Dissipative Solitons (DSs) and the FitzHugh-Nagumo (FN) Equation

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Workshop

Localized Multi-Dimensional Patterns in Dissipative Systems:

Theory, Modelling, and Experiments

Banff International Research Station (BIRS)

Banff 25.07. – 29.07.2011

Extended Version

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C. Radehaus
A.W. Liehr
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R. Schmeling
H. Willebrand
many others

Key referencse:

H.-G. Purwins, H. U. Bödeker and Sh. Amiranashvili,
Dissipative solitons, **Advances in Physics, vol. 59, pp. 485-701 (2010)**

http://www.uni-muenster.de/Physik.AP/Purwins/Research-Summary.

1. What is a Dissipative Soliton (DS)?

The Particle Concept

the particle concept

- **division of objects of natural perception into subunits**
- **homogeneous space**
- **small number, simple (short range) interaction**
- **most successful example: concept of the atom**
- **usually particles exist in closed systems**
- **particle description by ordinary differential equations**
- **other point of view: localized solitary deviation of some field variable on an otherwise homogeneous background**
- **relatively new aspect: localized structures in certain classes of dissipative systems behave like particles in many respect**

Dissipative Solitons (DSs) I:

The Ideal Object

- DSs are localized deviations of one or more state variable from an otherwise stationary homogeneous background.

- DSs or ensembles of them are attractors of a stationary or periodically driven spatially extended dissipative systems that are homogeneous by construction.

- Provided they are sufficiently far away from each other and from the boundary, for a given set of parameters, DSs can show up in any number in one or more distinct classes with same size, shape and dynamical properties within a given class.

- DSs coexist with some stable stationary homogeneous state that coincides with the background state far away from any DS.

- DSs undergo interaction, such that their individuality is retained to large extent, alternatively they are annihilated or generated as a whole.

Bode et al., Physica D 86, p. 53 (1995); N. Akhmediev et al, *Dissipative Solitons*, Springer, Berlin, 2005; Akhmediev et al., Dissipative Solitons, ibid (2010); Purwins et al., Advances in Physics, 59, p. 485 (2010)

Dissipative Solitons (DSs) II:

The Real Object

- Usually DSs exist only in some 1- or 2-dimensional subspace.
- DSs interact (weakly) with the boundary and with each other.
- Perturbations are present in the form of temporal and spatial noise.
- Well defined inhomogeneities may be present e.g. in the form of gradients as well as local or periodic, distortions.
- The background may be weakly modulated periodically by self-organization.
- One may deal with a discrete system.
- In the presence of global restrictions the background may depend weakly on the number of LSs.
- Possibly, in the case of different classes of DSs interaction may give rise to a change from one to another class.

2. The Emergence of Dissipative Solitons (DSs)

Detection of Nerve Fibres and Measurement of the Speed of Pulse Propagation

- Hermann Ludwig Ferdinand von Helmholtz (1812 - 1894)
  medical doctor and physicist

- perhaps first quantitative experimental characterization of a DS

- 1842 : detection of nerve fibres connecting ganglia
  (clusters of cells)

•1850: first measurement of speed of pulse propagation
on a nerve fibre for a frog and human being: $v \approx 30$ m/s

- because of the slow propagation Helmholtz thought, that
  some substance should propagate

Nerve Pulse Propagation and Reaction-Diffusion Systems

- Robert Thomas Dietrich Luther (1868 – 1945)
  theoretical chemist, scholar of Oswald
- 1906: meeting of the Bunsengesellschaft
- presentation of various experiments demonstrating waves in chemical reaction systems being homogeneous by construction
- Presentation of the result of the theoretical analysis of a reaction-diffusion equation (which equation he analysed is not known)
  c = speed of propagation
  D = diffusion constant
  τ = time constant of the chemical reaction

$$c = const. \times \sqrt{D/\tau}$$

- in the discussion with Nernst, Luther speculated that nerve pulses may be the result of the propagation of chemical reactions

The Zeldovich-Frank-Kamenetsky (ZF) Equation I: The Equation

$$u_t = Du_{xx} + f(u),$$

- showed up in the1930s in relation to genetic diffusion (Fischer (1937); Kolmogorov, Petrocsky, Piscounoff (1937))

- Zeldovich and Frank- Kamenetsky (1938) investigated the equation with

$$f(u) = \lambda u - u^3 + \kappa_1, \quad \lambda = 1/\tau, \quad \lambda \geq 0$$

- equation is referred to as the ZF equation

- 1940: discussion of nerve pulse propagation in relation to the Zeldovich-Frank-Kamenetsky equation

The Zeldovich-Frank-Kamenetsky (ZFK) Equation II: Analytical Expression for Kink Solutions in $\Re^1$

condition: two stable stationary solution of the system

$$u_t = Du_{xx} + \lambda u - u^3 + \kappa_1$$

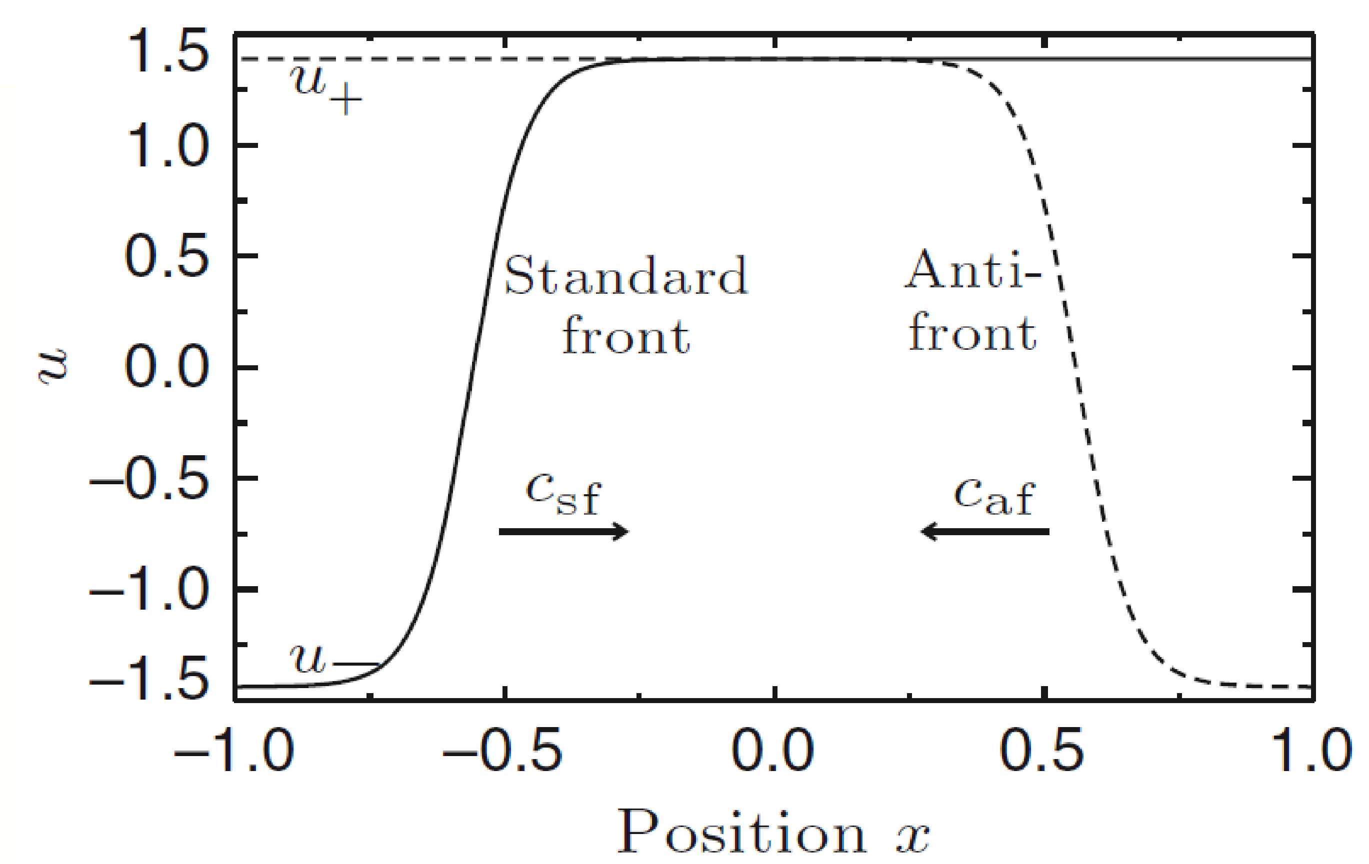
$$u(x,t) = \frac{1}{2}\left[(u_+ + u_-) + (u_+ - u_-)\tanh k(x - ct)\right]$$

$$c = -(3/2)\sqrt{2D}(u_+ + u_-), \quad k = (u_+ - u_-)/\sqrt{8D}$$

$$|c|_{\max} = \sqrt{\frac{3}{2}D\lambda} = const.\times\sqrt{\frac{D}{\tau}}, \quad \tau = \frac{1}{\lambda}$$

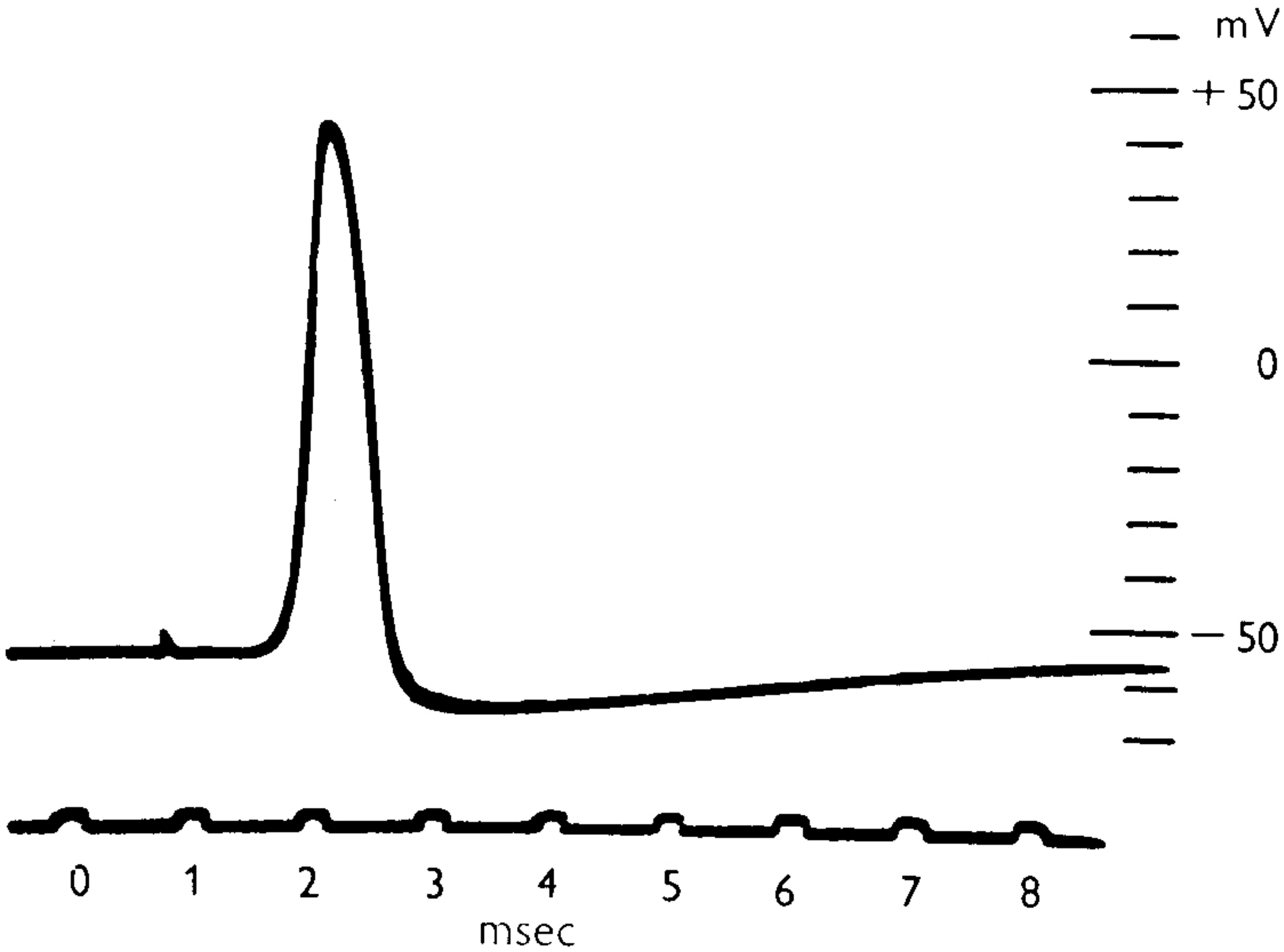
$u_{+,}\ u_- =$ **largest and smallest zeros of f(u) in the case of a total of three zeros of f(u)**
zeros depend on λ (> 1) and κ_1

The Zeldovich-Frank-Kamenetsky (ZFK) Equation IV: Illustration of Kinks and Localized Solutions in $\Re^1$



Purwins et al., Advances in Physics, 59, p. 485 (2010)

Nerve Pulse of an Octopus: Electrical Potential at a Given Position of the Fibre as a Function of Time



Baker et al. , Nature 190, p. 885 (1961)

Properties of Nerve Fibers

- **Nerve fiber (axon)**

  connected to neuron via the axon hillock for outgoing signals
  diameter = 0.5 – 10 μm
  length = 1 μm – 1 m and more
  speed of pulses propagation v ≈ 1 – 100 m/s

- **Dendrites**
  connecting nerve fibers via synapses for incoming signals
  up to 100 000 to 200 000 per neuron

The FitzHugh-Nagumo (FN) Equation I: The Equation

- 1952: Hodgkin and Huxley set up a system of 5 equations to describe nerve pulse excitation in the fibre

- 1962: FitzHugh and Nagumo reduced the system to two variables

$$u_t = d_u^2 u_{xx} + \lambda u - u^3 - v + \kappa_1,$$

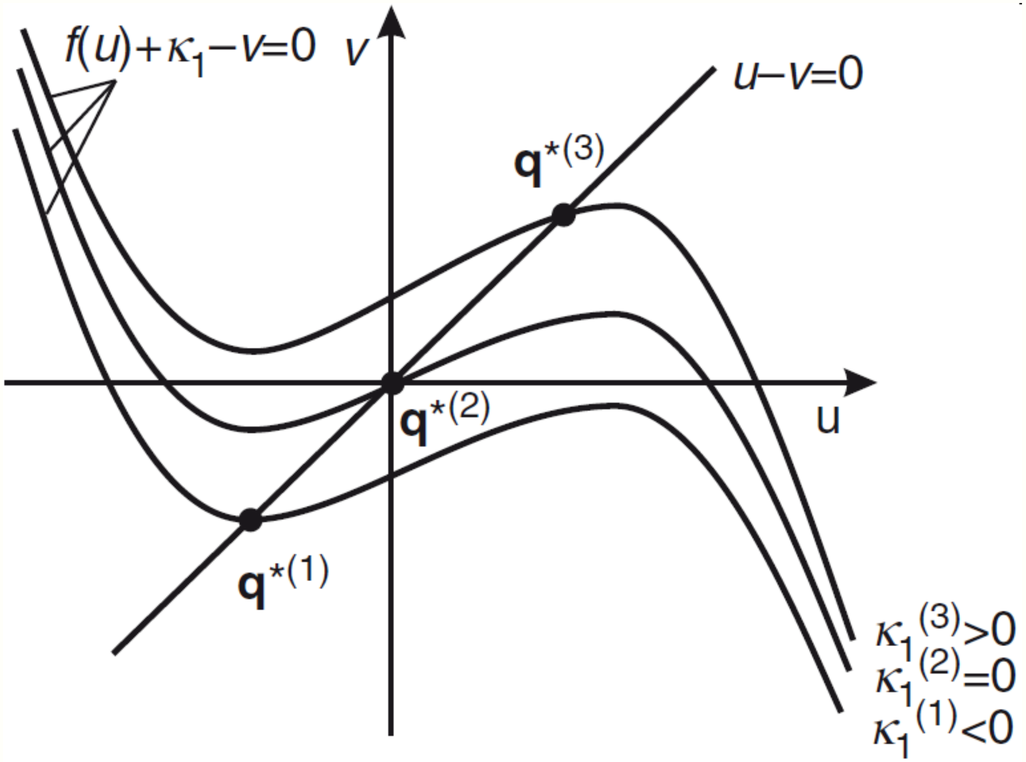
$$\tau v_t = d_v^2 v_{xx} + u - v$$

$$\lambda \geq 0, \tau >> 1$$

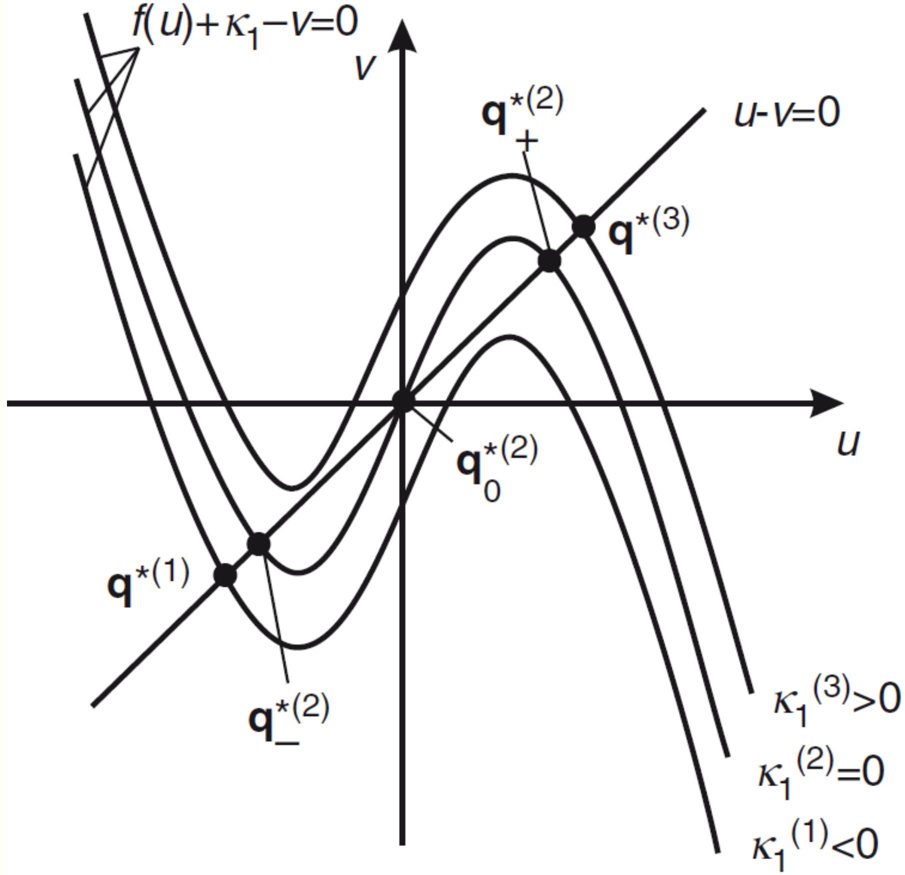
u = **transmembrane voltage**

v = **recovery variable (related to the potassium current)**

The FitzHugh-Nagumo (FN) Equation II: The Space Clamped System



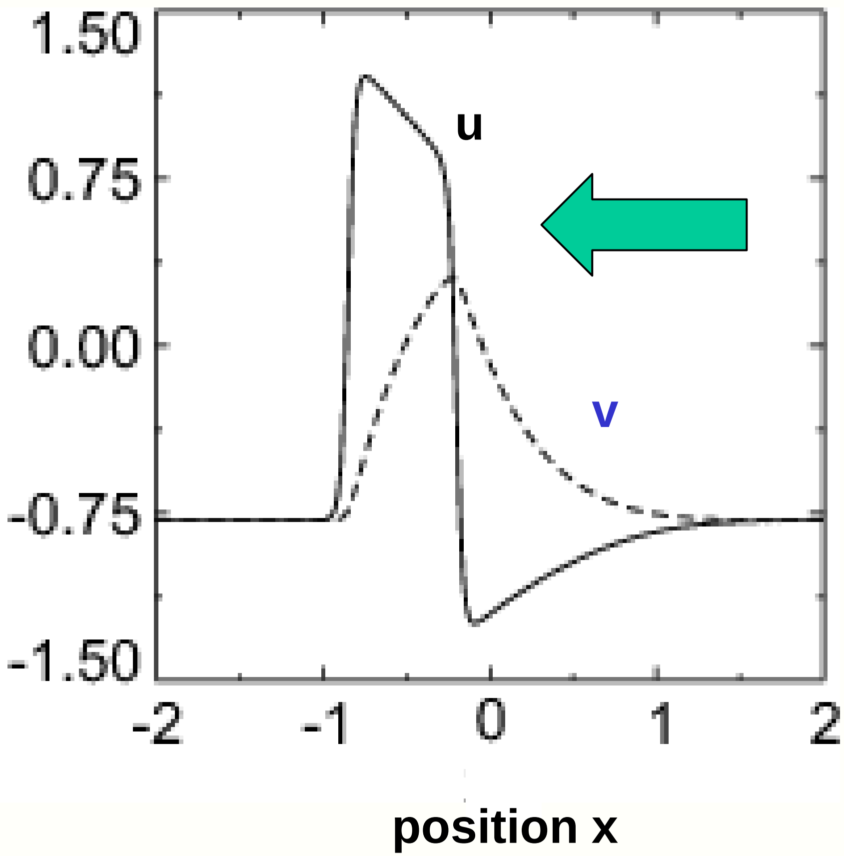
$\lambda < 1$



$\lambda > 1$

Purwins et al., Advances in Physics, 59, p. 485 (2010)

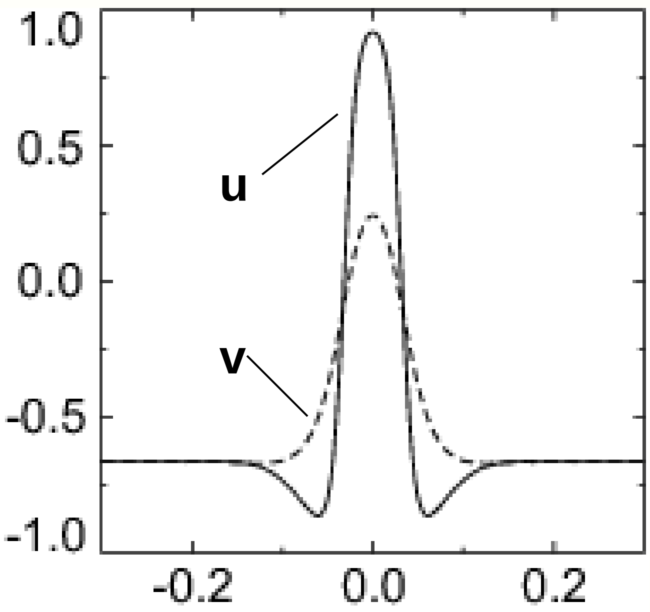
The FitzHugh-Nagumo (FN) Equation III: Example for a Numerical Solution for a Travelling DS in $\Re^1$ (Nerve Pulse)



Purwins et al., Advances in Physics, 59, p. 485 (2010)

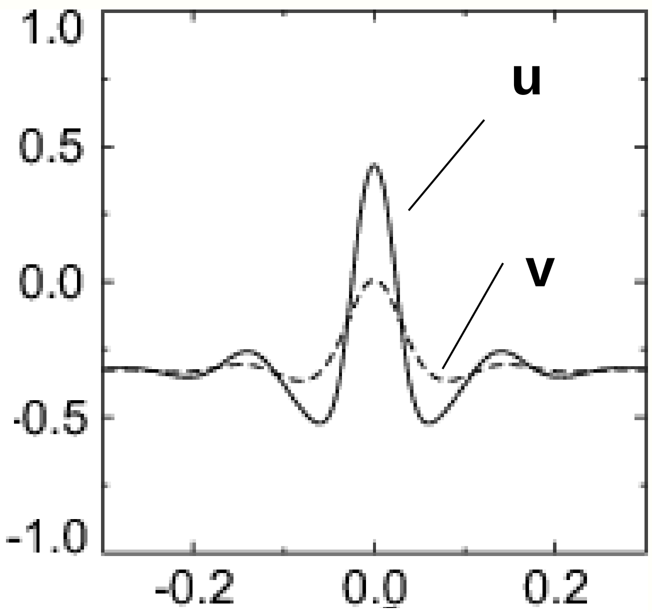
The FitzHugh-Nagumo (FN) Equation IV: Examples for Numerical Solutions for Stationary DSs in $\Re^1$

non-oscillatory tails



position x

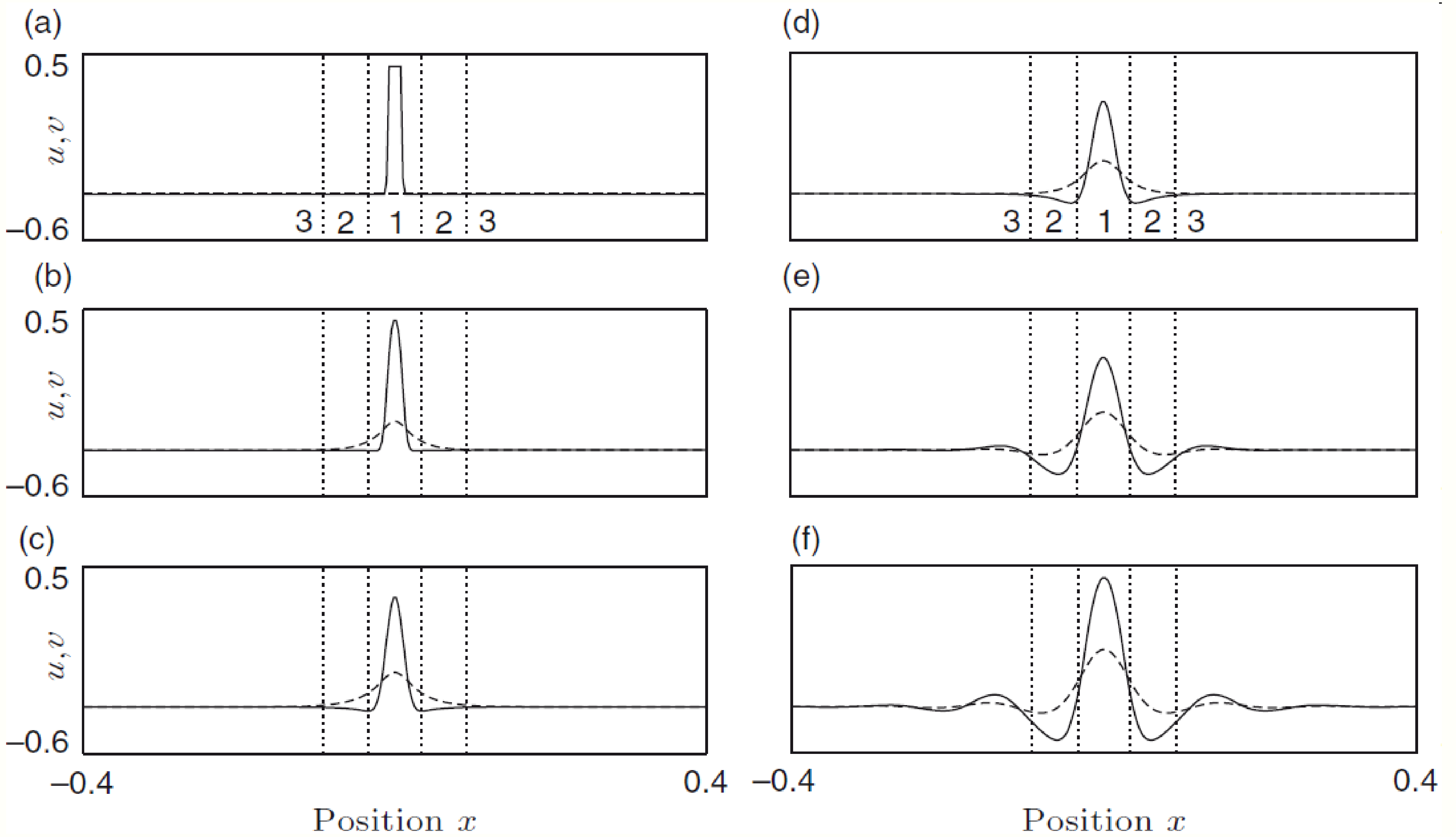
oscillatory tails



position x

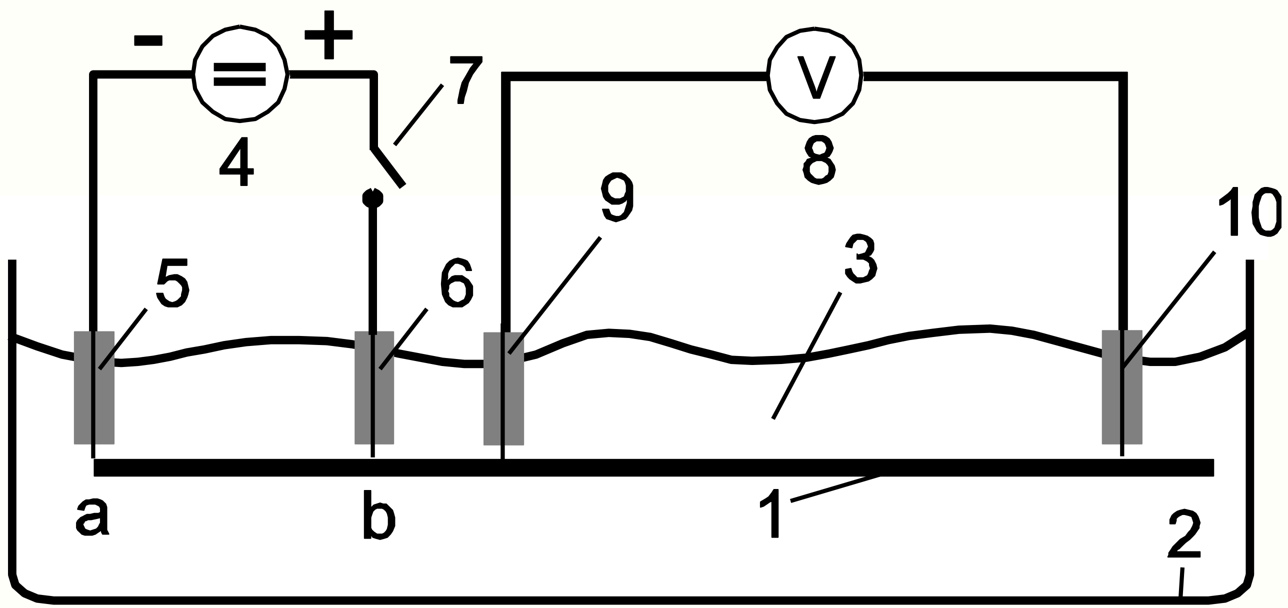
Purwins et al., Advances in Physics, 59, p. 485 (2010)

The FitzHugh-Nagumo (FN) Equation V: Illustration of the Formation of a Stationary DS in $\Re^1$



Purwins et al., Advances in Physics, 59, p. 485 (2010)

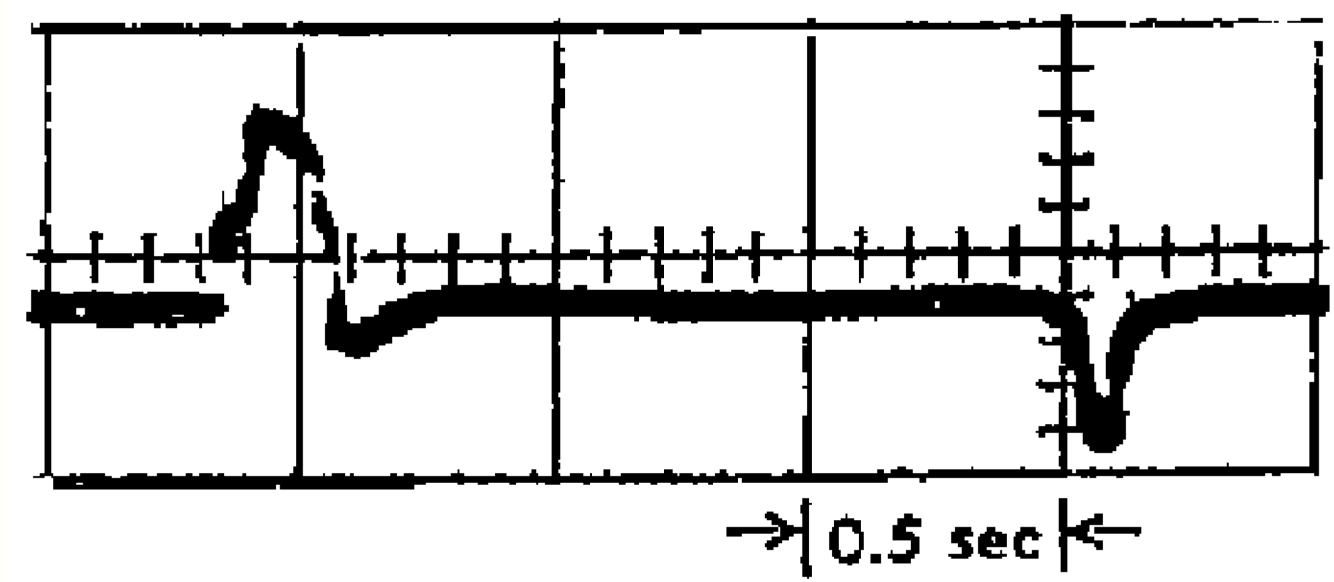
Pulse Propagation on an Iron Wire in Nitric Acid I: Experimental Set-Up



1. iron wire 2. dish 3. nitric acid 4. voltage source 5., 6. Pt electrodes

7. switch 8. potential meter 9., 10. fixed potential probes

Pulse Propagation on an Iron Wire in Nitric Acid II: Electrical Potential in Dependence of Time at Fixed Position of the Wire



potential difference between the probes 9. and 10. as a function of time

propagation of a well defined section of naked iron on an otherwise oxidised wire after an electrical perturbation initially deoxdising the wire locally

Suzuki, Adv. Biophys. 9 p. 115 (1976)

Remark on Russian Contributions

- at the end of the 80th s of the last century interesting contributions
- Rosanov in optics and here in relation with wave equation
- Osipov and Kerner in the field of electrical transport systems
- in particular the work of the Osipov and Kerner is in close relation to the 2-component FN equation

Rosanov, *Spatial Hysteresis and Optical Patterns*, Springer, Berlin, 2002;
Kerner, Osipov, "*Autosolitons*", Kluwer Academic Publishers, Dordrecht, 1994

3. A Special Class of Electrical Transport Systems and the Generalize FitzHugh-Nagumo (FN) Equation

FN Equation with Additional Global Coupling

$$u_t = d_u^2 u_{xx} + f(u) - v + \kappa_1 \left[-\kappa_2 \int_\Omega u\,dx \right],$$

$$\tau v_t = d_v^2 v_{xx} + u - v,$$

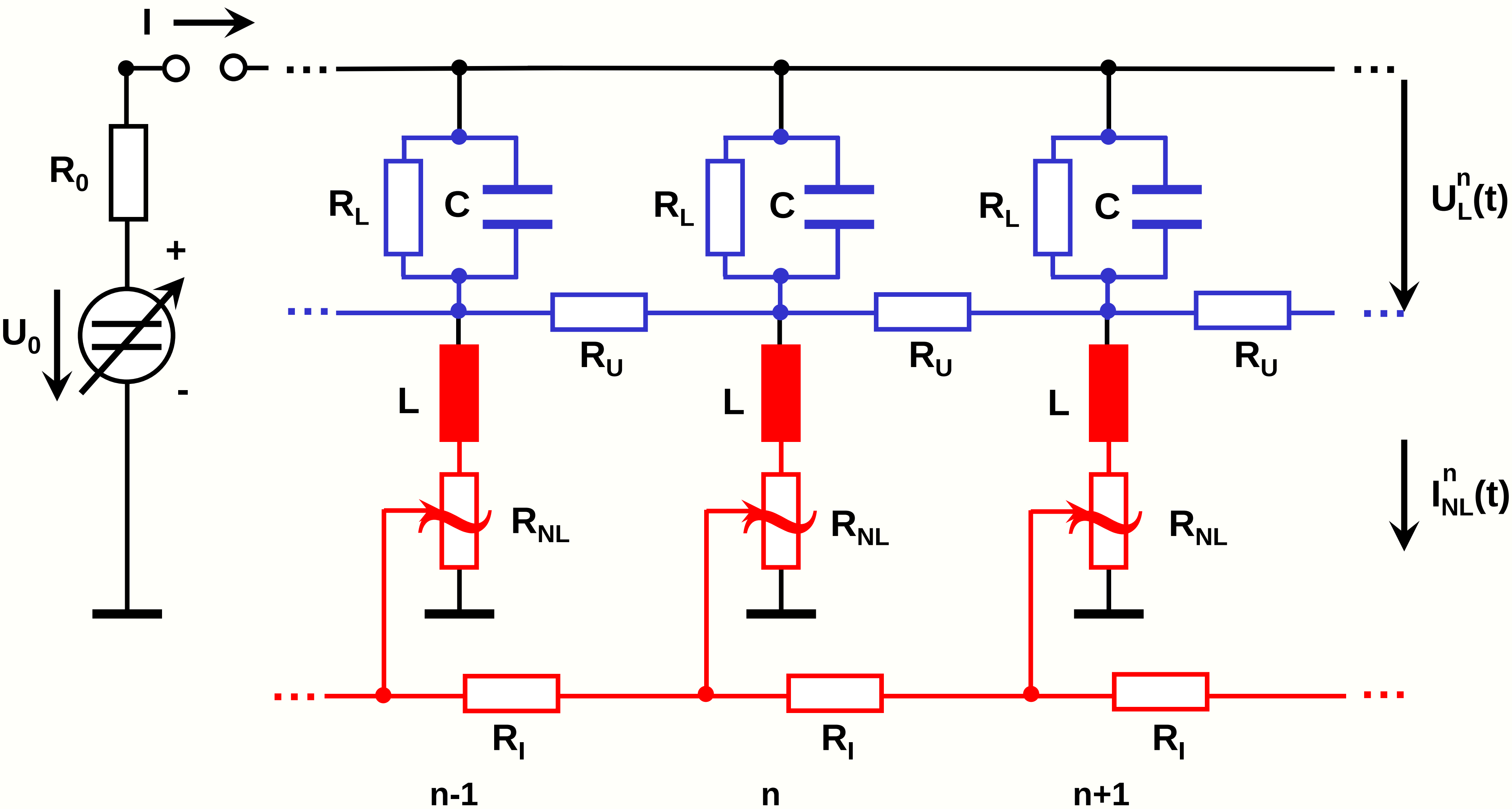
$$u = u(x,t),\ v = v(x,t),$$

$$f(u) = \lambda u - u^3,$$

$$d_u, d_v, \tau, \kappa_2, \lambda \geq 0$$

Purwins et al., Festkörperprobleme 27, p. 27 (1987)

Electrical Equivalent Circuit for the FN Equation I: The 1-Dimensional Circuit

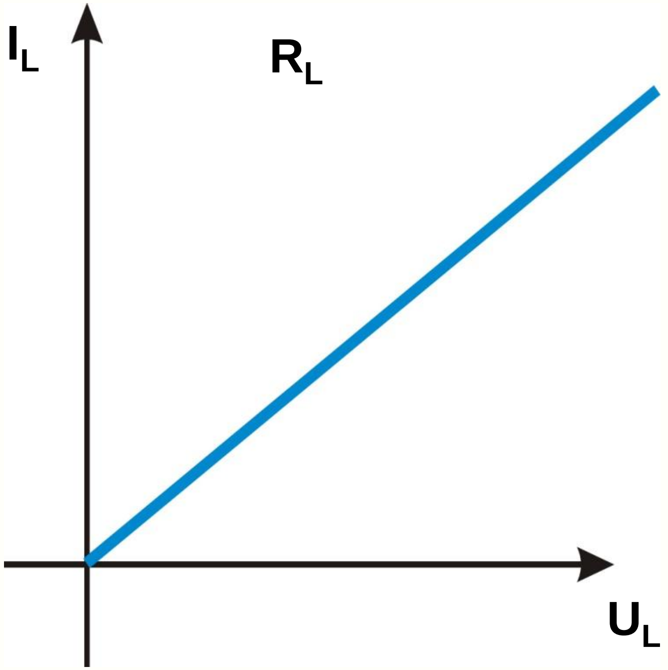


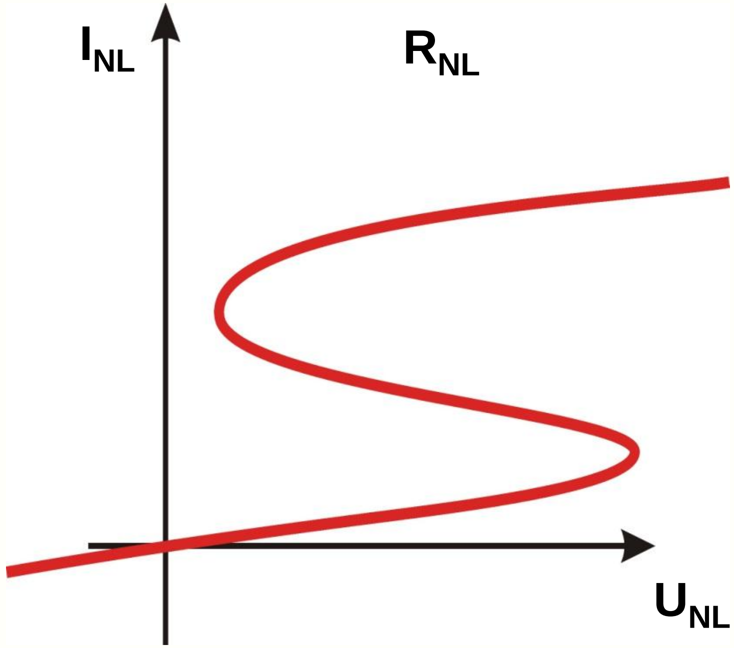
Berkemeier et al., Z. Phys. B-Condensed Matter 65, p. 255 (1986);
Purwins et al., Festkörperprobleme 27, p. 27 (1987)

Electrical Equivalent Circuit for the FN Equation II: (Current)-(Voltage) Characteristics of the Linear and the Nonlinear Resistance

monotonic
(approximative linear)

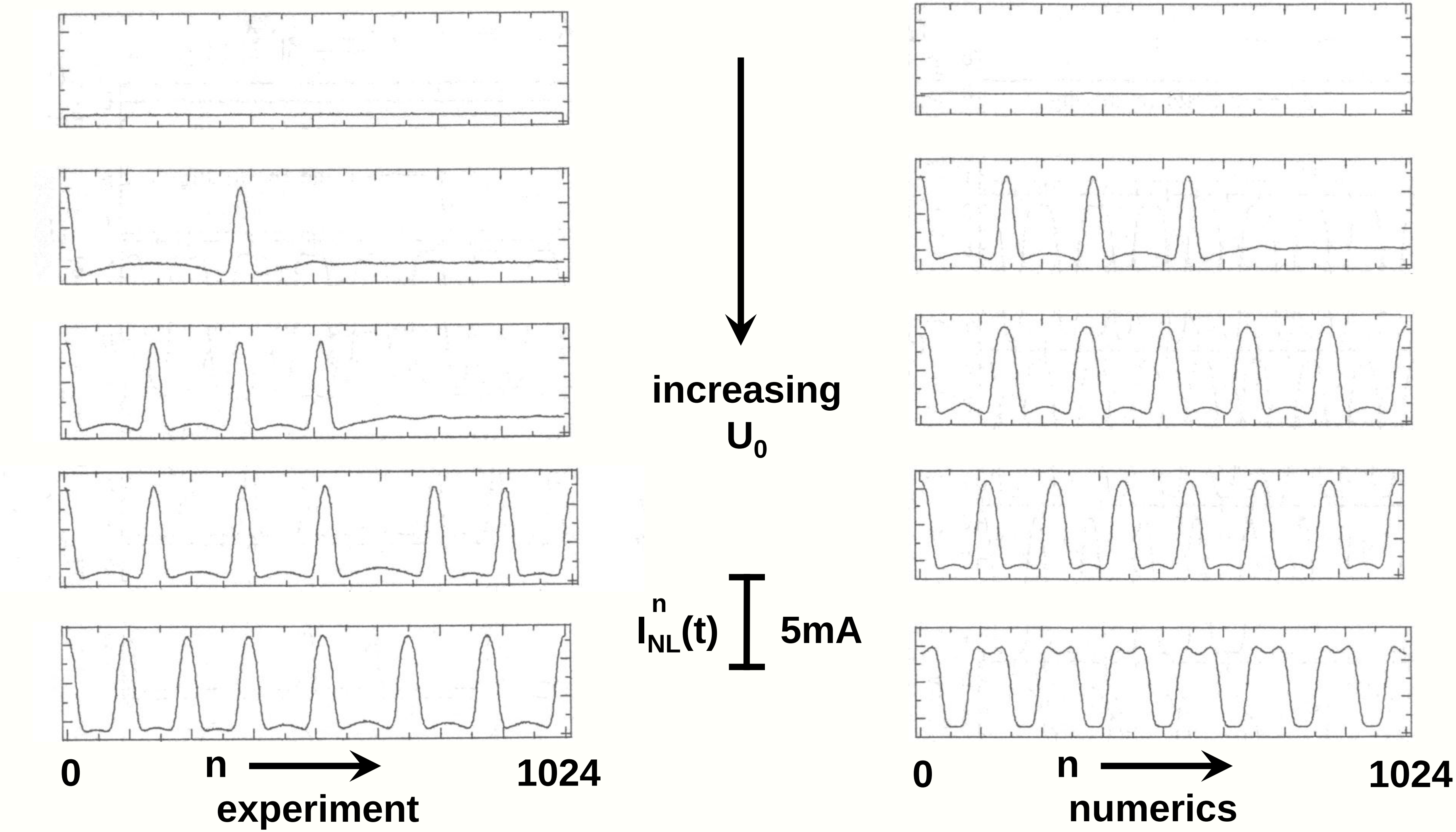
S-shaped
(strongly nonlinear)





