.
Can get two interference free channels.
Interference neutralization for ZZ and ZS networks (Mohajer, Diggavi, Tse ’09)
AF can do better than CF and DF.
Set \(X_3 = Y_3\) and \(X_4 = -Y_4\).
Can get two interference free channels.
Interference neutralization for ZZ and ZS networks (Mohajer, Diggavi, Tse ’09)
Aligned interference neutralization (Gou, Jafar, Jeon, C ’11)
Interference Neutralization

- **AF can do better than CF and DF.**
- Set $X_3 = Y_3$ and $X_4 = -Y_4$.
- Can get two interference free channels.
- Interference neutralization for ZZ and ZS networks (Mohajer, Diggavi, Tse '09)
- Aligned interference neutralization (Gou, Jafar, Jeon, C '11)
- Opportunistic interference neutralization (Jeon, C, Jafar, Allerton '09, IT '11)
Conclusions

- Multisource multicast with a single destination set
  - A big MAC problem
  - If a constant gap is good enough, then
    - Structured codes are not essential
    - Use noisy network coding
Conclusions

- Multisource multicast with a single destination set
  - A big MAC problem
  - If a constant gap is good enough, then
    - Structured codes are not essential
    - Use noisy network coding

- Multiple unicast
Conclusions

▶ Multisource multicast with a single destination set
  ▶ A big MAC problem
  ▶ If a constant gap is good enough, then
    ▶ Structured codes are not essential
    ▶ Use noisy network coding

▶ Multiple unicast
  ▶ Structured codes can help a lot