Lattice Based Structuring to Combat Interference in Simple Wireless Networks

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Algebraic Structure in Network Information Theory

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Based on joint studies with: A. Sanderovich, M. Gastpar, B. Nazer and M. Peleg

Outline

- Introduction
- Uplink Wyner Cellular Model
- Distributed Interference Mitigation
- Concluding Remarks

Introduction

Introduction

- Structured codes play an increasing role in various multi-terminal information theoretic settings
- Not as an alternative to random coding but as a basic paradigm that provides inherent advantages [Korner-Marton '79].
- Lattice codes were shown to be an appealing option in many linear Gaussian (and other) models.
- Recent overviews: "Can Structure Beat Shannon? - The Story of Lattice Codes," [Zamir, ISIT'10].
  Compute-and-Forward: Harnessing Interference through Structured Codes [Nazer-Gastpar '11].

Introduction

- Cellular network: a ubiquitous interference network. Interference managing is a MUST for high spectral efficiency.
- Simple (naive ?) Wyner model with finite backhaul connectivity
- Application of lattice codes as a means to mitigate the interference impact and enhance performance in terms of reliable symmetric rates.

Introduction

- Interfered relay assisted reception: a user wishes to send a message to a remote destination via helping relays.
- The relays are interfered by an external uncoordinated interference.
- The relays are connected to the destination by fixed links.
- Application of lattice codes as means of filtering (some of) the interference.

Some Related Work

Related work

- Decentralized processing for communication [Sanderovich, Shamai, Steinberg and Kramer '08, Sanderovich-Peleg-Shamai '08,].
- Distributed Source Coding [Krithivasan-Pradhan '08]
- Distributed MIMO: non decoding relays. [Sanderovich, Shamai, and Steinberg '07].
- Errors due to intentional jamming [Hedby '94].
- Gaussian two-way wiretap - cooperative jamming [Tekin & Yener '07].
- Interference Channels [Bresler-Parekh-Tse '07, Sridharan-Jafarian-Vishwanath-Jafar-Shamai '08, Khina-Hitron-Erez '11]
- Two-Way Relay Channel [Wilson-Narayanan-Pfister-Sprintson '07, '08, Nam-Chung-Lee '08]

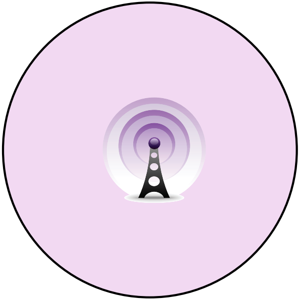
Related work

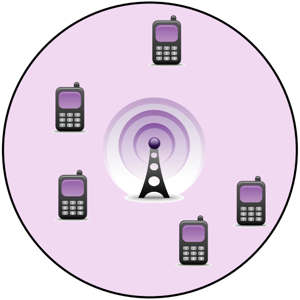
- Distributed Function Compression [Körner-Marton '79, Krithivasan-Pradhan '07, Wagner '08]
- Interference management using nonlinear relaying [Khormuji, Zaidi, and Skoglund '10].
- Gaussian diamond network with adversarial jammer [Mohajer and Diggavi '10].
- A Nonlinear Approach to Interference Alignment [Razaghi and Caire '11].
- Interference Channels with a Cognitive Relay [Rini, Tuninetti and Devroye '11].

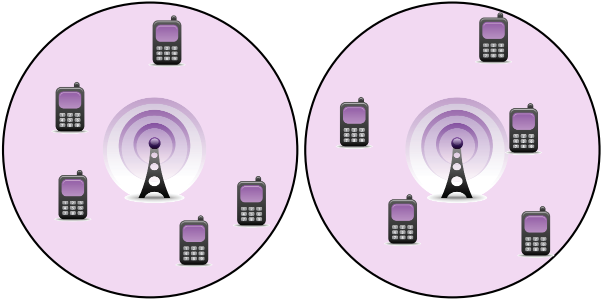
Related work - Lattices

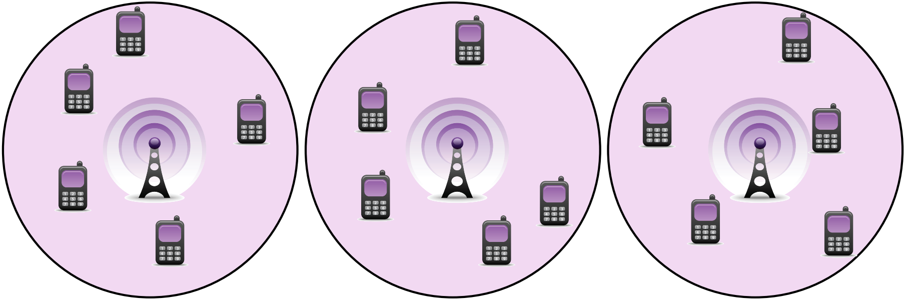
- Cancelling known interference with Lattices [Erez, Shamai and Zamir '05]
- Compute-and-Forward: Harnessing Interference through Structured Codes [Nazer & Gastpar '11]
- The helper node problem [Philosof, Khisti, Erez and Zamir '07]
- Practical Code Design for Compute-and-Forward [Ordentlich, Zhan, Erez, Nazer and Gastpar 2011]
- Secrecy [He-Yener '08, '09]
- The Degrees of Freedom of Compute-and-Forward [Niesen-Whiting '11]
- Can Structure Beat Shannon? - The Story of Lattice Codes, Plenary Talk [Zamir '10]

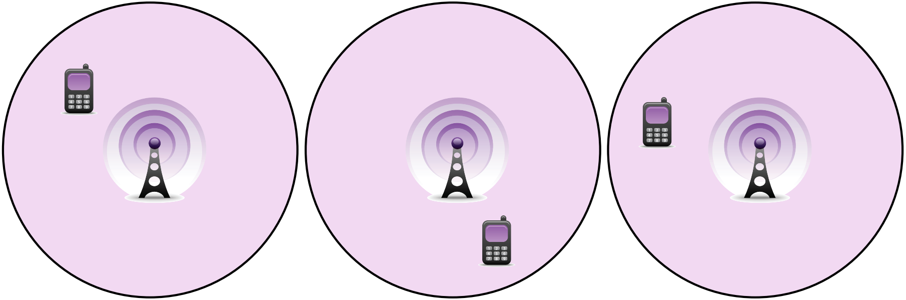
Uplink channel of the Wyner cellular model



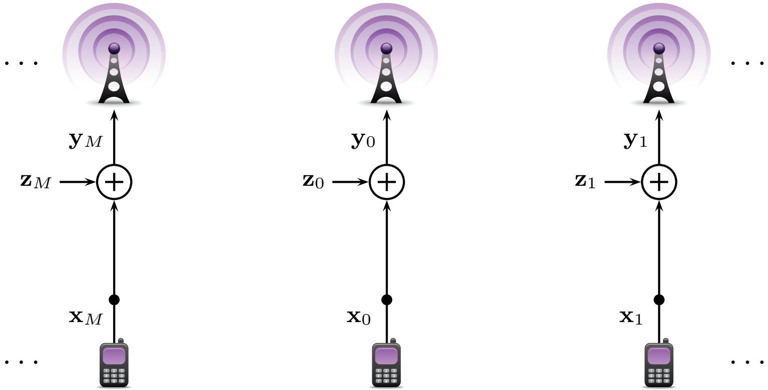








Inter-Cell Interference Model



Power constraint: $\frac{1}{n}\|\mathbf{x}_m\|^2 \leq \mathsf{SNR}$, i.i.d. noise: $\mathbf{z}_m \sim \mathcal{CN}(0, \mathbf{I}^{n\times n})$

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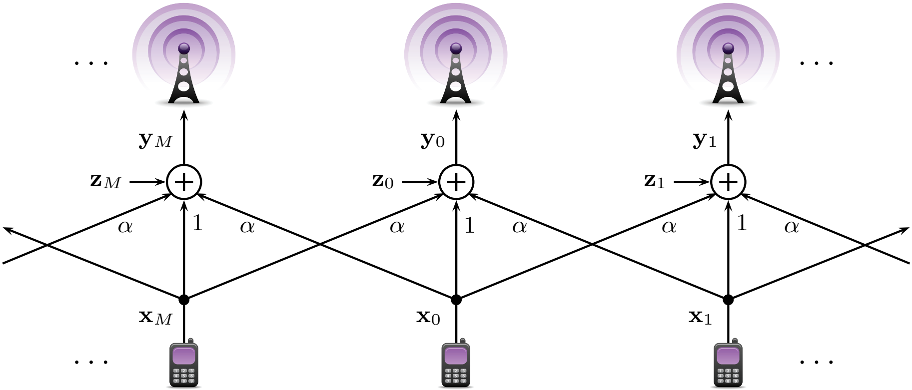


Power constraint: $\frac{1}{n}\|\mathbf{x}_m\|^2 \leq \text{SNR}$, i.i.d. noise: $\mathbf{z}_m \sim \mathcal{CN}(0, \mathbf{I}^{n\times n})$