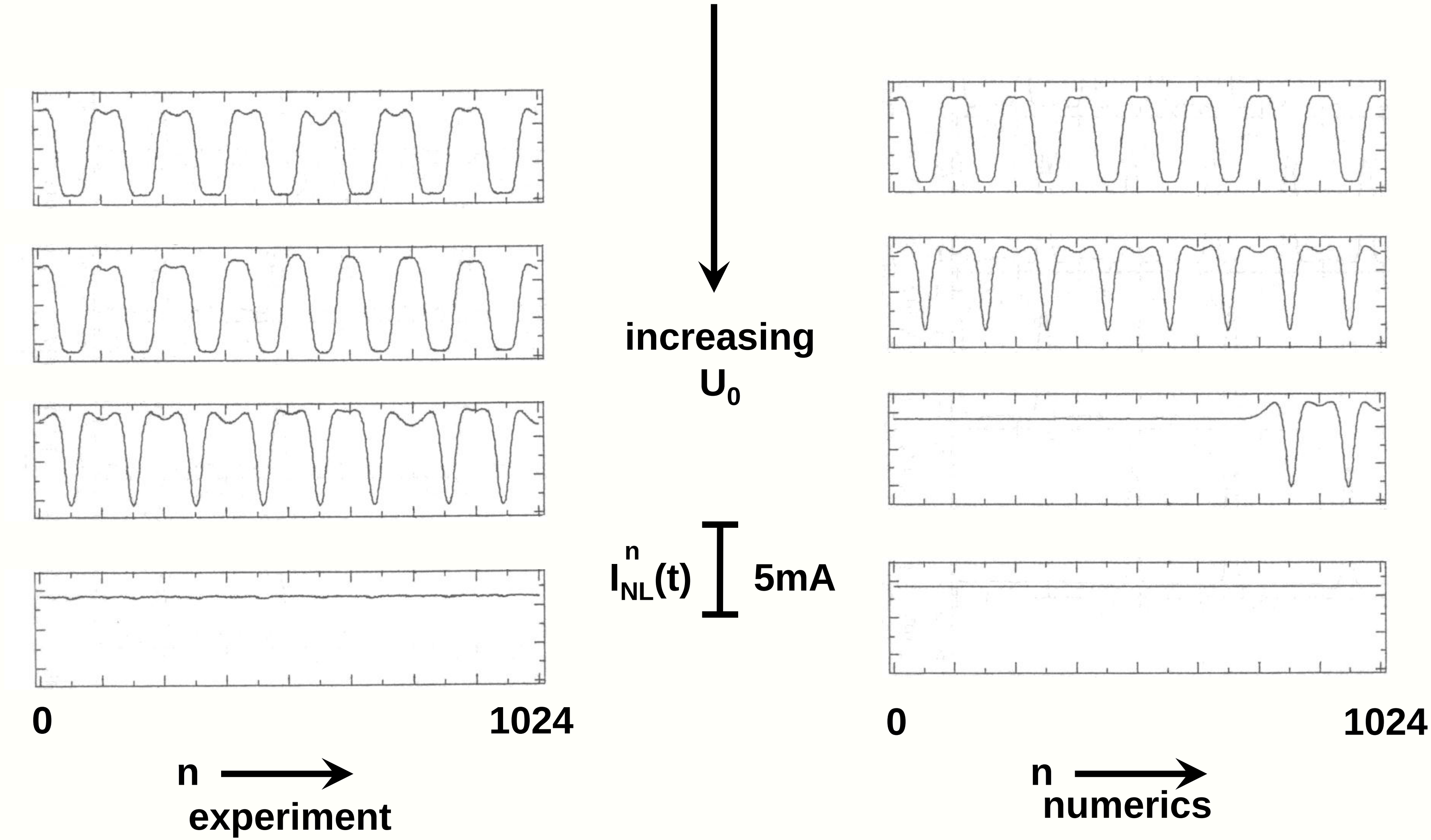
Berkemeier et al., Z. Phys. B-Condensed Matter 65, p. 255 (1986);
Purwins et al., Festkörperprobleme 27, p. 27 (1987)

Experimental Result for the 1-Dimensional Electrical Equivalent Circuit and Comparison to Numerical Solutions of the Corresponding Network Equation I



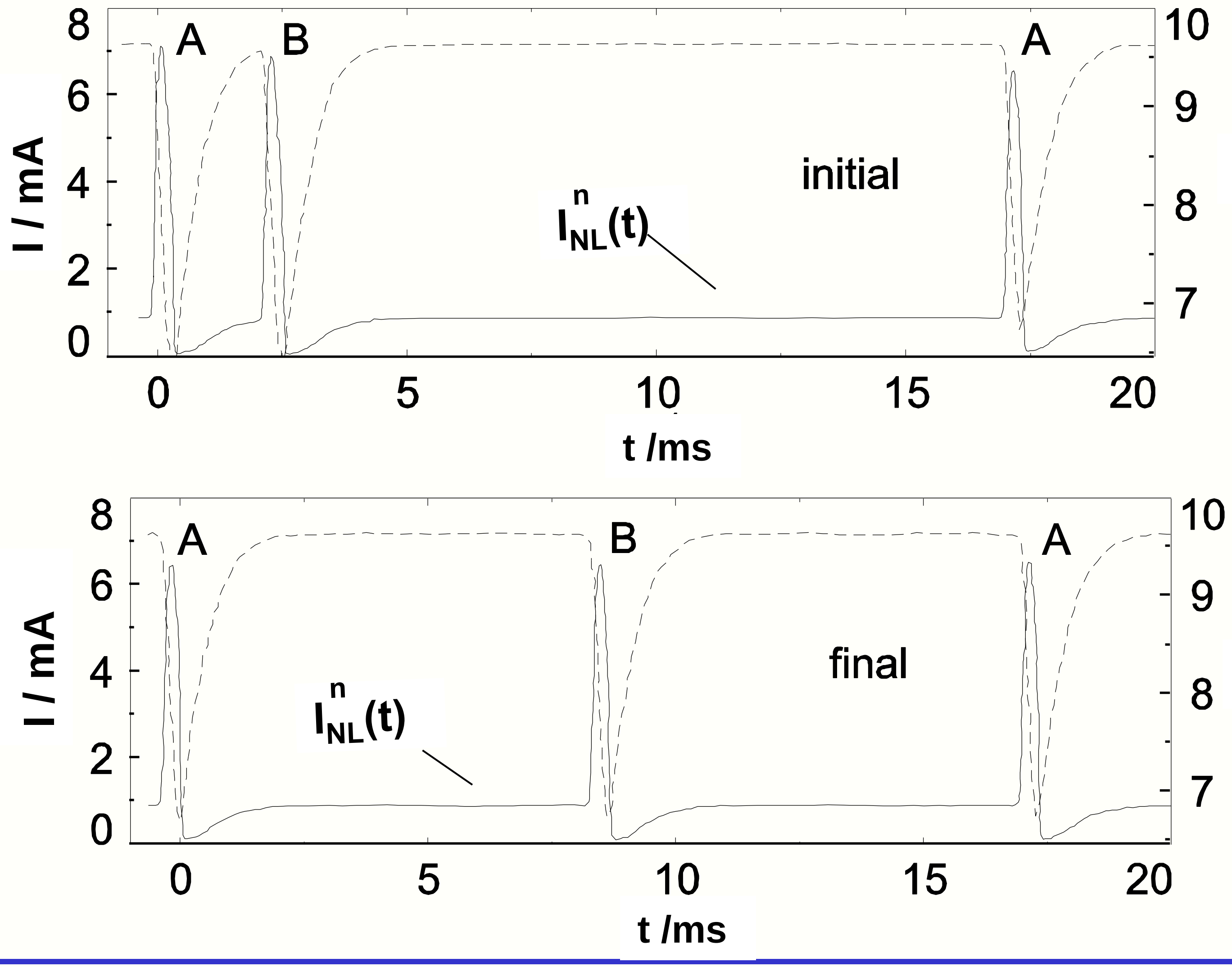
Schmeling, PhD Thesis, Universität Münster (1994); Purwins et al., Advances in Physics, 59, p. 485 (2010); http://www.unimuenster.de/Physik.AP/Purwins/Research-Summary

Experimental Result for the 1-Dimensional Electrical Equivalent Circuit and Comparison to Numerical Solutions of the Corresponding Network Equation II



Schmeling, PhD Thesis, Universität Münster (1994); Purwins et al., Advances in Physics, 59, p. 485 (2010); http://www.unimuenster.de/Physik.AP/Purwins/Research-Summary

Experimental Results for the 1-Dimensional Electrical Equivalent Circuit Closed to a Ring: Repulsive Interaction of Two Travelling DSs

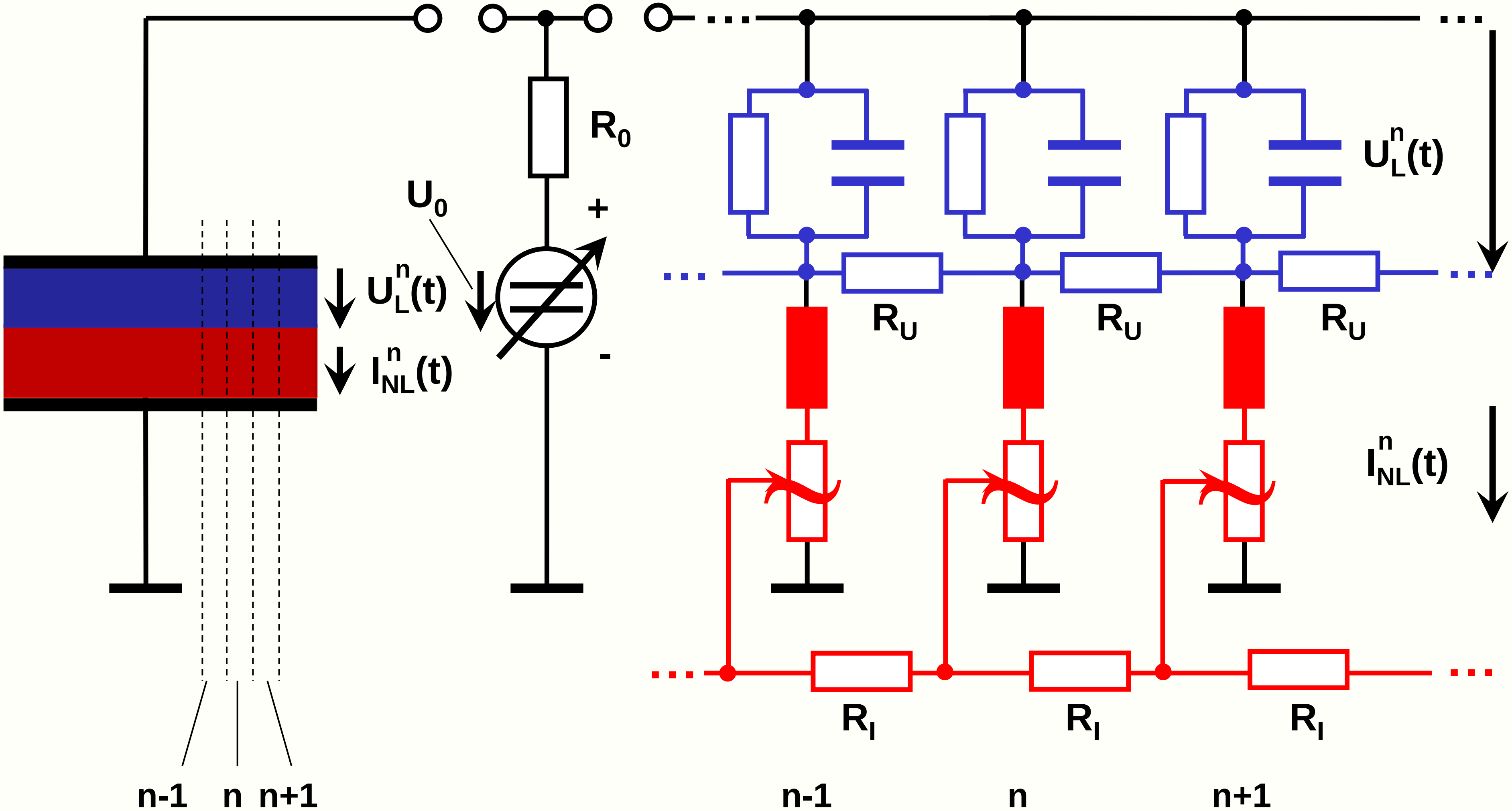


the current (continuous line) is measured at a given cell; initially two DSs are ignited close to each other and travel in the same direction; in the course of time the interaction leads to separation, finally corresponding to an angle of 180^0 on the ring

Schmeling, PhD Thesis, Universität Münster (1994); Purwins et al., Advances in Physics, 59, p. 485 (2010); http://www.uni-muenster.de/Physik.AP/Purwins/Research-Summary

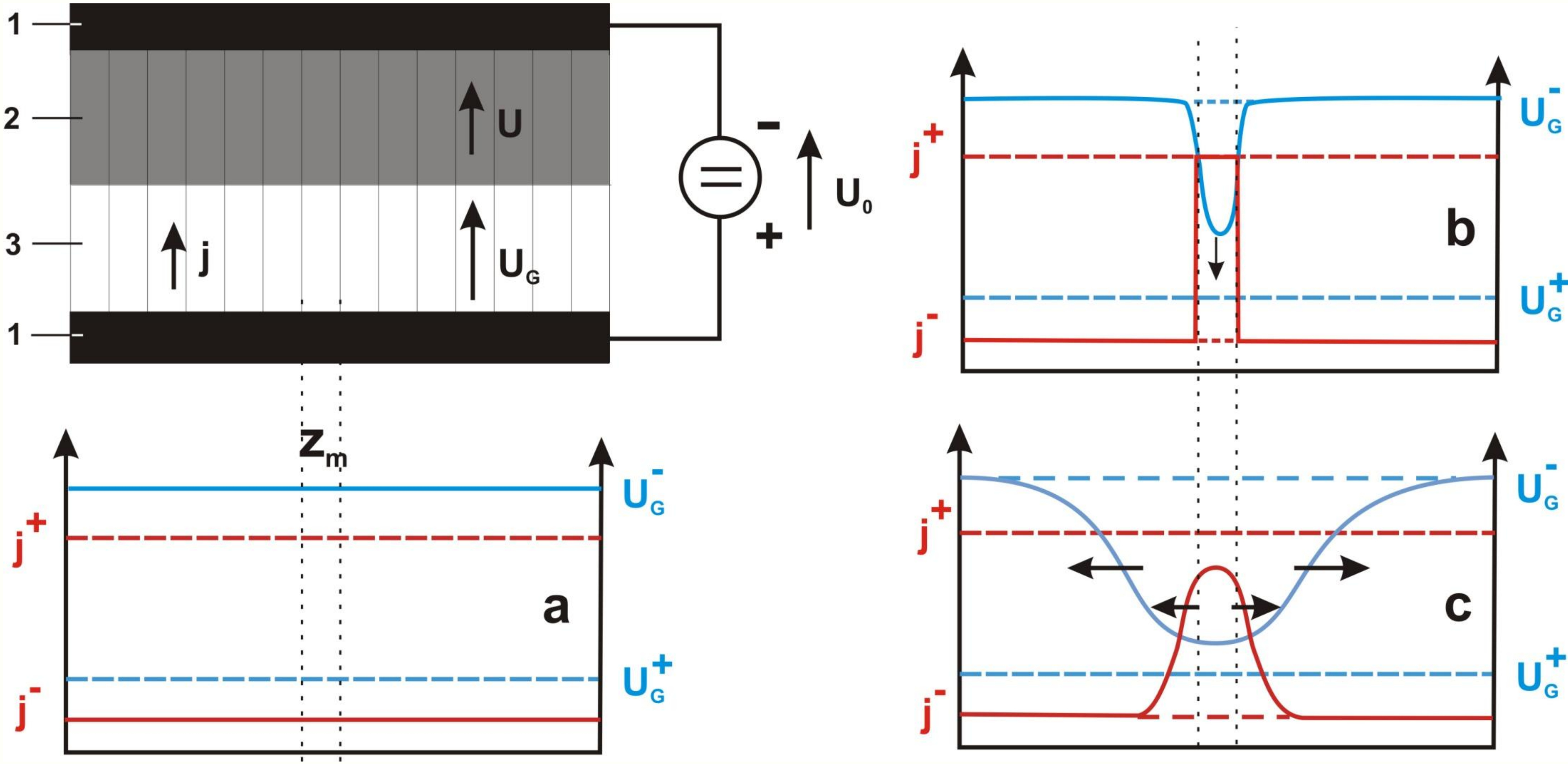
Pu 26.07.2011

The 1-Dimensional Network Equation as a Model Equation for a Double Layer Continuum System I: The System



Berkemeier et al. Z. Phys. B-Condensed Matter 65, p. 255 (1986);
Purwins et al., Festkörperprobleme 27, p. 27 (1987)

Illustration of the Formation of a Stable Current Filament in the 1-Dimensional Double Layer Continuum System I



Purwins et al., Phys. J. 6, 21 (2007)

Illustration of the Formation of a Stable Current Filament in the 1-Dimensional Double Layer Continuum System II

presuppositions for the FH equation:

- $u = j$, $v = U$
- **slow relaxation of** $u = j$ **with respect to** $v = U$ **:** $\tau < 1$
- **slow diffusion of** $u = j$ **with respect to** $v = U$ **:** $d_u < d_v$
- $\lambda > 1$ **and appropriate** κ_1 **in order to assure a stable stationary low curenent state** $(u^-, v^-) = (j^-, U^-)$ **and a stable stationary high current state** $(u^+, v^+) = (j^+, U^+)$ **(see Fig.I a)**
- **note:** $U^- = U_o - U_G^-$ **and** $U^+ = U_o - U_G^+$
- **for simplicity :** $\kappa_2 = 0$

Illustration of the Mechanism for the Formation and Stabilization of a Stable Current Filament in the 1-Dimensional Double Layer Continuum System III

Fig. I b Initial condition. The current density $j = u$ is set to j^{+} in the cell Z_m and to j^{-} elsewhere, the voltage drop $U = v$ is set to U^{-} everywhere.

Evolution of the system. As a result of the large current density j^{+} at Z_m the voltage drop U increases (this can also be seen from the second FH equation), consequently the corresponding voltage drop U_G decreases; this is a relatively, fast process due to the fast relaxation of U and U_G

Fig. I c. Some later time, due to moderate diffusion of j, the initial distribution broadenes; however, somewhat faster also the voltage drop distribution broadens due to large pseudo diffusion.

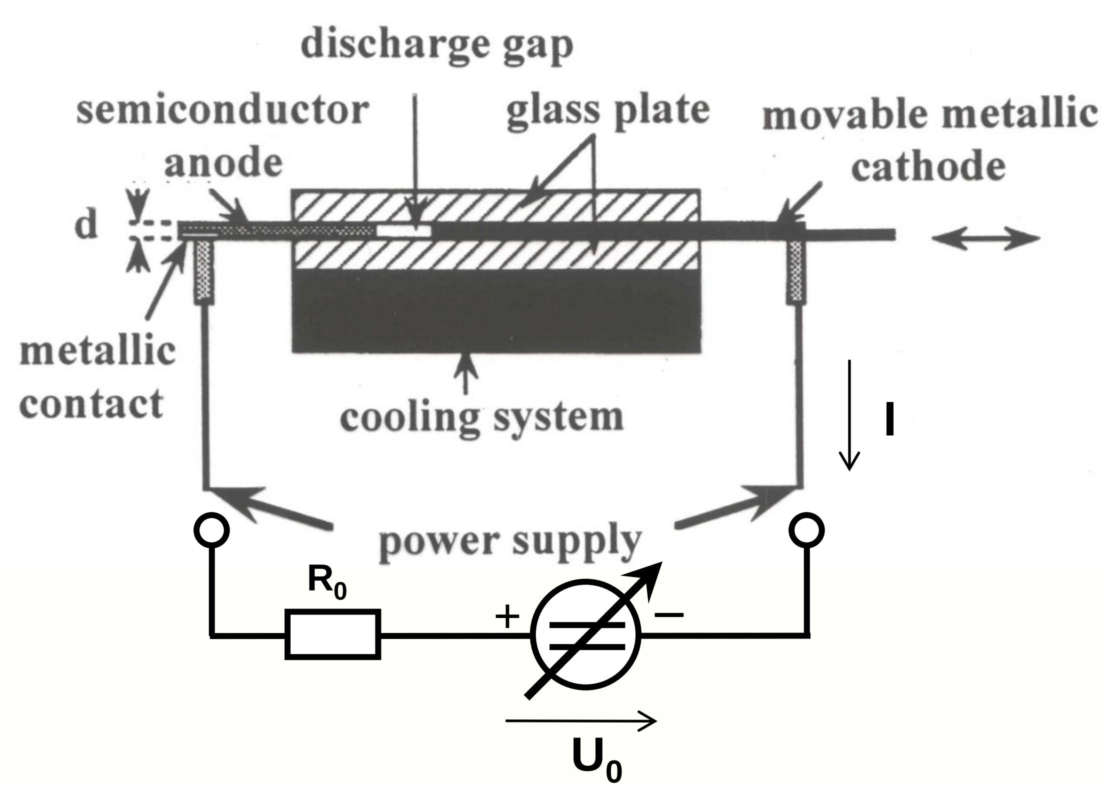
One may now look upon the dynamics as a competition of two phenomena: Two current density fronts travel in opposite direction, such, that the region of breakdown extends – however, two faster U_G voltage fronts, propagating also in opposite direction, lead to an obstruction of further extension of the region of breakdown.

Eventually the two pairs of fronts come to a rest in the final end.

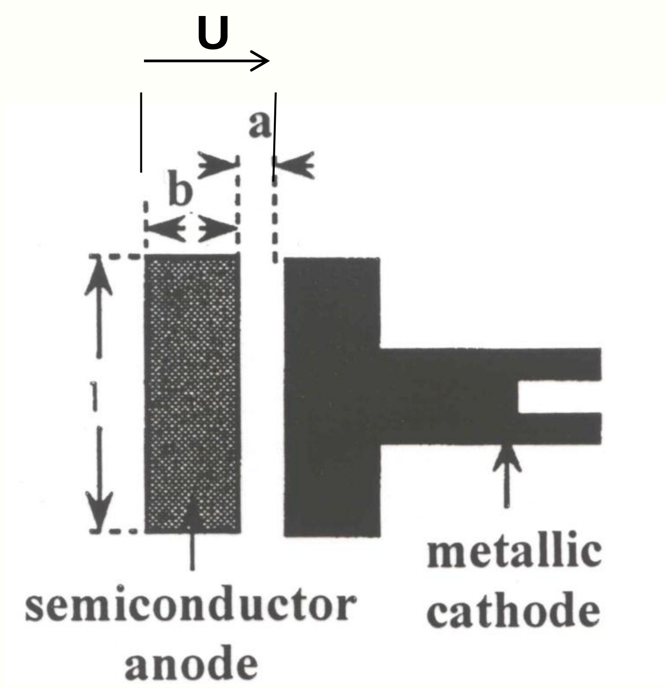
Purwins et al., Phys. J. 6, 21 (2007)

Quasi 1-Dimensional DC Gas-Discharge Systems: Experimental Set-Up

cross section



top view

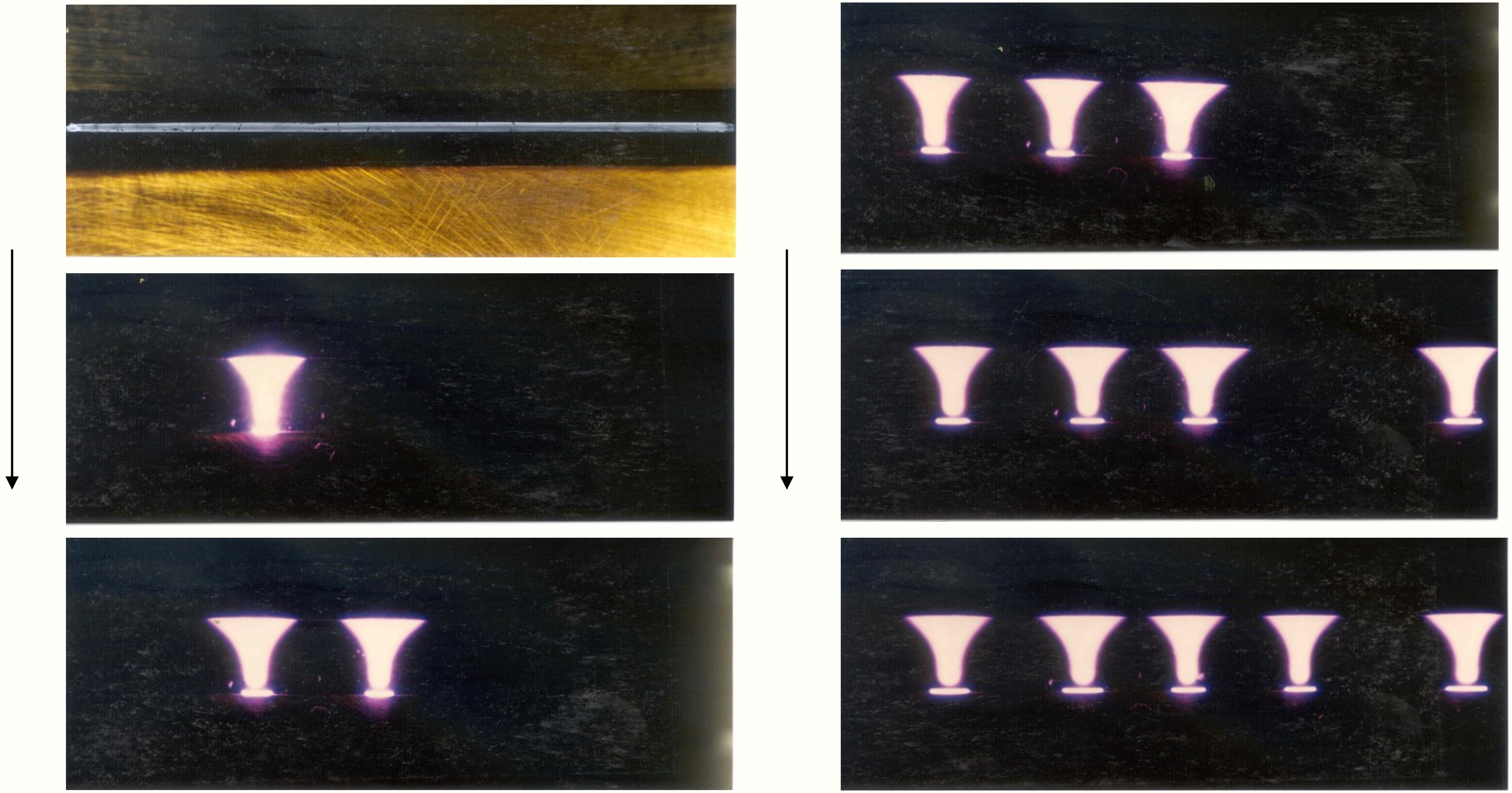


U_0 = 0 – 1500 V, R_0 = 20 – 1000 kΩ, a = 3 mm, b = 10 mm, d = 0.3mm,
ρ_{Si} = 0.9 – 2.6 MΩ cm, gas: Ar, He + air, p = 40 – 100 hPa, l = 20 – 50 mm

Radehaus et al., Z. Phys. B-Condensed Matter 65, p. 515 (1987); Purwins et al., Festkörperprobleme p. 27 (1987); Willebrand et al Phys. Lett. A 149, p. 131 (1990)

Quasi-1-Dimensional DC Gas-Discharge System: Experimental Cascade with Increasing Number of Stationary Current Filaments (A)

luminescence radiation distribution in the discharge slid for increasing driving voltage U_0

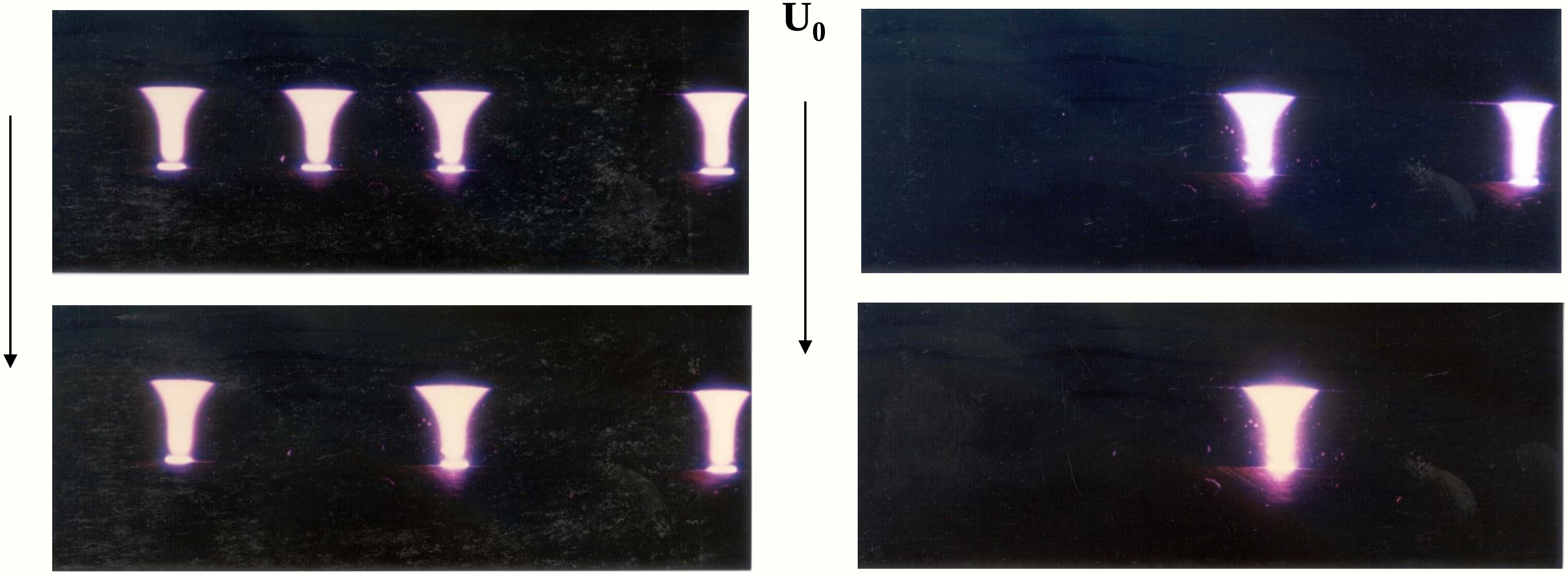


position in the discharge gap →

Radehaus et al., Z. Phys. B-Condensed Matter 65, p. 515 (1987); Willebrand, PhD Thesis, Universität Münster (1992); Bode, Purwins, Physica D 86, p. 53 (1995)

Quasi-1-Dimensional DC Gas-Discharge System: Experimental Cascade with Decreasing Number of Stationary Current Filaments (B)

luminescence radiation distribution in the discharge space for decreasing driving voltage



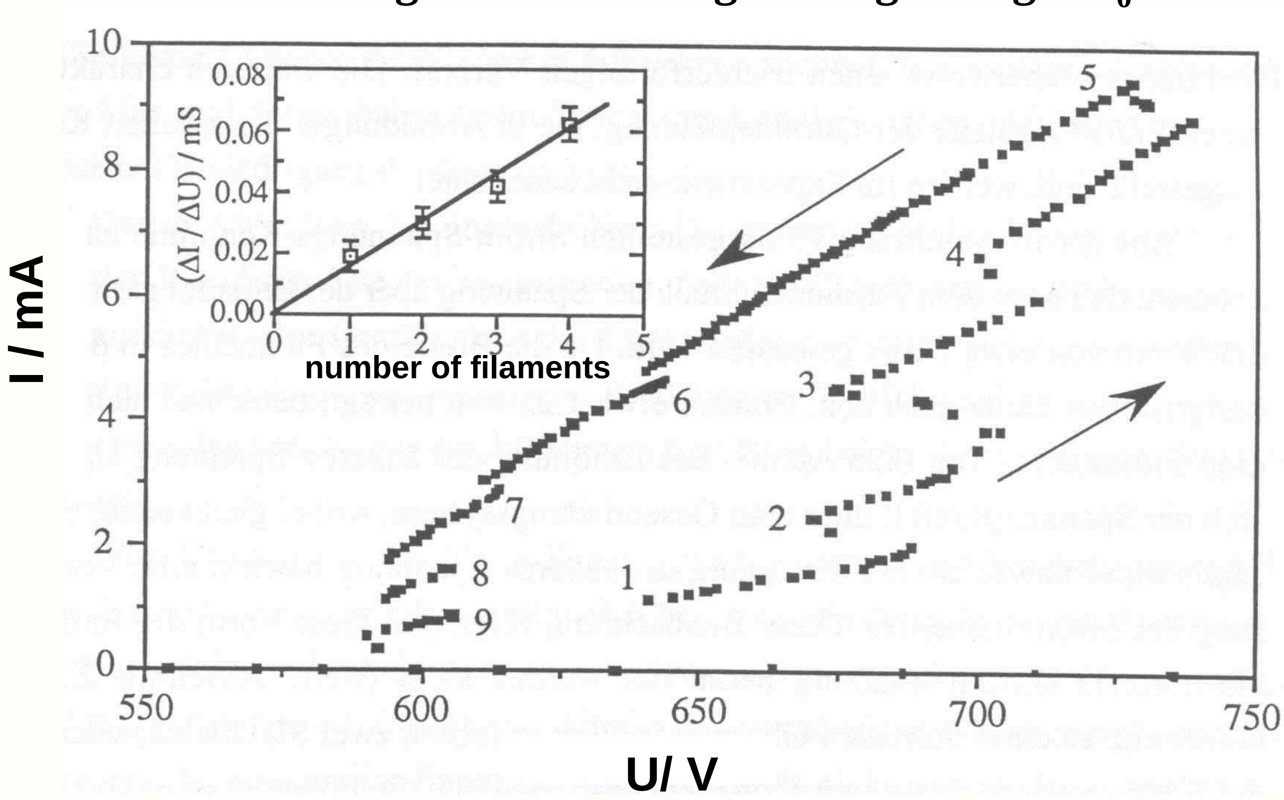
position in the discharge gap →

cathode: metal, anode: Si, ρ=1.5kΩ cm, gas: Ar, p=170hPa, d=3.75 mm, R_0=164kΩ, U_0=640->700->590V

Radehaus et al., Z. Phys. B-Condensed Matter 65, p. 515 (1987); Willebrand, PhD Thesis, Universität Münster (1992); Bode, Purwins, Physica D 86, p. 53 (1995)

Quasi-1-Dimensional DC Gas-Discharge System IV: Current-Voltage Characteristic for Increasing and Decreasing Number of Stationary Current Filaments (C)

(total current)-(voltage drop at the device) characteristic for increasing and decreasing driving voltage U_0



Semiconductor: Si ρ=1kΩ cm l_{gap}=38mm d_{gap}=3.75mm

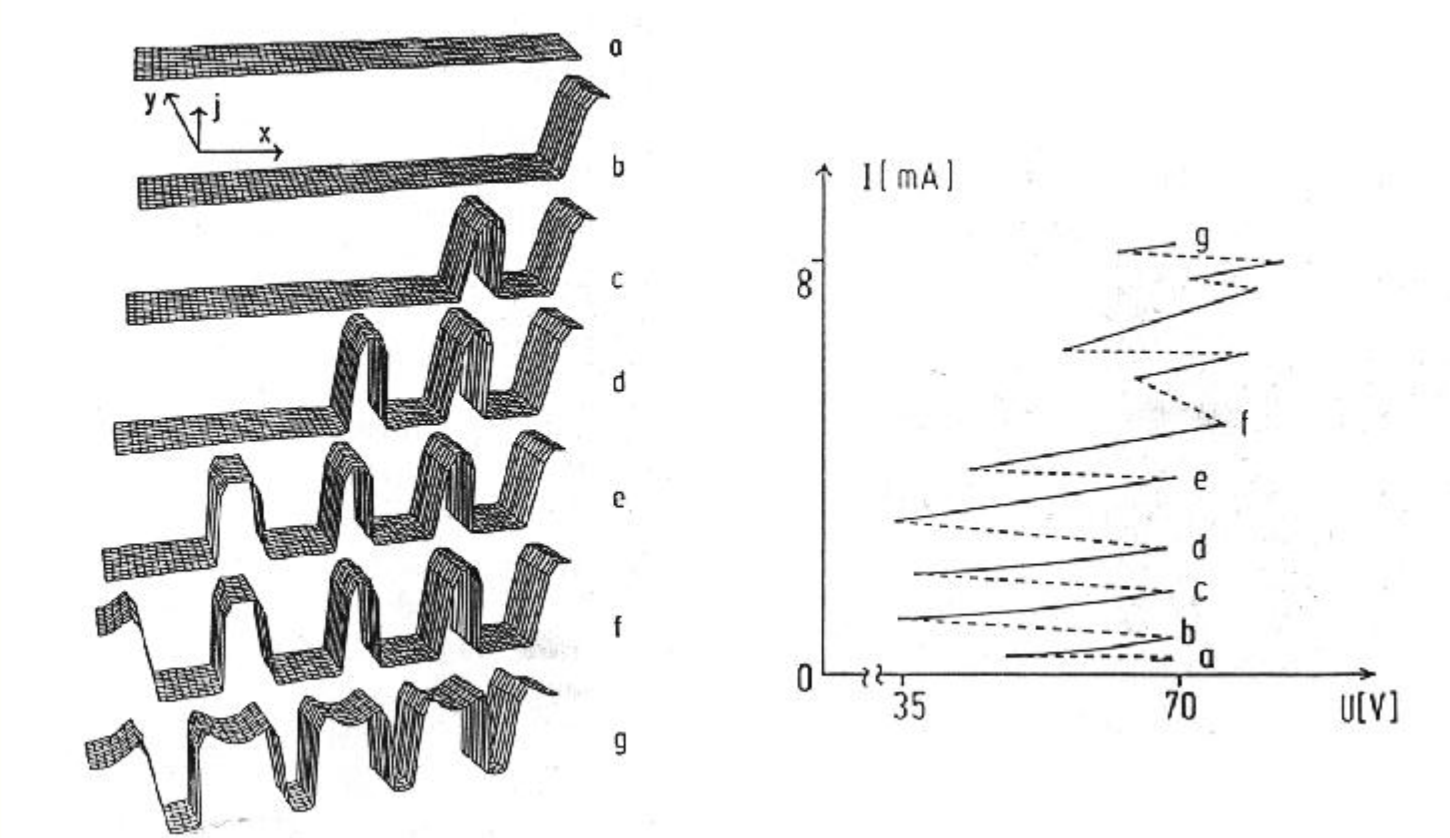
d_{SC}=0.3mm h_{SC}=10mm gas: 170hPa Ar + 20hPa air

Willebrand et al. Phys. Lett. A 149, p. 131 (1990); Willebrand, PhD Thesis, Universität Münster (1992); Purwins et al., Advances in Physics, 59, p. 485 (2010)

Numerical Solution of the Network Equation in $\Re^1$: Increasing Number of Stationary DSs when Increasing U_0

A)

B)

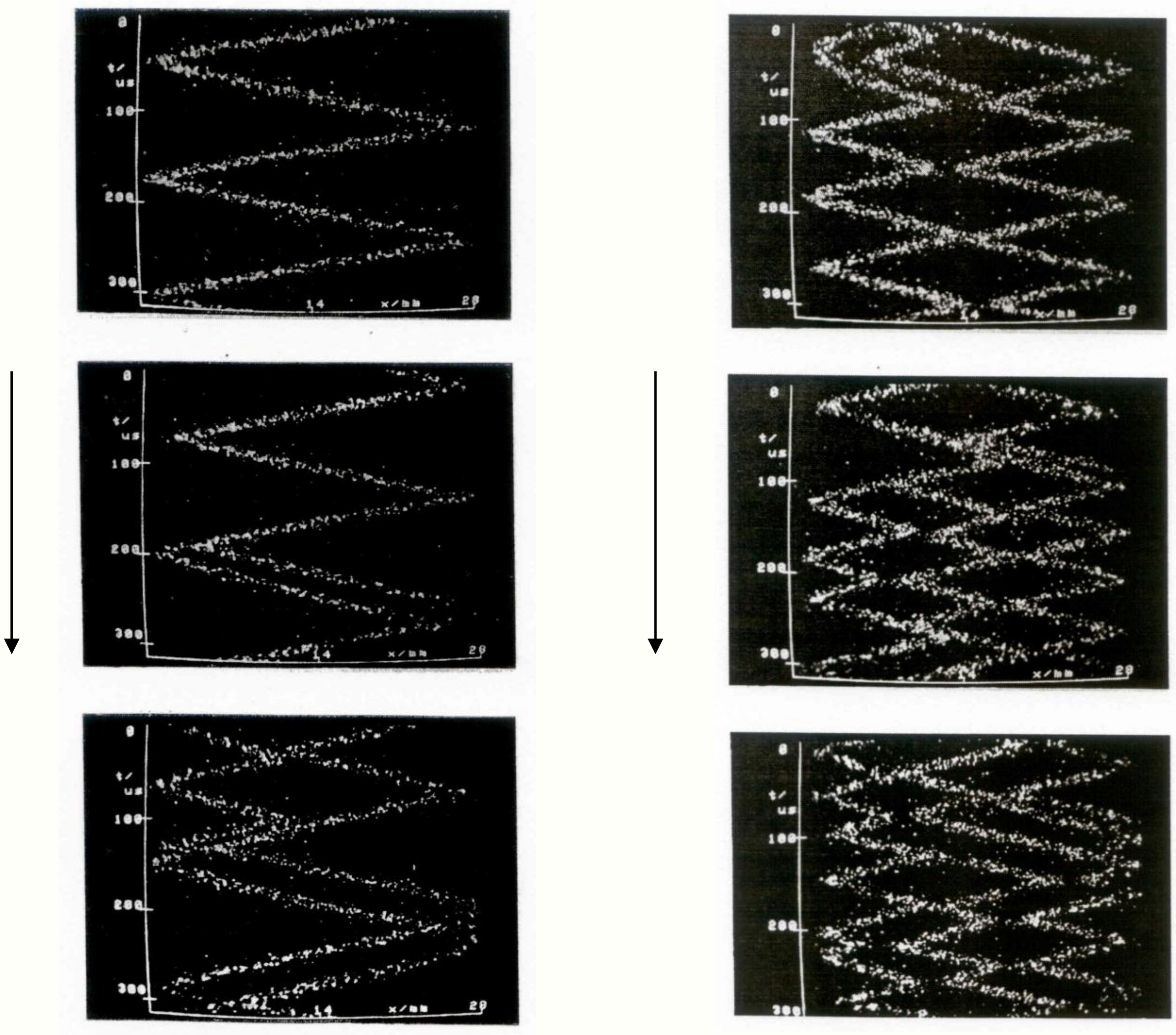


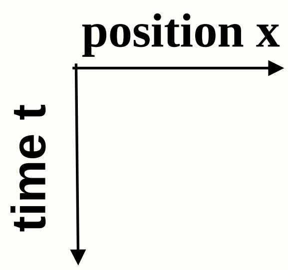
stationary patterns of the current density for U_0 increasing from a to g

(total current)-(voltage drop at the device) characteristic for increasing U_0

Purwins et al., Z. Naturforsch. 43a, p. 17 (1988);
see also: http://www.uni-muenster.de/Physik.AP/Purwins/Research-Summary

Quasi-1-Dimensional DC Gas-Discharge System: Cascade of Traveling Current Filaments

