Mitigating Interference with Integer-Forcing Architectures

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MIMO Channel with External Interference

- M x M MIMO with K dimensional external interference
- Each antenna encodes an independent message (or data stream) at rate $R/M$ using power $P$
- Channel state information is known only at the receiver

$y = Hx + Jv + z$
Previously proposed Integer-Forcing Linear Receiver for MIMO channels without external interference [Zhan-Nazer-Erez-Gastpar ISIT ’10]

Show that It can be used to mitigated external interference
Traditional Linear Receivers

\[ y = Hx + Jv + z \]

- Project received signal: \( \tilde{y} = By \)
- Each stream \( \tilde{y}_m \) is fed into a separate decoder that attempts to recover a message \( w_m \)
Traditional Linear Receivers

\[ y = Hx + Jv + z \]

- **Null Interference**: Transmit M-K data streams and then project output unto space orthogonal to external interference
- **Treat Interference as Noise**: Transmit M streams and then use Decorrelator or linear MMSE estimator
Integer-Forcing Architecture

- Project received signal: $\tilde{y} = By$
- Each stream $\tilde{y}_m$ is fed into a separate decoder that attempts to recover an equation $u_m$
- Equations can be digitally solved for the original messages
- Compute-and-Forward is used to decode equations [Nazer-Gastpar IT ’11]
Integer-Forcing Architecture

- Freedom to choose $\mathbf{A}$ to be any full rank integer matrix
- Choosing $\mathbf{A} = \mathbf{I}$ reduces to traditional linear receivers

\[
\mathbf{u}_m = \sum_{i=1}^{M} a_{i,j} \mathbf{w}_{i,j} \quad \mathbf{y} = \mathbf{Hx} + \mathbf{Jv} + \mathbf{z}
\]
Choosing Equations

\[
\sigma_{\text{EFFECTIVE},m} \leq \lambda_{\text{MAX}}^2 \|a_m\|^2 + \|\tilde{J}^T a_m\|^2 \text{INR}
\]
Mitigating Interference

After Preprocessing:

Null Interference
- One Data Stream
- No External Interference after preprocessing

Treat Interference as Noise
- Two Data Streams
- External Interference Mostly Unmitigated

Integer-Forcing
- Two Linear combinations of Data Streams
- External Interference Mostly Mitigated
2% Outage Rate

\[ \alpha = \frac{\log \text{INR}}{\log \text{SNR}} \]

\[ \alpha = 0.2 \]
Consider the $M \times M$ MIMO channel with $K$ dimensional Interference.

Let $\alpha = \frac{\log \text{INR}}{\log \text{SNR}}$. The integer-forcing linear receiver achieves the generalized degrees of freedom:

$$d_{\text{INT}} = M - K\alpha$$

If $H \in \mathbb{R}^{M \times M}$ and $J \in \mathbb{R}^{M \times K}$ are such that the rows of $H^{-1}J$ are rationally independent.

$$d_{\text{JOINT}} = M - K\alpha$$

$$d_{\text{MMSE,M-K}} = M - K$$

$$d_{\text{MMSE,M}} = M - M\alpha$$
Generalized Degrees of Freedom

Interference Strength \( \alpha \)

Generalized Degrees of Freedom (d)

- Joint (\( N = M \))
- Integer (\( N = M \))
- MMSE (\( N = M \))
- MMSE (\( N = M - K \))
Generalized Degrees of Freedom

Transmit \( M - K \) Data Streams
Null Interference

Transmit \( M \) Data Streams
Treat Interference as Noise

Integer-Forcing

\[ d = M - K \]
\[ d = M(1 - \alpha) \]
\[ d = M - K\alpha \]
Proof Ideas

• Choose \( \mathbf{a}_1 \cdots \mathbf{a}_M \) to minimize:

\[
\max_m \lambda_{\text{MAX}}^2 (H^{-1}) \| \mathbf{a}_m \|^2 + \| \tilde{J}^T \mathbf{a}_m \|^2 I NR
\]

Find \( M \) linearly independent integer vectors \( (\mathbf{q}_1, \mathbf{p}) \cdots (\mathbf{q}_M, \mathbf{p}_M) \in \mathbb{Z}^{M-K} \times \mathbb{Z}^K \) that minimizes:

\[
\| \mathbf{q}_m \|^2 + \| \mathbf{X} \mathbf{q}_m - \mathbf{p}_m \|^2 I NR
\]

where \( \mathbf{X} \in \mathbb{R}^{K \times (M-K)} \)
Dirichlet’s Approximation Theorem

- In the scalar case, the tradeoff is:

  \[ |q| \leq Q \quad \|qx - p\| \leq \frac{1}{Q} \]

- Proof is by Pigeon hole principle

**Lemma (Dirichlets)**

For any \( X \in \mathbb{R}^{K \times (M-K)} \) and any \( Q \in \mathbb{Z}_+ \), there exists a \((q, p) \in \mathbb{Z}^{M-K} \times \mathbb{Z}^K \setminus \{0\} \) with \( \|q\|_{\infty} \leq Q \) such that

\[ \|Xq - p\| \leq \frac{1}{Q^{\frac{M-K}{K}}} \]

- What happens with multiple linearly independent approximations?
Diophantine Approximations

• In the scalar case, the tradeoff becomes:

\[ |q| \leq \Theta \left( (\log Q)^2 Q \right) \quad \|qx - p\| \leq \Theta \left( \frac{(\log Q)^2}{Q} \right) \]

• Proof uses Khintchine, Minkowski and Lagrange Multipliers

**Lemma**

Let \( \mathbf{X} \in \mathbb{R}^{K \times (M-K)} \) where \( |x_{i,j}| \leq 1 \) and \( x_{i,j} \) are rationally independent for all \( i, j \). There exists an \( Q' \in \mathbb{N} \) such that all for \( Q > Q' \), there exist \( M \) linearly independent integer vectors \( (q_1, p), \cdots (q_M, p_M) \in \mathbb{Z}^K \times \mathbb{Z}^{M-k} \) such that

\[
\|\mathbf{X}q_m - p_m\| \leq \frac{C(\log Q)^2}{Q^{M-K}K} \quad \|q_m\|_\infty \leq CQ(\log Q)^2
\]

where \( C \) is a constant that is independent of \( Q \).
Some Connections

- **Lattice Reduction:** Yao-Wornell ’02, Taherzadeh-Mobasher-Khandani ’07, Jalden-Elia ’09

- **Lattices for AWGN Capacity:** Erez-Zamir ’04

- **Lattices for DMT:** El Gamal-Caire-Damen ’04

- **Practical compute-and-forward:** Feng-Silva-Kschischang ’10, Hern and Narayanan ’11, Ordentlich and Erez ’10, Osmane and Belfiore ’11

- **The Degrees of Freedom of Compute-and-Forward:** Niesen-Whiting ’11