Symmetric interference between adjacent cell-sites (**Wyner '94**):

$$\mathbf{y}_m = \mathbf{x}_m + \alpha\ \mathbf{x}_{[m-1]_M} + \alpha\ \mathbf{x}_{[m+1]_M} + \mathbf{z}_m \qquad \alpha \in [0,1]$$

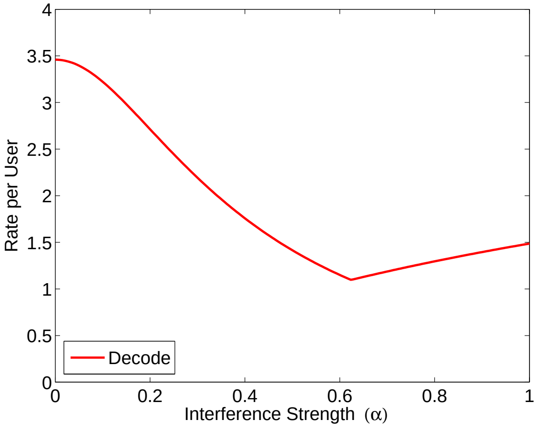
Decode-and-Forward

Decode-and-Forward: Each cell-site can treat other signals as noise or decode and remove them:

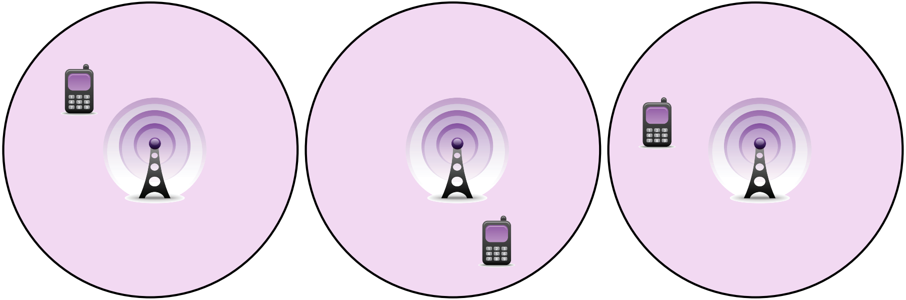
$$R_{\mathsf{DF}} = \max\left(\log\left(1 + \frac{\mathsf{SNR}}{1 + 2\alpha^2\mathsf{SNR}}\right), \min\left(\frac{1}{2}\log\left(1 + 2\alpha^2\mathsf{SNR}\right), \frac{1}{3}\log\left(1 + (1 + 2\alpha^2)\mathsf{SNR}\right)\right)\right)$$

Decode-and-Forward

SNR $= 10$dB



Full Cooperation using Infinite Backhaul Rate



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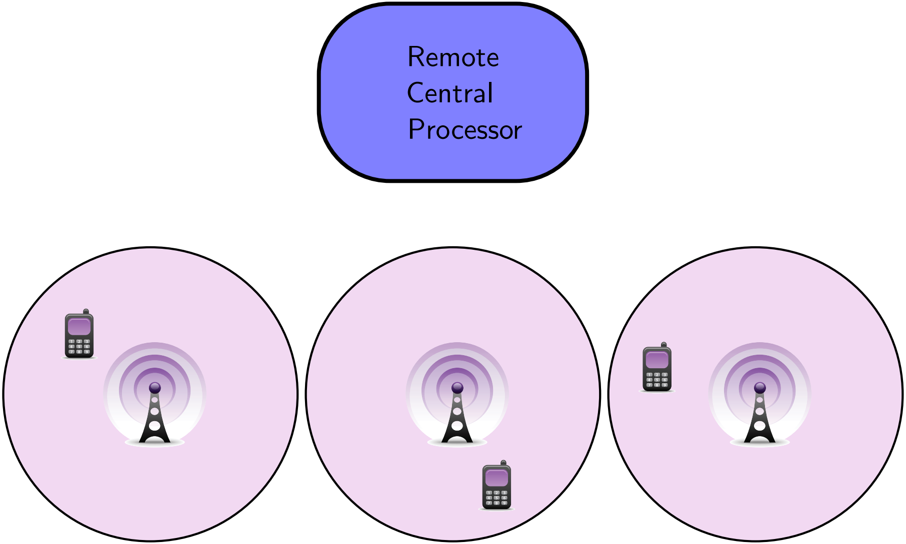

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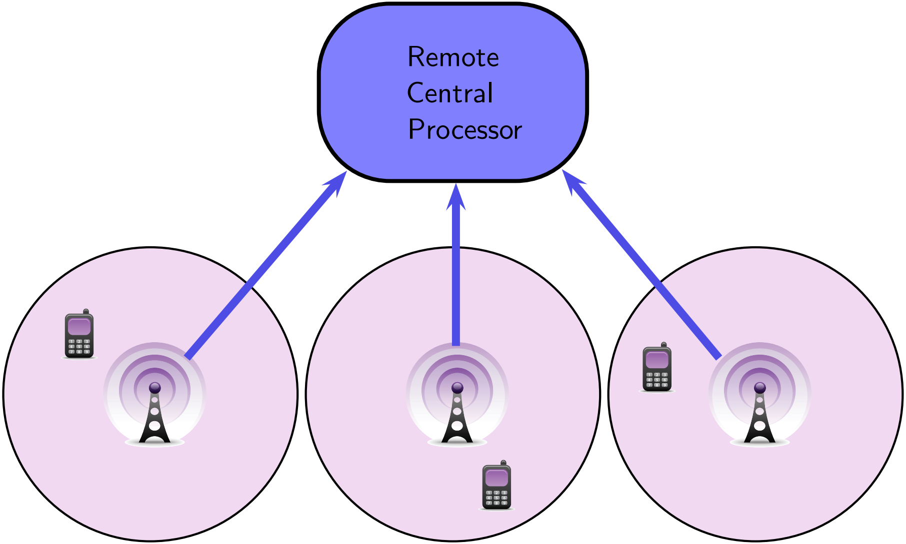


Full Cooperation

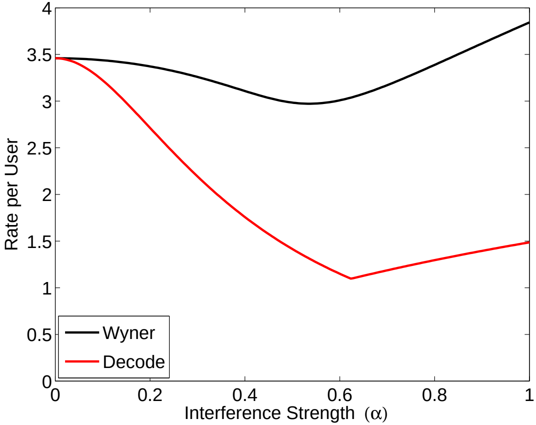
Wyner '94: Proposed full cooperation model and found capacity as number of cell-sites tends to infinity:

$$R_{\mathsf{WYNER}} = \int_0^1 \log\left(1 + \mathsf{SNR}(1 + 2\alpha\cos 2\pi\theta)^2\right) d\theta.$$

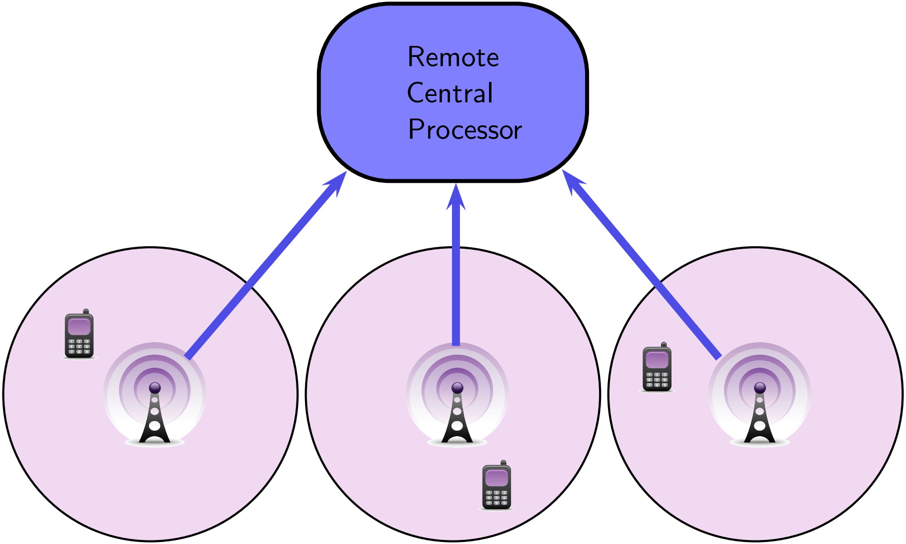
Excellent performance but infinite backhaul rate is not realistic.