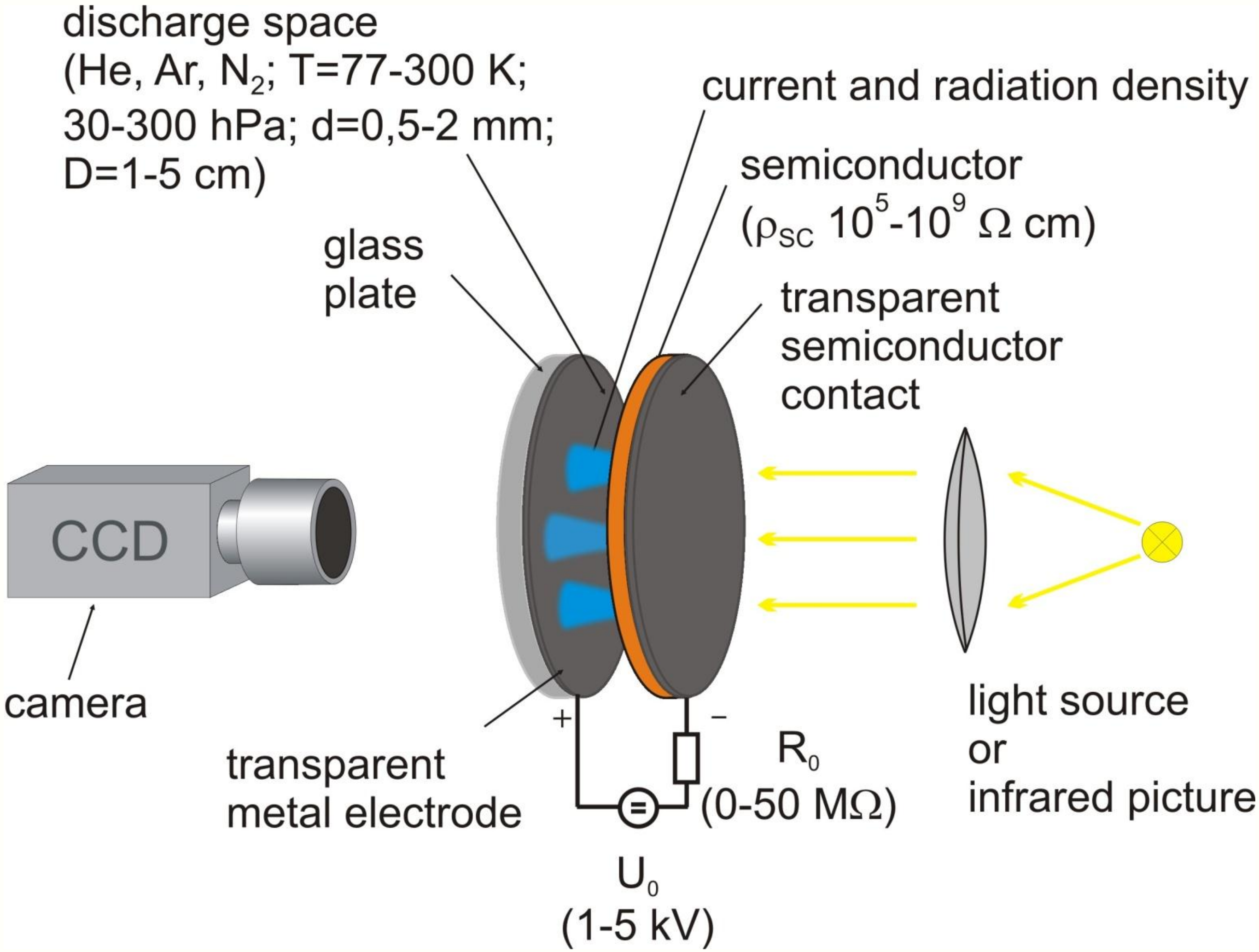




luminescence radiation distribution in the discharge space for increasing driving voltage U_0

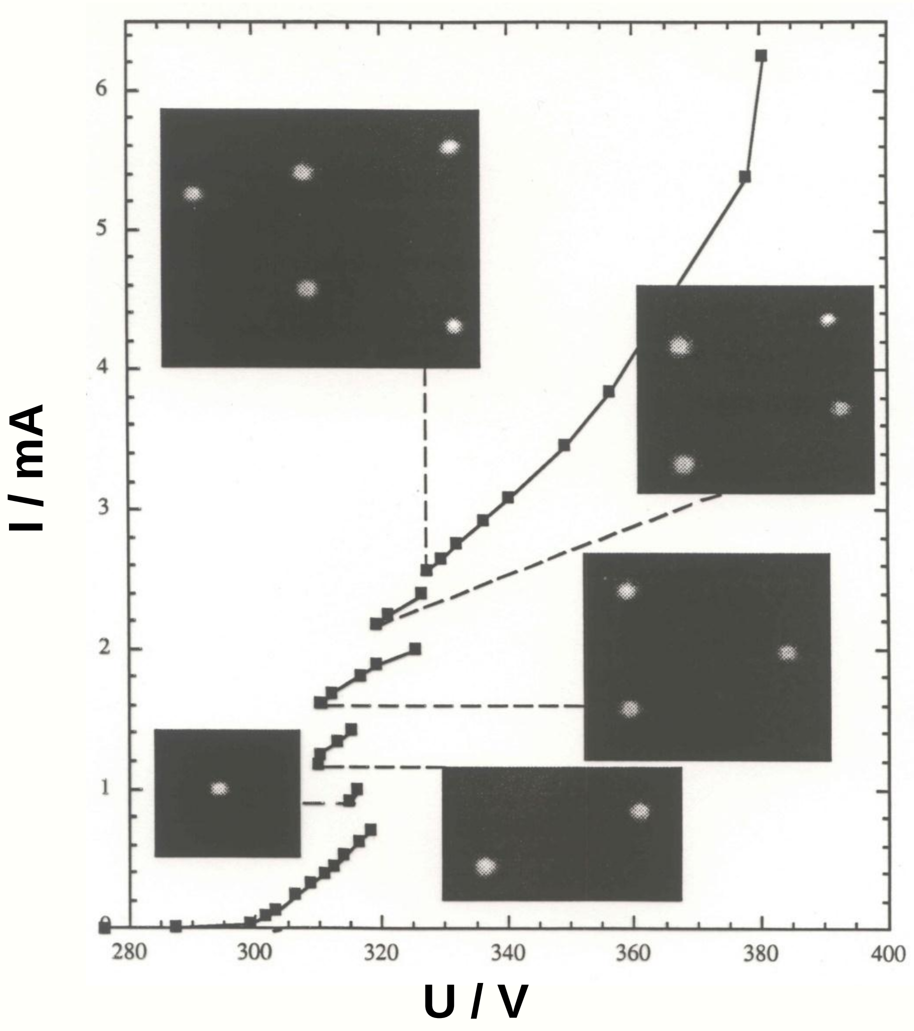
Willebrand et al., Phys. Rev. A 45, p. 8766 (1992)

Quasi 2-Dimensional DC Gas-Discharge System: Experimental Set-Up



Willebrand et al., J. Phys. D Appl. Phys. 27, p. 2354 (1994);
Ammelt, PhD Thesis, Universität Münster (1995)

Quasi 2-Dimensional DC Gas-Discharge System: Cascade with Increasing Number of Stationary Current Filaments when Increasing U_0



the driving voltage U_0 is increased

I = total current

U = voltage drop at the device

anode: Si compensated with Au

ρ_{SC}=150 kΩ cm, a_{SC}=0.5mm, T_{SC}=4°C,

cathode: optically transparent ITO,

gas: He, p=300hPa, d=1.5mm,

l_x=l_y=11mm, R_0=86kΩ, t_{exp}=20ms

inserted images:
luminescence radiation distribution in the discharge space

Becker, Diplomarbeit, Universität Münster (1994); Ammelt, PhD Thesis, Universität Münster (1995);
Purwins, AIP Conference Proceedings, Vol. 993. p. 67, Melville, New York, 2008

Quasi 2-Dimensional DC Gas-Discharge System: Travelling Isolated Current Filament (Movie)

luminescence radiation distribution in the discharge plane

1 cm

real time

if not linked: start movie “Filament.avi” in the folder

parameters: U_0=2,7 kV, ρ_{SC}=4,95 MΩ cm, R_0=20 MΩ, Gas: N_2, T=100 K, p=280 hPa, D=30 mm, d=250 µm, a_{SC}=1 mm, I=46 µA

Bödeker et al., Physical Review E, 67, p. 056220-1 (2003);
Liehr et al., New J. Phys., 5, p. 89.1 (2003)

The Generalized FN Equation I: The Equation

$$u_t = d_u^2 \Delta u + f(u) - \kappa_3 v - \kappa_4 w + \kappa_1,$$

$$\tau v_t = d_v^2 \Delta v + u - v,$$

$$\theta w_t = d_w^2 \Delta w + u - w,$$

$$u = u(x, y, z, t),$$

$$v = v(x, y, z, t),$$

$$w = w(x, y, z, t),$$

$$f(u) = \lambda u - u^3$$

$$d_u, d_v, d_w, \tau, \theta, \kappa_3, \kappa_4 \geq 0, \lambda > 1$$

Woesler et al., Physica D 91, p. 376 (1996); Schenk et al., Phys. Rev. Lett. 78, p. 3781 (1997); Purwins et al., Advances in Physics, 59, p. 485 (2010)

The Generalized FN Equation II: Physical Meaning of Dependent Variables and Parameters in the Case of Gas-Discharge

u current density

v voltage drop at the high ohmic layer

w surface charge? temperature?

τ dielectric relaxaction time normalized to reaction time of charge carriers

θ relacation time normalized to reaction time of charge carriers

d_u ambipolar diffusion length

d_v electrical pseudo diffusion length

λ parameter describingthecurrendensitsvoltragecharacteristic

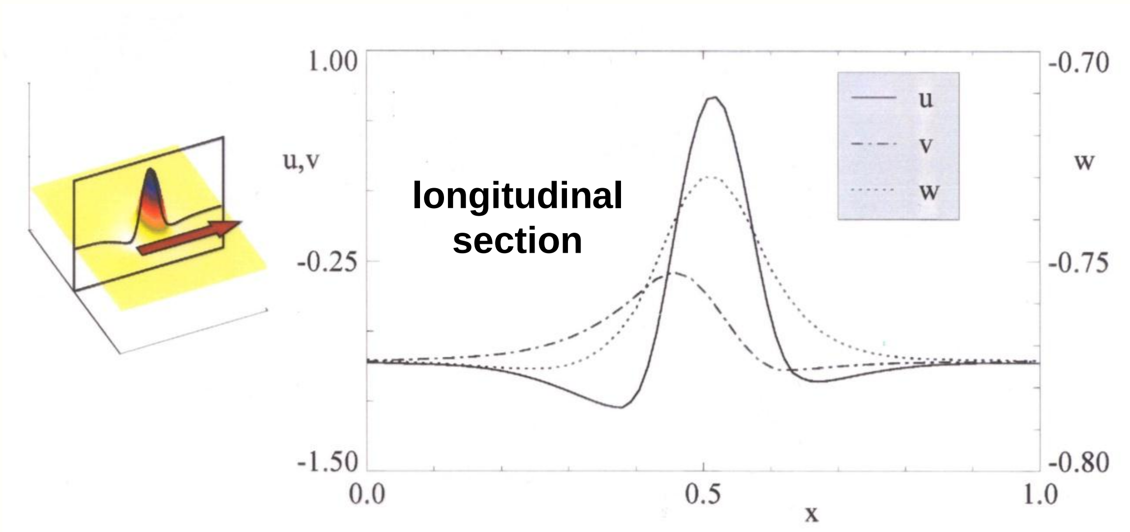
d_w diffusion length u

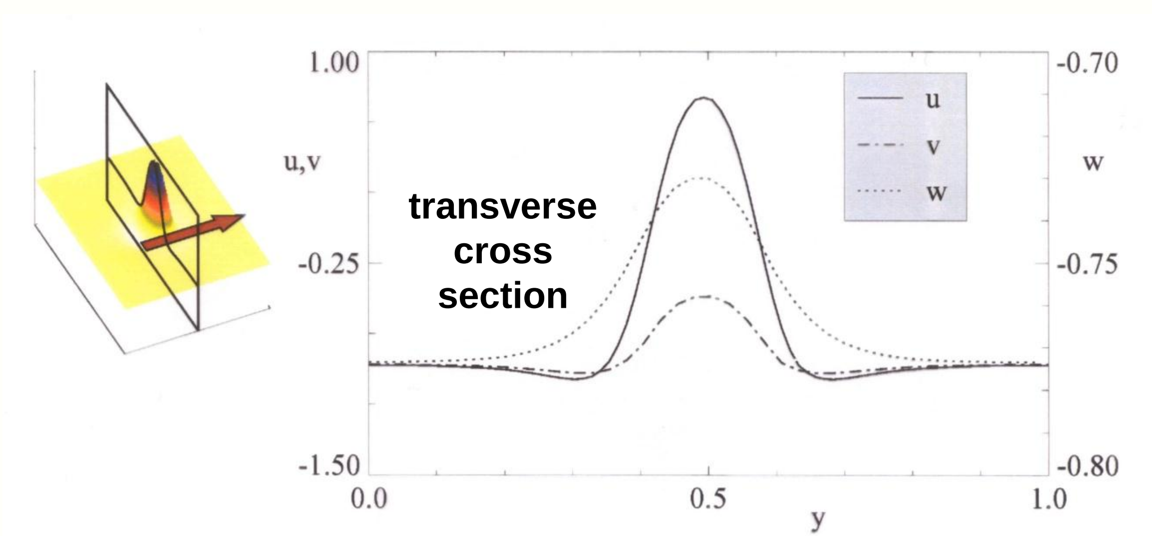
κ_1 driving voltage

κ_3, κ_4 physical origin not well known

Woesler et al., Physica D 91, p. 376 (1996); Schenk et al., Phys. Rev. Lett. 78, p. 3781 (1997); Purwins et al., Advances in Physics, 59, p. 485 (2010)

Numerical Solution for the Generalized FH Equation in $\Re^2$: Isolated Travelling DS





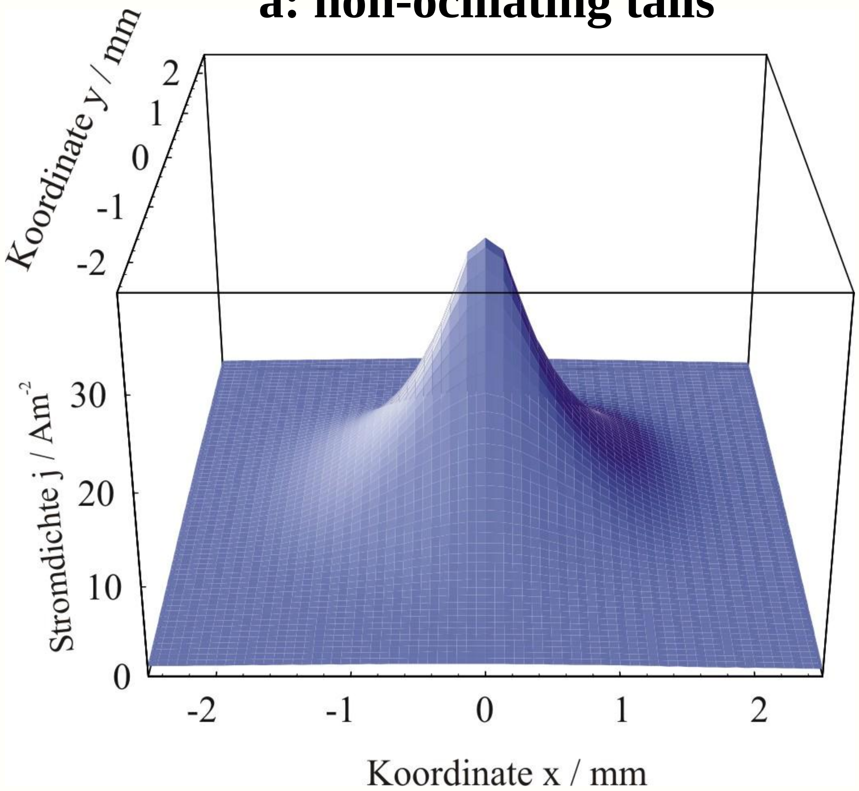
parameters: $D_u=10^{-3}$, $D_v=1.25*10^{-3}$, $D_w=6.4*10^{-2}$, $\kappa_1=-6.92$, $\kappa_3=1$, $\kappa_4=8.5$, $\lambda=2$, $\tau=25$, $\theta=1$, $\Delta x=1/38$

Schenk et al., Phys. Rev. Lett. 78, p. 3781 (1997)

4. Additional Results on Dissipative Solitons (DSs) in DC Gas-Discharge Systems and Comparison with Solutions of the Generalized FitzHugh-Nagumo (FN) Equation

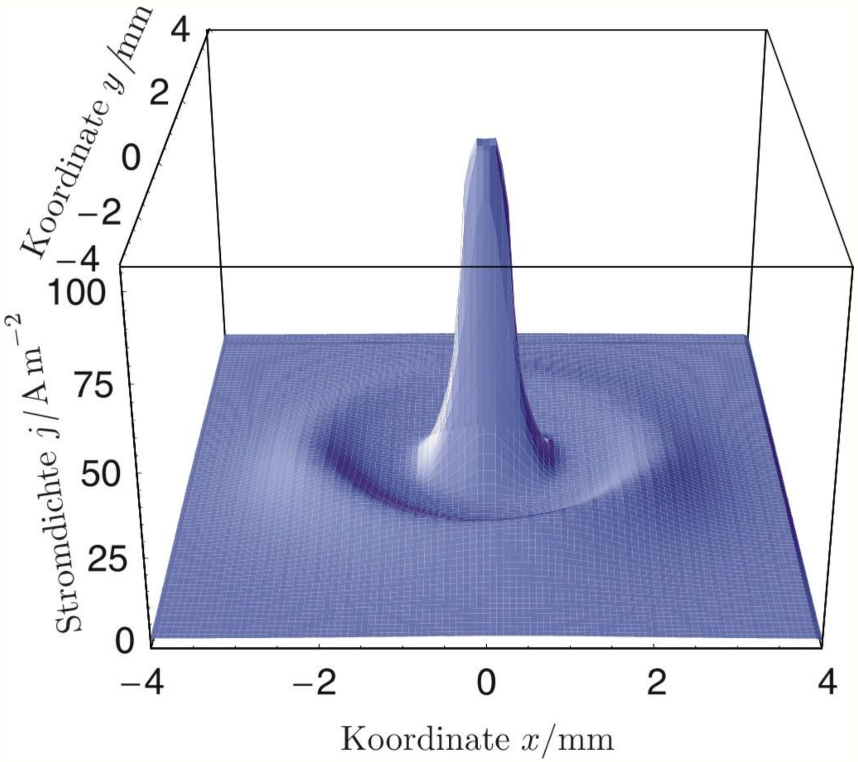
Experimental Quasi 2-Dimensional DC Gas-Discharge System: Current Filaments with Non-Oscillating and Oscillating Tails

a: non-ocillating tails



parameters: U_0 = 2,74 kV, ρ_{SC} = 4,95 MΩ cm, R_0 = 20 MΩ, Gas: N_2, T=100 K, p = 280 hPa, D=30 mm, d = 250 µm, a_{SC}=1 mm, I = 46 µA, t_{exp}=20 ms

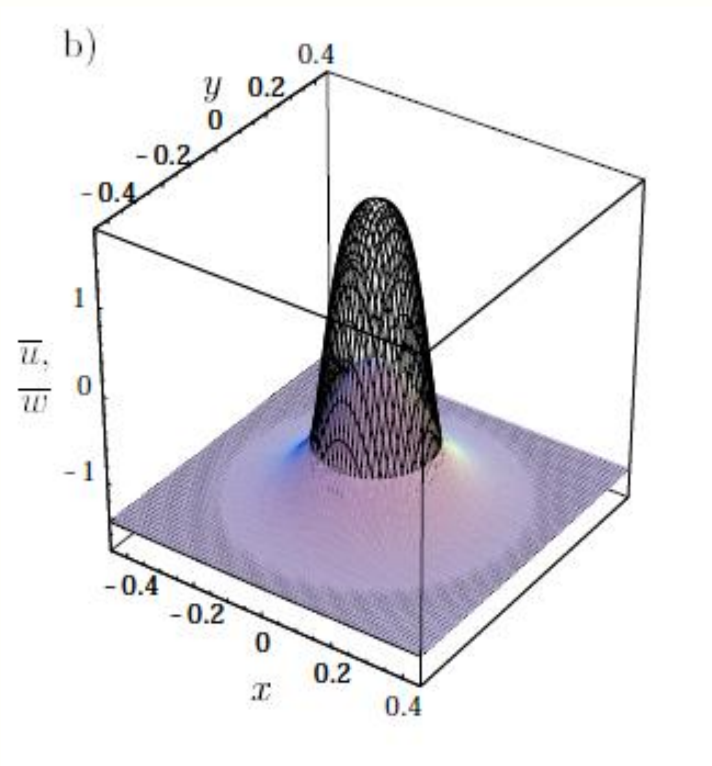
b: oscillating tails



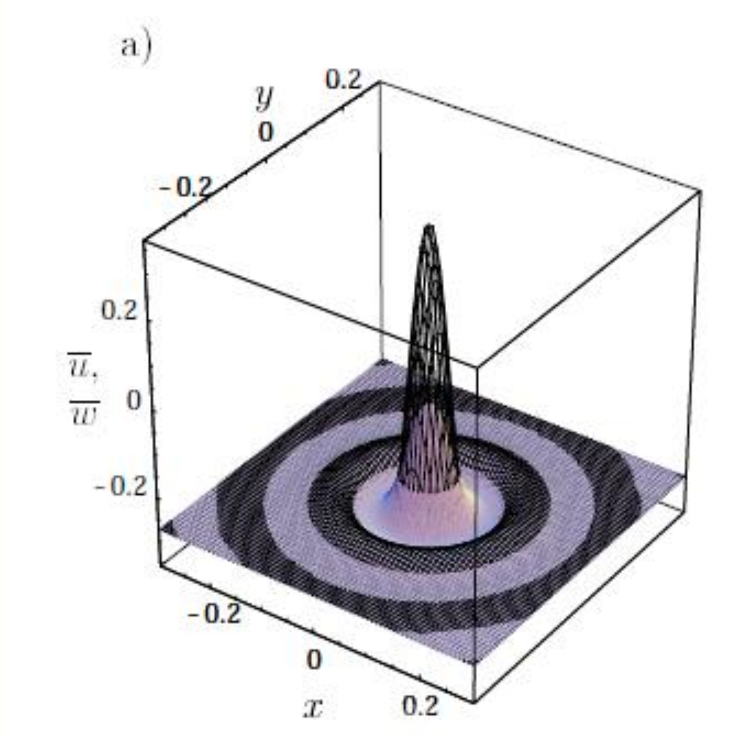
parameters: U_0 = 3,6 kV,ρ_{SC} = 3,05 MΩ cm, R_0 = 4,4 MΩ, Gas: N_2, T=100 K, p = 279 hPa, D=30 mm, d = 500 µm, a_{SC}=1 mm, I = 200 µA, t_{exp}=20 ms

Bödeker et al., New J. Phys., 6, p. 62 (2004) (b);
Purwins et al. 2005 in: *Dissipative Solitons*, Springer Lectures Notes in Physics 661, p. 267 (2005) (a, b)

Numerical Solution for the Generalized FH Equation in $\Re^2$: Stationary DSs with Non-Oscillating and Oscillating Tails I



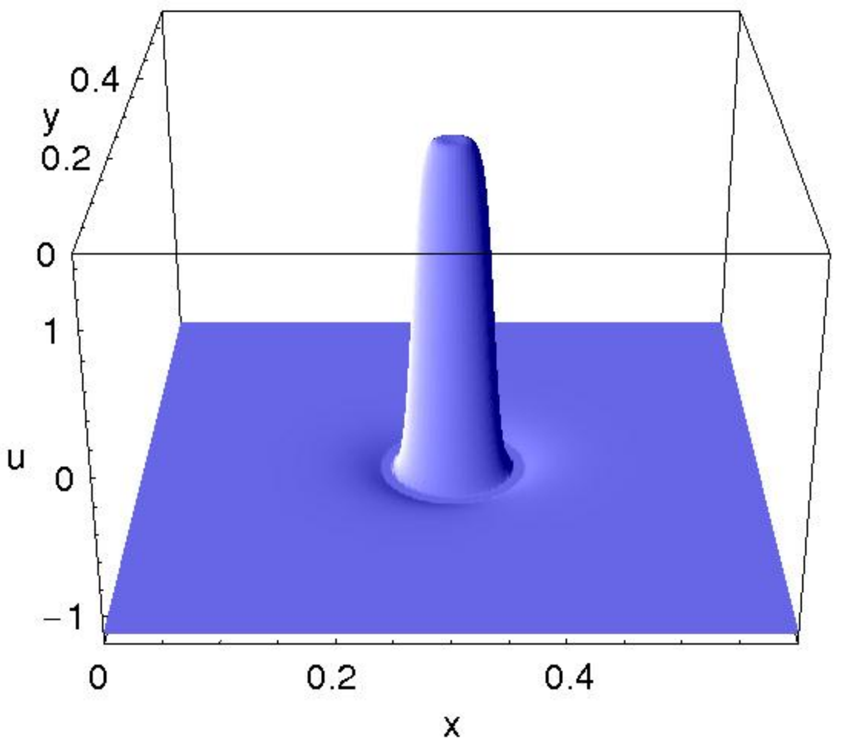
a: non-ocillating tails



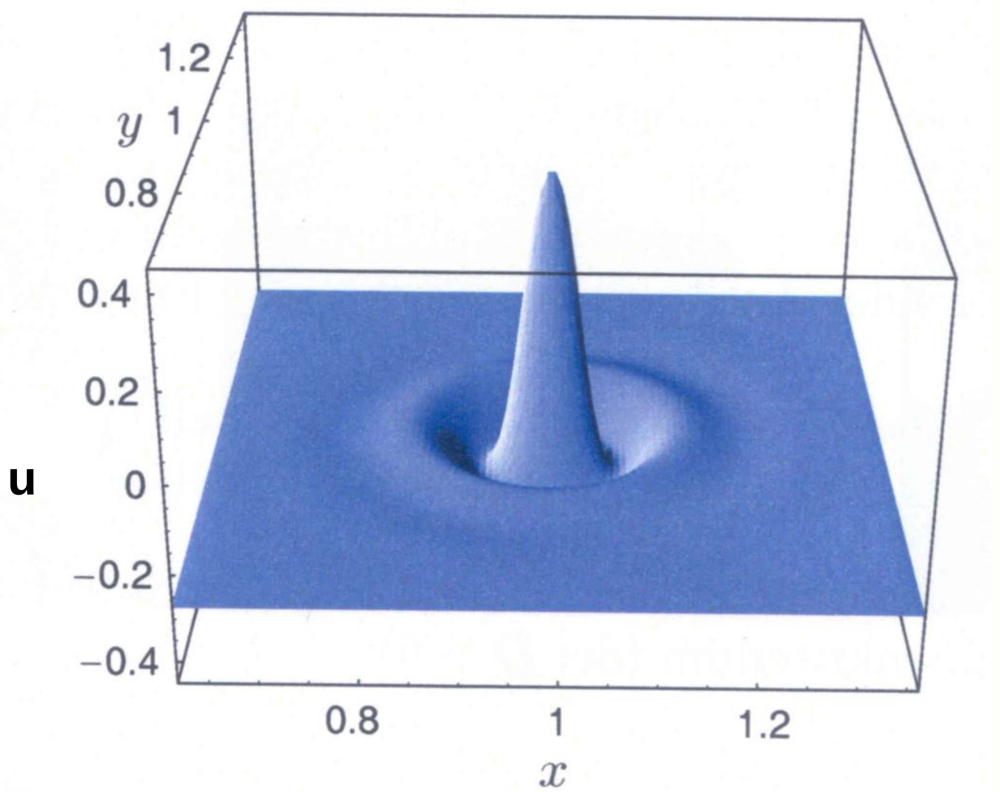
b: oscillating tails

Moskalenko, Diplomarbeit, Universität Münster (2002) Abb. 1.5

Numerical Solution for the Generalized FH Equation in $\Re^2$: Stationary DSs with Non-Oscillating and Oscillating Tails I



a: non-ocillating tails

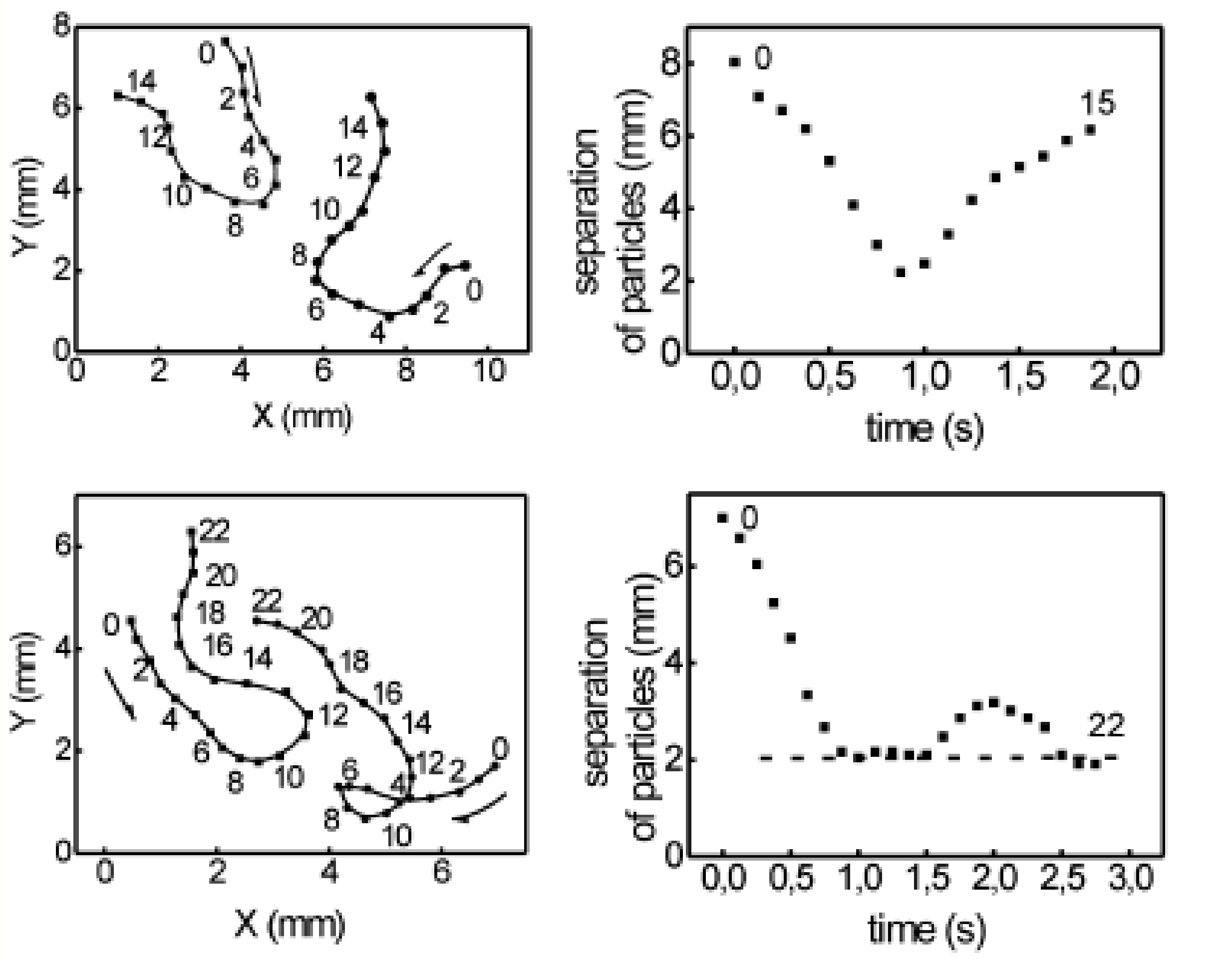


b: oscillating tails

Bödeker et al., New J. Phys., 6, p. 62 (2004) (b);
Purwins et al. 2005 in: *Dissipative Solitons*, Springer Lectures Notes in Physics 661, p. 267 (2005) (a, b)

Experimental Quasi 2-Dimensional DC Gas-Discharge System: Examples for Travelling, Scattering and Cluster Formation of Filaments (“Molecules”)

trajectories | **distance**

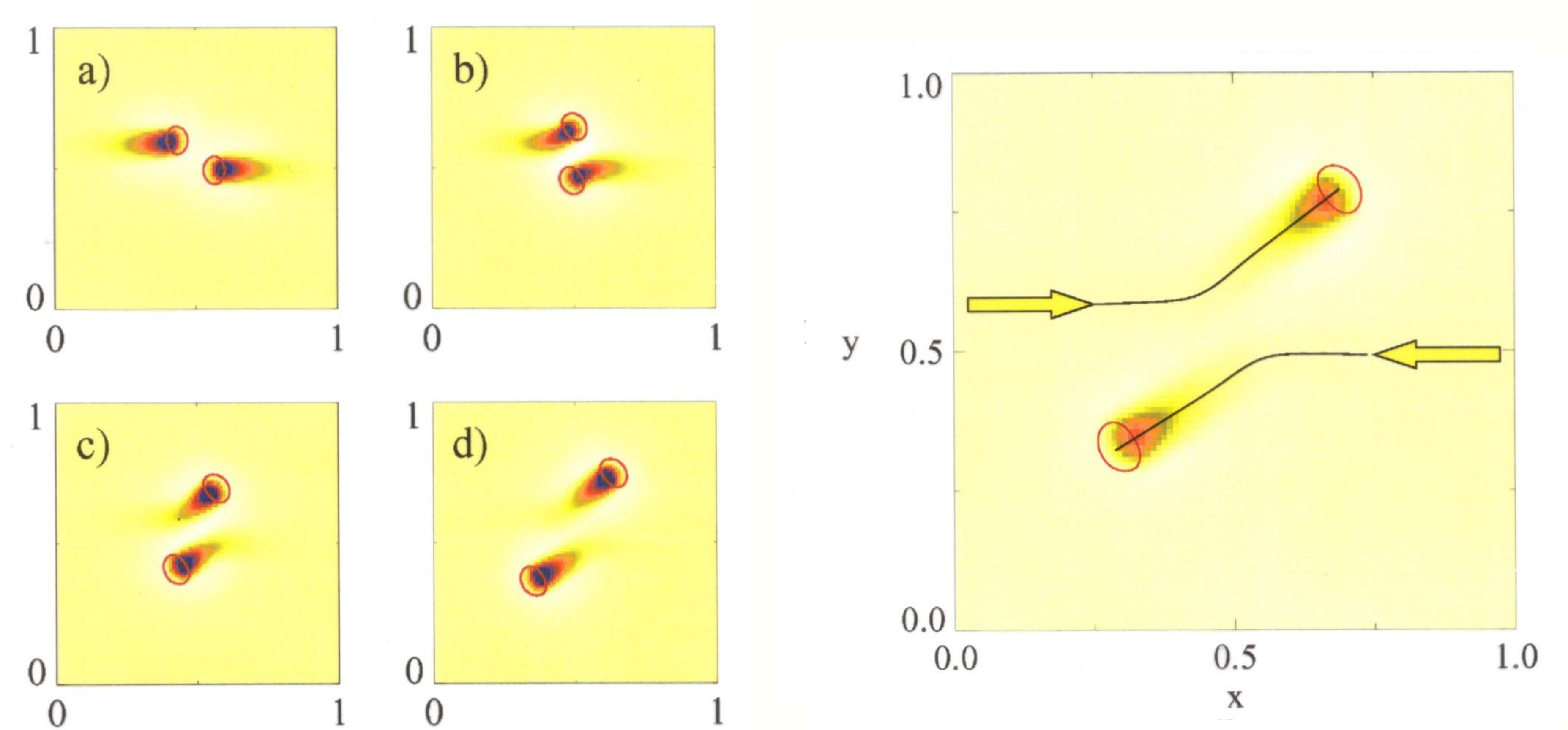


scattering

cluster formation

Astrov et al., Phys.Lett.A, 283, p. 349 (2001)

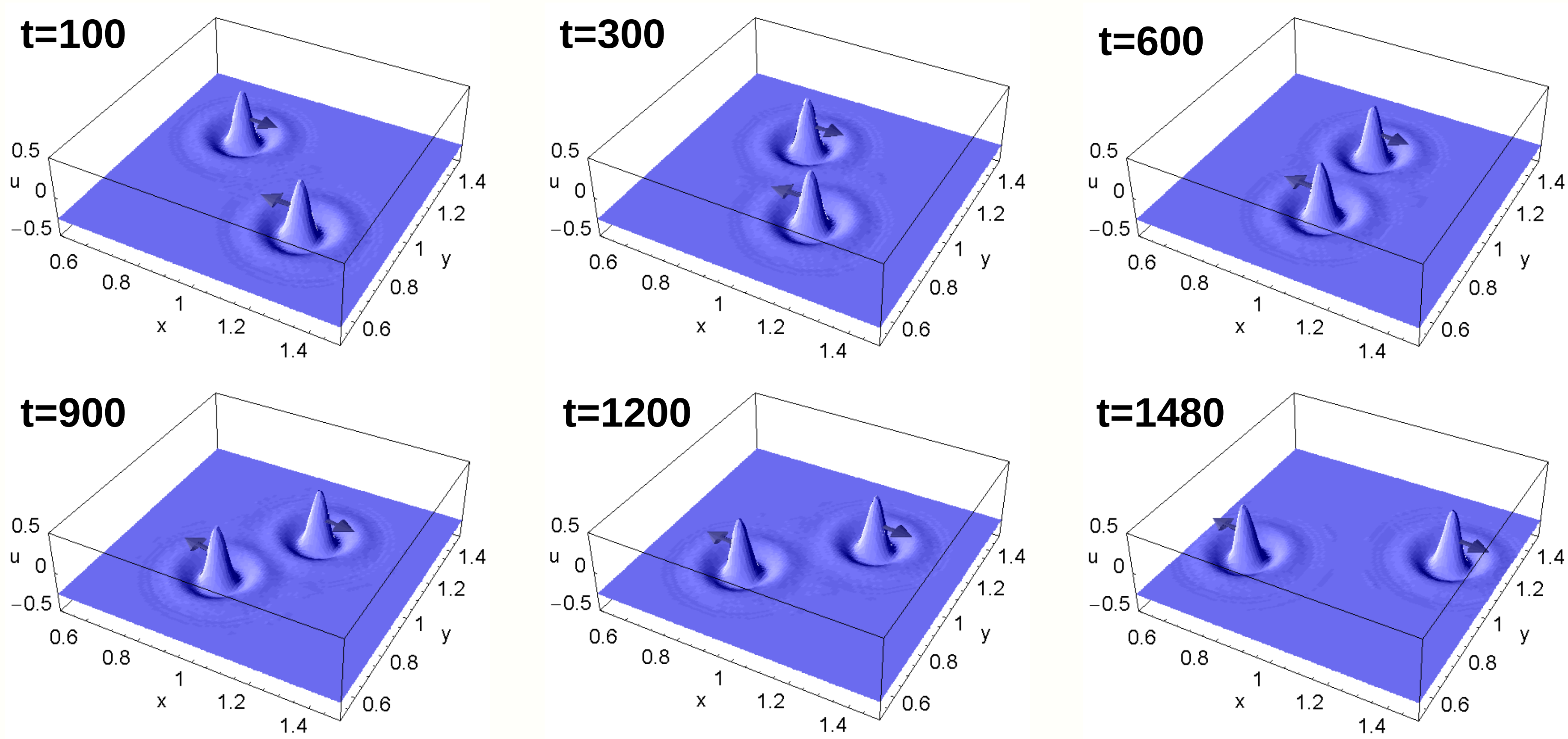
Numerical Solution for the Generalized FH Equation in $\Re^2$: Scattering of 2 DSs (Weak Interaction I)



parameters: D_u=1.55*10^{-4}, D_v=1.95*10^{-4}, D_w=0.05, κ_1=-8.715, κ_3=1, κ_4=8.44, λ=2, τ=48, θ=0.5, Δx=1/100

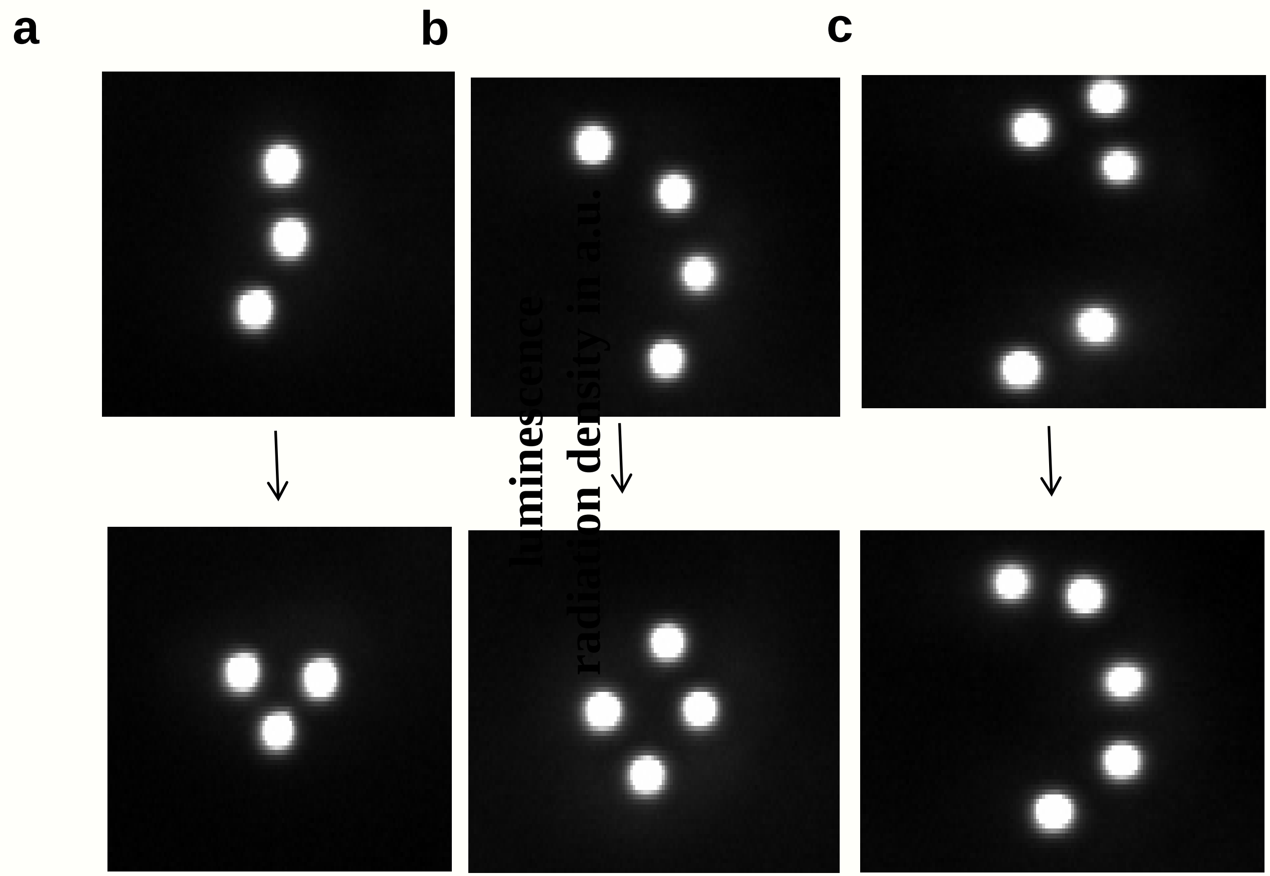
Schenk et al., Phys. Rev. Lett. 78, p. 3781 (1997)

Numerical Solution for the Generalized FH Equation in $\Re^2$: Scattering of 2 DSs (Weak Interaction II)



τ=3.35, θ=0, D_u=1.1*10^{-4}, D_v=0, D_w=9.64*10^{-4}, λ=1.01, κ_1=-0.1, κ_3=0.3, κ_4=1.0
Ω=[0,1]x[0,1], Δx=5*10^{-3}, Δt=0.1 .

Experimental Quasi 2-Dimensional DC Gas-Discharge System: Formation of Clusters of Current Filaments (“Molecules)

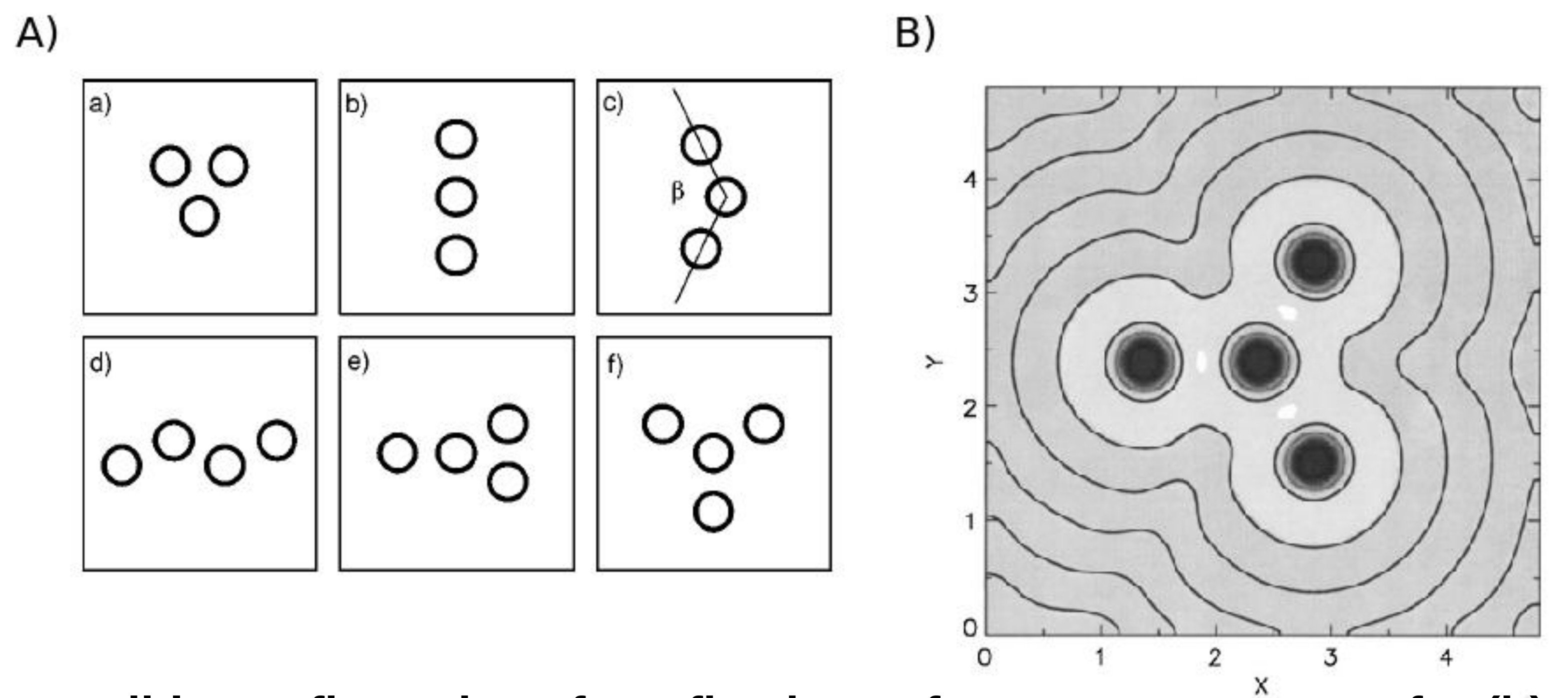


luminescence radiation distribution in the discharge plane

for supply voltage increasing from a to c clusters are observed with increasing number of filaments; for a given set of parameters the number of filaments is retained the but their configuration may change in time

Bödeker. Diplomarbeit, Universität Münster (2003); Bödeker et al. in “Anomalous Fluctuation Phenomena in Complex Systems”, *Ed: C. Riccardi, H. E. Roman* Research Signpost, Kerala, India

Numerical Solution for the Generalized FH Equation in $\Re^2$: Formation of Clusters of DSs (“Molecules”) I



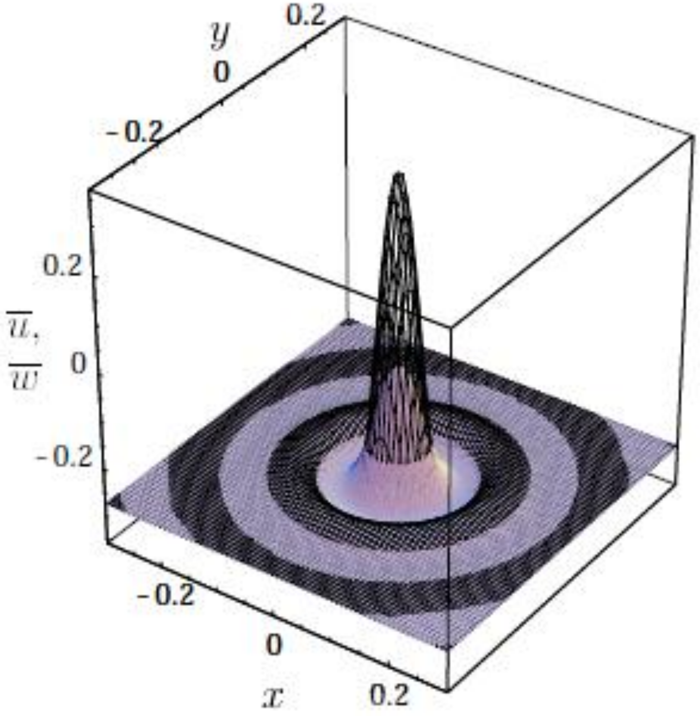
A: possible configurations for a fixed set of parameters; except for (b) the solutions are stable

B: u distribution for the case (f) in (A) reflecting the oscillatory behavior in the surrounding of individual DSs and related lock-in distances

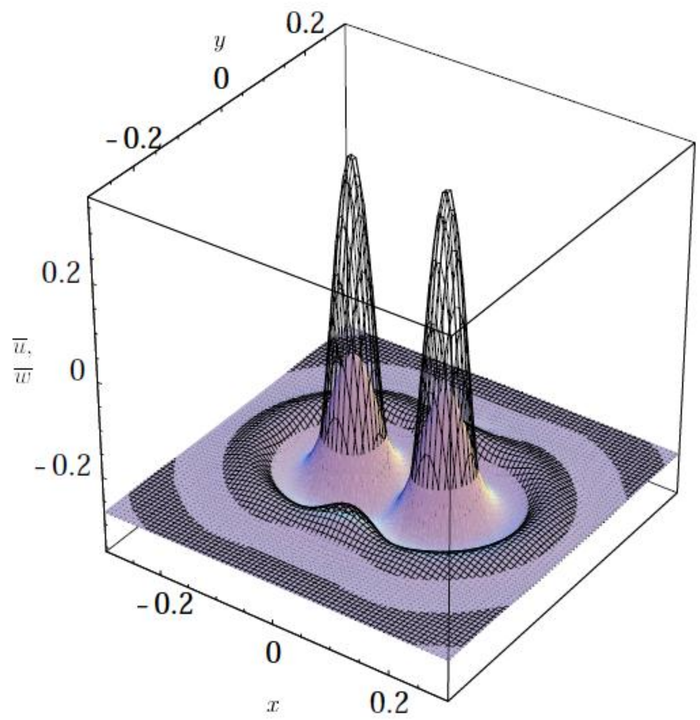
Schenk et al., Phys. Rev. E 57, 6480-6486 (1998)

Numerical Solution for the Generalized FH Equation in $\Re^2$: Formation of Clusters of DSs ("Molecules") II

a: single DS



b: 2-DS-molecule

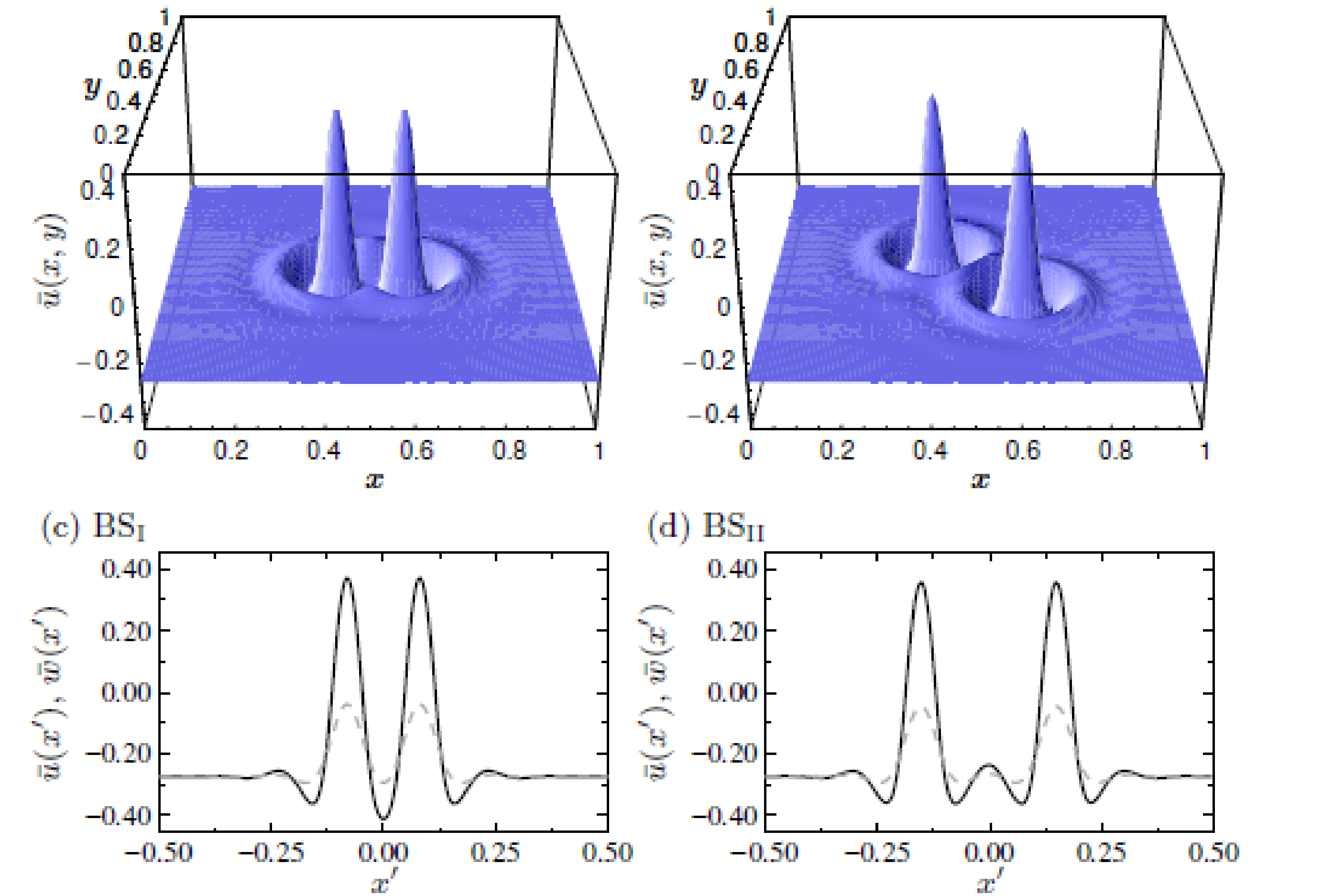


same parameters

Moskalenko, Diplomarbeit, Universität Münster (2002) Abb. 1.5/2.3; oder: Moskalenko et al. Europhysics Letters, 63, 361 (2003) Fig. 1

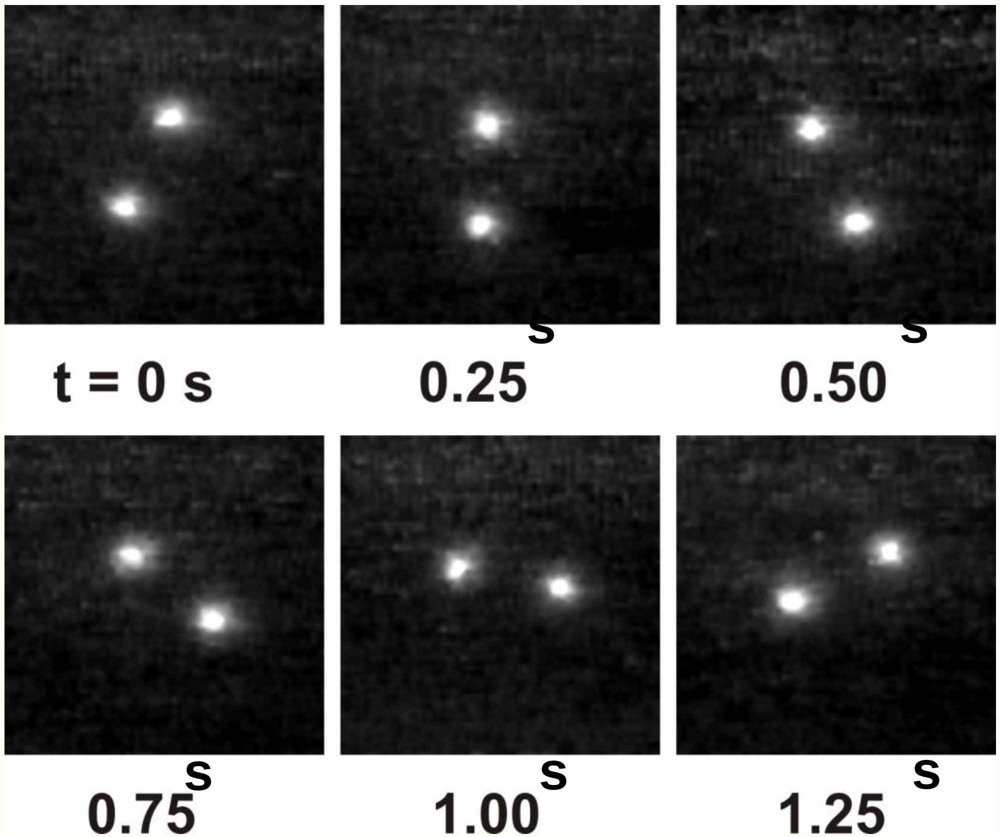
Numerical Solution for the Generalized FH Equation in $\Re^2$: Formation of Clusters of DSs ("Molecules") III

stable stationary clusters with different distance at a given set of parameters



Experimental Quasi 2-Dimensional DC Gas-Discharge System: Rotating Cluster of 2 Current Filaments („Molecule“)

luminescence radiation distribution in the discharge plane

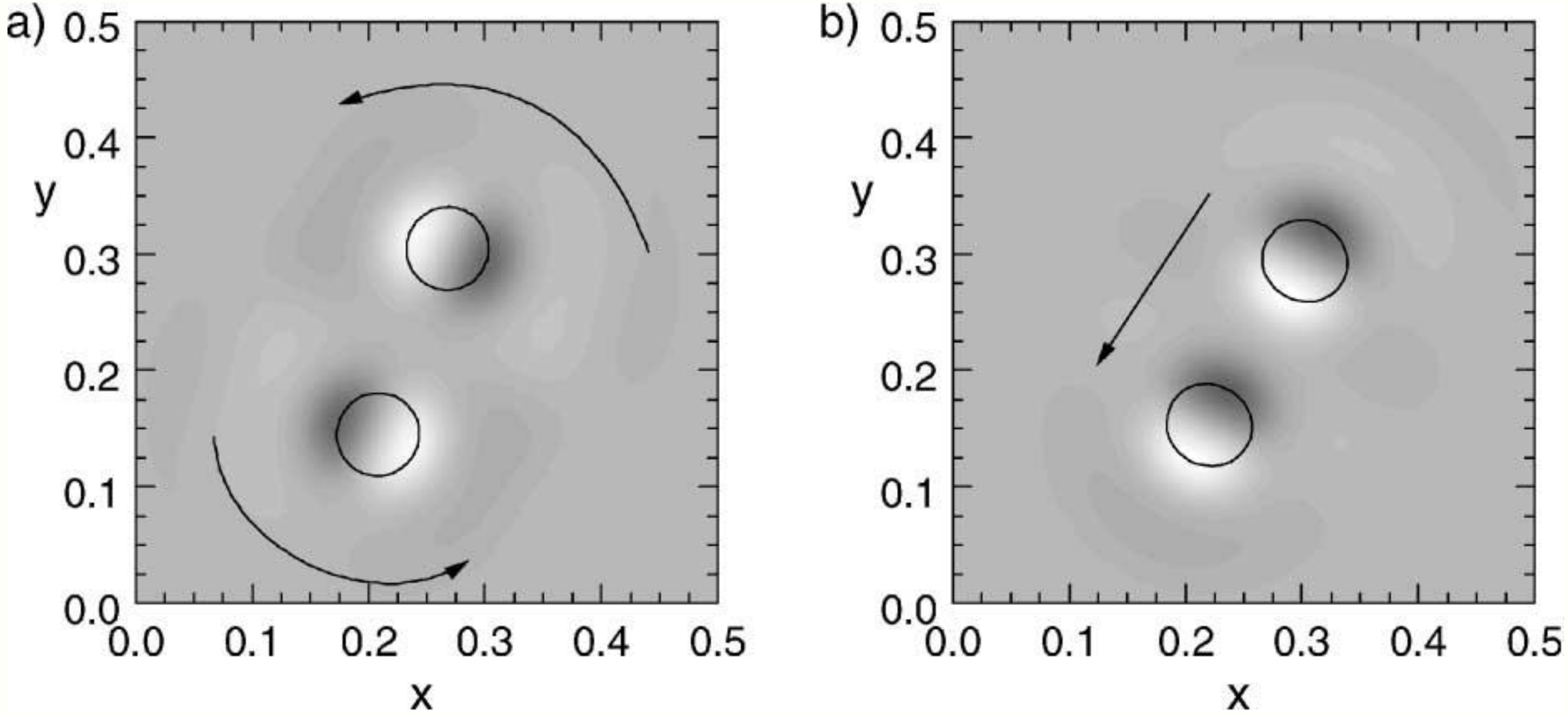


parameters: U_0 = 1.9kV, ρ_{SC} = 22.2MΩ cm, gas: N_2, T=90 K, p = 2hPa, d_{gap}=0.8 mm, d_{SC} = 1 mm, a_{SC}=20 mm, t_{exp}=20 ms

Liehr et al., Eur. Phys. J. B37, 199 (2004)

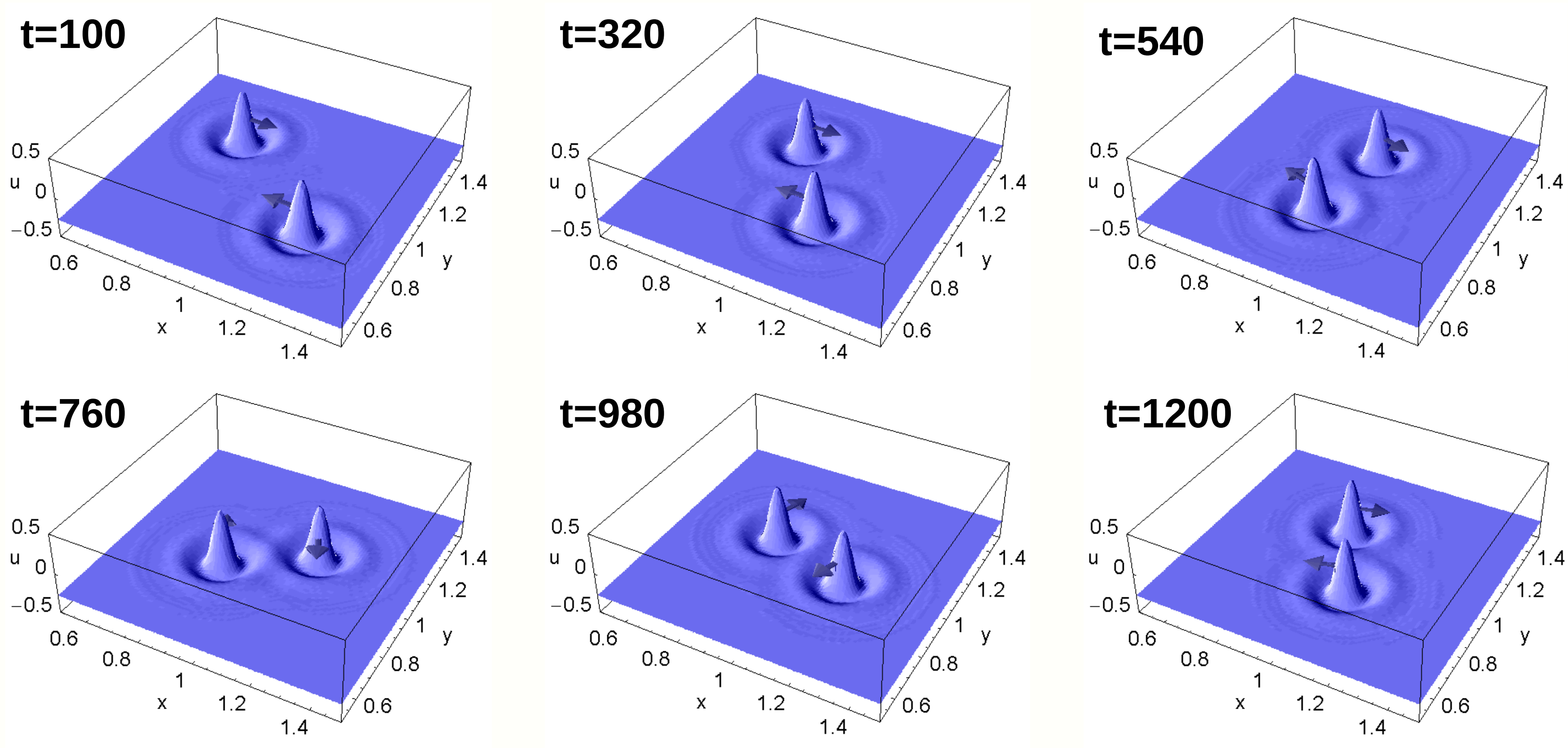
Numerical Solution for the Generalized FH Equation in $\Re^2$: Rotating and Travelling Cluster of DSs (“Molecules”)

depending on the details of the collision process clusters with different dynamics may occur for a given set of parameters



Bode et al., Physica D, 161, 45-66 (2002)

Numerical Solution for the Generalized FH Equation in $\Re^2$: Formation of a Rotating Cluster of 2 DSs (Molecule) in the Course of Collision

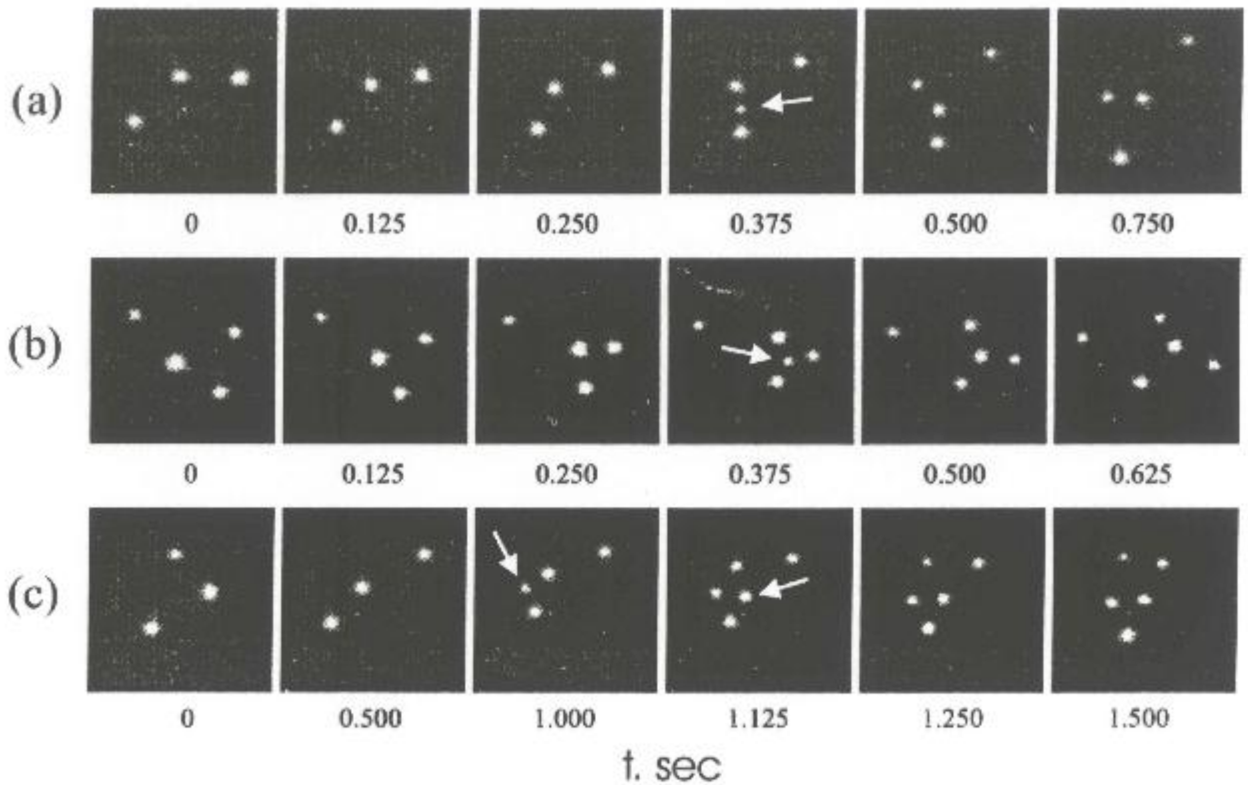


τ=3.35, θ=0, D_u=1.1*10^{-4}, D_v=0, D_w=9.64*10^{-4}, λ=1.01, κ_1=-0.1, κ_3=0.3, κ_4=1.0, Ω=[0,1]x[0,1], Δx=5*10^{-3}, Δt=0.1 .

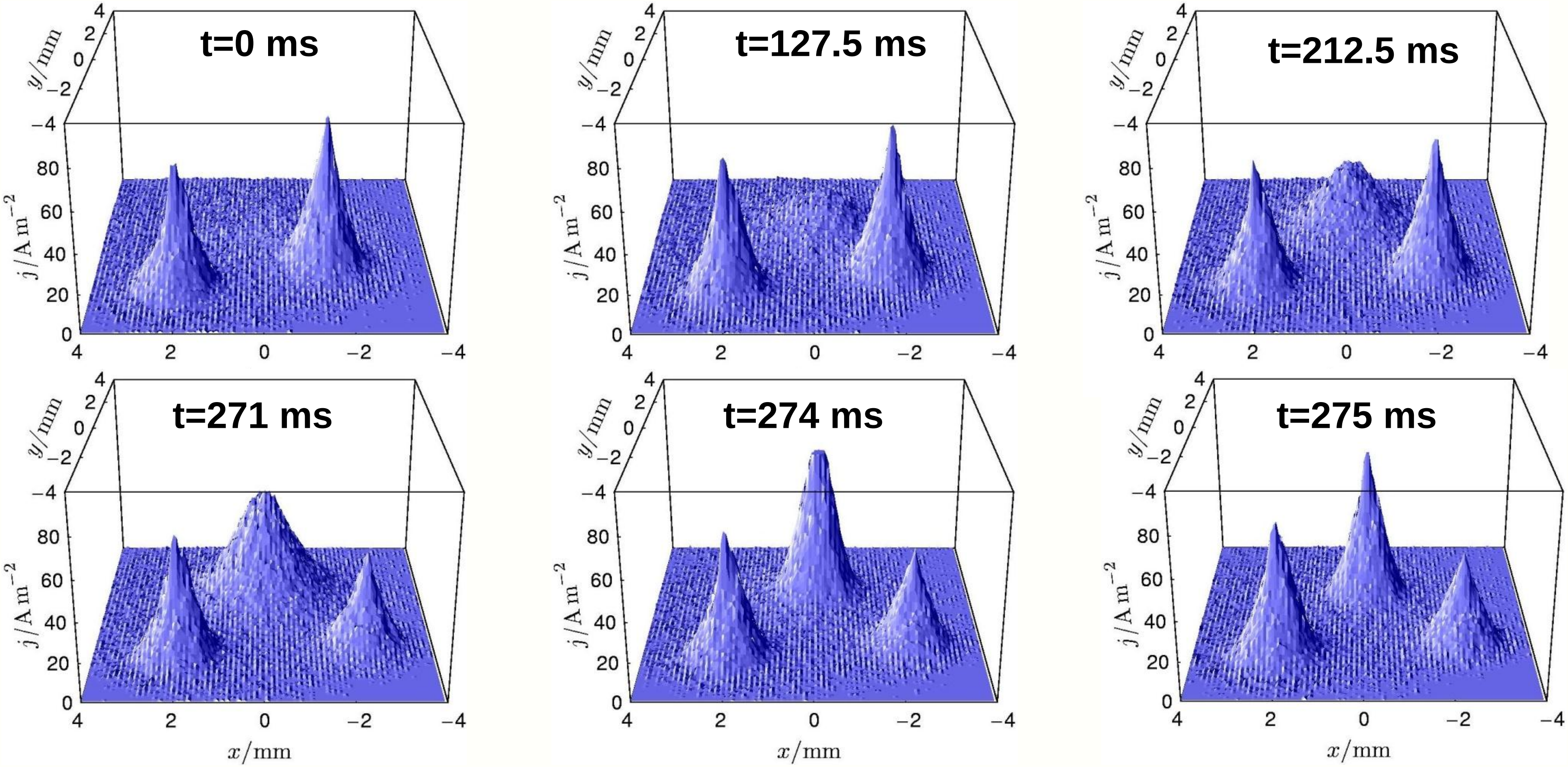
Purwins et al., in: *Dissipative Solitons*, Springer, Lectures Notes in Physics 661, p. 267 (2005)

Experimental Quasi 2-Dimensional DC Gas-Discharge System: Current Filament Generation in the Course of Collision I

luminescence radiation distribution in the discharge plane



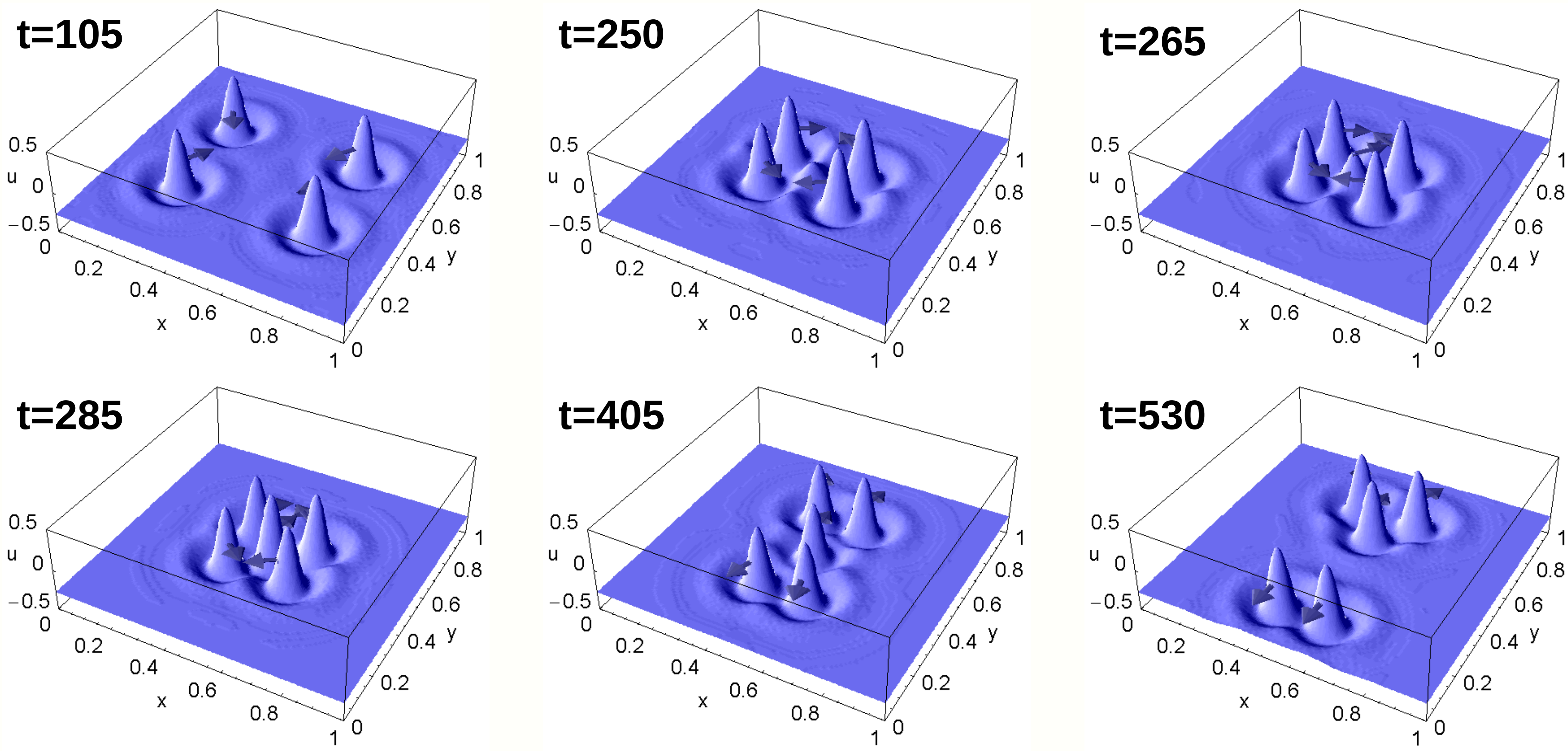
Experimental Quasi 2-Dimensional DC Gas-Discharge System: Current Filament Generation in the Course of Collision I



parameters: U_0=3,8 kV, ρ_{SC}=4,14 MΩ cm, R_0=20 MΩ, Gas: N_2, T=100 K, p=290 hPa, D=30 mm, d=500 μm, a_{SC}=1 mm, I=100-250 μA, t_{exp}=0,2 ms, f_{rep}=2 kHz

Bödeker, Diplomarbeit, Universität Münster (2003); Purwins, H.-G., H.U. Bödeker, and A.W. Liehr in: *Dissipative Solitons*, Springer, Lectures Notes in Physics 661, p. 267 (2005)

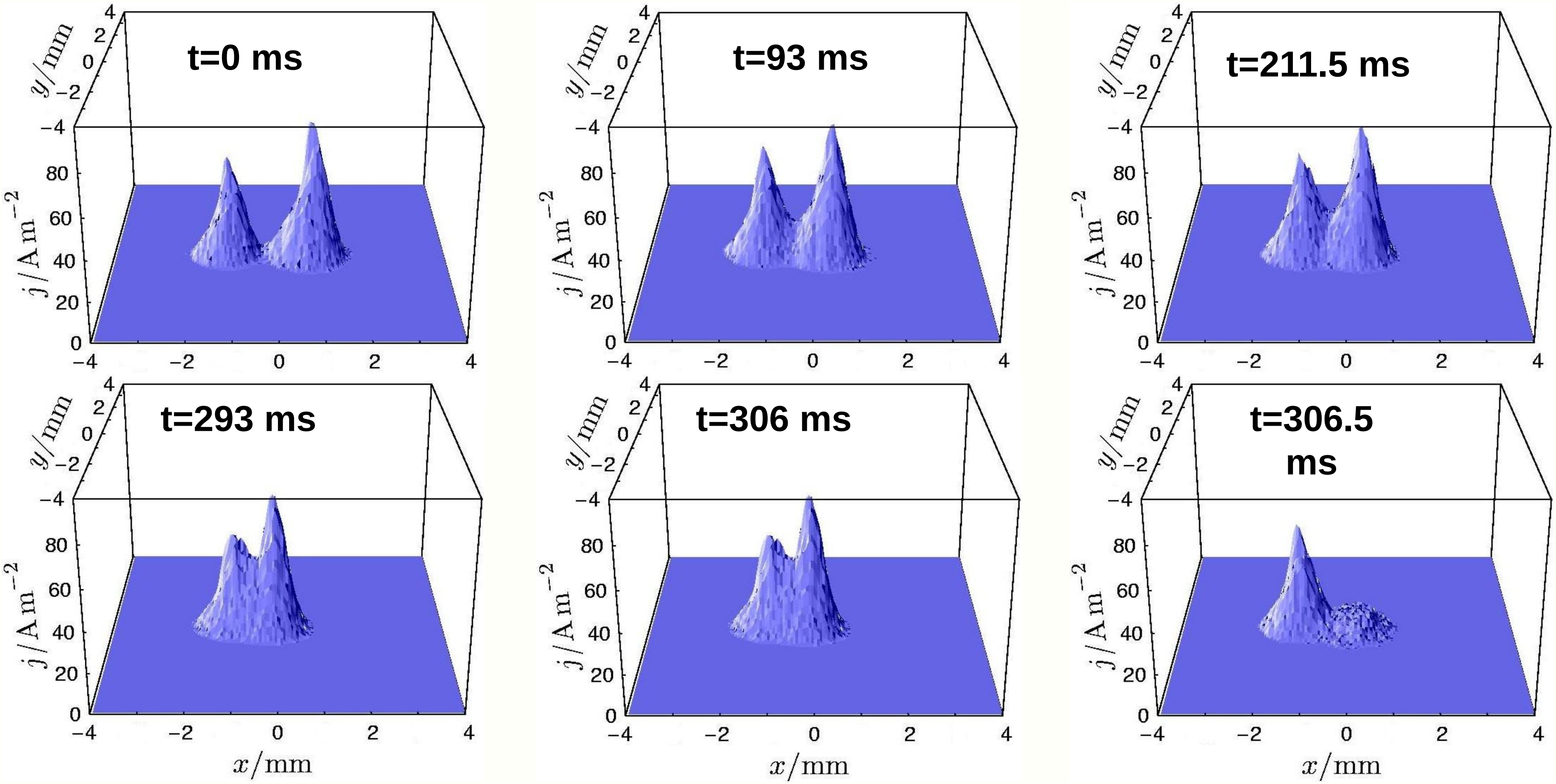
Numerical Solution for the Generalized FH Equation in $\Re^2$: DS Generation in the Course of Collision



$\tau=3.47$, $\theta=0$, $D_u=1.1*10^{-4}$, $D_v=0$, $D_w=9.64*10^{-4}$, $\lambda=1.01$, $\kappa_1=-0.1$, $\kappa_3=0.3$, $\kappa_4=1.0$, $\Omega=[0,1]x[0,1]$, $\Delta x=5*10^{-3}$, $\Delta t=0.1$.

Liehr et al., In: *High Performance Computing in Science and Engineering '02*, Krause,E., Jäger, W. (eds.) (2002); Purwins et al., in: *Dissipative Solitons*, Springer, Lectures Notes in Physics 661, p. 267 (2005)

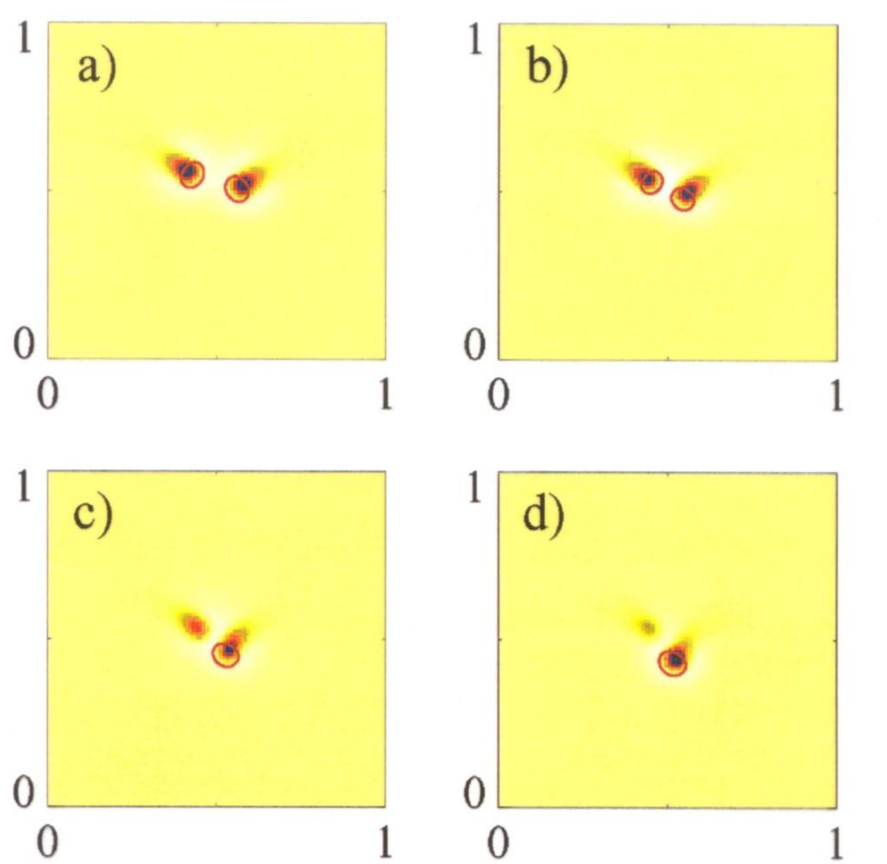
Experimental Quasi 2-Dimensional DC Gas-Discharge System: Current Filament Annihilation in the Course of Collision

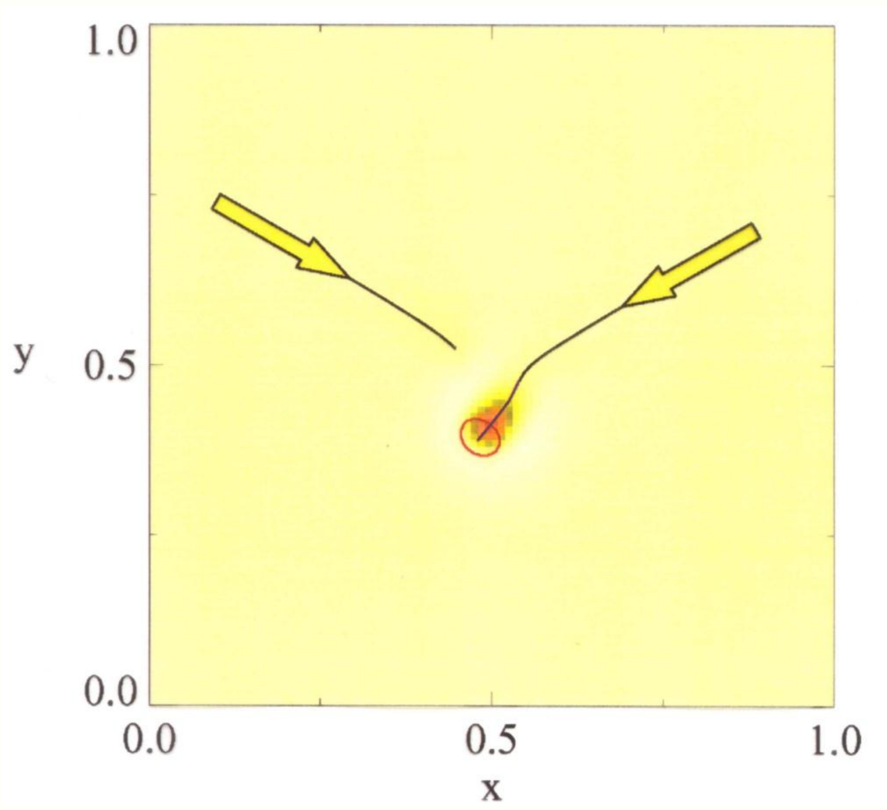


parameters: U_0=3,8 kV, ρ_{SC}=4,14 MΩ cm, R_0=20 MΩ, Gas: N_2, T=100 K, p=290 hPa, D=30 mm, d=500 µm, a_{SC}=1 mm, I=100-250 µA, t_{exp}=0,2 ms, f_{rep}=2 kHz

Bödeker, Diplomarbeit, Universität Münster (2003); Purwins et al., in: *Dissipative Solitons*, Springer, Lectures Notes in Physics 661, p. 267 (2005)