Full Cooperation

$\mathsf{SNR} = 10\mathsf{dB}$



Partial Cooperation using Rate-Limited Backhaul



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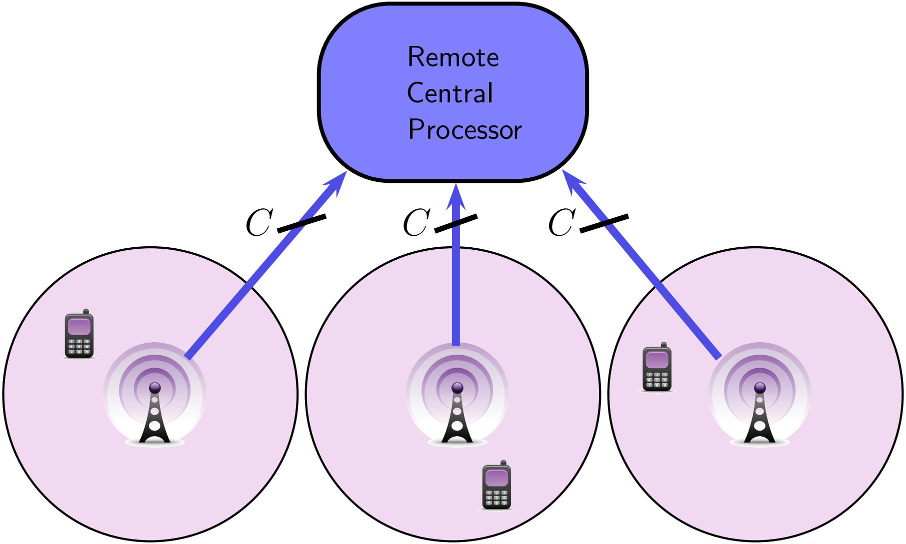


Compress-and-Forward

Can approximate full cooperation using vector quantizers at the cell-sites.

Sanderovich-Somekh-Shamai-Poor '09: Proposed rate-limited cooperation model. Derived compress-and-forward rate as number of cell-sites tends to infinity:

$$R_{\mathsf{CF}} = F(r^*) \text{ where } r^* \text{ is the solution of } F(r^*) = C - r^*,$$
$$F(r) = \int_0^1 \log\left(1 + \mathsf{SNR}(1 - 2^{-r})(1 + 2\alpha\cos 2\pi\theta)^2\right) d\theta.$$

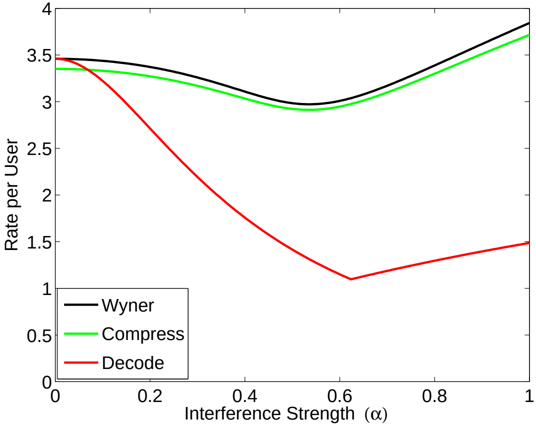
Does not require knowledge of codebooks at the cell-sites.

Optimal if C or SNR tends to infinity.

Compress-and-Forward

$$\text{SNR} = 10\text{dB}$$

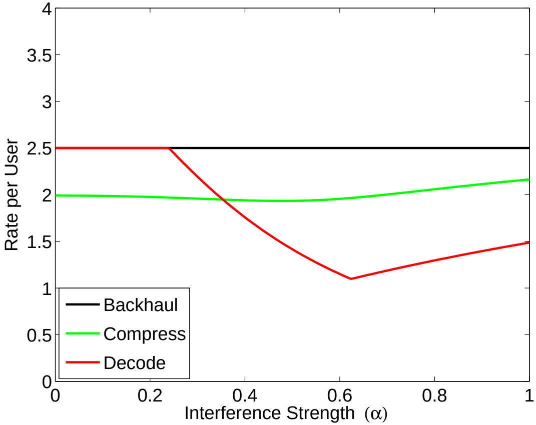
High Backhaul Rate, $C = 8$

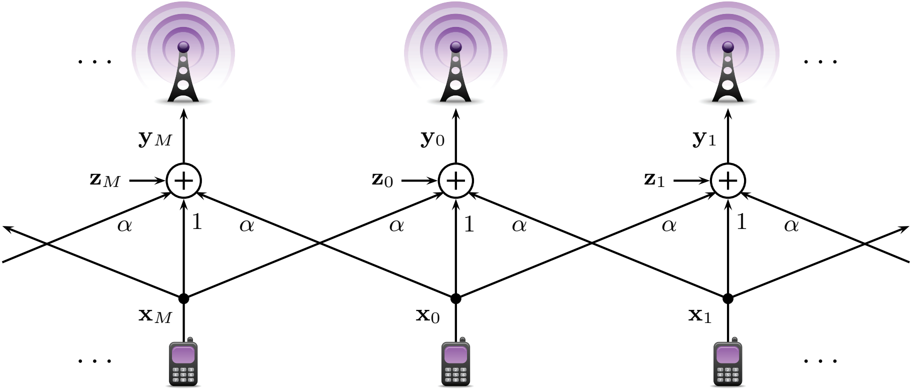


Compress-and-Forward

$\text{SNR} = 10\text{dB}$

Low Backhaul Rate, $C = 2.5$

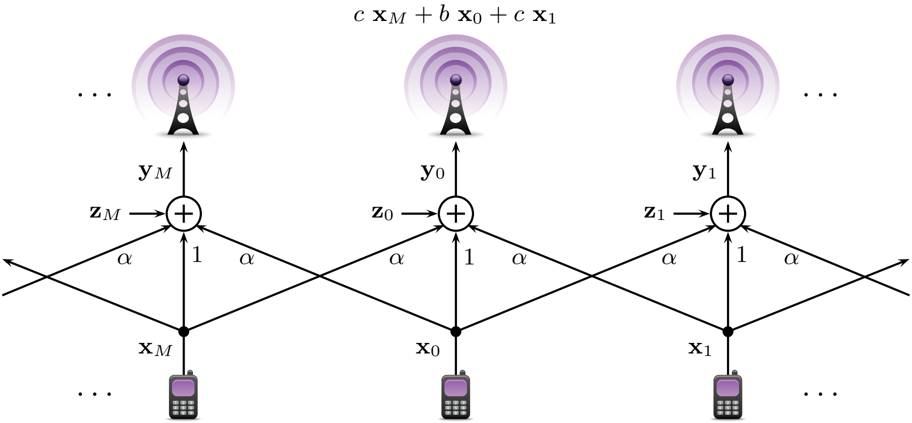




Nazer-Gastpar '08: Can decode a linear function of the codewords.

Problem symmetry guarantees full rank system if $b, c \neq 0$.

Compute-and-Forward



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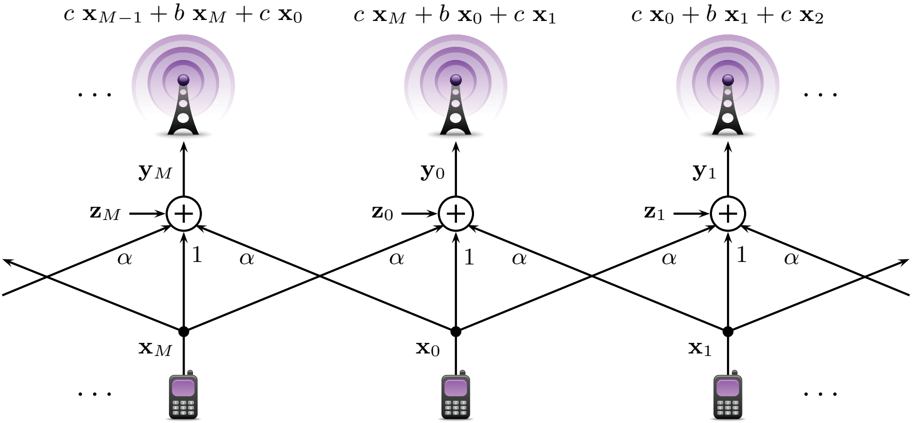

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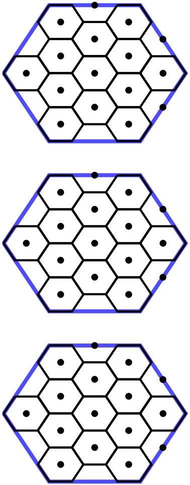
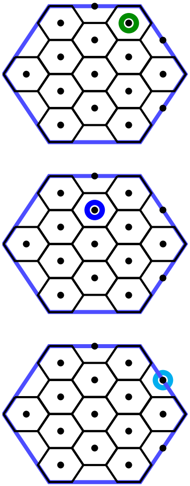
Computation Coding

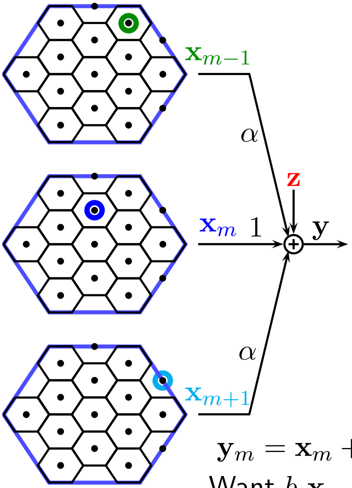
Choose good nested lattice code (**Erez-Zamir '04**).
All users employ the same codebook:



Each user maps its message to a lattice point:



Transmit lattice points over the channel:



$$\mathbf{y}_m = \mathbf{x}_m + \alpha\ \mathbf{x}_{m-1} + \alpha\ \mathbf{x}_{m+1}, \quad \alpha \in [0,1]$$

Want $b\ \mathbf{x}_m + c\ \mathbf{x}_{m-1} + c\ \mathbf{x}_{m+1}, \quad b, c \in \mathbb{Z}$

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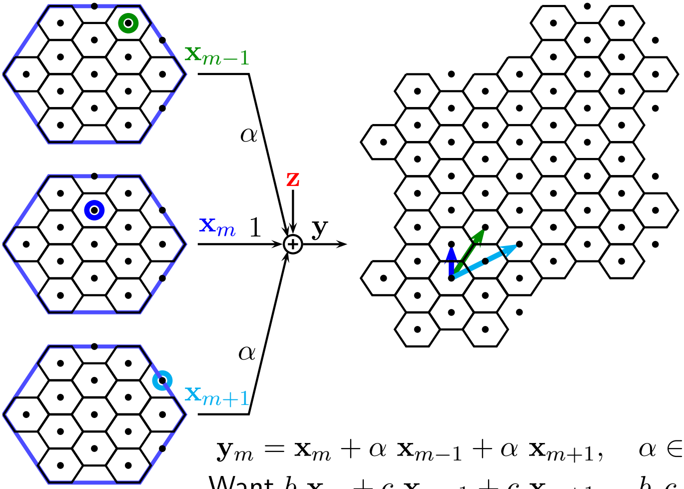

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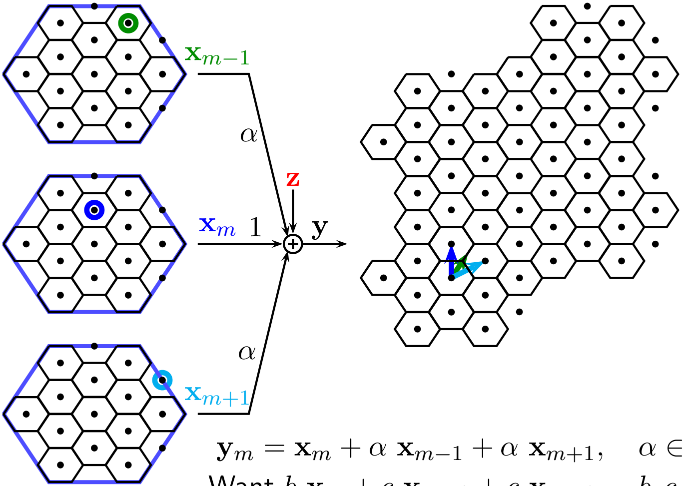
Lattice codewords are scaled by channel coefficients:



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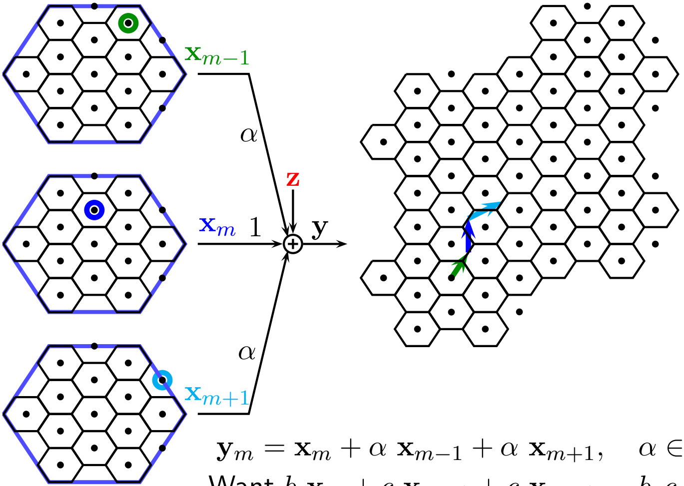
Scaled codewords added together plus noise:



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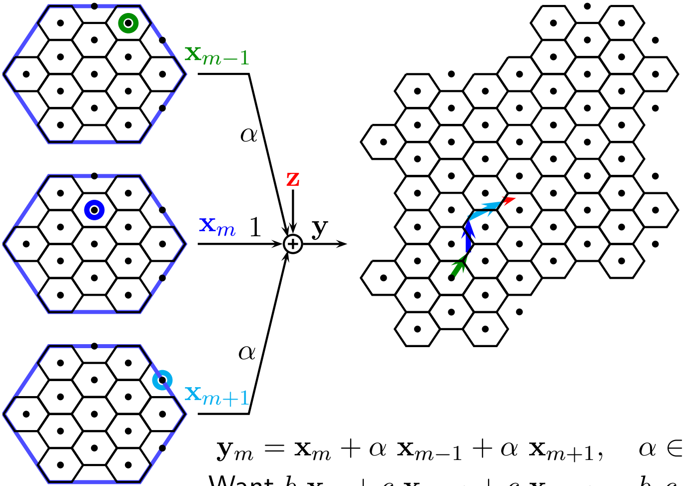

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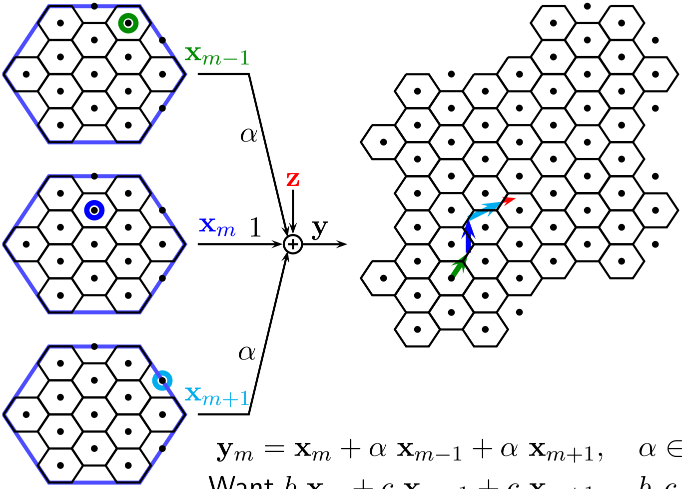
Computation Coding

Extra noise penalty for non-integer channel coefficients:



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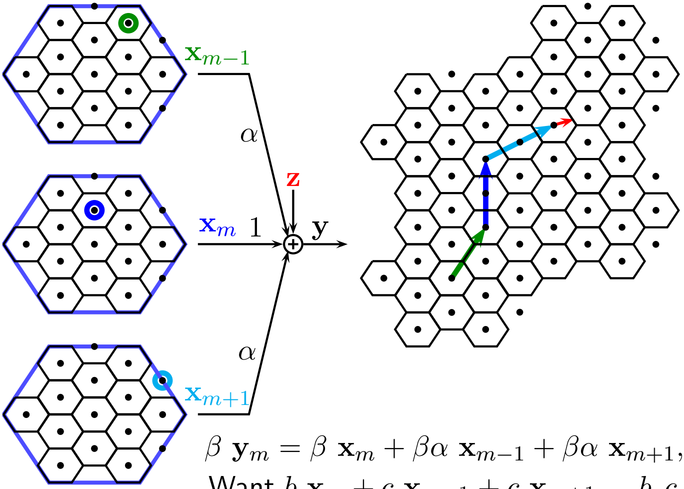
Computation Coding

Scale output by β to reduce non-integer noise penalty:



$\beta\ \mathbf{y}_m = \beta\ \mathbf{x}_m + \beta\alpha\ \mathbf{x}_{m-1} + \beta\alpha\ \mathbf{x}_{m+1},\ \ \alpha \in [0,1]$

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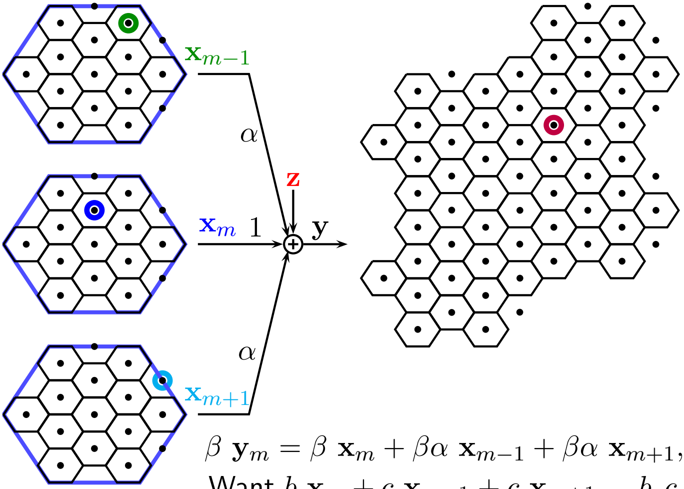
Decode to closest lattice point:



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Want $b\ \mathbf{x}_m + c\ \mathbf{x}_{m-1} + c\ \mathbf{x}_{m+1}, \quad b, c \in \mathbb{Z}$