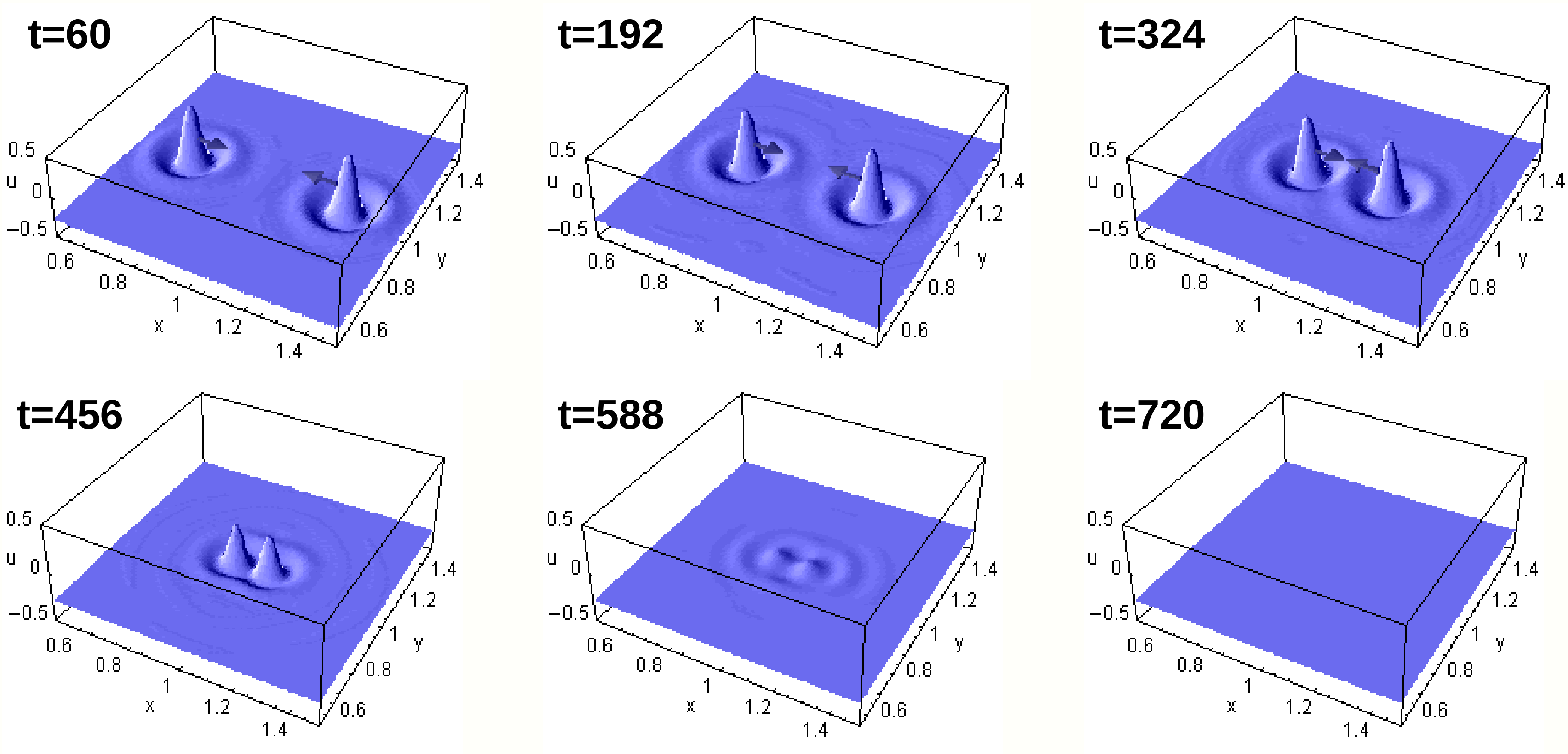
Numerical Solution for the Generalized FH Equation in $\Re^2$: DS Annihilation in the course of Collision I





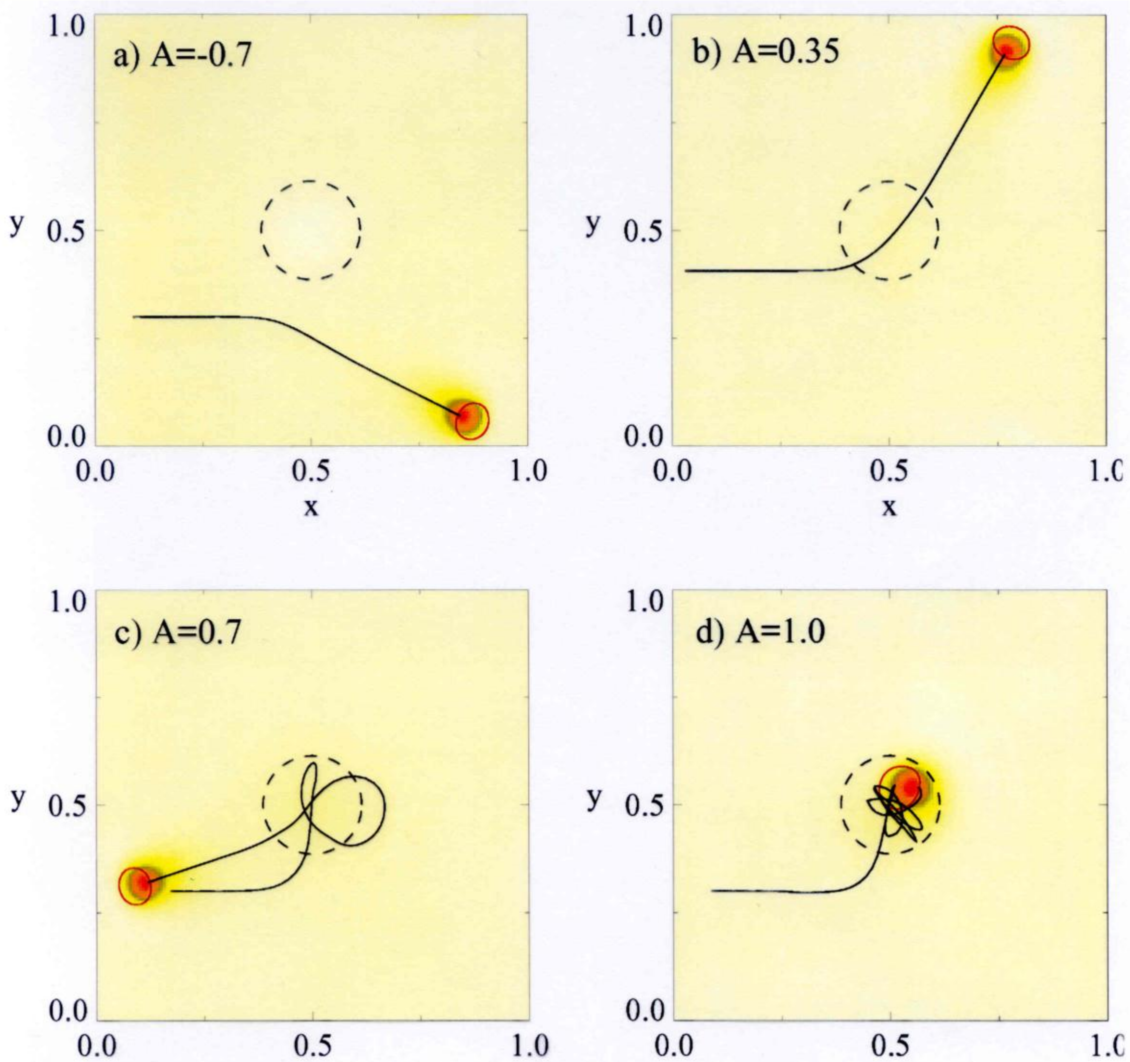
parameters: D_u=1.55*10^{-4}, D_v=1.95*10^{-4}, D_w=9.6*10^{-4}, κ_1=-8.92, κ_3=1, κ_4=8.5, λ=2, τ=25, θ=1, Δx=1/100

Numerical Solution for the Generalized FH Equation in $\Re^2$: DS Annihilation in the course of Collision II



$\tau=3.59$, $\theta=0$, $D_u=1.1*10^{-4}$, $D_v=0$, $D_w=9.64*10^{-4}$, $\lambda=1.01$, $\kappa_1=-0.1$, $\kappa_3=0.3$, $\kappa_4=1.0$, $\Omega=[0,1]x[0,1]$, $\Delta x=5*10^{-3}$, $\Delta t=0.1$.

Numerical Solution for the Generalized FH Equation in $\Re^2$: Scattering of a DS at an Inhomogeneity

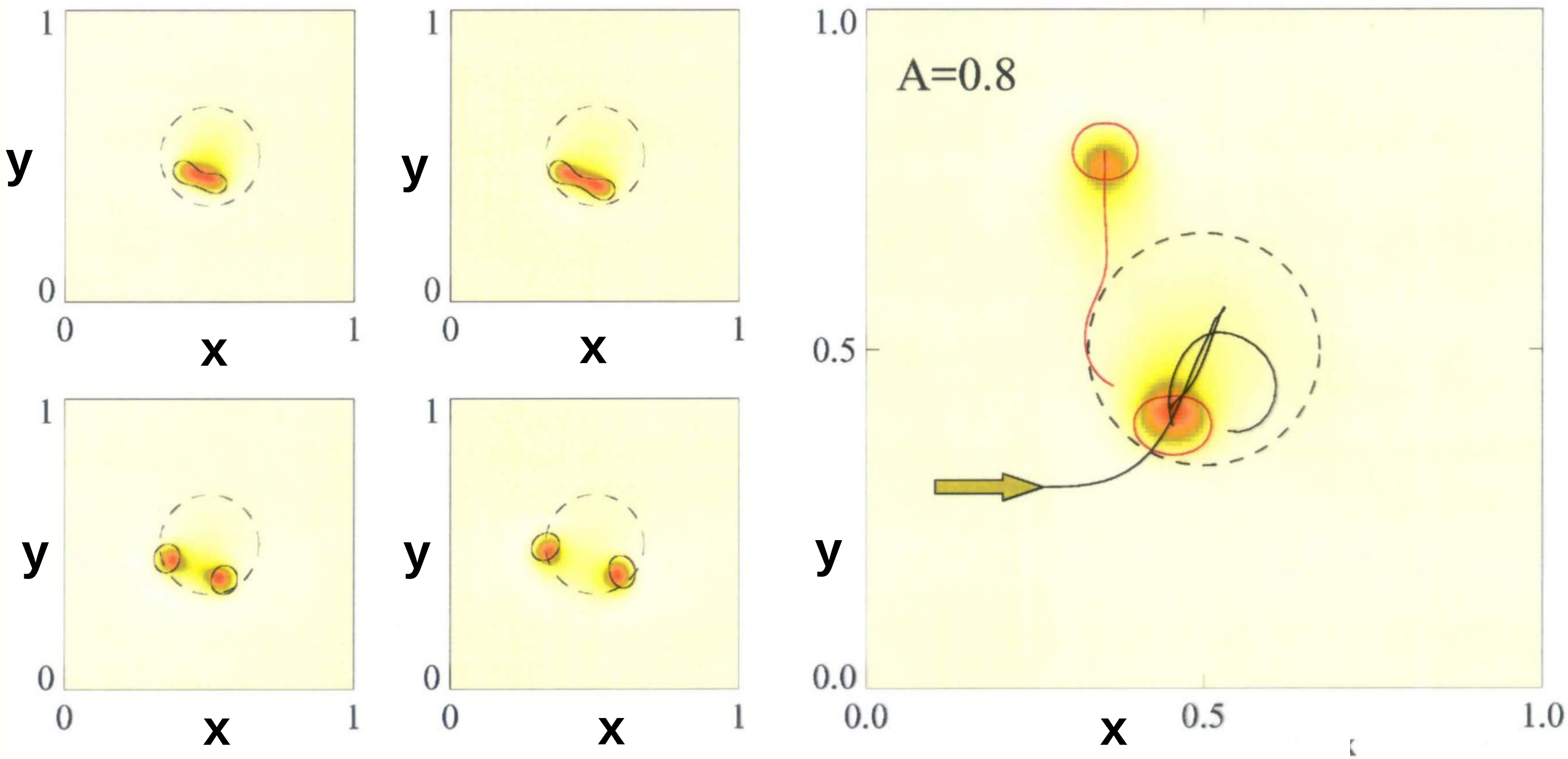


parameters: $D_u=1.21*10^{-4}$, $D_v=1.70*10^{-3}$, $D_w=1.46*10^{-2}$, $\kappa_3=8$, $\lambda=4.3$, $\tau=60$, $\theta=0$, $\Delta x=1/160$

$\kappa_1=-12.34+A\exp(-53.3((x-0.5)^2+(y-0.5)^2))$

Numerical Solution for the Generalized FH Equation in $\Re^2$: Generation of a DS at an Inhomogeneity

inhomogeneity as particle generator



D_u=1.21 10^{-4}, D_v=1.70 10^{-3}, D_w=1.46 10^{-2}, λ=4.3, κ_3=8, τ=60, θ=0, Δx=1/160

κ_1=-12.34+ A exp(-35.5 $((x-0.5)^2+(y-0.5)^2)$)

Bode et al., Physica D, 161, p. 45 (2002)

5. Theoretical Foundation of a Particle Concept

Particle Equation for Interacting DSs Derived from the Generalized FH Equation near to the Travelling Bifurcation Point

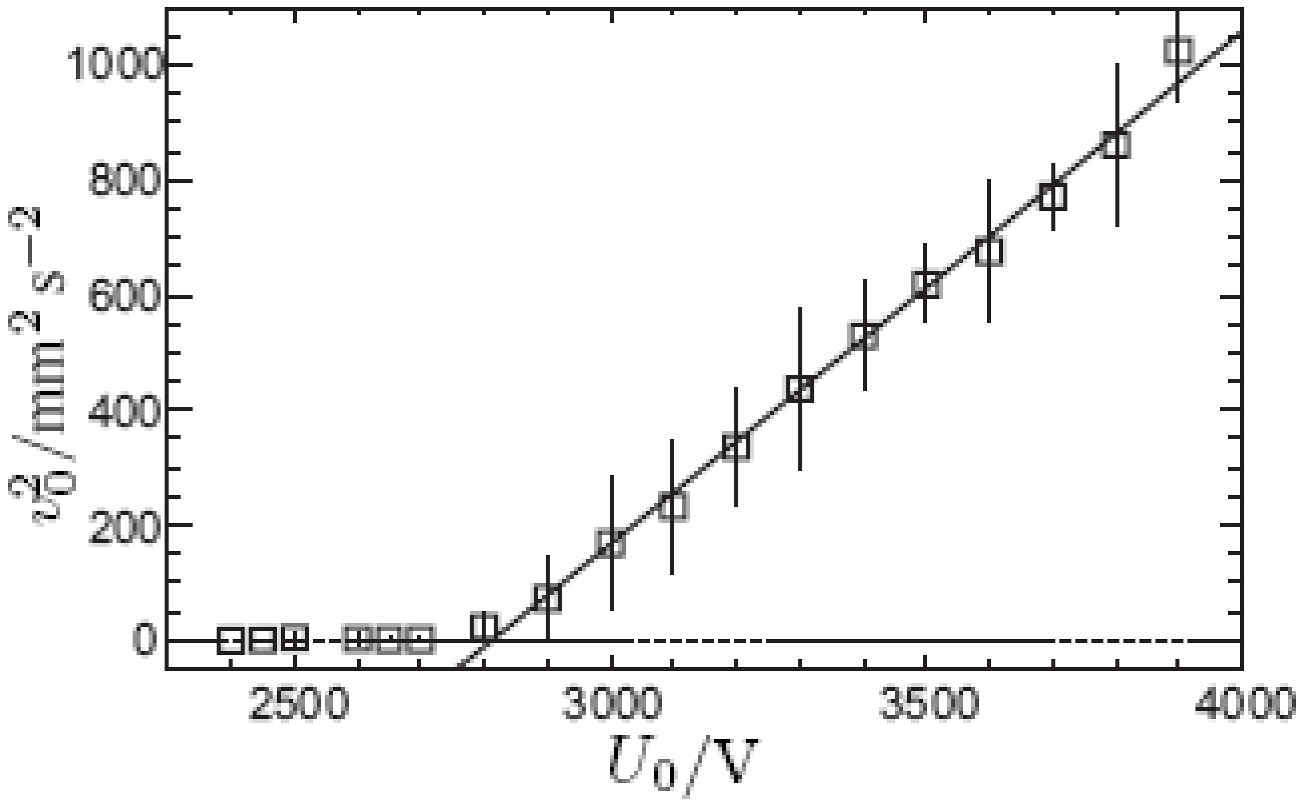
$$\partial_t \underline{p}_i = \underline{\alpha}_i + \frac{1}{\kappa_3} F\left(\left|\underline{p}_i - \underline{p}_j\right|\right)\frac{\underline{p}_i - \underline{p}_j}{\left|\underline{p}_i - \underline{p}_j\right|},$$

$$\partial_t \underline{\alpha}_i = (\tau - \tau_c)\kappa_3^2 \underline{\alpha}_i + \frac{1}{\kappa_3}\frac{\left\langle\left(\partial_x^2 u_0\right)^2\right\rangle}{\left\langle\left(\partial_x u_0\right)^2\right\rangle}\left|\underline{\alpha}_i\right|^2 \underline{\alpha}_i + F\left(\left|\underline{p}_i - \underline{p}_j\right|\right)\frac{\underline{p}_i - \underline{p}_j}{\left|\underline{p}_i - \underline{p}_j\right|},$$

$\underline{p}_i$	= position coordinate of the ith DS
$\underline{\alpha}_i$	= displacement of the slow inhibitor with respect to the activator
u_0	= solution for the activator component at the bifurcation point
$F\left(\left|\underline{p}_i - \underline{p}_j\right|\right)$	= interaction function

Bode et al., Physica D, 161, p. 45 (2002);
Purwins et al., Advances in Physics, 59, p. 485 (2010)

Experimental Quasi 2-Dimensional DC Gas-Discharge System: Comparison of the Experimental to the Theoretical Scaling Law for the Velocity of DSs I

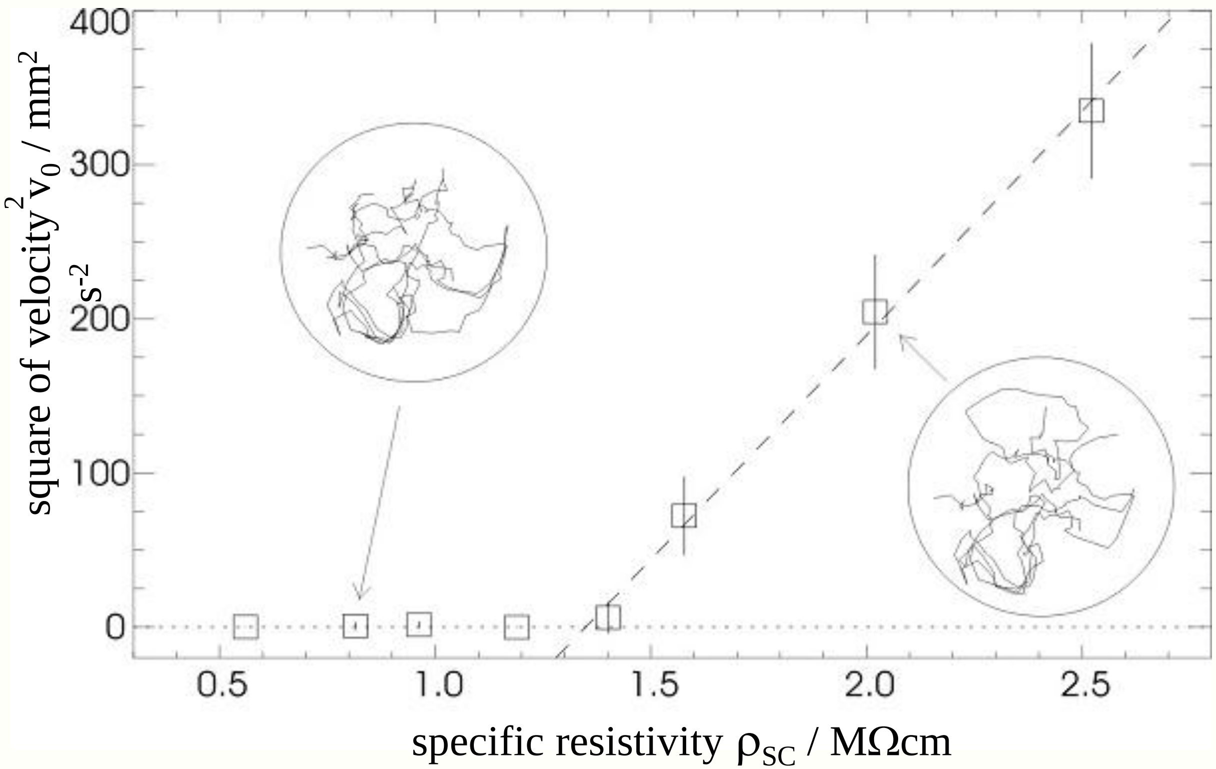


parameters:
ρ_0=1,83 MΩ, R_0=10 MΩ, Gas: N_2, T=100 K, p=280 hPa, d=500 μm, t_{exp}=20 ms

square of the intrinsic velocity of a single filament plotted as a function of the driving voltage U_0 (points); the velocity is evaluated by advanced statistical date processing; the straight line is deduced from the generalized FN equation

Or-Guil et al., Phys. Rev. E. 57, p. 6432 (1998);
Gurevich et al. Physica D 199, p. 115 (2004)

Experimental Quasi 2-Dimensional DC Gas-Discharge System: Comparison of the Experimental to the Theoretical Scaling Law for the Velocity of DSs II



parameters:
U_0=3,7 kV, R_0=10 MΩ,
Gas: N_2, T=100 K,
p=286 hPa, D=30 mm,
d=750 µm, a_{SC}=1 mm,
I=107 µA, t_{exp}=20 ms,
f_{rep}=50 Hz

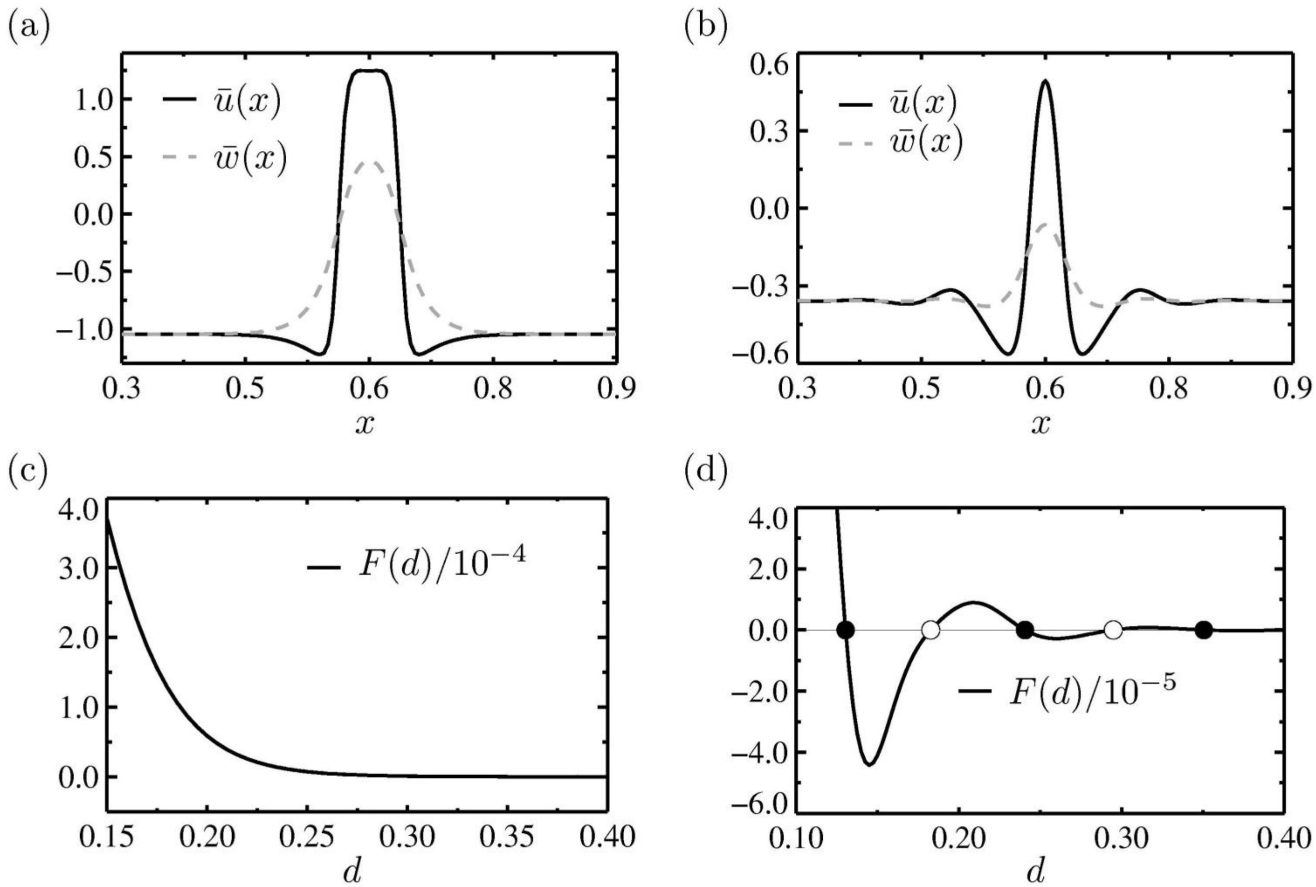
square of the intrinsic velocity of a single filament plotted as a function of the specific resistivity of the semiconductor wafer (points); the velocity is evaluated by advanced statistical date processing; the straight line is deduced from the generalized FN equation; typical experimental trajectories are displayed in the insert

Or-Guil et al., Phys. Rev. E. 57, p.6432 (1998); Liehr, et al., New J. Phys., 5, 89.1 (2003); Bödeker et al., Physical Review E, 67, p. 056220-1 (2003)

Numerical Solution for the Generalized FH Equation in $\Re^1$: Interaction Function for Dissipative Solitons

$\bar{u}(x) = \bar{v}(x)$

$F(d) = F(|\vec{p}_2 - \vec{p}_1|)$



a+c: τ=0, θ=0, D_u=0.5*10^{-4}, D_v=0, D_w=10^{-3}, λ=3.0, κ_1=-0.1, κ_3=1.0, κ_4=1.0, Ω=[0,1.2], Δx=5*10^{-3},

b+d: D_u=1.1*10^{-4}, D_w=9.64*10^{-4}, λ=1.71, κ_1=-0.15, Δx=2.5*10^{-3}, others as in a+c.

Purwins et al. 2005 in: *Dissipative Solitons*, Springer Lectures Notes in Physics 661, p. 267 (2005);
Purwins et al., Advances in Physics, 59, p. 485 (2010)

Selected Additional Theoretical Work on the Generalized FitzHugh-Nagumo Equation

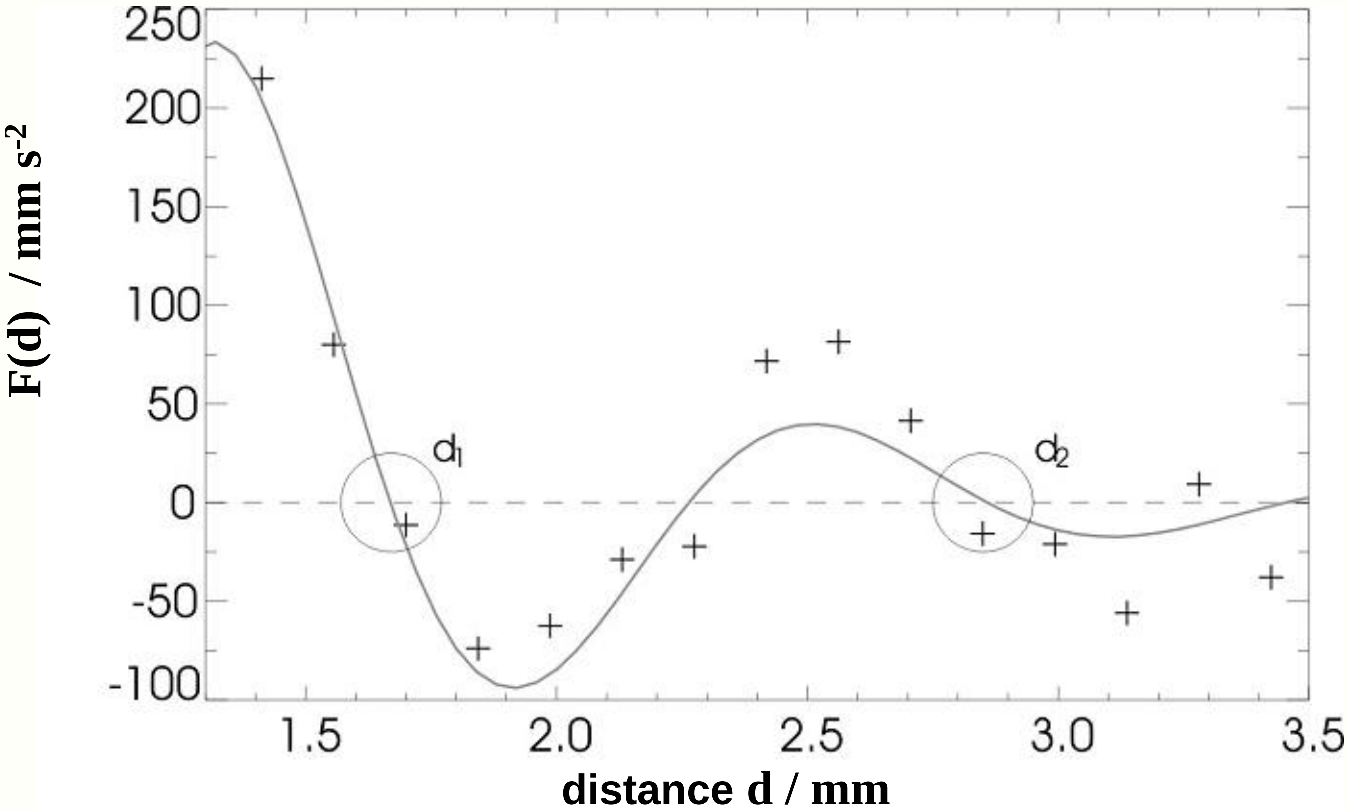
M. Krupa, B. Sandstede, P. Szmolyan, "Fast and slow waves in the FitzHugh-Nagumo equation",
J. Differential Equations, vol. 133, p. 49 (1997)

Y. Nishiura et al., "Dynamics of Travelling Pulses in Heterogeneous Media",
Chaos vol. 17, p. 037104 (2007), see also references therein

P. v. Heijster, A. Doelman, T. S. Kaper , "Pulse dynamics in 3-component systems: Stability and bifurcations",
Physica D vol. 237 p. 3335 (2008), see also references therein

P. v. Heijster, B. Sandstede, "Planar radial spots in a three component FitzHugh Nagumo equation",
math.bu.edu/people/heijster/PAPERS/planar spots.pdf, see also references therein

Experimental Quasi 2-Dimensional DC Gas-Discharge System: Determination of the Interaction Function of the Particle Equation from Experimental Trajectories



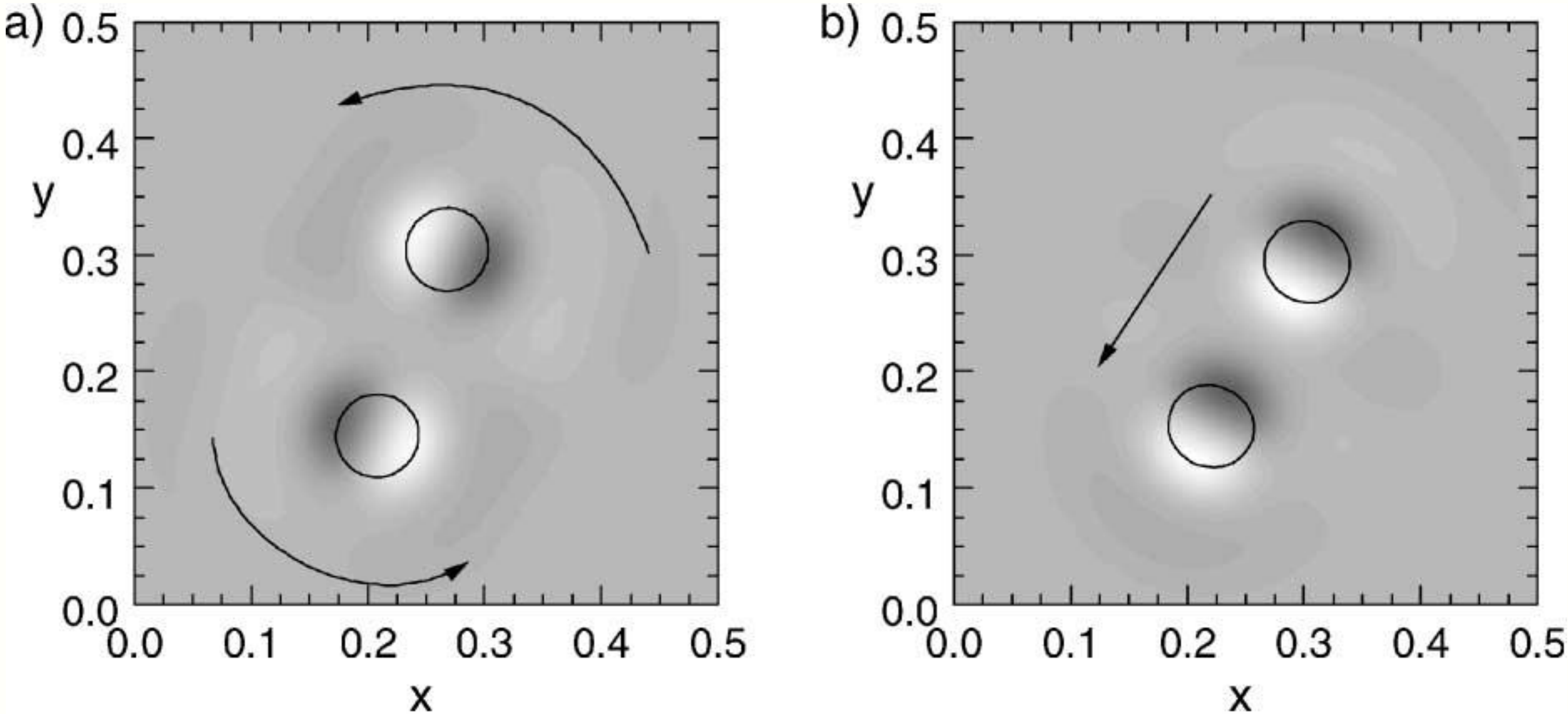
parameters:

- U_0 = 4600 V
- ρ_{SC} = 3,55 MΩcm
- R_0 = 4,4 MΩ,
- Gas: N_2
- T = 100 K
- p = 283 hPa
- D = 30 mm,
- d = 550 µm
- a_{SC} = 1 mm
- I = 233µA
- t_{exp} = 20 ms,
- f_{rep} = 50 Hz

the interaction function $F(d) = F(|\underline{p}_i - \underline{p}_j|)$ is evaluated by advanced statistical date processing; the continuous line is deduced from the generalized FN equation

Numerical Solution for the Particle Equation in $\Re^2$: Scattering Diagram for a Fixed Set of Parameters III

solution of the generalized FN equation:



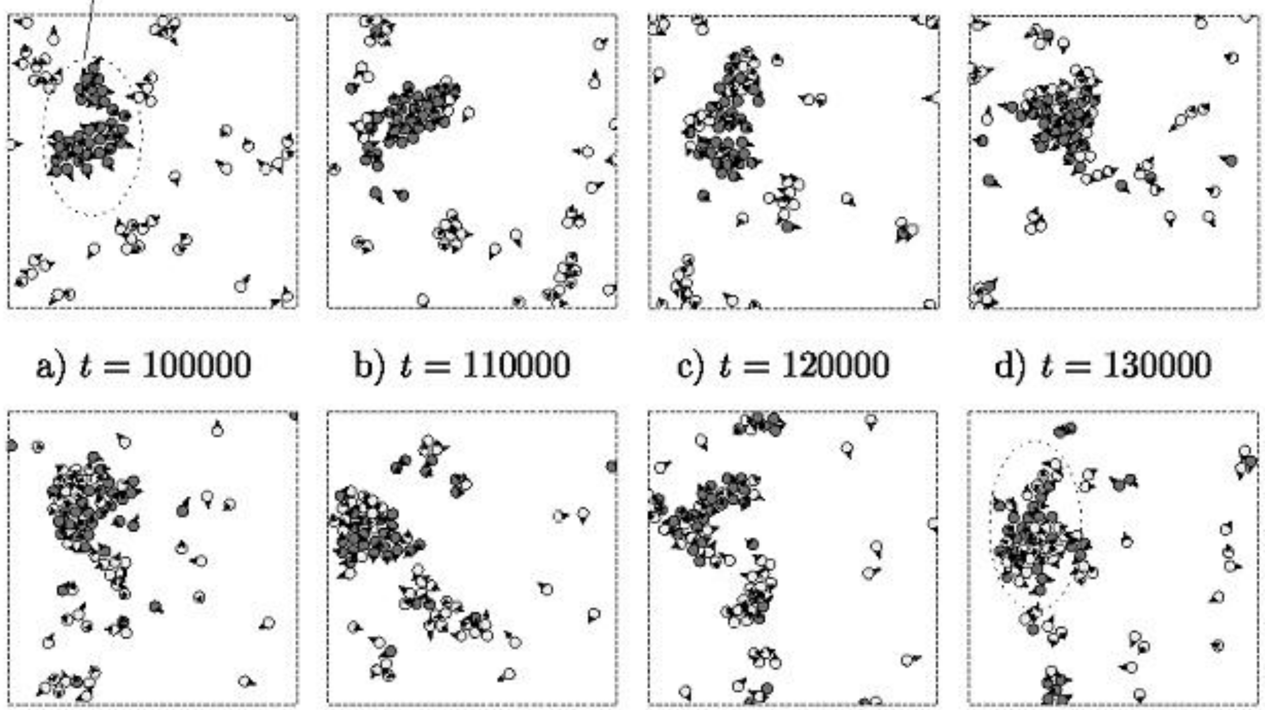
parameters of the original equation:

$D_u=1.1*10^{-4}$, $D_v=0$, $D_w=9.64*10^{-4}$, $\kappa_1=-0.1$, $\kappa_3=0.3$, $\kappa_4=1$, $\lambda=1.01$, $\tau=3.43$, $\theta=0$

Numerical Solution for the Particle Equation in $\mathfrak{R}^2$: Possible Behaviour of a Many-DS System

marked solitons in the largest cluster at starting time

N=81



a) $t = 100000$ b) $t = 110000$ c) $t = 120000$ d) $t = 130000$

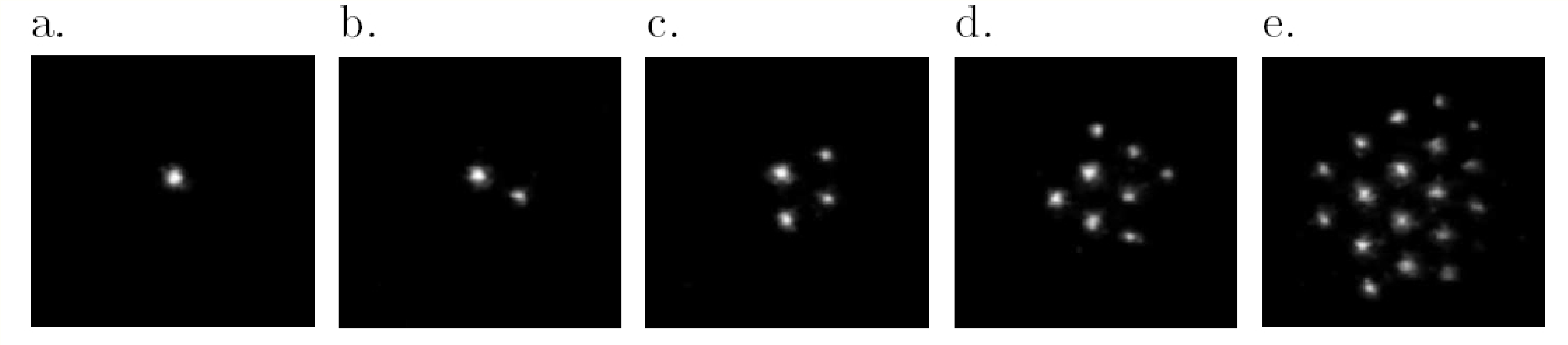
e) $t = 140000$ f) $t = 150000$ g) $t = 160000$ h) $t = 170000$

τ=3.34, θ=0, D_u=1.1*10^{-4}, D_v=0, D_w=9.64*10^{-4}, λ=1.01, κ_1=-0.1, κ_3=0.3, κ_4=1.0, Ω=[-2,2]x[-2,2], Δx=5*10^{-3}, Δt=0.1; F(d), Q = 1950.