Compute-and-Forward: Achievable Rates

Each cell-site can decode $b\ \mathbf{x}_m + c\ \mathbf{x}_{[m-1]_M} + c\ \mathbf{x}_{[m+1]_M}$ at rate:

$$R(b,c) = \max_{\beta} \log\left(\frac{\mathsf{SNR}}{\beta^2 + \mathsf{SNR}(\beta - b)^2 + 2\mathsf{SNR}(\beta\alpha - c)^2}\right)$$

Remote central processor solves for original messages:

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$$R_{\mathsf{COMP}} = \min\left\{ \max_{b,c\in\mathbb{Z}} R(b,c), C \right\}$$

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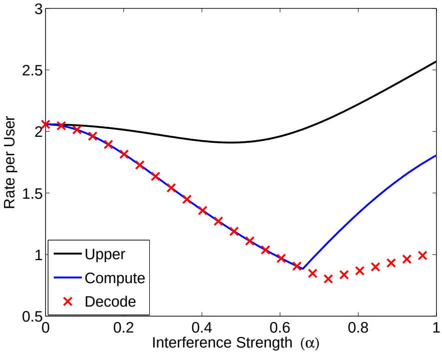
Remote central processor solves for original messages:

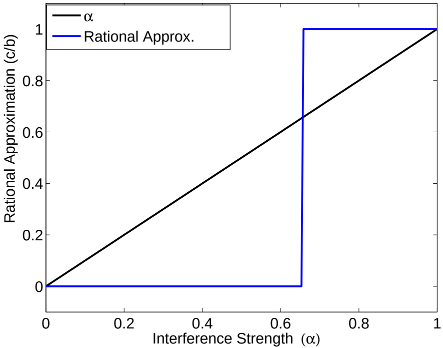
$$R_{\mathsf{COMP}} = \min\left\{\max_{b,c\in\mathbb{Z}} R(b,c), C\right\}$$

Only need to search over $b^2 + 2c^2 \leq 1 + \mathsf{SNR}(1+2\alpha^2)$

Compute-and-Forward: Performance

SNR $= 5$dB



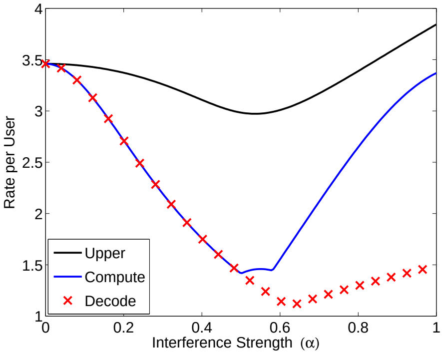


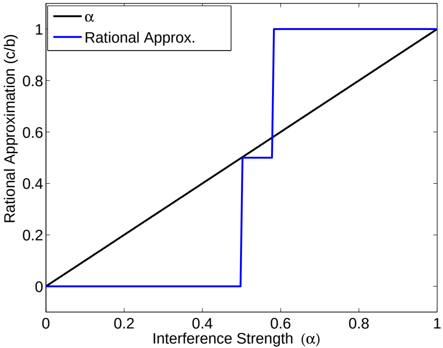
Significant performance variation due to integer effects.

In a rate "valley", would rather have higher or lower α.

Compute-and-Forward: Performance

$\mathsf{SNR} = 10\text{dB}$





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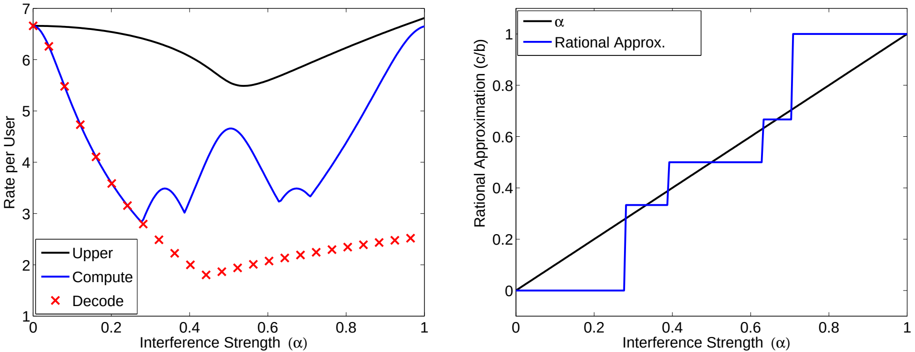
Compute-and-Forward: Performance

$\text{SNR} = 20\text{dB}$



Significant performance variation due to integer effects.

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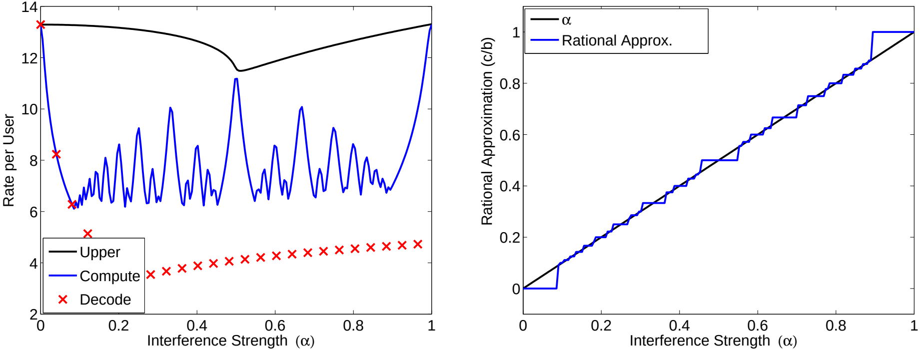
Compute-and-Forward: Performance

$\text{SNR} = 40\text{dB}$



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- Superposition usually used to send more than one message from a single transmitter (**Cover '72**).

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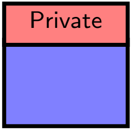
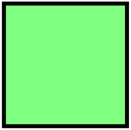
- Here, we will use superposition to change the interference structure of the problem.
- Half the users will superimpose a private message onto the lattice codeword.
- Other half of the users stay the same.
- Asymmetry will move all users out of the rate "valley."

Structured Superposition

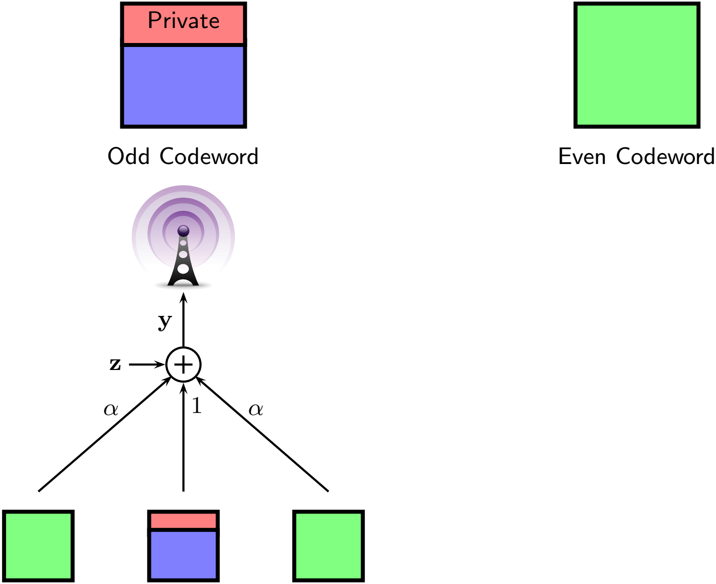


Odd Codeword

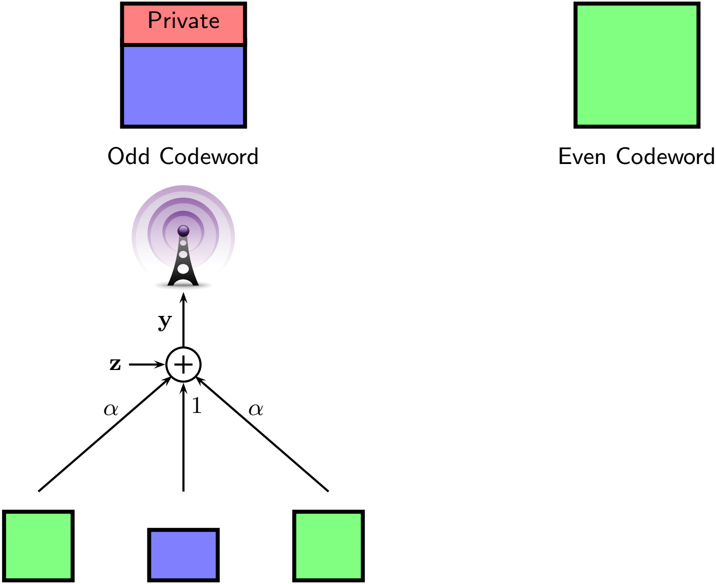


Even Codeword

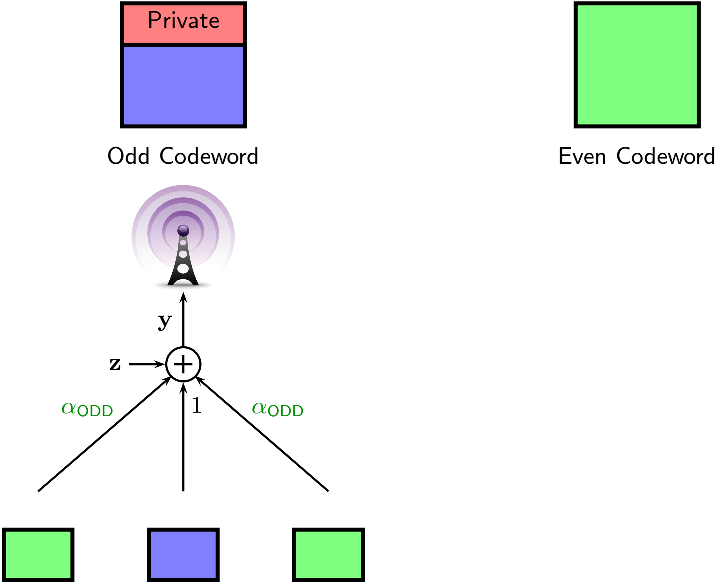
Structured Superposition



Structured Superposition



Structured Superposition



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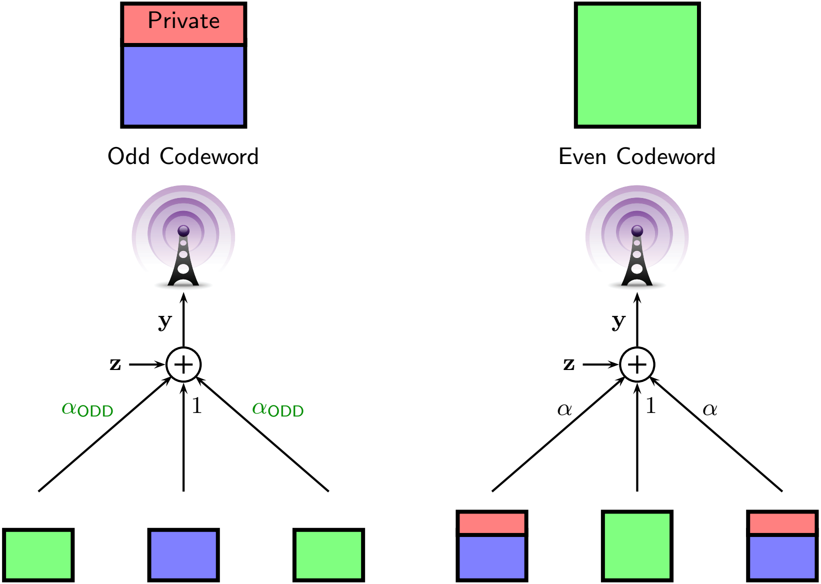

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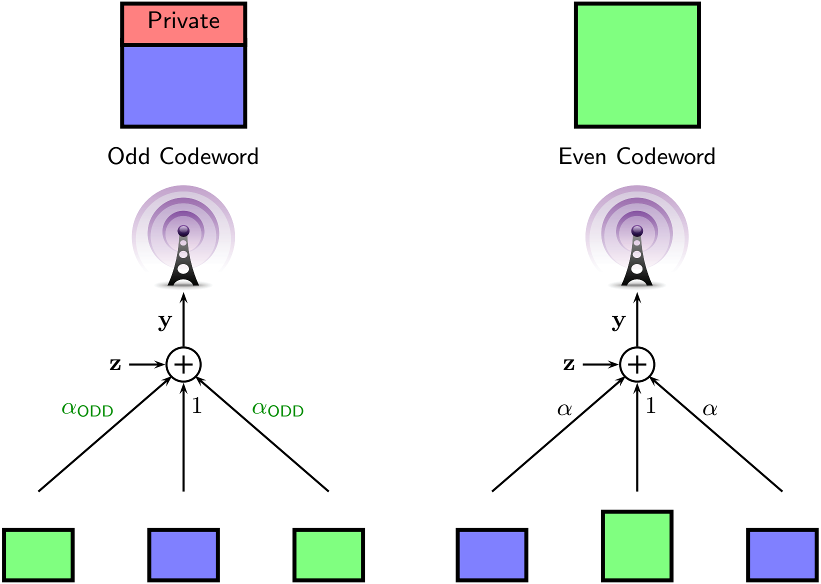

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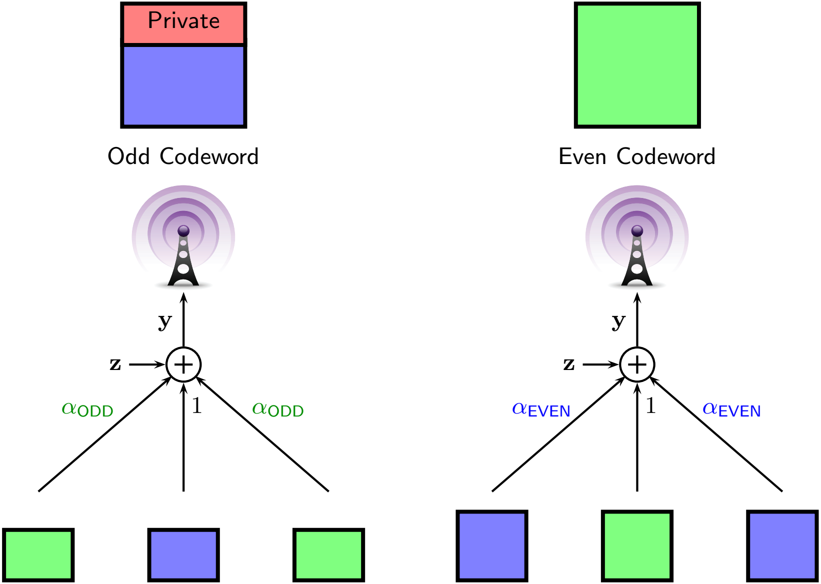

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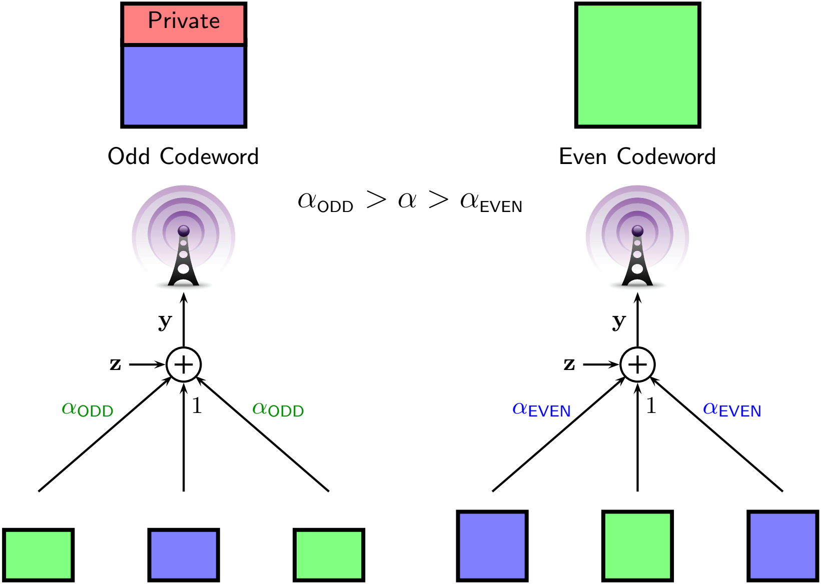
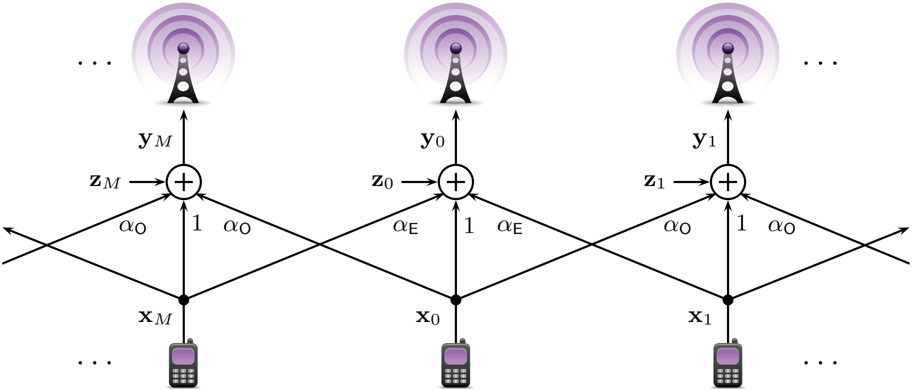
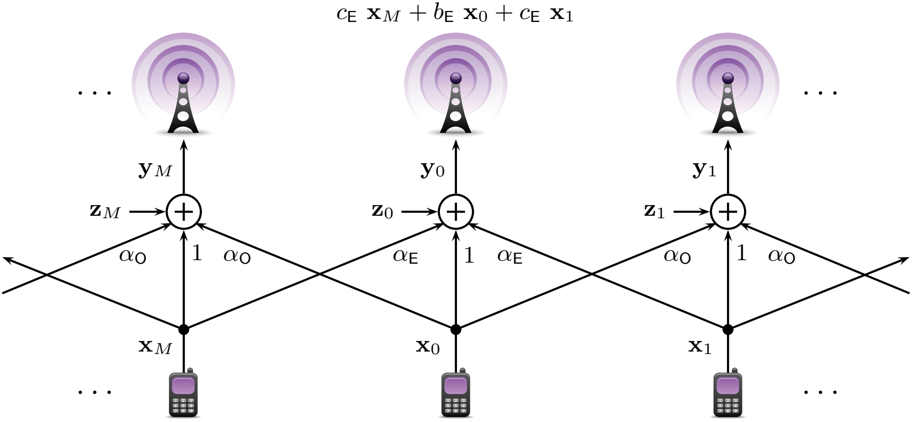




Each cell-site sees either α_E or α_O which is strictly better than α.