Röttger, Diplomarbeit, Universität Münster (2003);
Purwins et al., Advances in Physics, 59, p. 485 (2010)

Experimental Quasi 2-Dimensional DC Gas-Discharge System: Self Completion I

luminescence radiation distribution in the discharge plane

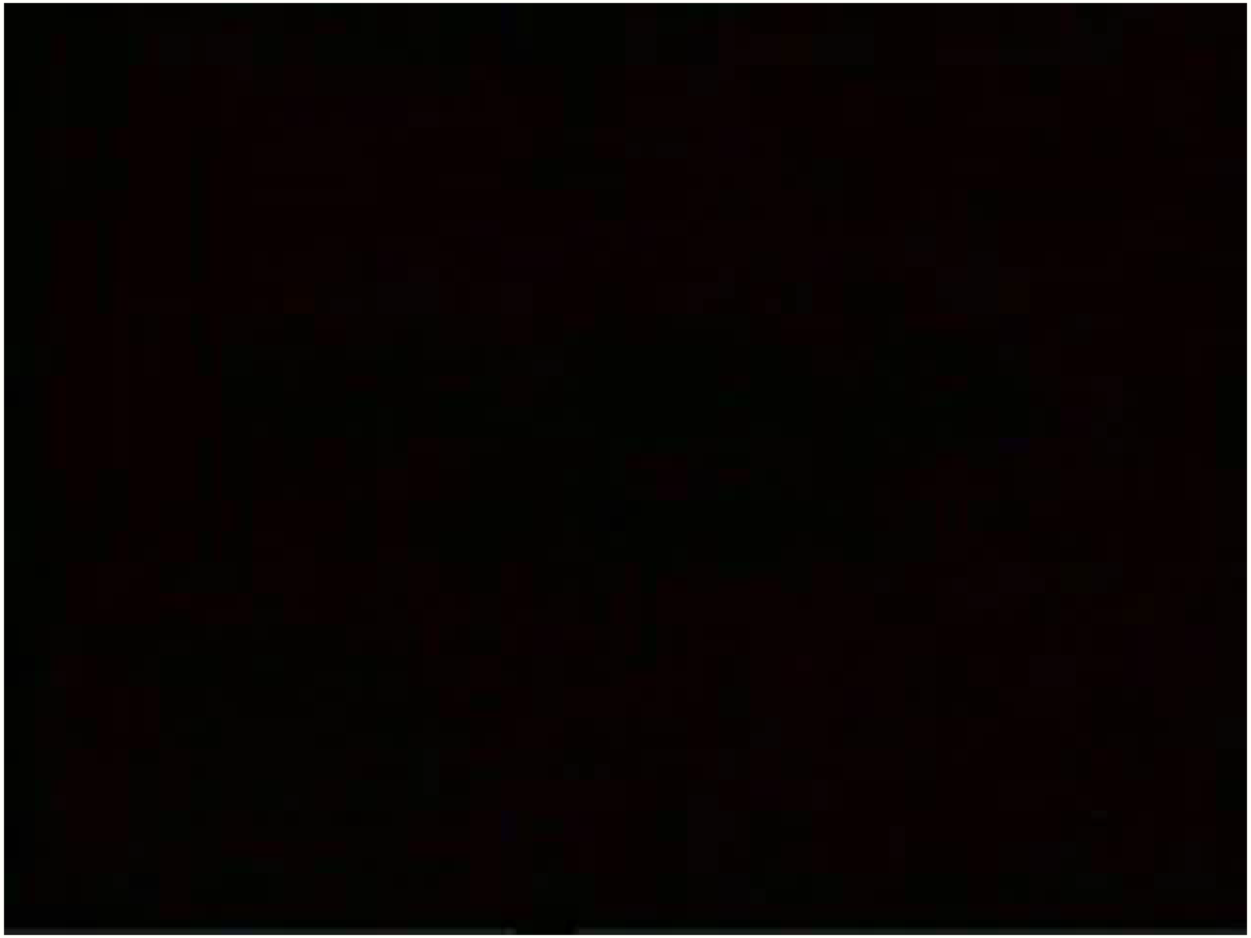


at fixed parameter in the course of time near to an existing localized structure an increasing number of such objects is generated at well defined distance to existing ones until the plain is fully covered with a hexagonal pattern

Ammelt et al. 1998, unpublished; see also: Astrov et al., Phys. Rev. Lett. 79 p. 2983 (1997); Purwins et al., Advances in Physics, 59, p. 485 (2010)

Experimental Quasi 2-Dimensional DC Gas-Discharge System: Self Completion II (Movie)

luminescence radiation distribution in the discharge plane

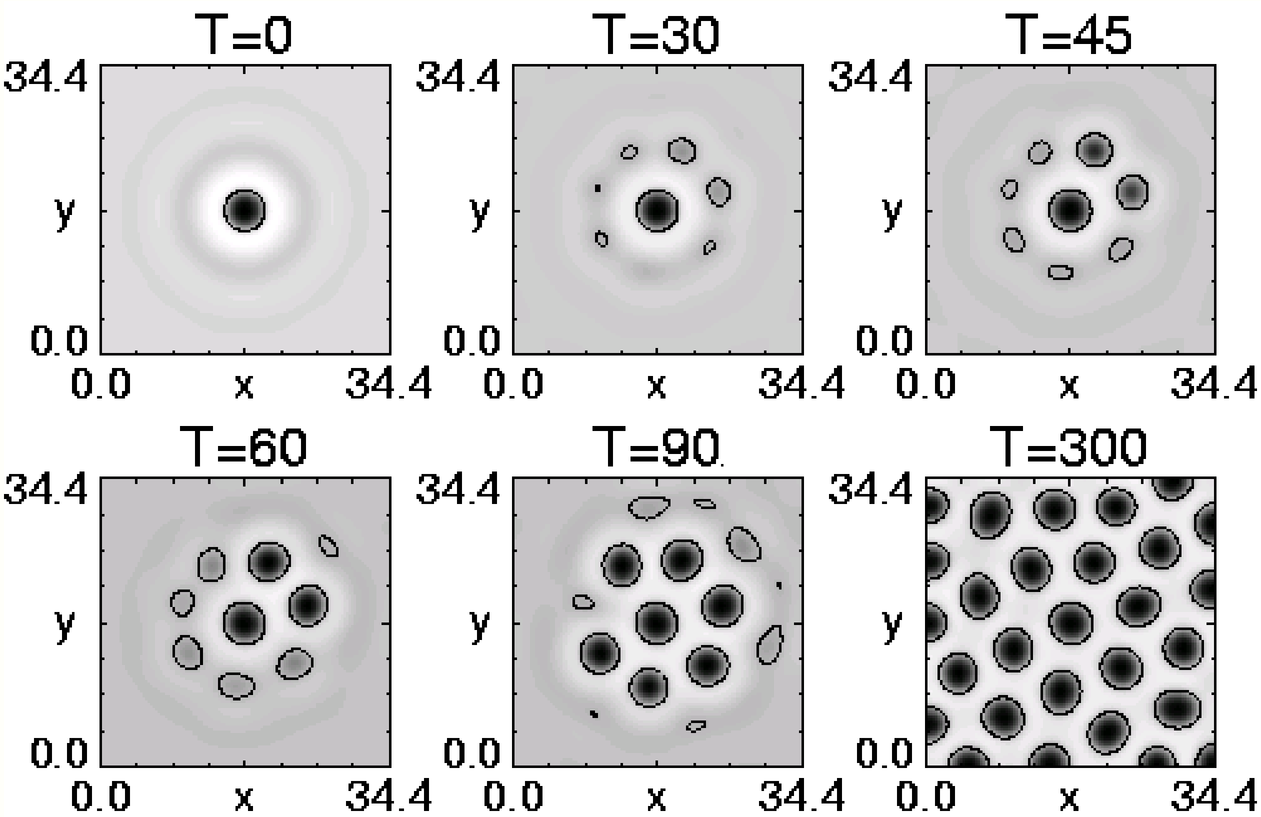


radiation distribution in the discharge plane

if not linked: start movie "self completion.wmv" in the folder

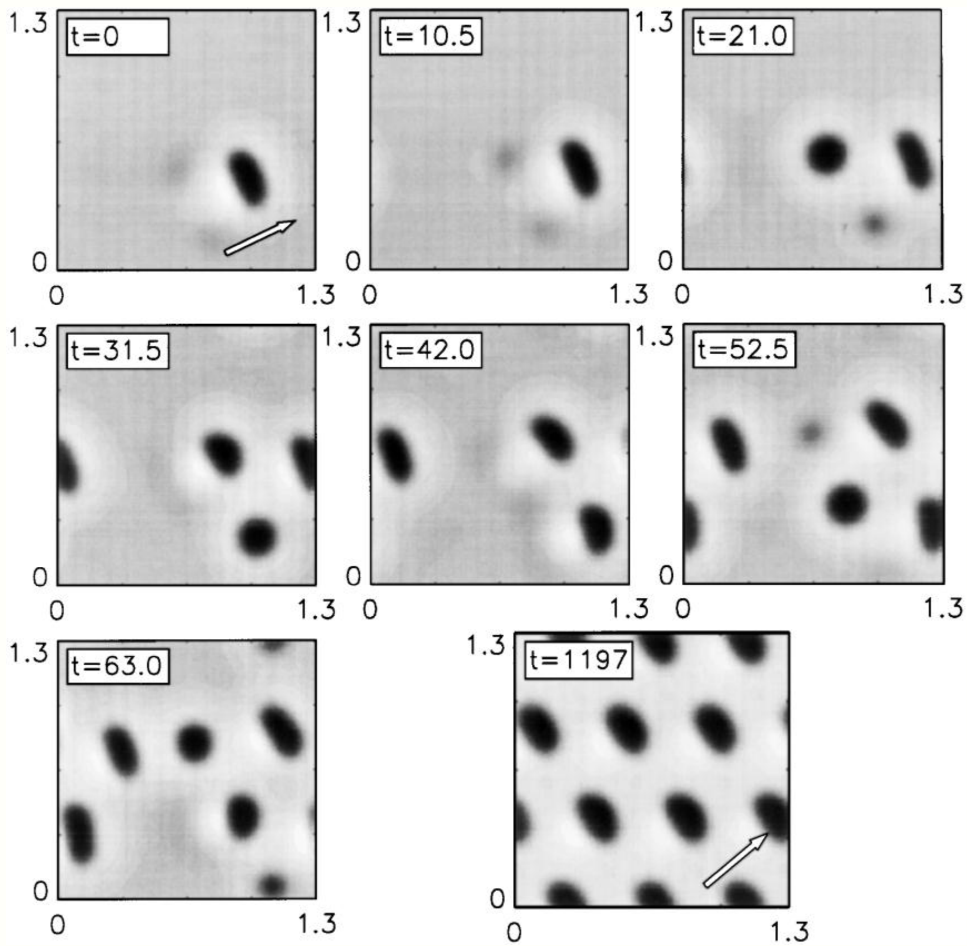
Ammelt et al. 1998, unpublished

Numerical Solutions for the Generalized FH Equation in $\Re^2$: Self-Completion I

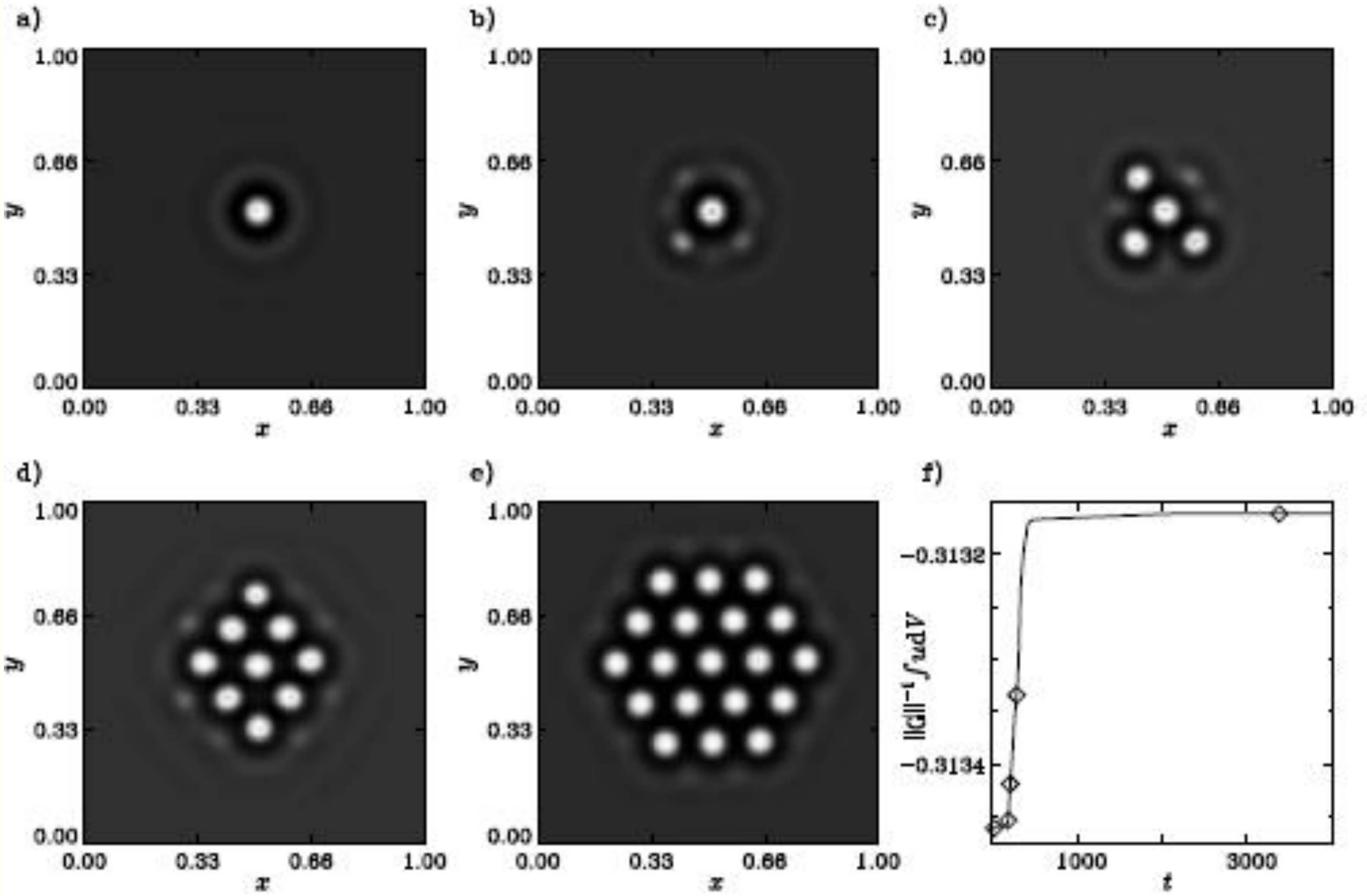


Schenk, PhD Thesis, Universität Münster (1999)

Numerical Solutions for the Generalized FH Equation in $\Re^2$: Self-Completion II

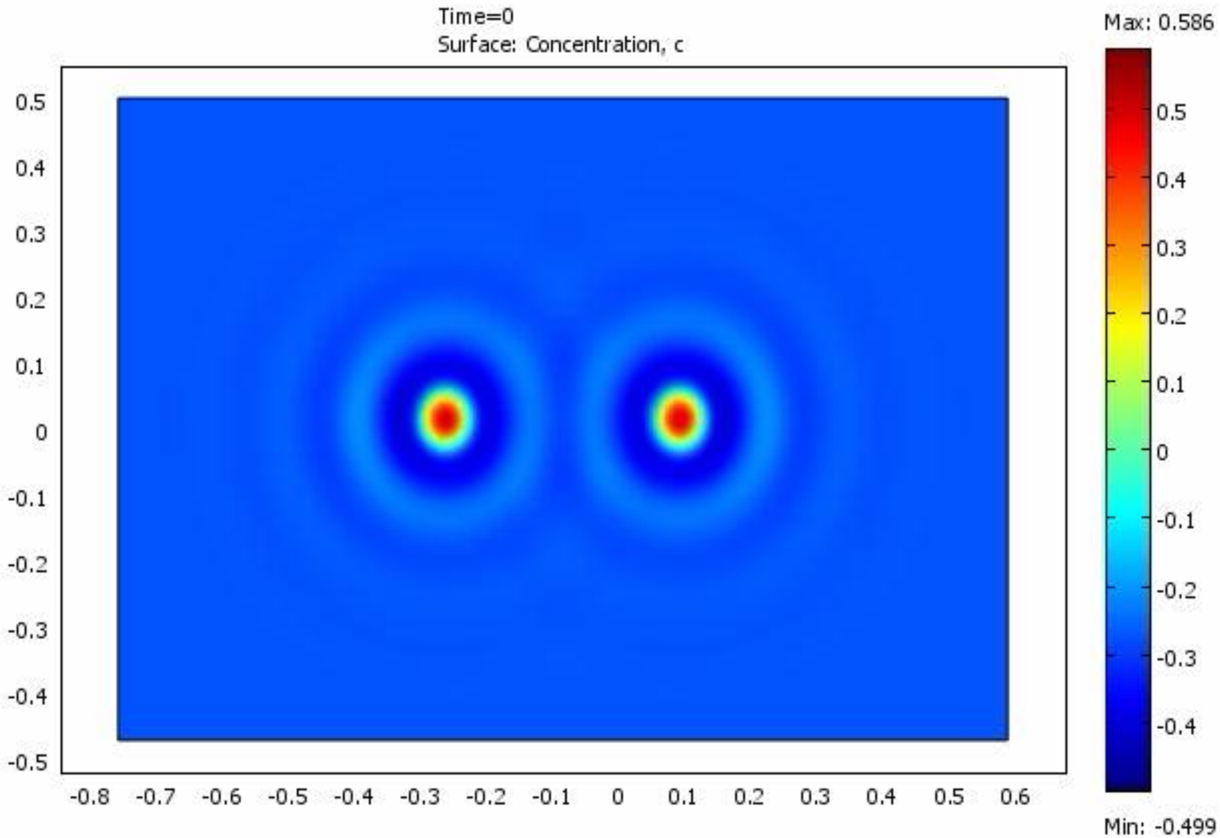


Numerical Solutions for the Generalized FH Equation in $\Re^2$: Self-Completion III



Liehr et al., in: *High Performance Computing in Science and Engineering 2000,* Krause,E., Jäger,W. (eds.) (2001)

Numerical Solutions for the Generalized FH Equation in $\Re^2$: Self-Completion IV (Movie)

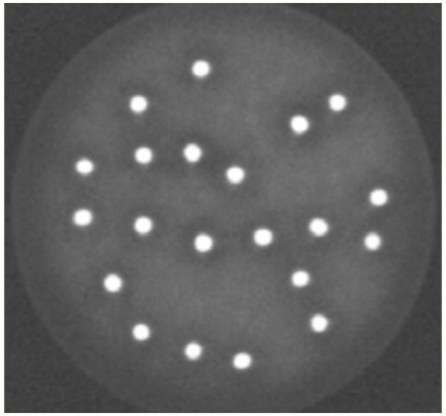


if not linked: start movie “self_completion. avi” in the folder

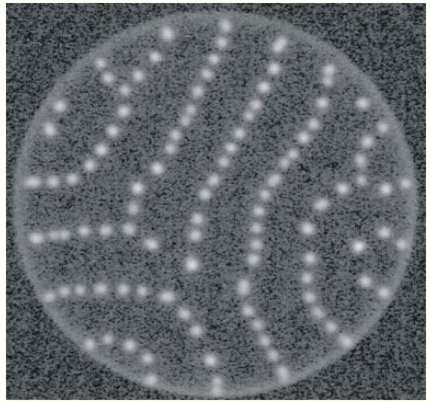
Bödeker et al (2007), unpublished

Experimental Quasi 2-Dimensional DC Gas-Discharge System: Various Many-Filament Patterns I

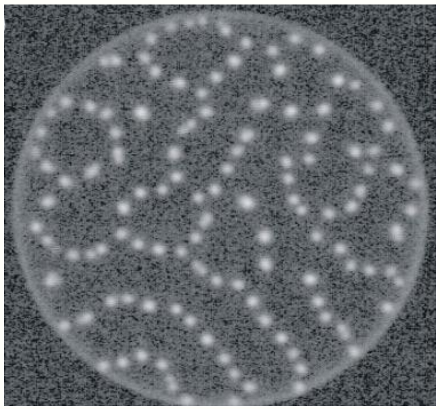
„gas“



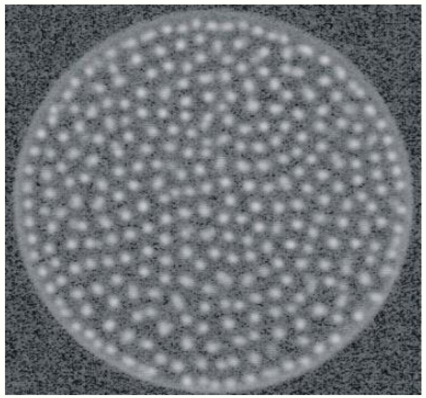
dynamic chains



dynamic net



„liquid“

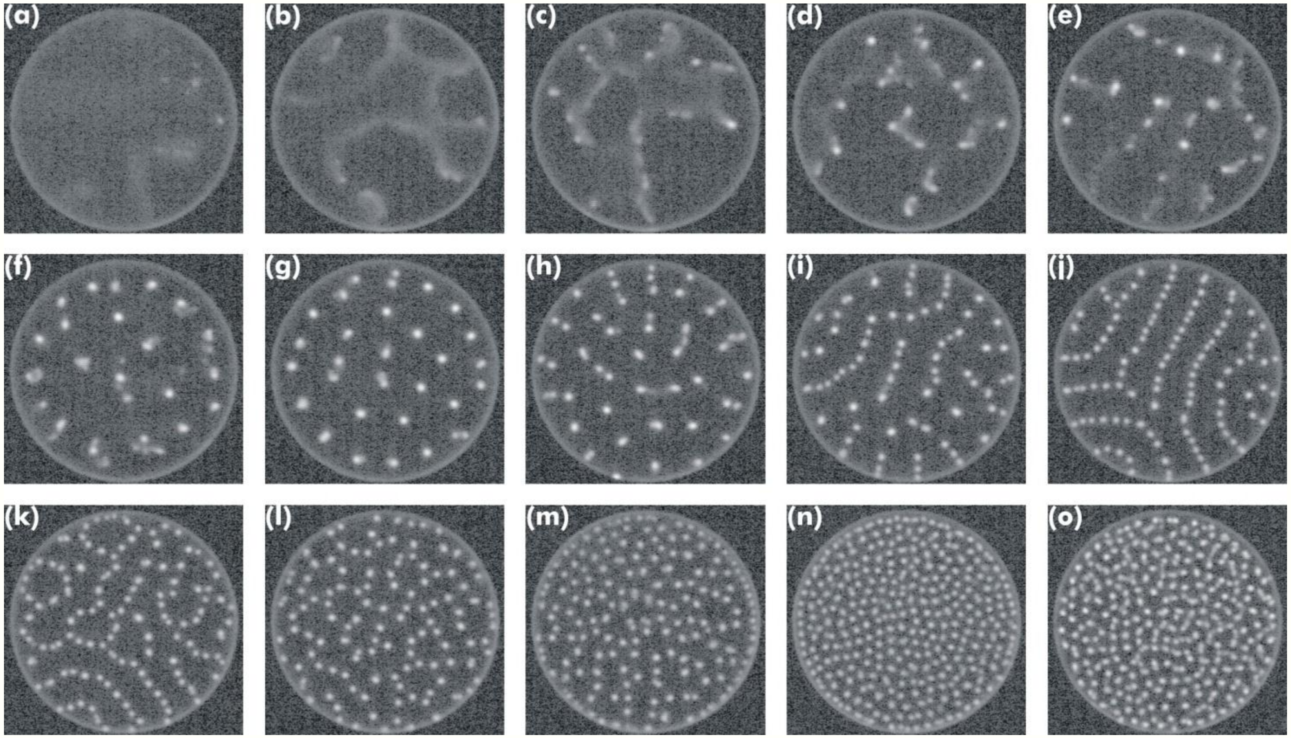


luminescence radiation distribution in the discharge plane

Strümpel et al., Phys. Rev. E, 63, p. 026409-1(2001)

Experimental Quasi 2-Dimensional DC Gas-Discharge System: Various Many-Filament Patterns II

sequence of patterns: luminescence radiation distribution in the discharge plane



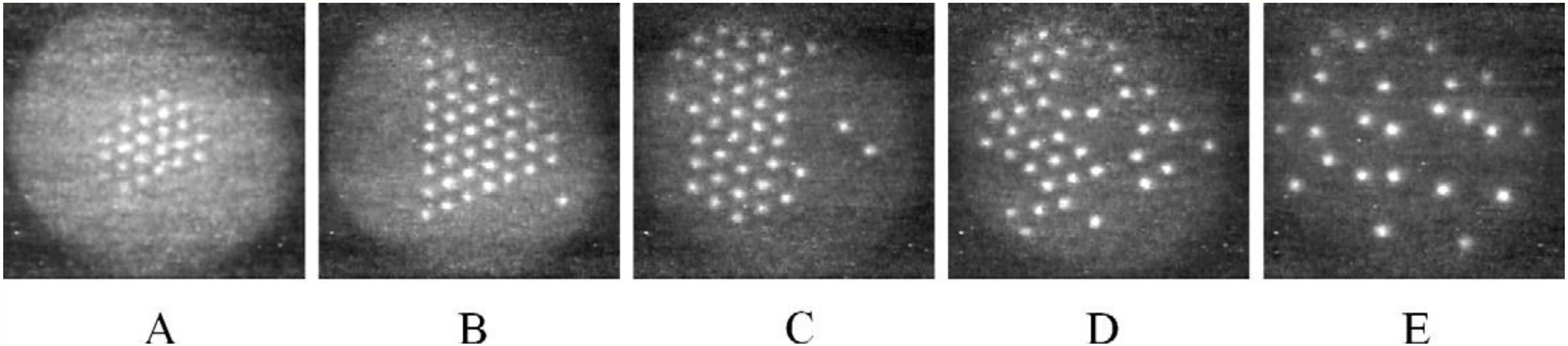
parameters: increasing voltage from 572 V to 1208 V for a) to o)
p=41 hPa, D=3 cm, d=0.05 cm, t_{exp}=40 ms

Strümpel et al. 2001

Experimental Quasi 2 Dimensional DC Gas-Discharge System: Various Many-Filament Patterns III

luminescence radiation distribution in the discharge plane:

equilibrium between „crystal“- and „liquid“-many-filament patterns



radiation distribution in the discharge plane

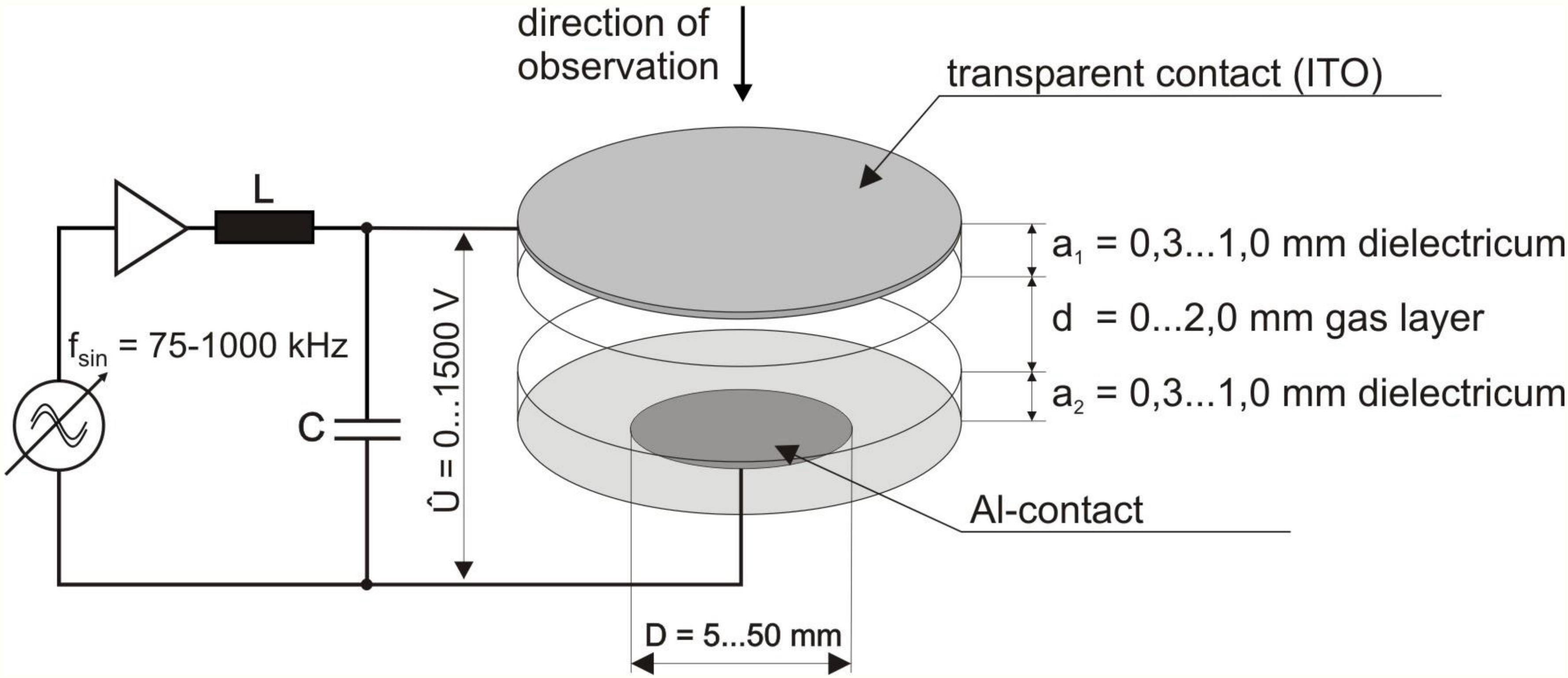
→

increasing conductivity of the semiconductor wafer

Astrov et al., Phys.Lett.A, 283, p. 349 (2001) Pu 80

6. Experimental Results on Dissipative Soliton (DS) in AC Gas-Discharge Systems

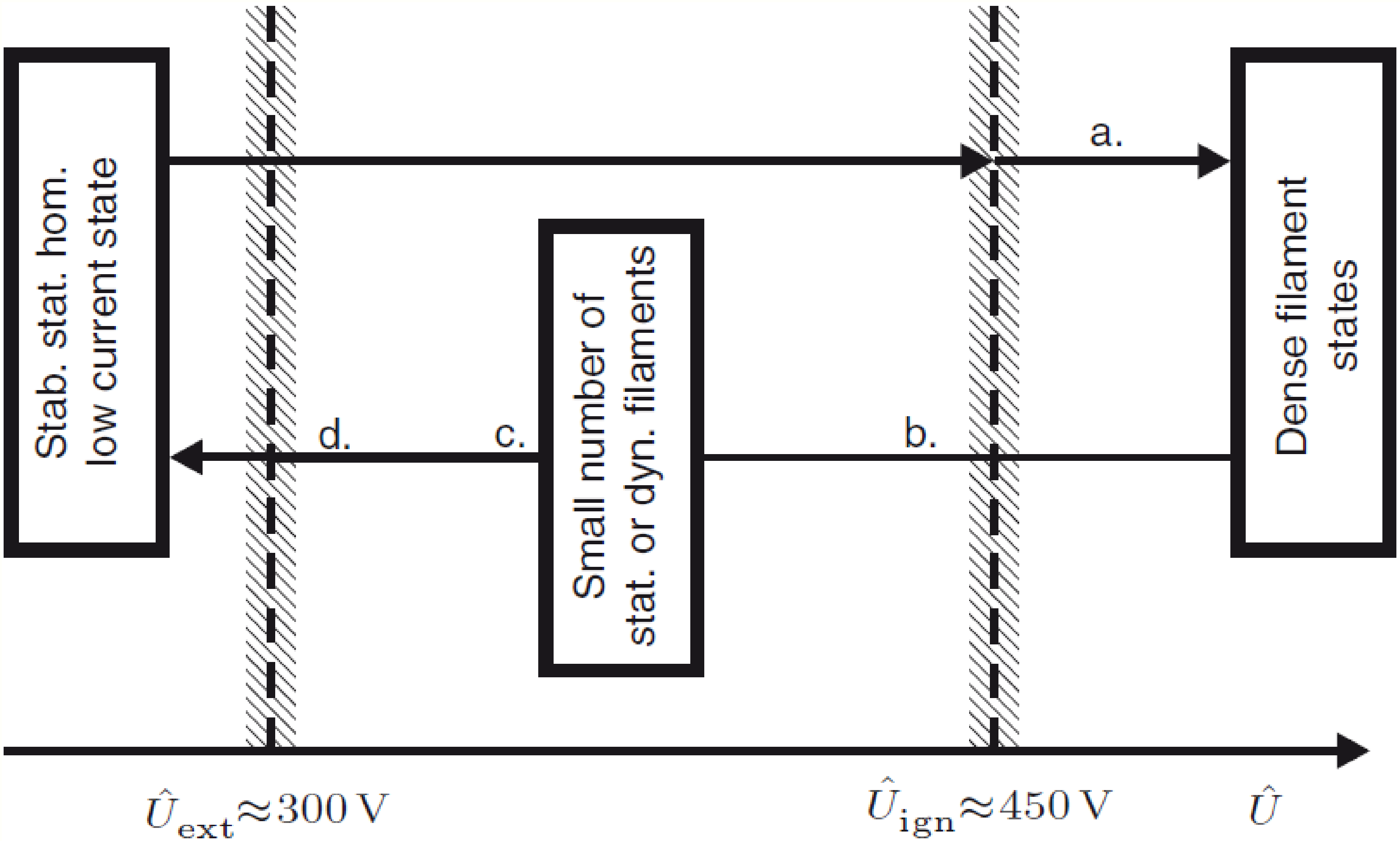
Experimental Quasi 2-Dimensional AC Gas-Discharge System: Experimental Set-Up



Ammelt et al. Physics Lettters, A 179 p. 348 (1993)

Experimental Quasi 2-Dimensional AC Gas-Discharge System: Bifurcation Sequence of Current Filament Patterns I

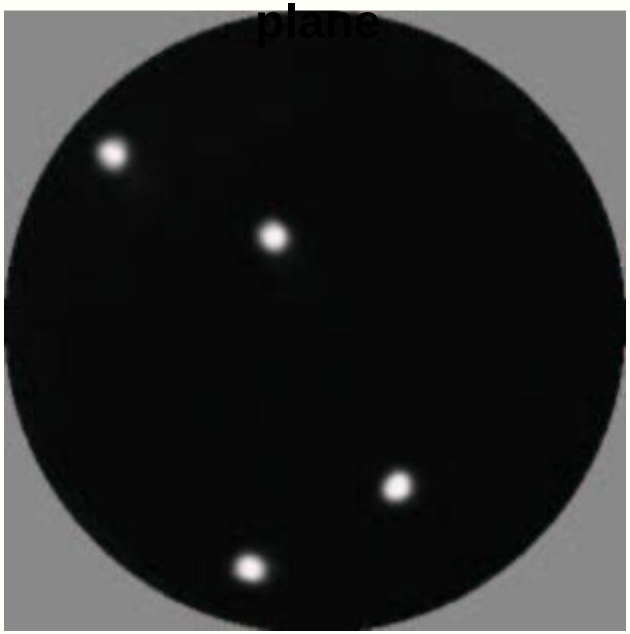
schematic formation of patterns for increasing and decreasing driver amplitude



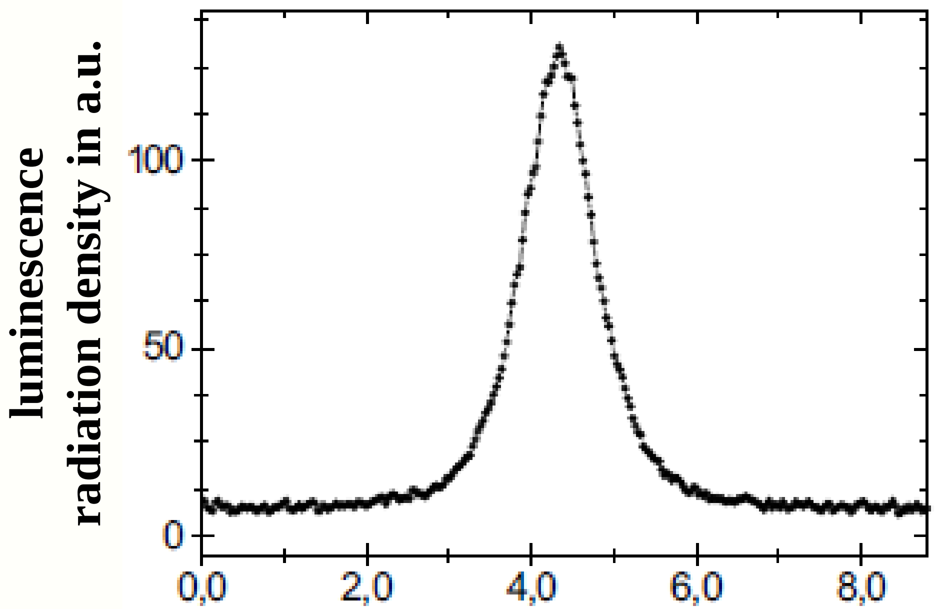
Brauer, PhD Thesis, Universität Münster (2000); Stollenwerk et al. Eur. Phys.J.42, p. 273 (2007); Purwins et al., Advances in Physics, 59, p. 485 (2010)

Experimental Quasi 2-Dimensional AC Gas-Discharge System: Isolated Stationary Current Filaments

luminescence radiation distribution in the discharge plane



cross section of a filament

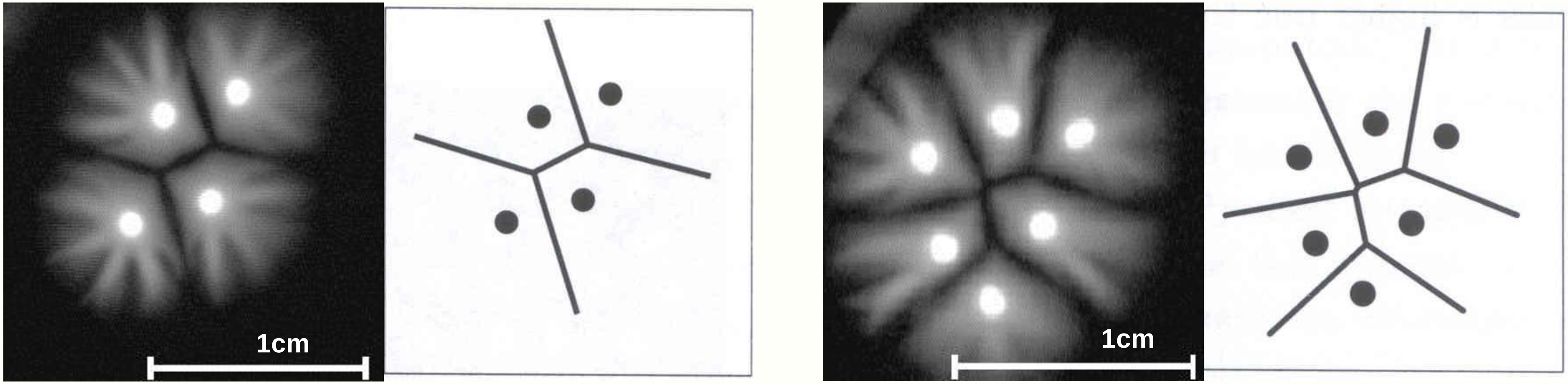


position in mm

electrodes: a transparent ITO glass plate and a borosilicate glass plate (0.5 mm thick); room temperature, gas: He, p=133hPa, d~0.5mm, D~20mm, f=200kHz, U=380V, t_{exp}=40ms

Experimental Quasi 2-Dimensional AC Gas-Discharge System: Self-Organized Voronoi Diagrams I

luminescence radiation distribution in the discharge plane

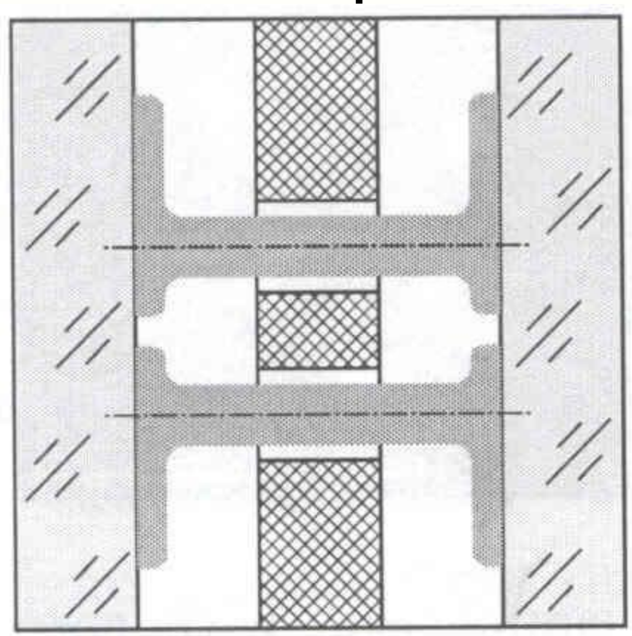


parameters: gas: N_2, p=122 hPa, d=2.6 mm, D=40mm, a_1=0.5 mm, a_2=1 mm, Û=2050 V, t_{exp}=40 ms, f=50 kHz

Zanin et al., Applied Physics Letters, 81, p. 3338 (2002)
de Lacy Costello et al., Int. J. Bif. Chaos., 14, p. 2187 (2004)

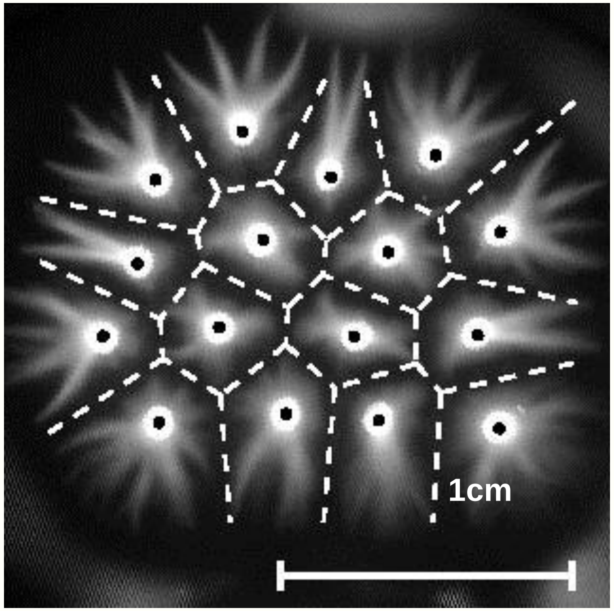
Experimental Quasi 2-Dimensional AC Gas-Discharge System: Self-Organized Voronoi Diagrams II

cross section of the set-up



the positions of filaments are predefining

luminescence radiation distribution in the discharge plane



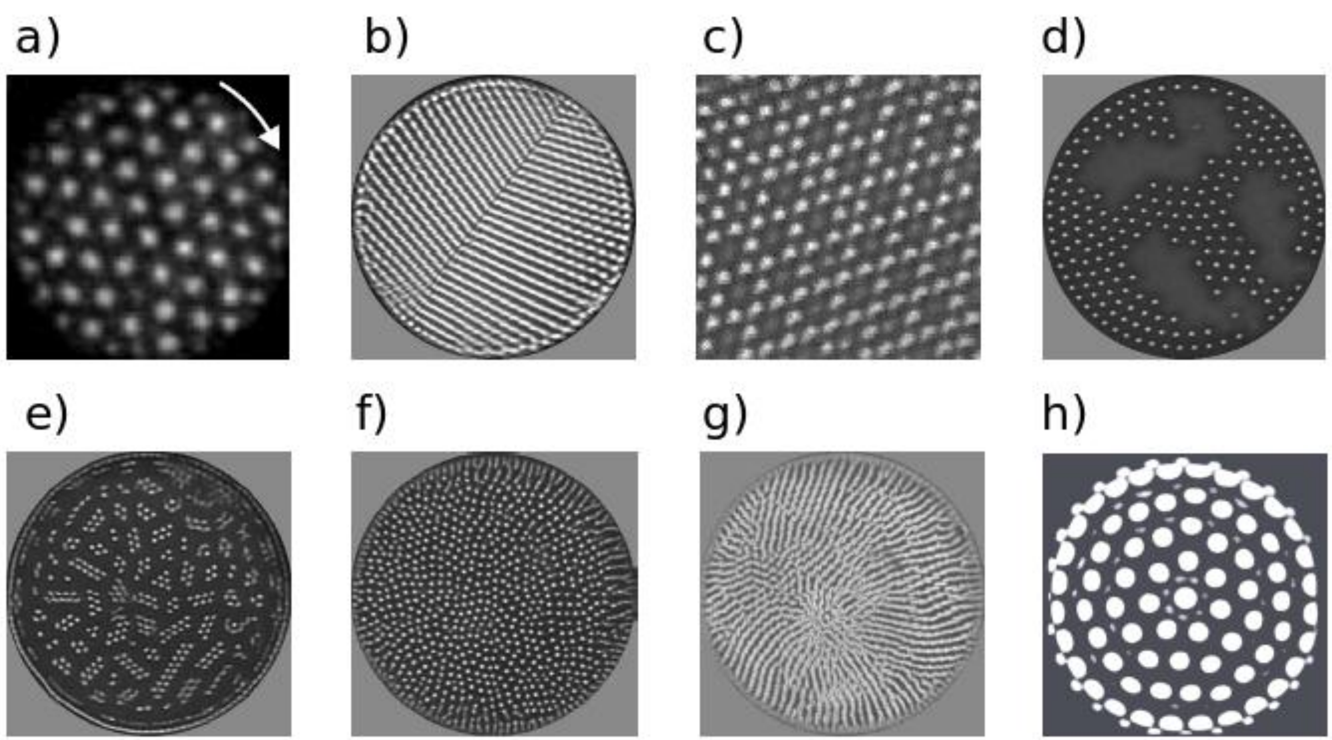
top view

parameters: gas: N_2, p=122 hPa, d=3.0 mm (1mm inner isolating layer), D=40mm, a_1=0.7 mm, a_2=0.1 mm, Û=2100 V, t_{exp}=40 ms, f=50 kHz

Zanin et al., Applied Physics Letters, 81, p. 3338 (2002);
de Lacy Costello et al., Int. J. Bif. Chaos., 14 (7), p. 2187 (2004)

Experimental Quasi 2-Dimensional AC Gas-Discharge Systems: Selection of Snap-Shots of Dynamic Many-Filament Patterns

luminescence radiation distribution in the discharge plane



(a) rotating hexagonal pattern with "point-defects"; (b) travelling hexagonal pattern with a "grain boundary"; (c) superlattice with hexagonal symmetry; (d) pattern consisting of domains made of filaments; (e) pattern in the form of dynamical clusters ("molecular gas"); (f, g) generation of filaments at the boundary and annihilation while travelling to the centre, short (f) and long (g) exposure time; (h) filaments travelling on concentric rings with constant angular velocity, neighbouring rings may rotate in opposite direction

Experimental Quasi 2-Dimensional AC Gas-Discharge Systems: Travelling Filaments and Annihilation (Movie)

luminescence radiation distribution in the discharge plane

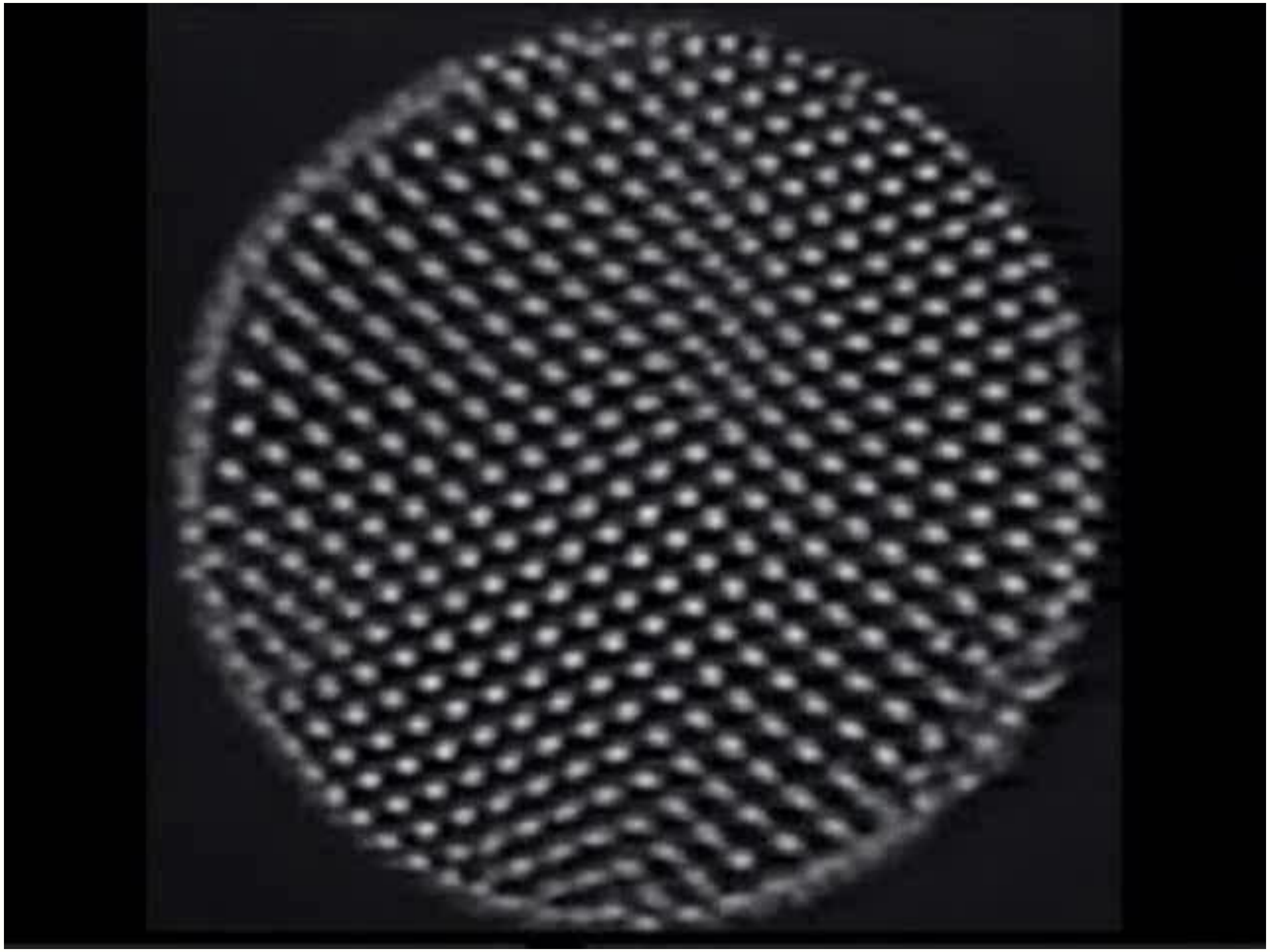
if not linked: start movie "Spots and their Interaction.wmv" In the folder

Brauer, PhD Thesis, Universität Münster (2000)

Experimental Quasi 2-Dimensional AC Gas-Discharge Systems: Dynamic Hexagonal Filament Pattern (Movie)

luminescence radiation distribution in the discharge plane

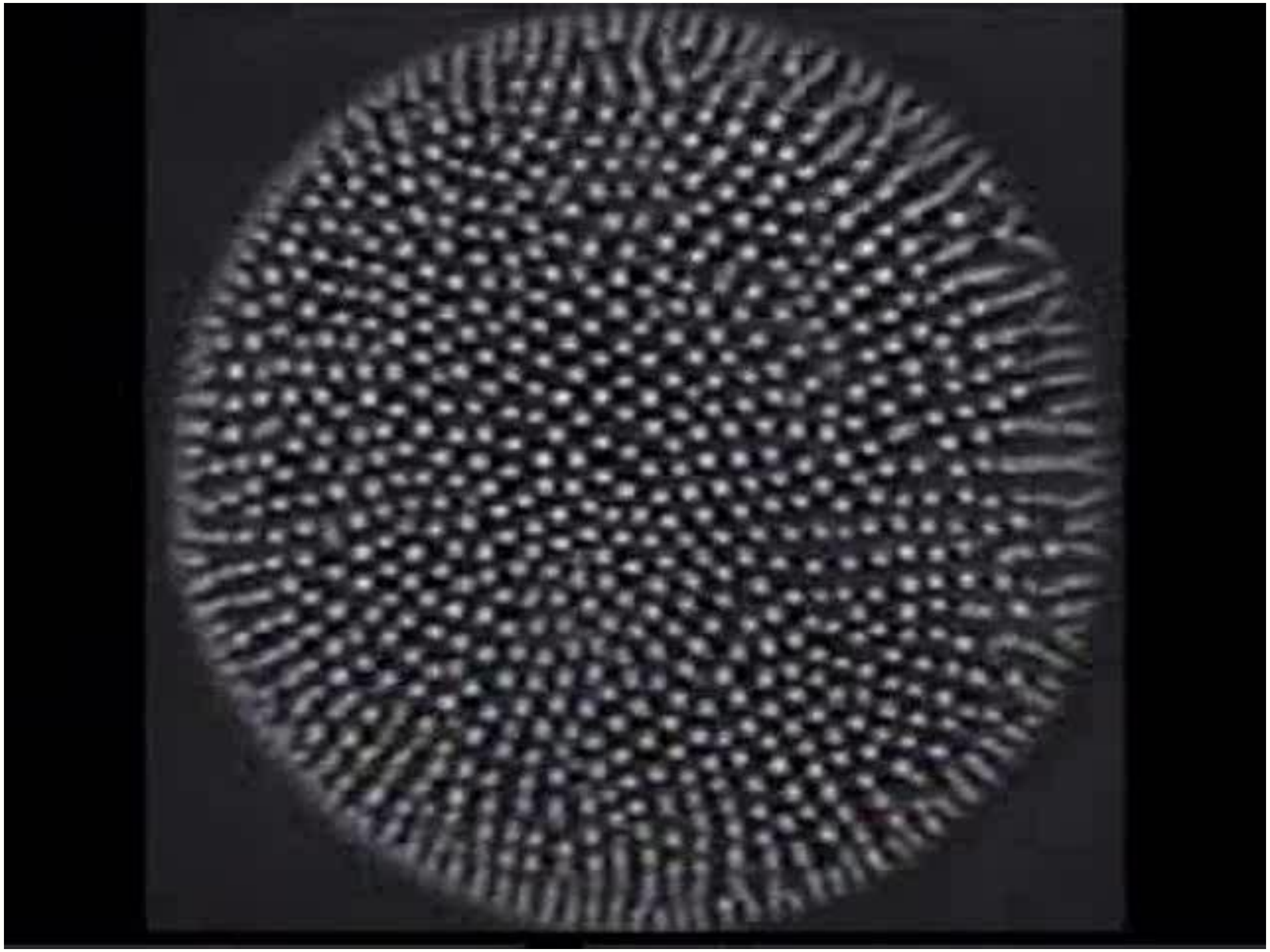
if not linked: start movie “Interaction of Filaments at High Density.wmv” in the folder



Experimental Quasi 2-Dimensional AC Gas-Discharge Systems: Many-Filament Pattern with Generation and Annihilation (Movie)

luminescence radiation distribution in the discharge plane

if not linked: start movie "Generation and Annihilation.wmv" in the folder



Ammelt et al. (1997), unpublished;
see also http://www.uni-muenster.de/Physik.AP/Purwins/Research-Summary

Experimental Quasi 2-Dimensional AC Gas-Discharge Systems: Dynamic Many-Filament Pattern with Domain Structure (Movie)

luminescence radiation distribution in the discharge plane

start movie from folder "Domains of High Density Of Filament Patterns.wmv"

Ammelt et al. (1997), unpublished;
see also http://www.uni-muenster.de/Physik.AP/Purwins/Research-Summary

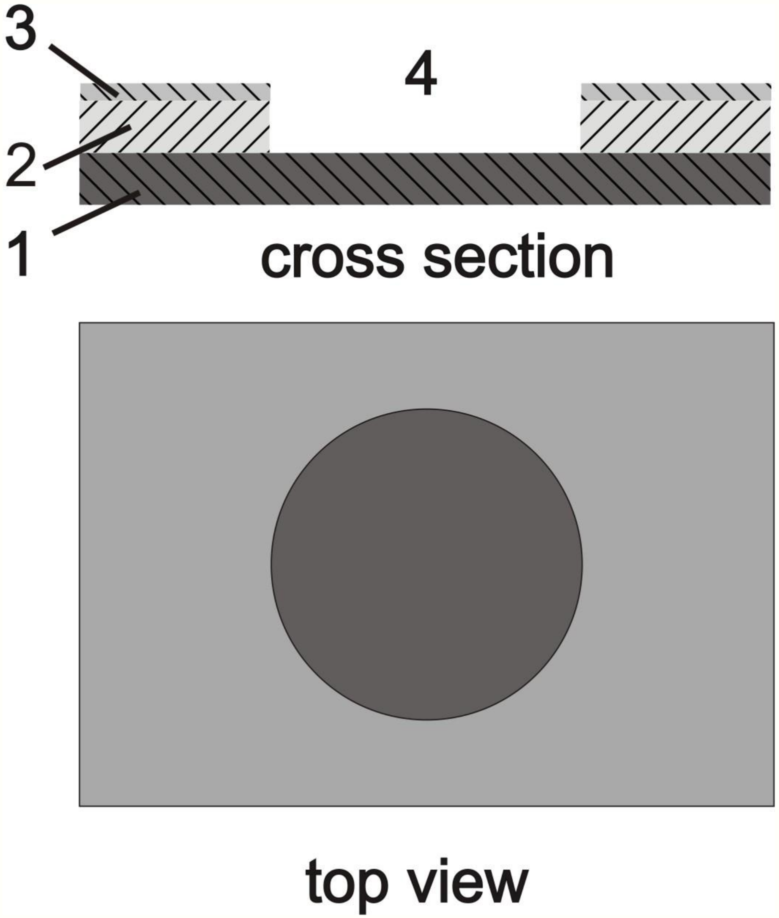
Experimental Quasi 2-Dimensional AC Gas-Discharge Systems: Dynamic Many-Filament Pattern Resembling a “Molecule Gas” (Movie)

luminescence radiation distribution in the discharge plane

if not linked: start movie “Interaction of Clusters of High Density.wmv” in the folder

Ammelt et al. (1997), unpublished;
see also http://www.uni-muenster.de/Physik.AP/Purwins/Research-Summary

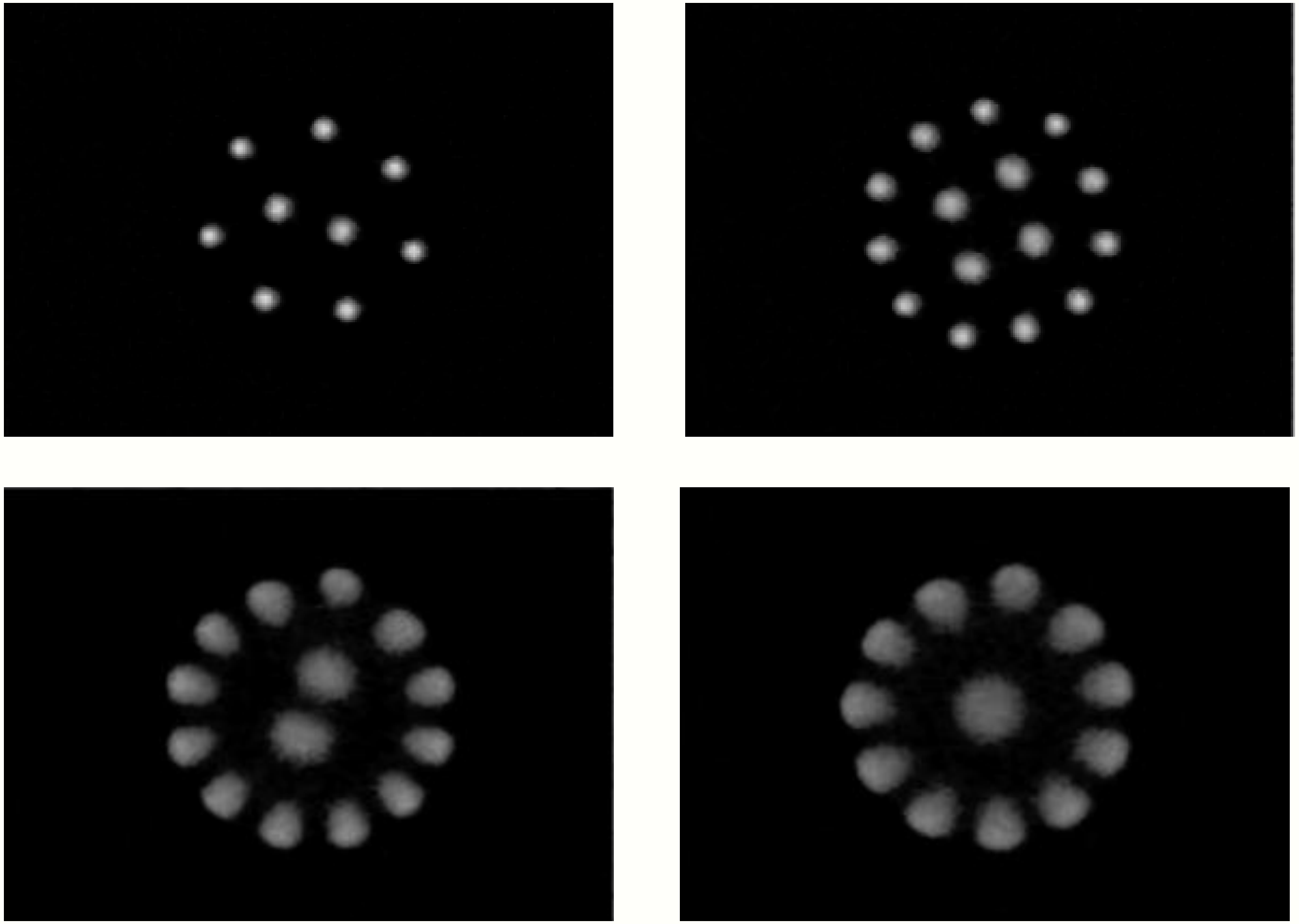
Cathode Spots in Gas-Discharge Systems I: Experimental Set-Up



1. cathode
2. dielectric spacer
3. hollow anode
4. gas space

Schoenbach et al., Plasma Sources Sci. Technol. 13 p. 177 (2004)

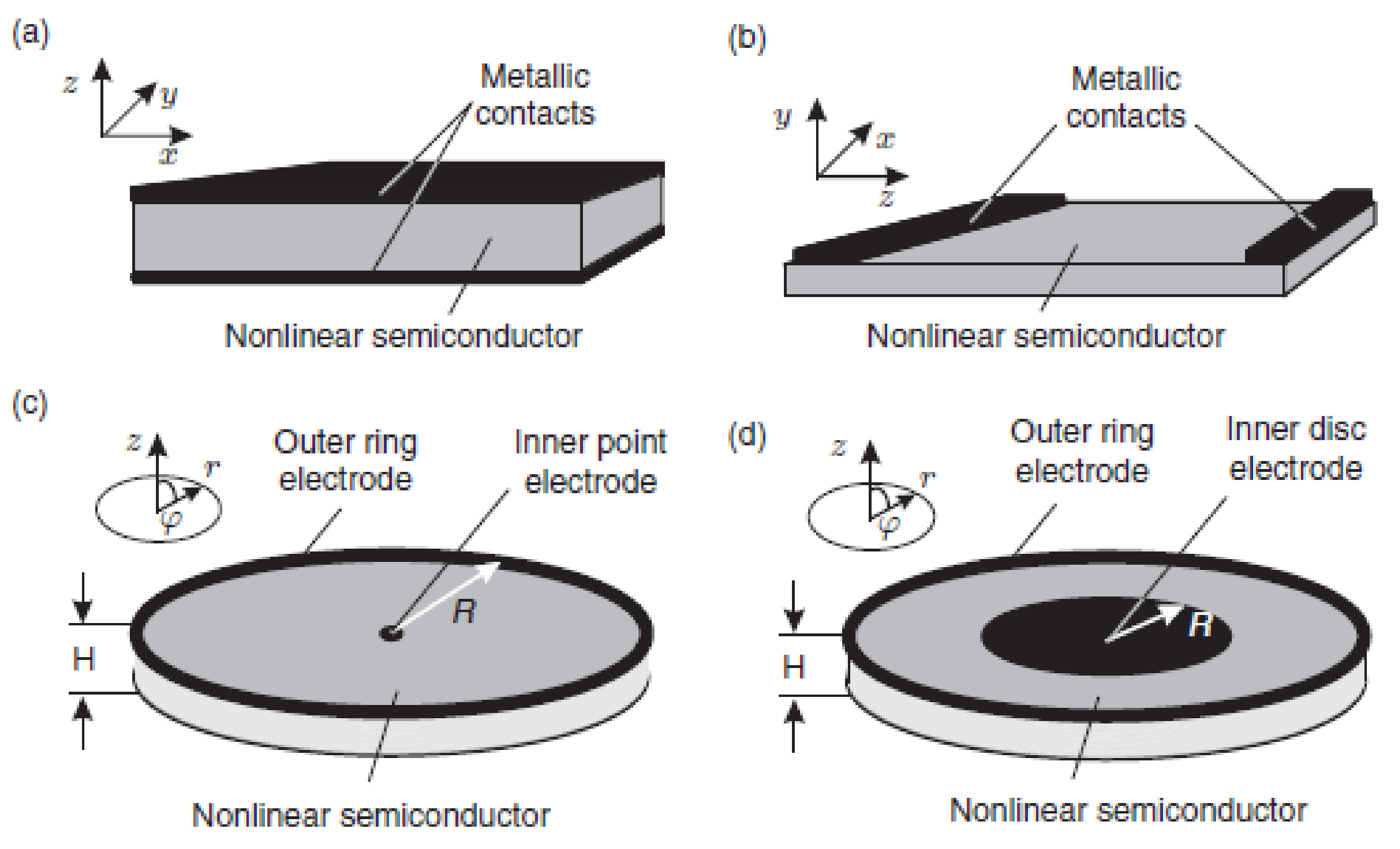
Cathode Spots in Gas-Discharge Systems II: Observed Spot Patterns



Schoenbach et al., Plasma Sources Sci. Technol. 13 p. 177 (2004)

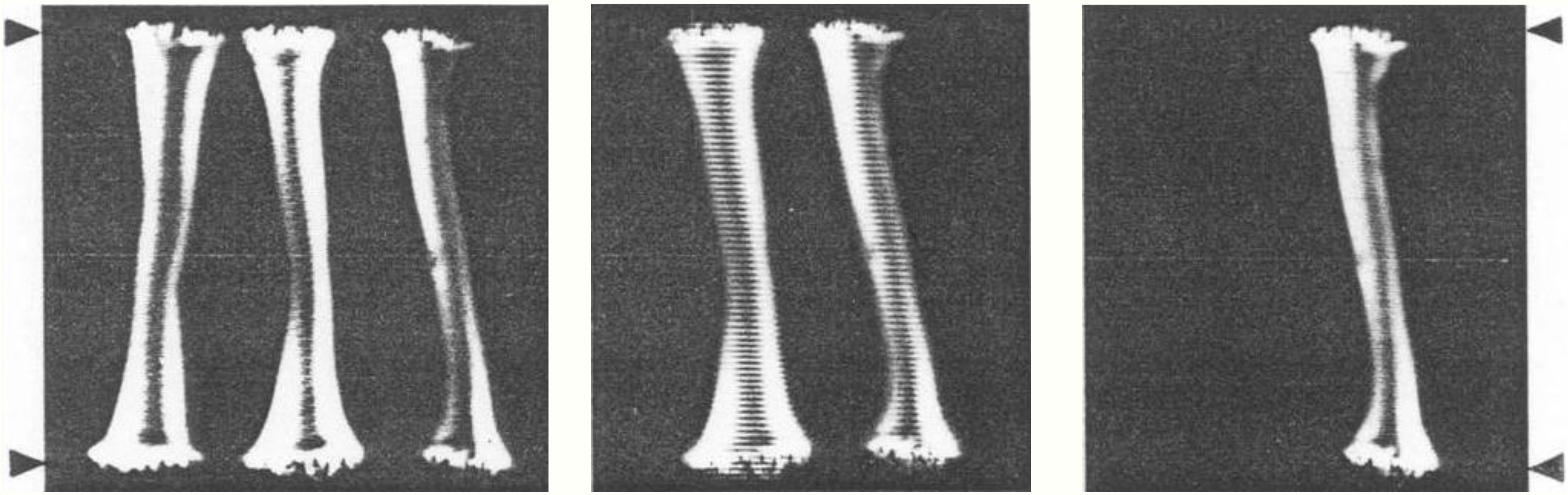
7. Experimental Results on Dissipative Solitons (DS) in Semiconductors

Current Filaments in Semiconductors I: Experimentally Investigated Configurations



Currnt Filaments in Semiconductors II: Bifurcation Cascade of Stationary Current Filaments for Decreasing Driving Voltage

electron microscope image of a film of n-GaAs at T = 4.2 K



for Decreasing Driving Voltage in a Film

Mayer, et al., Z. Phys. B Condensed Matter 71 p. 171 (1988)

8. Quantitative Modelling of AC Gas-Discharge by Drift-Diffusion Equations

Typical Time Scales for Low Temperature Gas Discharge Systems (He, p ≈ 100hPa, T_e = 1eV) I

time scale	value	comments
times for electrons		
• free path times	10^{-12}s	average time between collisions
• drift time	10^{-9}s	travel time between electrodes
• diffusion time	10^{-7}s	diffusion over electrode gap
times for ions		
• free path times	10^{-9}s	average time between collisions
• drift time	10^{-7}s	travel time between electrodes
• diffusion time	10^{-5}s	diffusion over electrode gap
times in experiment		
• ac driver period	≈ 10^{-5}s	
• transient time	≲ 10^{-1}s	

Drift-Diffusion Model for the Planar DC Semiconductor Gas-Discharge System I: Equations for the Gas

in the gas:

$$\partial_t \mathbf{n}_{e,i} = -\,\mathrm{div}\vec{\Gamma}_{e,i} + \mathbf{S}_{e,i},$$

$$\vec{\Gamma}_{e,i} = \mp \mu_{e,i}(\mathbf{E})\mathbf{n}_{e,i}\vec{\mathbf{E}} - \mathbf{D}_{e,i}\nabla \mathbf{n}_{e,i},$$

$$\Delta\varphi = (\mathbf{n}_i - \mathbf{n}_e)(|\mathbf{e}| / \varepsilon_0\varepsilon),$$

$$\vec{\mathbf{E}} = \nabla\varphi$$

$$\mathbf{S}_e = \mathbf{S}_i = \alpha(\mathbf{E})(\vec{\Gamma}_e / \mathbf{e})$$

Punset et al., J. Appl. Phys. 86 p. 124 (1999)

Drift-Diffusion Model for the Planar DC Semiconductor Gas-Discharge System II: Boundary Gas – Semiconductor at z=0

$$\partial_t \sigma - D_S \Delta\sigma = \vec{e}_z (\vec{j}_g - \vec{j}_{SC})_{z=0},$$

$$\frac{1}{\varepsilon_0}\sigma = (\varepsilon \vec{e}_z \vec{E})_{z=+0} - (\vec{e}_z \vec{E})_{z=-0},$$

$$(\vec{\Gamma}_p \vec{e}_z)_{z=-d} = (\mu_p n_p \vec{E}\vec{e}_z + \frac{1}{4} n_p \langle v_p \rangle)_{z=-0},$$

$$(\vec{\Gamma}_e \vec{e}_z)_{z=-0} = (-\mu_e n_e \vec{E}\vec{e}_z + \frac{1}{4} n_e \langle v_e \rangle - \gamma \vec{\Gamma}_p \vec{e}_z)_{z=-0},$$

$$\langle v_{e,p} \rangle = \sqrt{8kT_{e,p} / \pi m_{e,p}}.$$

$$(\varphi)_{z=-d} - (\varphi)_{z=d_{SC}} = U.$$

σ surface charge, dependence on surface properties

D_S diffusion constant of surface charge

$\vec{e}_z$ unity vector in z-direction

ε dielectric constant of the semiconductor

v_e, v_p thermal electron/ion speed

γ γ -Townsend-coefficient

k Boltzmann-constant

m_e, m_p electron/ion mass

U voltage drop at the component

z=0 gas/semiconductor surface

z=-d metallic anode

Punset et al., J. Appl. Phys. 86 p. 124 (1999)

Drift-Diffusion Model for the Planar DC Semiconductor Gas-Discharge System III: Semiconductor Wafer ($0 < z < d_{SC}$)

$$\vec{j}_{SC} = \lambda \vec{E},$$
$$\vec{E} = -\vec{\nabla}\varphi,$$
$$\mathrm{div}(\varepsilon \vec{\nabla}\varphi) = 0.$$

$\vec{j}_{SC}$	global electrical current density
λ	specific electrical conductivity
$\vec{E}$	electrical field
φ	electrical potential
ε	dielectric constant of the semiconductor

Punset et al., J. Appl. Phys. 86 p. 124 (1999)

Drift-Diffusion Model for the Planar AC Semiconductor Gas-Discharge System I: Equations

equations similar to those of the DC system

Numerical Solutions of the Drift-Diffusion Equation for the Quasi-2-Dimensional AC Gas-Discharge System Hexagonal Pattern II: Parameters

experimental (theoretical) parameters

gas: He

p = 300hPa (300)

a = 0.5mm (0.5)

d = 0.5mm (0.5)

D = 8mm (8)

f_{sin}=200kHz (200)

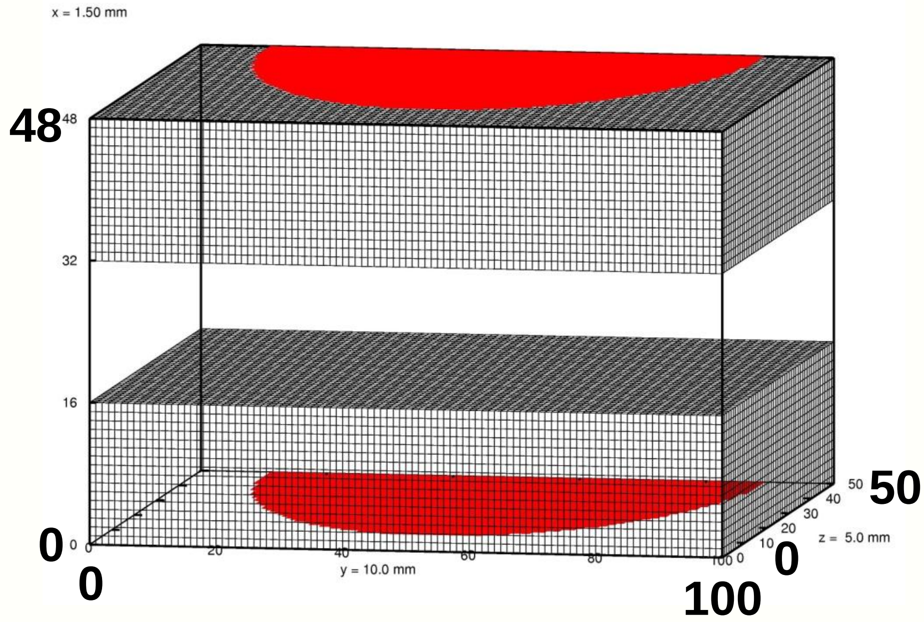
Û=675V (700)

t_{exp}=2μs

(γ= 0,05)

(α, μ from tables)

Numerical parameters



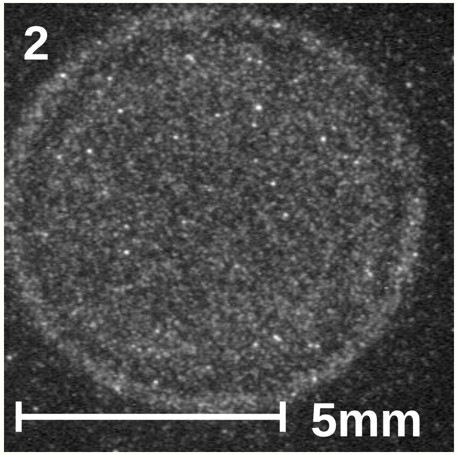
grid points 100 x 50 x 48

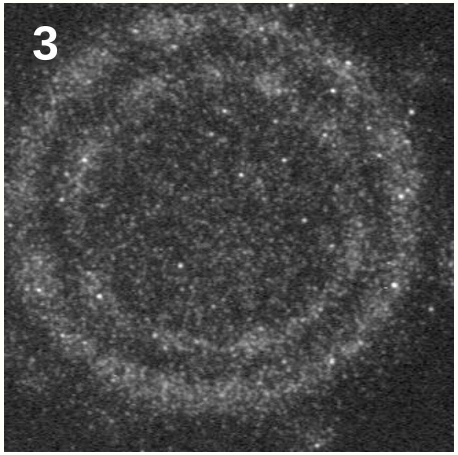
10mm x 5mm x 1,5mm

Neumann boundary conditions

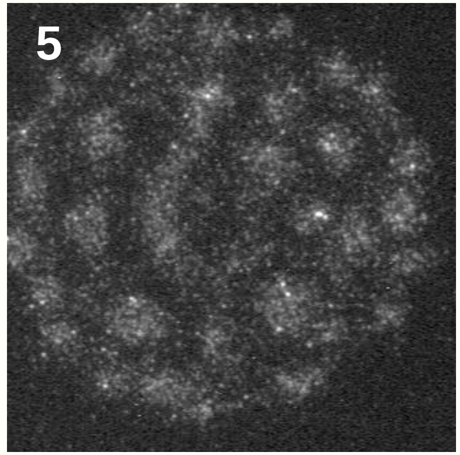
Stollenwerk et al., Phys. Rev. Lett. 96, p. 255001-1 (2006)

Numerical Solutions of the Drift-Diffusion Equation for the Quasi-2-Dimensional AC Gas-Discharge System Hexagonal Pattern III: Experiment versus Theory (a)

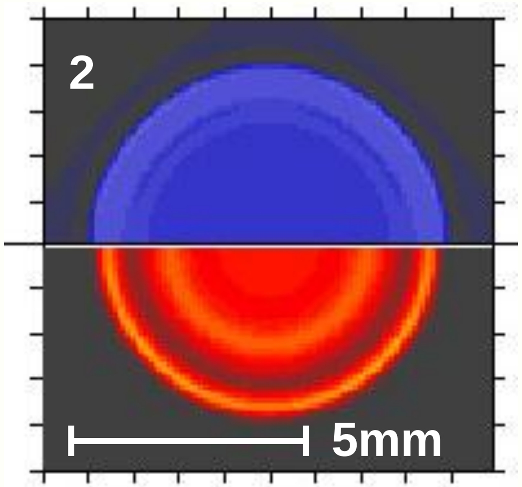




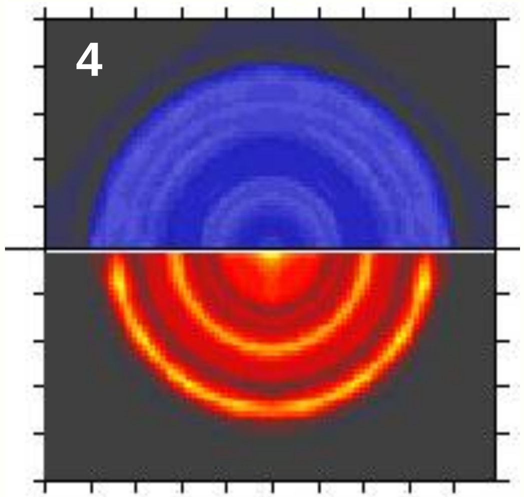
experiment



increasing number of breakdowns



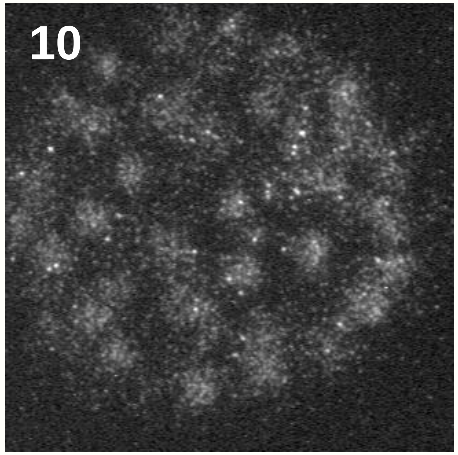
theory



Stollenwerk et al., Phys. Rev. Lett. 96, p. 255001-1 (2006)

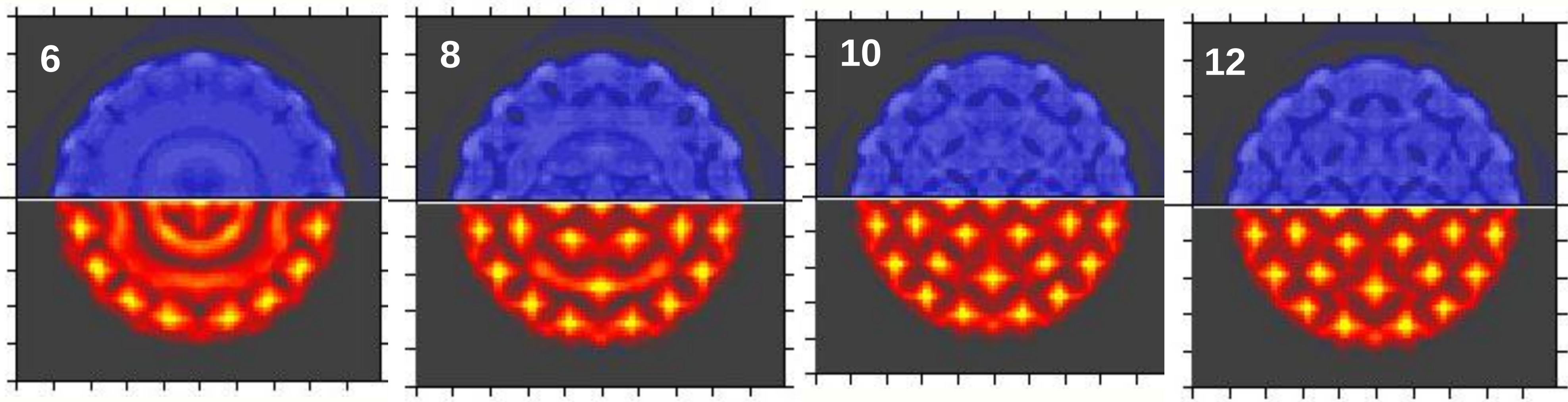
Numerical Solutions of the Drift-Diffusion Equation for the Quasi-2-Dimensional AC Gas-Discharge System Hexagonal Pattern IV: Experiment versus Theory (b)

experiment →



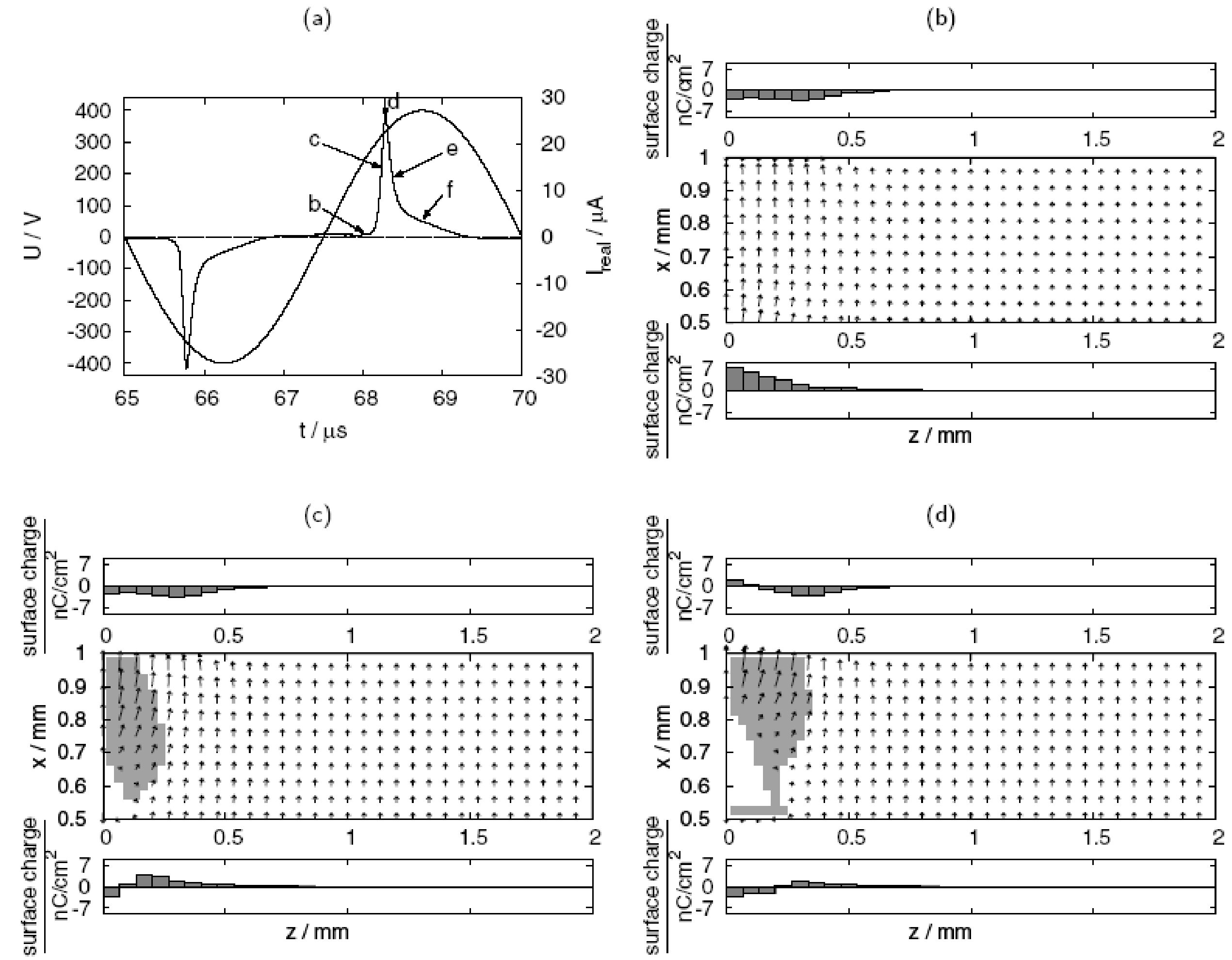
theory ↘

—— increasing number of breakdowns ——→



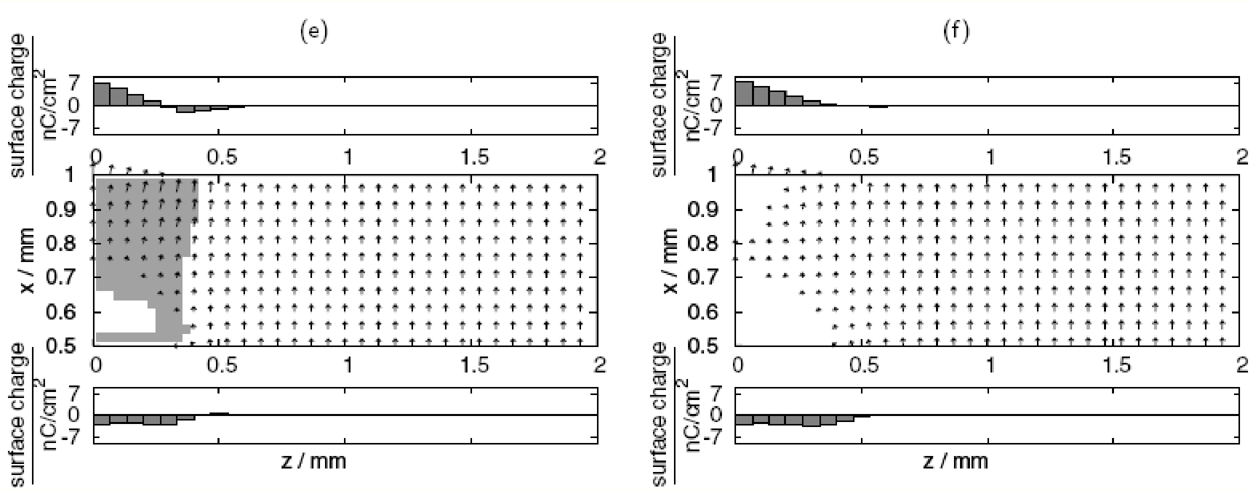
Stollenwerk et al., Phys. Rev. Lett. 96, p. 255001-1 (2006)

Numerical Solutions of the Drift-Diffusion Equation for the Quasi-2-Dimensional AC Gas-Discharge System: Evolution of a Stationary Filament I



Stollenwerk et al., Eur. Phys. J. D 44, p. 133 (2007)

Numerical Solutions of the Drift-Diffusion Equation for the Quasi-2-Dimensional AC Gas-Discharge System: Evolution of a Stationary Filament II

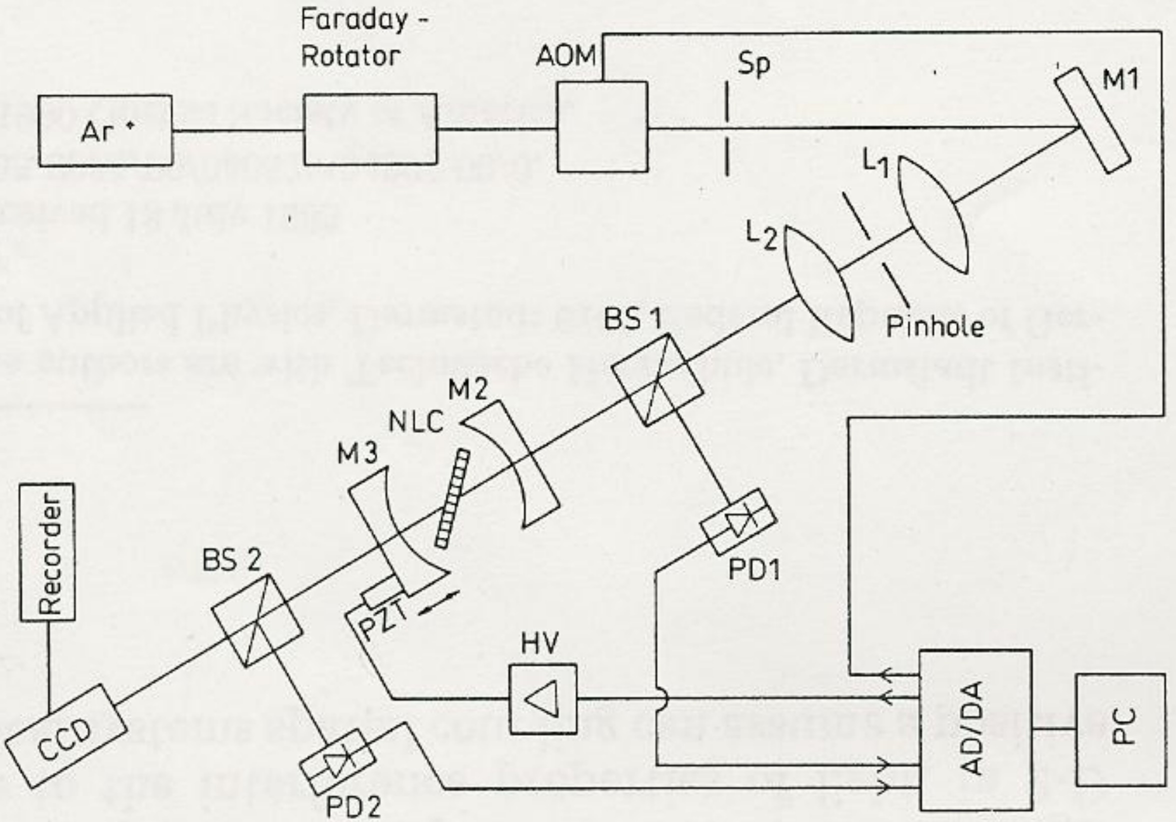


Stollenwerk et al., Eur .Phys.J. D 44, p. 133 (2007)

9. Various Dissipative Soliton Carrying Experimental Systems:

Other than Electrical Transport Systems

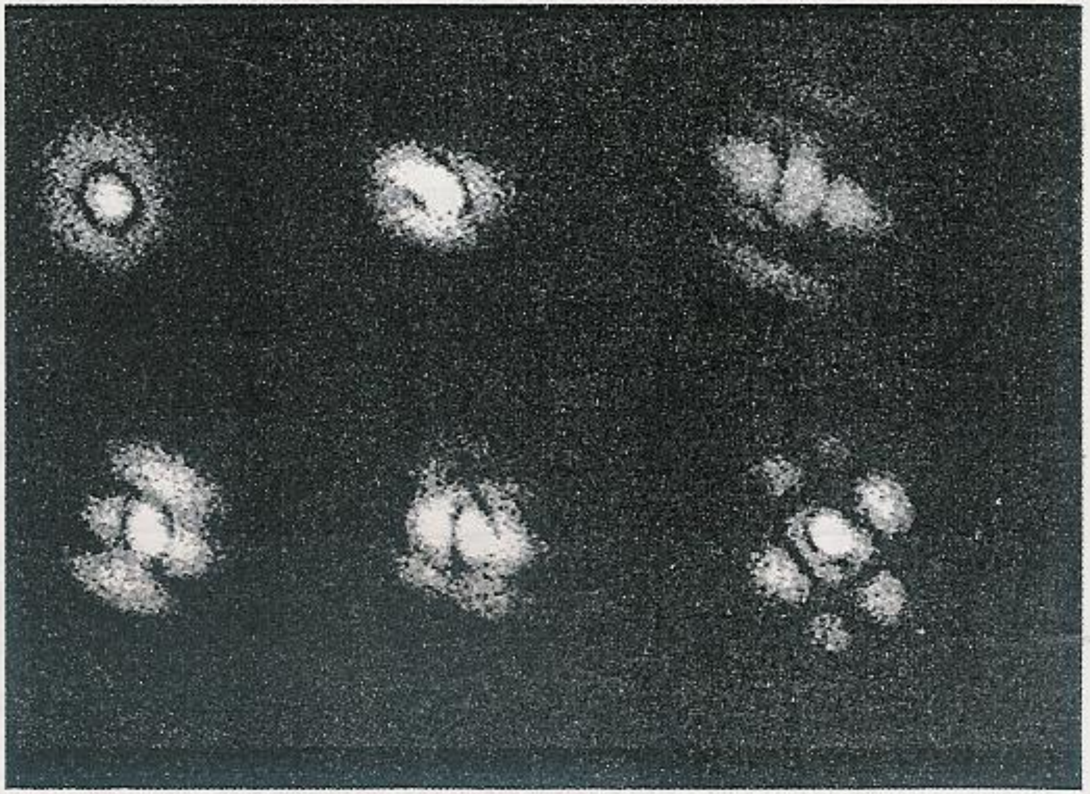
Optical Filaments in a Fabry-Perot Resonator with a Liquid Crystal as Kerr Medium I: Experimental Set-Up



Experimental setup.