Structured Superposition



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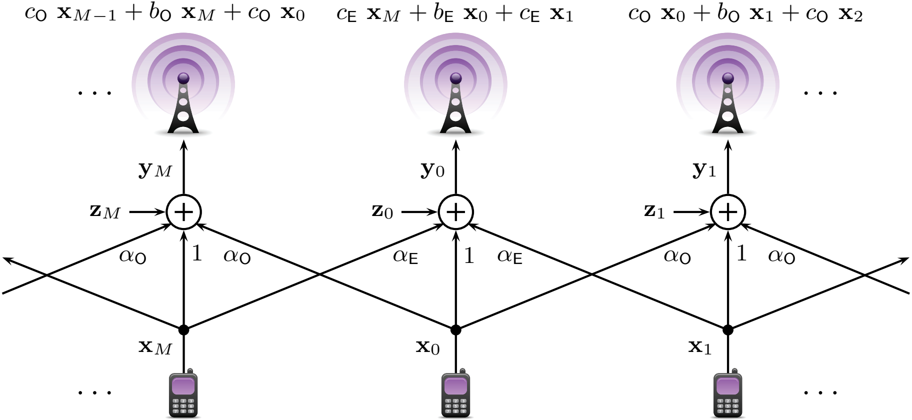

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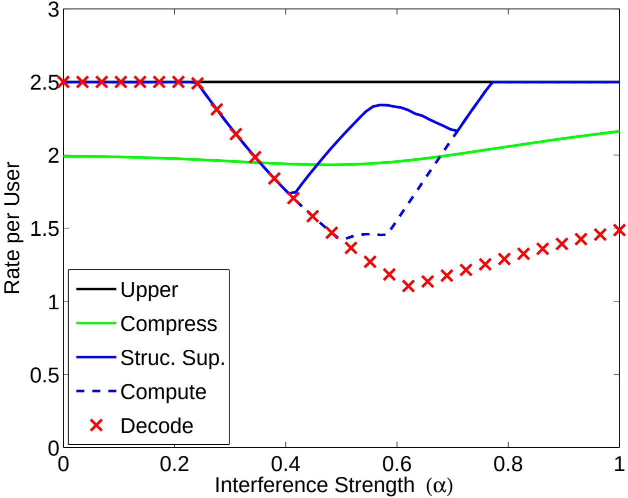
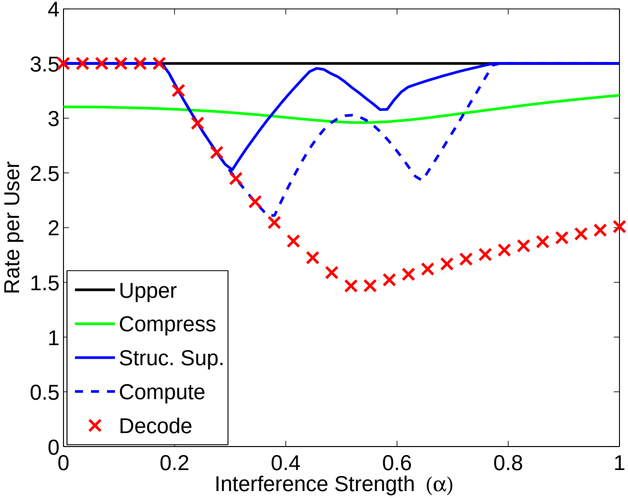
Structured Superposition: Performance

SNR $= 10$dB, Backhaul Rate $C = 2.5$



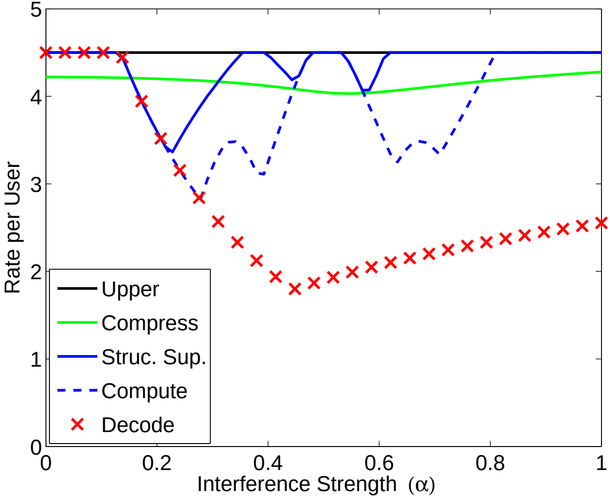
Structured Superposition: Performance

SNR $= 15$dB, Backhaul Rate $C = 3.5$



Structured Superposition: Performance

SNR $= 20$dB, Backhaul Rate $C = 4.5$



Part I Conclusions

- Developed a new scheme for cooperation in cellular uplink networks.
- Codes with algebraic structure can provide large gains.
- Superposition coding can adjust the interference structure to be more favorable for reliable computation.
- More advanced superposition structure could be used to overcome non-integer penalty for other networks.

Interfered transmission using multiple distributed reception

Background

- Distributed reception increases spectral efficiency, especially for fading channels.
- Two major issues with current ubiquitous wireless broadband (3G,4G, 802.11n) networks
  1. Intra/Inter cell Interferences.
  2. Limited backhaul links between access points/cell-sites and the backbone.
- Interference mitigation:
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  2. Interference reduction
     number of receiving antennas $\geq$ number of interferences $+ 1$

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- Caveat: very high connectivity is required.

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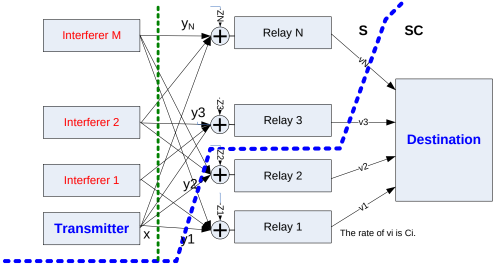
- Interference reduction hinges on spatially different interference and desired signal.
- Better performance for physically distributed receptions.
- Caveat: very high connectivity is required.
- Quantify the bandwidth that is required for distributed reception to effectively overcome interference.
- Essentially an open information theoretic problem.

Description

- Single transmitter X and M independent Gaussian interferers $\{J_i\}$.
- Destination D and N remote relays $\mathcal{R}_1, \ldots, \mathcal{R}_N$.
- Error-free links between relays and destination, with capacities $C_1, \ldots, C_N \longrightarrow$ Limited connectivity.



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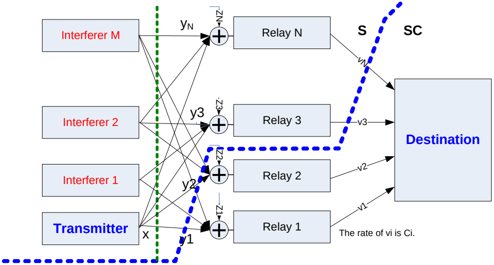


The problem

Quantify the required $C_1, \ldots, C_N$, for $R \sim \frac{1}{2}\log_2(P_X)$.

Description

- Full channel knowledge at $\mathcal{R}_1, \ldots, \mathcal{R}_N$ and D.
- Additive white Gaussian noise channel

$$\mathbf{Y} = \mathbf{h}\mathbf{X} + \mathbf{H}\mathbf{J} + \mathbf{Z}.$$

J, Z are Gaussian independent vectors with powers P_J, P_N.

- W is the transmitted message, along n channel uses.

$$\begin{aligned}
X^n(W) = \phi_S(W) \quad &: \quad I_{nR} \to \mathbb{R}^n \\
V_i(Y_i^n) = \phi_{R_i}(Y_i^n) \quad &: \quad \mathbb{R}^n \to I_{nC_i} \\
\hat{W}(V_1, \ldots, V_N) = \phi_D(V_1, \ldots, V_N) \quad &: \quad I_{nC_1} \times \cdots \times I_{nC_N} \to I_{nR}.
\end{aligned}$$

Description

- Channel model
$$\mathbf{Y} = \mathbf{h}\mathbf{X} + \mathbf{H}\mathbf{J} + \mathbf{Z}.$$
- $\mathbf{h}$ is the fixed channel coefficient vector of the transmitter.
- $\mathbf{H}$ is the fixed $N \times M$ channel coefficients matrix of the M interferers $\{J_i\}_{i=1}^{M}$.
- The interferers are identically independently distributed, uncorrelated and Gaussian. The are also unknown to neither transmitter or receivers.
- $\mathbf{Z}$ is the Gaussian additive noise.

Description

- Channel model

$$\mathbf{Y} = \mathbf{h}\mathbf{X} + \mathbf{H}\mathbf{J} + \mathbf{Z}.$$

- We consider only strong interferers $P_J >> P_X$.
- The interferers J may, in fact, be users transmitting to another destination.
- Robust relaying procedure: relays do not exploit the structure (codes-word or the like) of the interfering signals.

Description

Example

The Model

- For example, assume that the interferers transmit in a much higher rate than what the relays/destination can decode. Making the transmission similar to a plain strong Gaussian noise.

- Another example refers to oblivious operation of relays and destination which are not required to have the codebook knowledge of the interferers. This is especially true in modern communication systems where the association of a station is done only with a destination.

Description

- For the sake of the lower bound, the following condition is met, when a is a positive constant

$$\forall \mathcal{S} \subseteq \{1,\dots,N\}, N_s > M, \exists \mathbf{A}_{\mathcal{S}} \text{ s.t.}$$
$$|\mathbf{A}| = 1 \text{ and } \mathbf{A}_{\mathcal{S}}\mathbf{H}_{\mathcal{S}} = 0 \text{ and } |\mathbf{A}_{\mathcal{S}}\mathbf{h}_{\mathcal{S}}| \geq a,$$

- Basically: coefficients are non degenerated and allow zero forcing out of $\mathbf{Y}$
- When $\lambda_{\min}(\mathcal{S})$ is the smallest eigenvalue of $\mathbf{H}_{\mathcal{S}}\mathbf{H}_{\mathcal{S}}^T$, and $K > 0$ is some constant,

$$\forall \mathcal{S} \subseteq \{1,\dots,N\}, N_s \leq M : \ \lambda_{\min}(\mathcal{S}) \geq K.$$

- This condition ensures that a set of N_s antennas is effectively jammed by $M \geq N_s$ interferers.

Results

- An upper bound to the achievable rate is

$$R \lesssim \min\left\{\frac{1}{2}\log_2(1+P_X), \min_{\mathcal{S}\subseteq\{1,\ldots,N\}, s.t.\ N_s=N-M} \sum_{i\in\mathcal{S}} C_i\right\}$$

- A lower bound to the achievable rate when $N \geq M+1$, is

$$R \geq \frac{1}{2}\log_2\left(\frac{P_X}{\frac{1}{a^2}+\sum_{i=1}^{N}\frac{P_X}{2^{2C_i}-1}}\right).$$

Upper bound

- Using Fano's inequality, if R is reliable, then for any $\epsilon > 0$, there exists $n > 0$ such that

$$R \leq I(X; Y_{\mathcal{S}^C}) + \sum_{i \in \mathcal{S}} C_i + \epsilon. \quad (1)$$

- Since the interferers are strong $P_J >> P_X$, when $N_s = N - M$ there is no degree of freedom and we get

$$I(X; Y_{\mathcal{S}^C}) \sim 0.$$

- And thus for $N_s = N - M$

$$R \lesssim \sum_{i \in \mathcal{S}} C_i.$$

- When combining also the case of an empty $\mathcal{S}$ one gets to the upper bound.

Lower bound

- Use lattice code for the transmission, use modulo that lattice in the relays,
  the modulo filters out the unnecessary interference.
- Then use compression for the output of the modulo.
- Let $\mathbb{C}_2$ and ν_2 be the a good source lattice code, and its Voronoi region, respectively.
- Define $\mathbf{A}$ as the length N linear combing vector which nulls the M interferers from $\mathbf{Y}$, while receiving the signal $\mathbf{X}$ with gain a, so that $|\mathbf{A}|^2 = 1$.
- Transmit the information W as a codeword from a codebook, where every codeword in this codebook is randomly and independently generated by dividing it into many (multi-letter) entries, each generated uniformly i.i.d. over the Voronoi region ν_2.
- Add a pseudo random dithering $-U$, which is uniformly generated over ν_2 and known to all parties, to get: $X = V - U \mod \nu_2$.

Lower bound

At the i^{th} relay

- Multiply Y_i by $A_i\alpha$, apply $\mod \mathbb{C}_2$, and quantize the received signal using standard information theoretic techniques into $W_i = \alpha A_i Y_i \mod \mathbb{C}_2 + D_i$.
- The distortion D_i in W_i is Gaussian with zero mean and is independent with any other random variable.

The destination

- Decode W_i and calculate the $\sum_{i=1}^{N} W_i$.
- From the result, subtract the known pseudo random dither U and uses $\mod \mathbb{C}_2$.
- Then find the vector $\hat{V}$ which is jointly typical with the resulting outcomes of the modulo operation.

Properties

Considering the upper bound

- For any $M < N$ we have

$$\sum_{i=1}^{N} C_i \gtrsim R \frac{N}{N-M}.$$

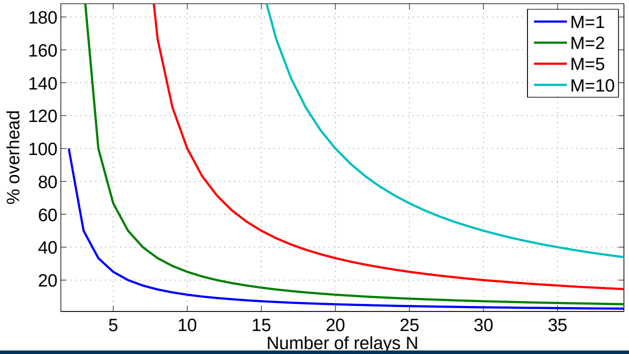
- For $M = N - 1$ the sum rate $\sum_{i=1}^{N} C_i \geq NR$: for each bit of desired information we have to transmit additional $N - 1$ nuisance information bits.

Properties

- The lower bound on the excess bit rate is

$$\frac{\sum_{i=1}^{N} C_i - R}{R} \gtrsim \frac{M}{N-M}$$

- The excess bit rate is lowered by adding relays.
- The excess bit rate ratio goes to zero as the ratio of number of relays to number of interferers increases.



The case of $M = 1, N = 3$

- The minimal excess bit rate is

$$\frac{\sum_{i=1}^{N} C_i - R}{R} \gtrsim \frac{M}{N - M} = 1/2.$$

- Need an achievable scheme to forward only 1.5 bits for any information bit.

Some intuition

- Assume no noise (asymptotically high SNR).
- Special channel coefficients.
- Generalization to scenario with noise, and general channel coefficients follows the same line.

The case of $M = 1, N = 3$

Example

Definitions

- Let the transmitter channel be $h = [\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2} + \frac{1}{2^K}]'$, where K is some positive integer.
- Let the interferer channel be $H = [1, 1, 1]'$.
- The links from relays to destination are $C_i = K$ bits/channel use.

Transmission scheme

- The transmitter encodes the information and then sends X into the channel, where $X = [0, 1, \ldots, 2^{2K} - 1]$.

The case of $M = 1, N = 3$

Relaying scheme

- The relays add a pseudo-random dithering U_i to the received signal, and then take the K least significant bits, in a fixed point representation.

$$W_i = round(Y_i + U_i \mod 2^K).$$

- Each relay then forwards W_i to the destination using the K bits/channel use link.
- Overall the three relays use $3K$ bits per channel use.

The case of $M = 1, N = 3$

Decoding scheme

- The destination decodes the information out of

$$\begin{aligned} I_1 &= W_1 - W_2 - U_1 + U_2 \mod 2^K = X + n_1 \mod 2^K \\ I_2 &= W_3 - W_2 - U_3 + U_2 \mod 2^K = X/2^K + n_2 \mod 2^K. \end{aligned}$$

- n_1, n_2 are due to the rounding in the relays along with the dithering.
- The encoding/decoding is successful if the information rate

$$R \leq I(X; I_1, I_2).$$

- Since the dithering is uncorrelated, we have that n_1, n_2 are independent with regards to all other random variables, and have also a finite variance, for any K.

The case of $M = 1, N = 3$

Decoding scheme

- This means that

$$\lim_{K\to\infty} \frac{R}{2K} = 1.$$

- We end up with a reliable scheme with asymptotically excess of 0.5 bit for any information bit, identical to the upper bound.

Results

- By taking $X = [0, 1, \ldots, 2^{2K} - 1]$ we lose a fixed amount of information bits, which for large K is negligible.
- For $K = 10$, MATLAB simulations gets an overall excess of 7 bits, which improves on 10 for two relays.

Part II conclusions

- A scheme for mitigation of multiple strong interference was suggested.
- Achievable rates and outer bound were presented.
- For special case of $N = M + 1$ the scaling laws were fully characterized.
- For N=3, M=1 an example was given to show that excess information to be relayed is determined by the ratio of relays to interferers.

Conclusions

- The effectiveness of lattice codes was demonstrated in two very common scenarios in wireless networks.
- Enhanced performance via superposition coding/combining random coding.
- Maintaining the lattice algebraic structure is the limiting factor of using the lattice codes. Integer coefficients are the best here.
- Lattices were utilized as filters for undesired interferences. Using their algebraic nature, they allow to compress a signal in a way that filters undesired signals while keeping required signal unharmed.

Thank You!