Kreuzer, et al., Applied Optics 29, p. 579 (1990)

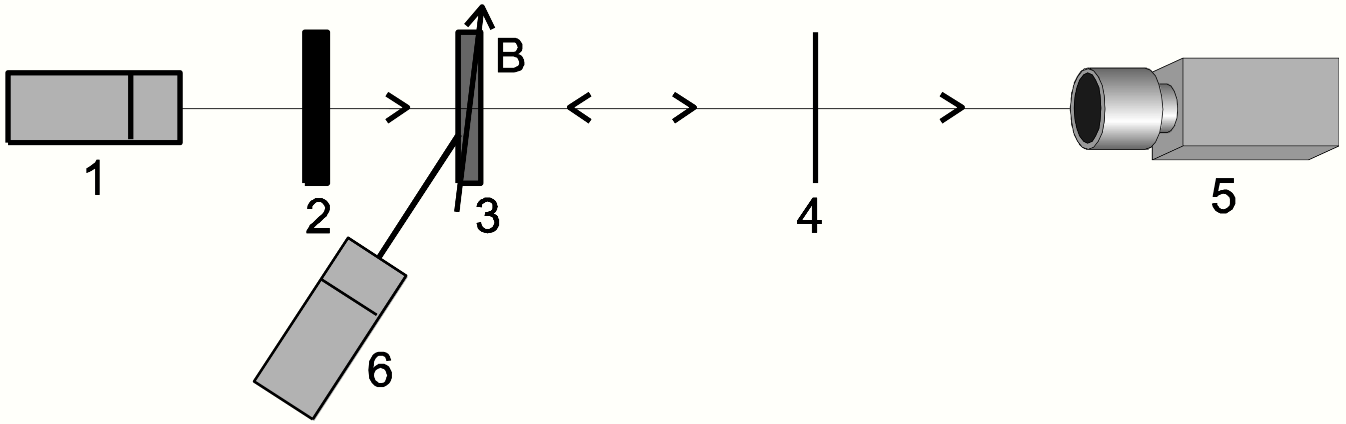
Optical Filaments in a Fabry-Perot Resonator with a Liquid Crystal as Kerr Medium II: Observation of Localized Structures



Sequence of patterns from a structure-forming bistable element.

Kreuzer, et al., Applied Optics 29, p. 579 (1990)

Optical Filaments in a Single Mirror Feedback Device with Na Vapour as Nonlinear Medium I: Experimental Set-Up



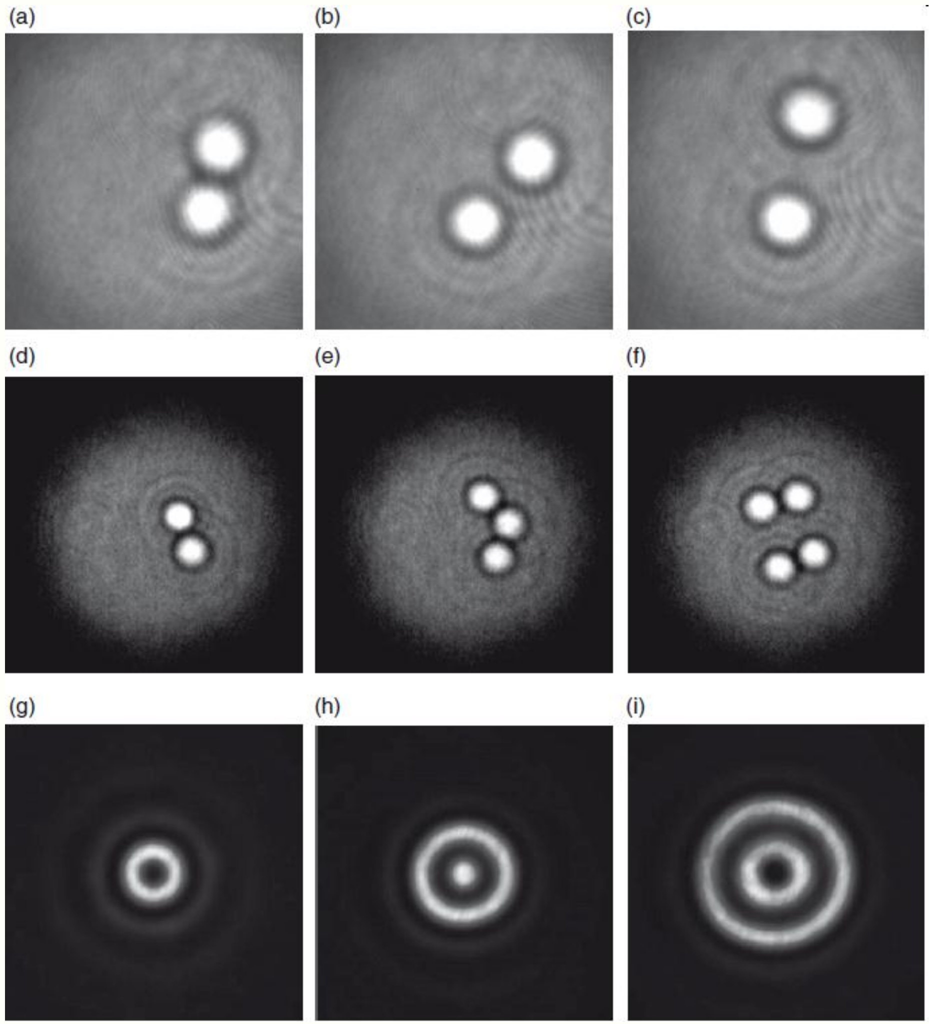
1. laser, 2. polarizing element, 3. cell with Na vapour beeing subject to a magnetic field, 4. semitransparent mirror, 5. camera, 6. laser for control beam

Optical Filaments in a Single Mirror Feedback Device with Na Vapour as Nonlinear Medium II: Multistability of Localized Structures

at a given set of parameters filaments may lock in at different distances forming clusters („molecules“)

at a given set of parameters different number of can exist

at a given set of parameters different kinds of filament can exist



Schäpers, PhD Thesis, Universität, Münster (2007);
Pesch, PhD Theses, Universität, Münster (2006)

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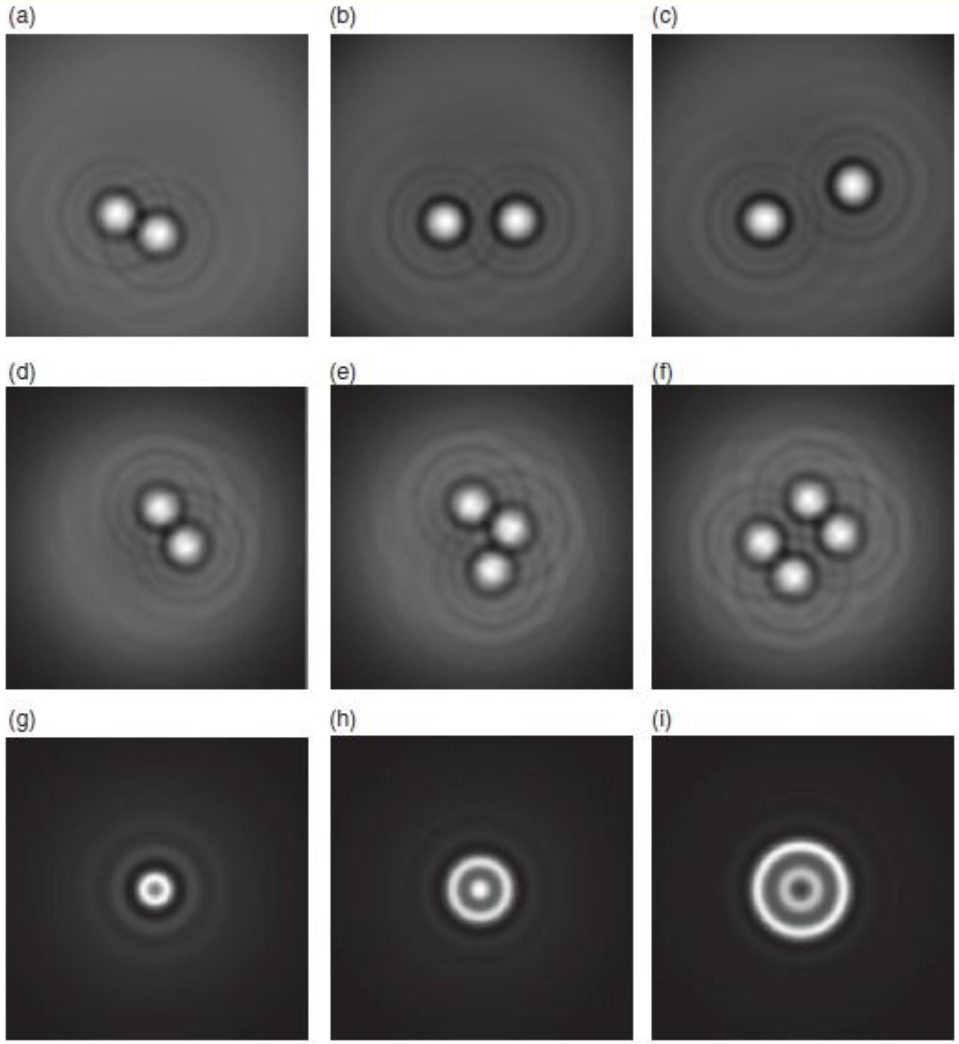
Optical Filaments in a Single Mirror Feedback Device with Na Vapour as Nonlinear Medium III: Numerical Solutions of the Model Equations

at a given set of parameters filaments may lock in at different distances forming clusters („molecules“)

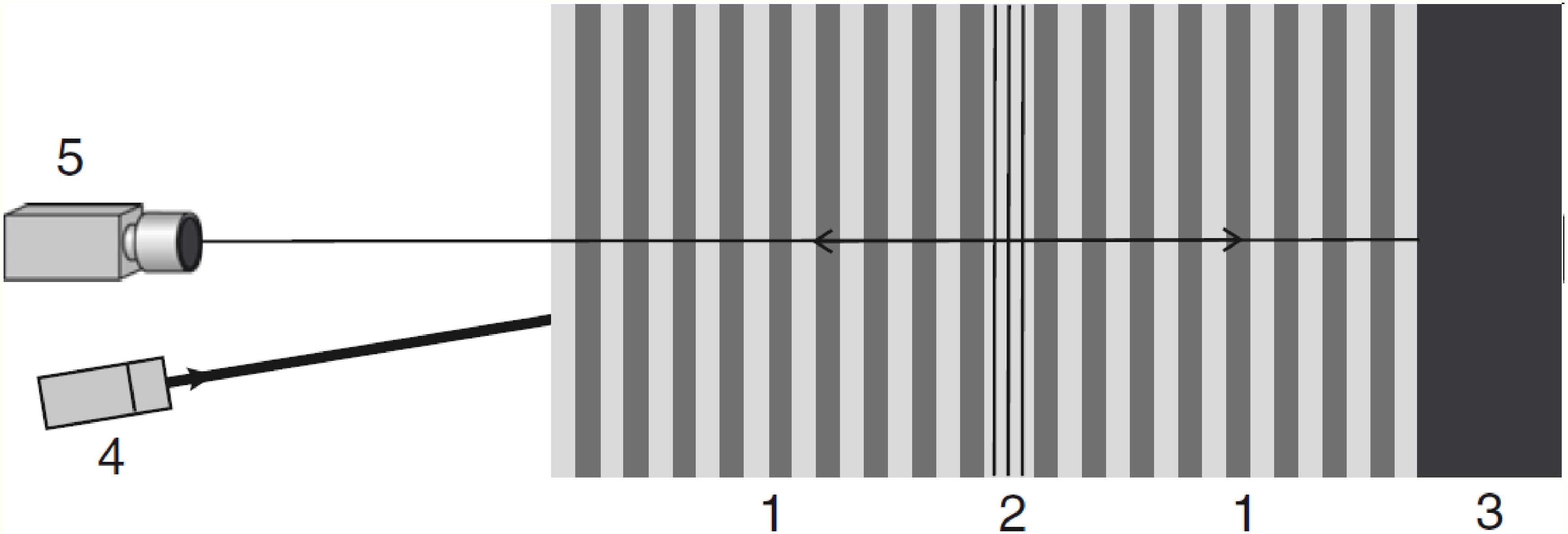
at a given set of parameters different number of can exist

at a given set of parameters different kinds of filament can exist



Schäpers, PhD Thesis, Universität, Münster (2007);
Pesch, PhD Theses, Universität, Münster (2006)

Optical Semiconductor Vertical Cavity Surface Emitting Lasers (VCSELs) I: Experimental Set-Up



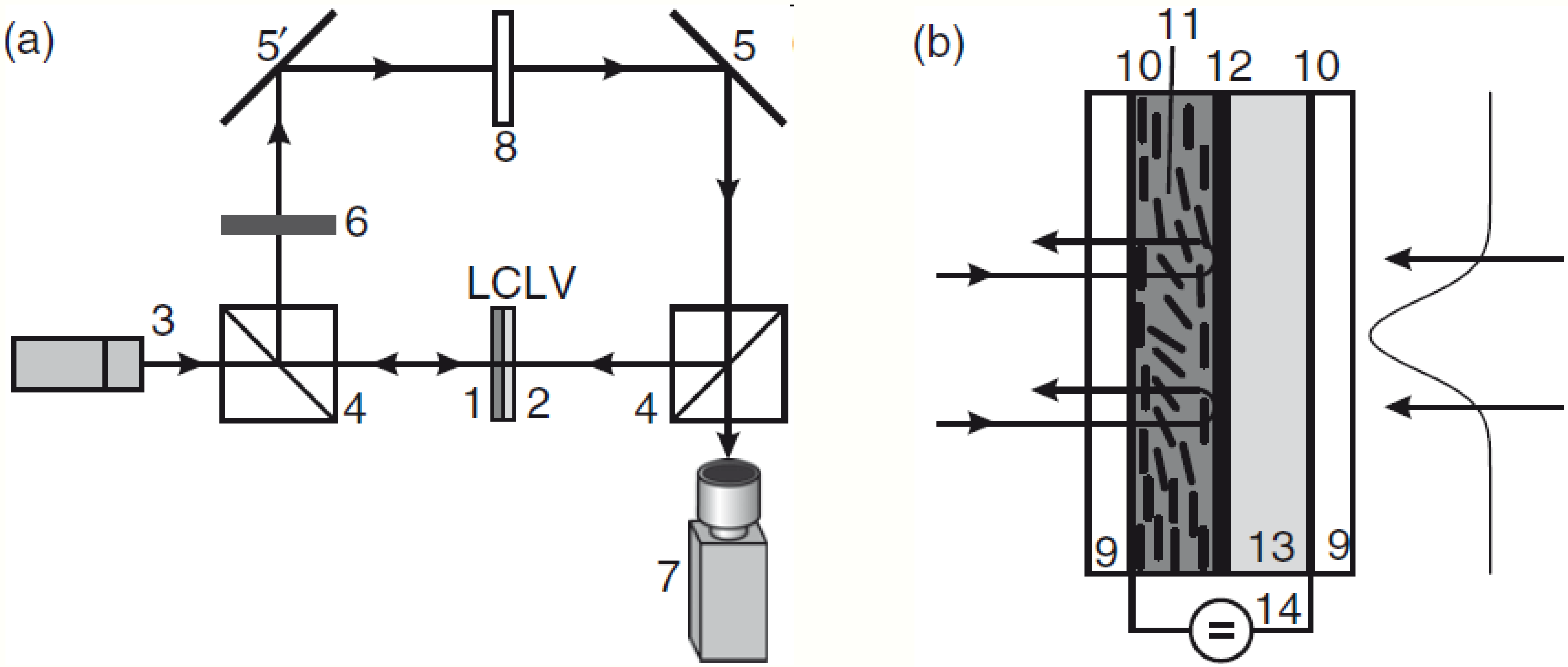
1 Bragg reflectors
2 layer of quantum wells
3 substrate
4 Laser
5 camera

Ackemann et al., Advances in Atomic, Molecular, and Optical Physics 57, p. 323 (2009)

Optical Semiconductor Vertical Cavity Surface Emitting Lasers (VCSELs) II: Stationary Localized structures in the Plane

Pedaci et al. Appl. Phys. Lett. 92, p. 011101 (2008)

Optical Liquid Crystal Light Valves S(LCLVs) I: Experimental Set-Up



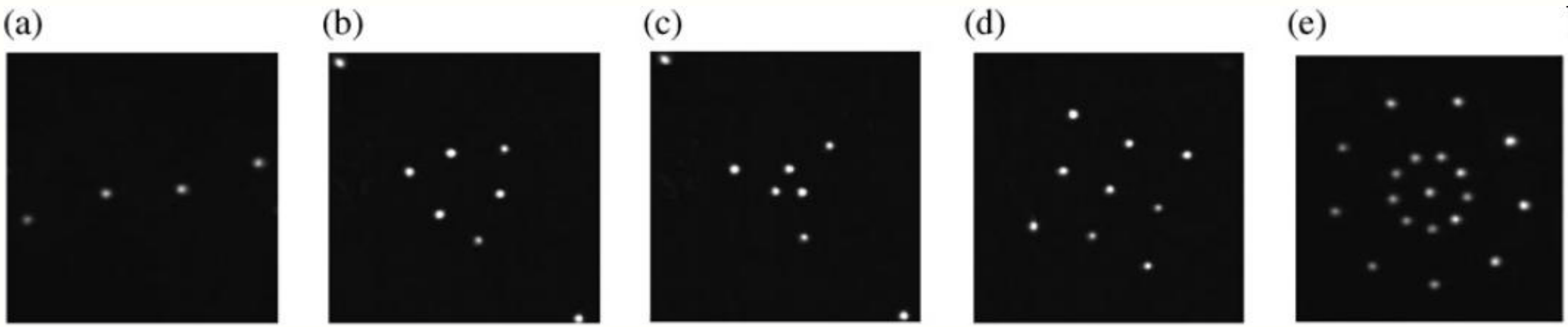
1 write side of the LCLV
2 read side of the LCLV
3 laser
4 beam splitter
5 mirror

6 polarising elements
7 camera
8 additional element for manipulation
9 glass plates

10 electrically cond. layer
11 liquid crystal layer
12 mirror
13 photocond. layer
14 voltage

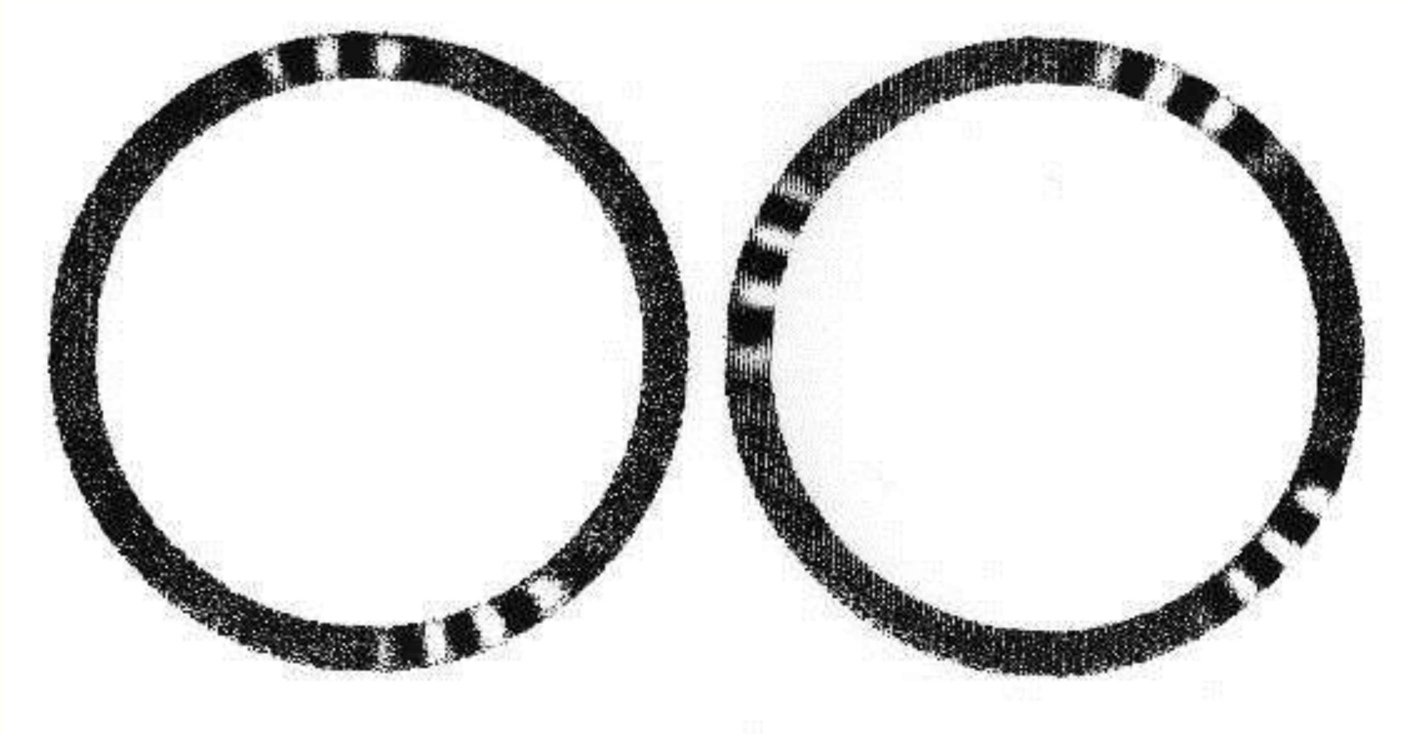
Neubecker et al., Phys. Rev. A 52 p. 791 (1995)

Optical Liquid Crystal Light Valves S(LCLVs) II: Stationary Localized Structures



Residori et al., Physica D 199), p. 149 (2004)

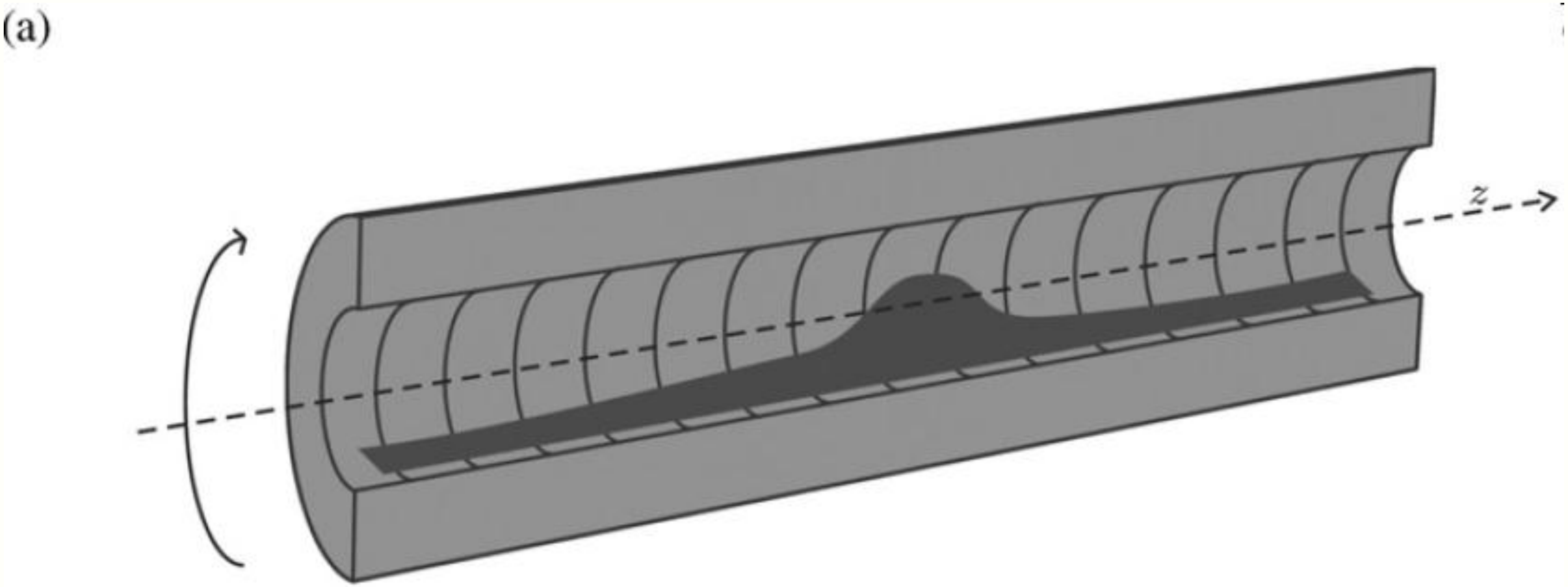
Intermediate Localization of Patterns in Hydrodynamics: Quasi 1-Dimensional Rayleigh-Benard Experiment



localized structures in the convection of an annulus filled with a mixture of ethanol and water and heated from below, the pulses are surrounded by non-convecting fluid

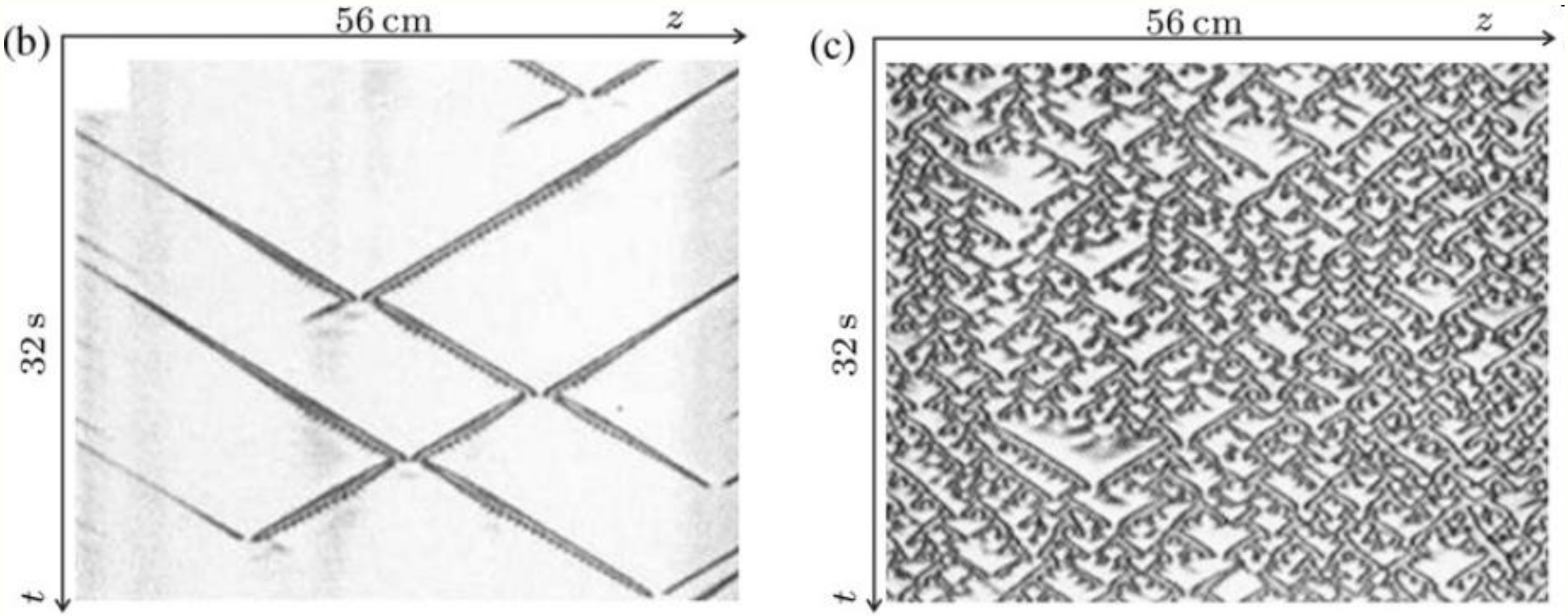
Ahlers, Physica D 51, p. 421 (1991)

Film Dragging in a Rotating Tube Partly Filled with a Fluid I: Experimental Set-Up with Indication of a Localized Structures



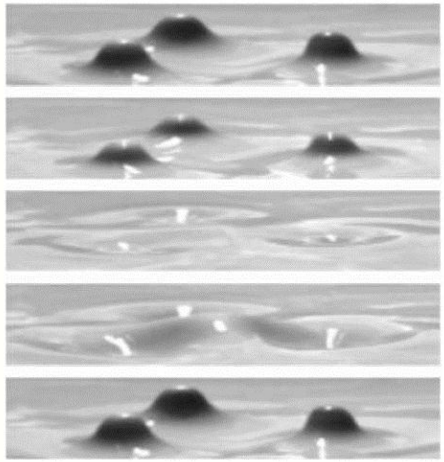
Melo et al., Phys. Rev. Lett. 71, (p. 3283 (1993)

Film Dragging in a Rotating Tube Partly Filled with a Fluid II: Travelling Localized Structures



Melo et al., Phys. Rev. Lett. 71, (p. 3283 (1993)

Vertically Driven Systems: Granular Material and Fluid



vertically driven granular medium

Umbanhowar et al., Nature 382 p. 793. (1996)

vertically driven liquid

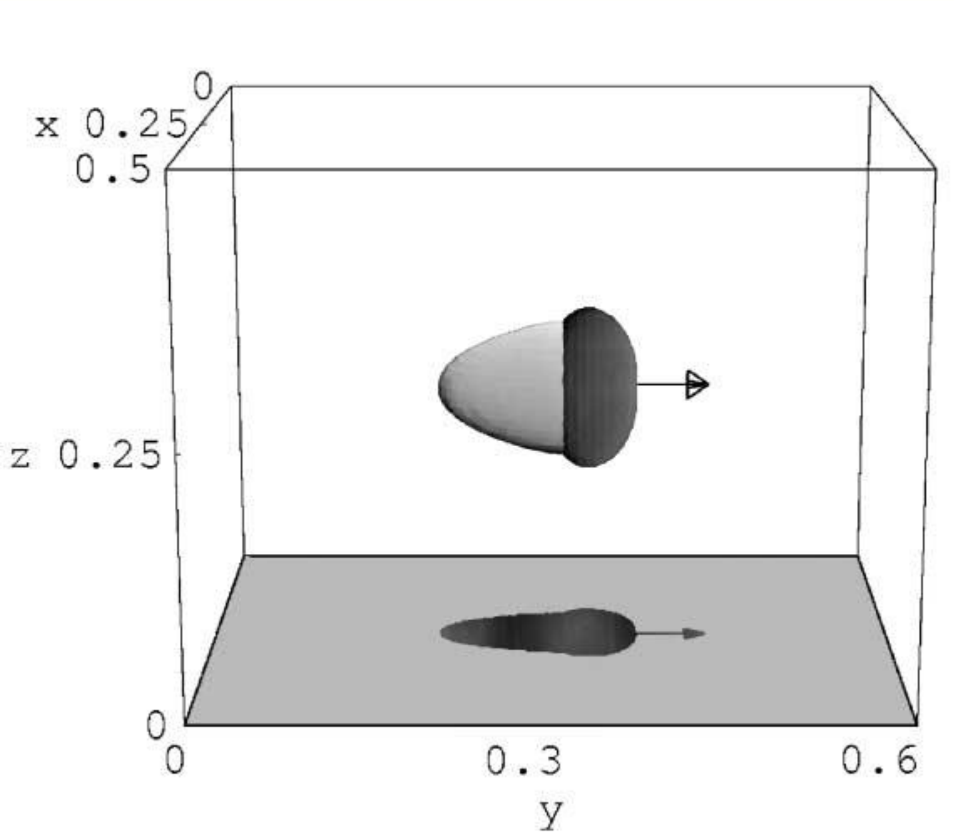
Lioubashevski et al., Phys. Rev. Lett. 83, p. 3190 (1999)

10. Dissipative Soliton (DS) Solutions of the Generalize FitzHugh-Nagumo (FN) Equation In 3-Dimensional Space

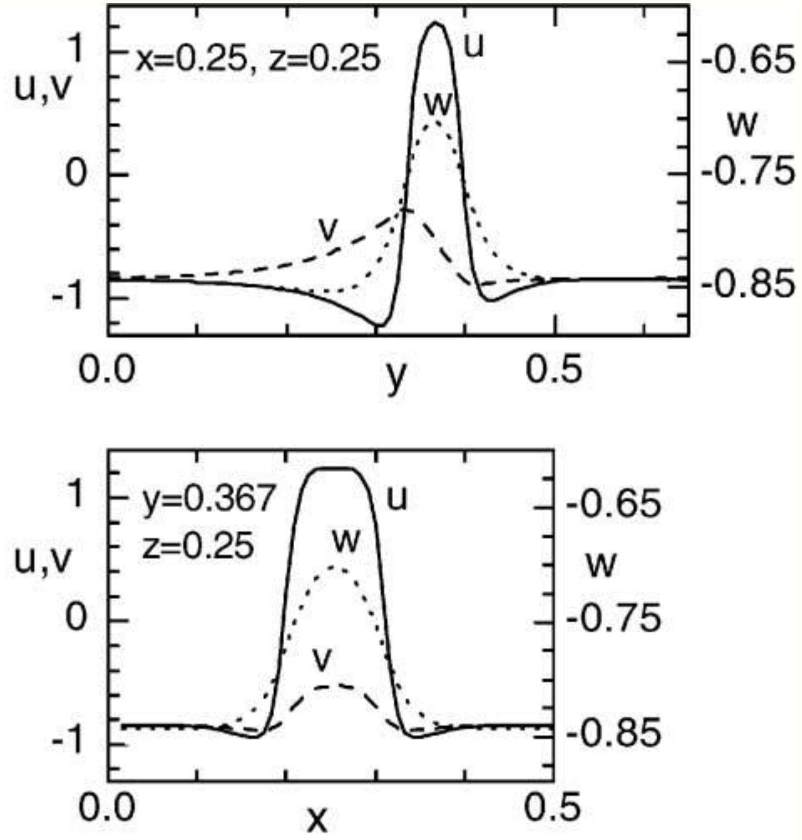
Numerical Solution of the Generalized FN Equation in 3-Dimensional Space: Isolated Travelling DS I

pseudo 3-dimensional representation

longitudinal and transverse cross section



(a)

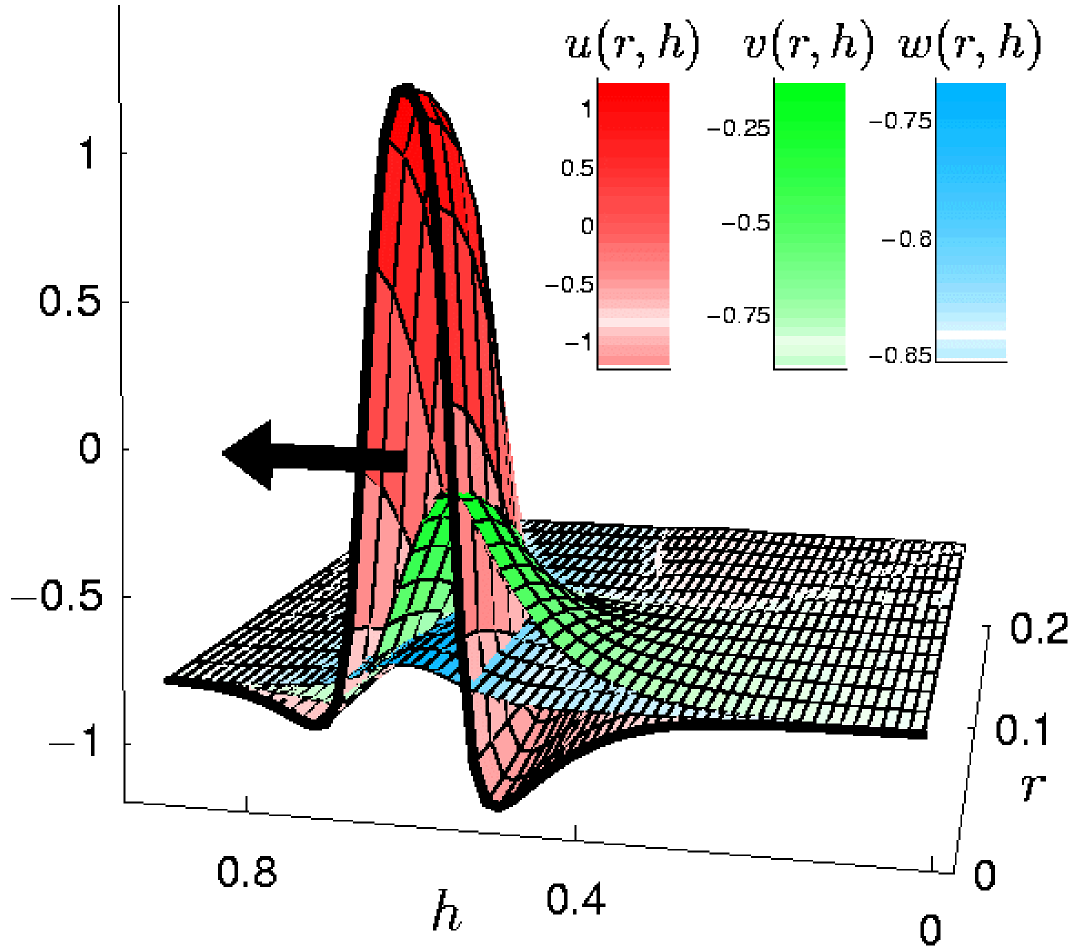


(b)

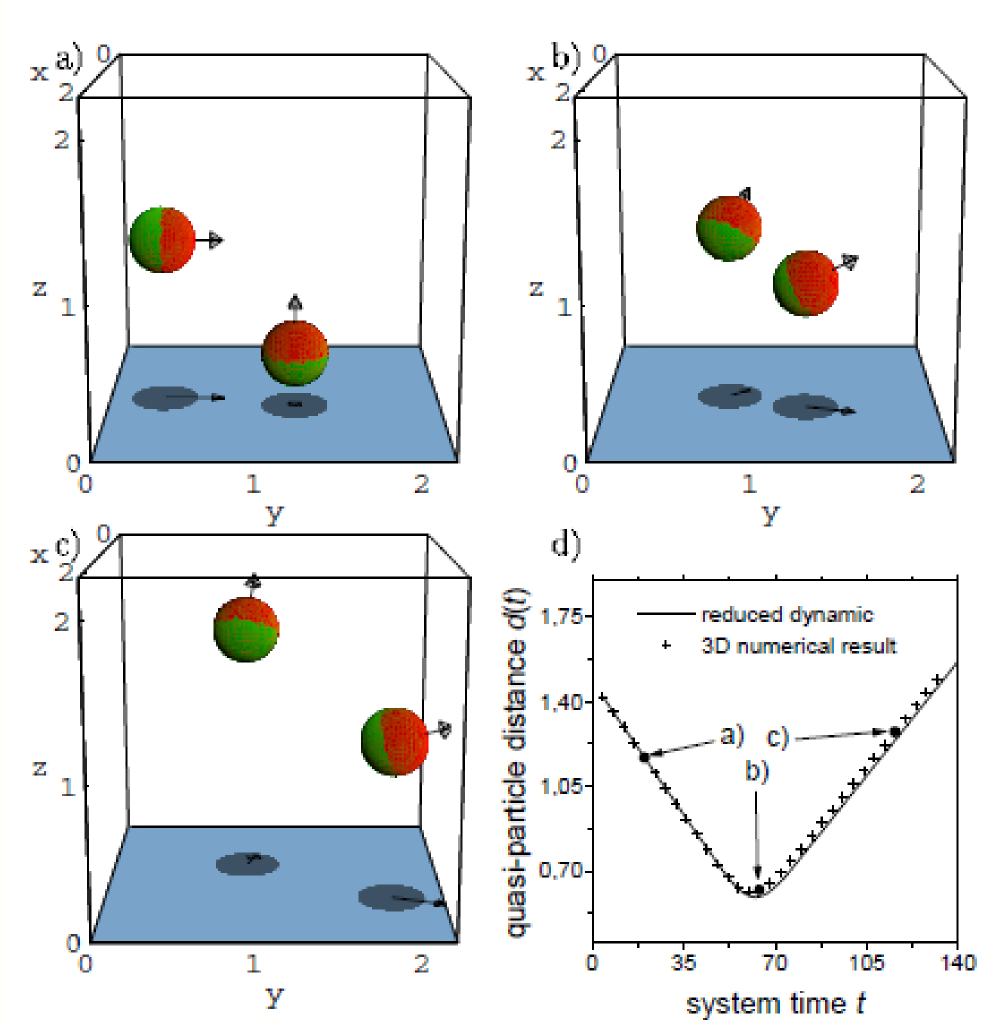
Schenk et al., in: *High Performance Computing in Science and Engineering `99,* E. Krause, W. Jäger (Eds.), p. 354 (2000); Bode et al., Physica D, 161, 45-66 (2002)

Numerical Solution of the Generalized FN Equation in 3-Dimensional Space: Isolated Travelling DS II

pseudo 3-dimensional representation

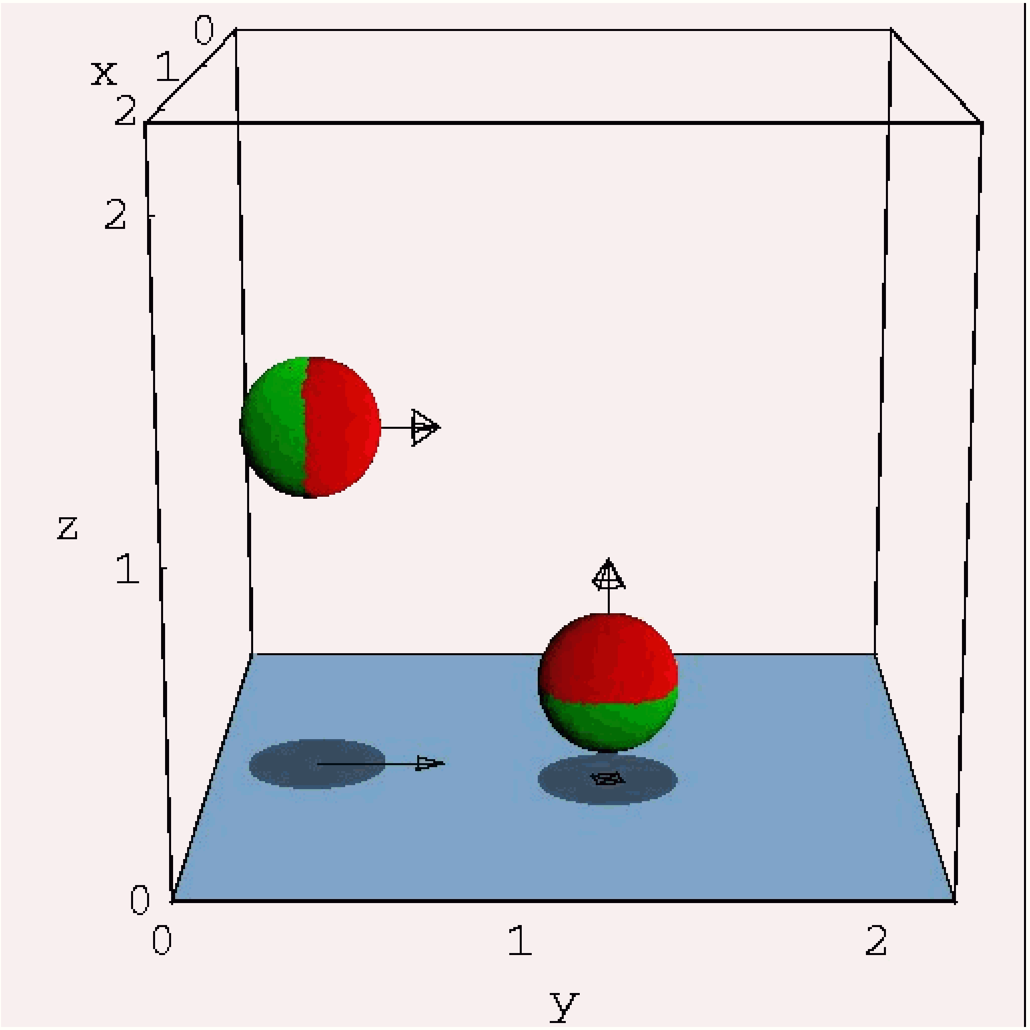


Numerical Solution of the Generalized FN Equation in 3-Dimensional Space: Weak Scattering od 2 DSs I



Liehr et al. in: *High Performance Computing in Science and Engineering 2000,* Krause,E.; Jäger,W. (eds.), Stuttgart (HLRS) (2001); http://www.uni-muenster.de/Physik.AP/Purwins/Research-Summary.

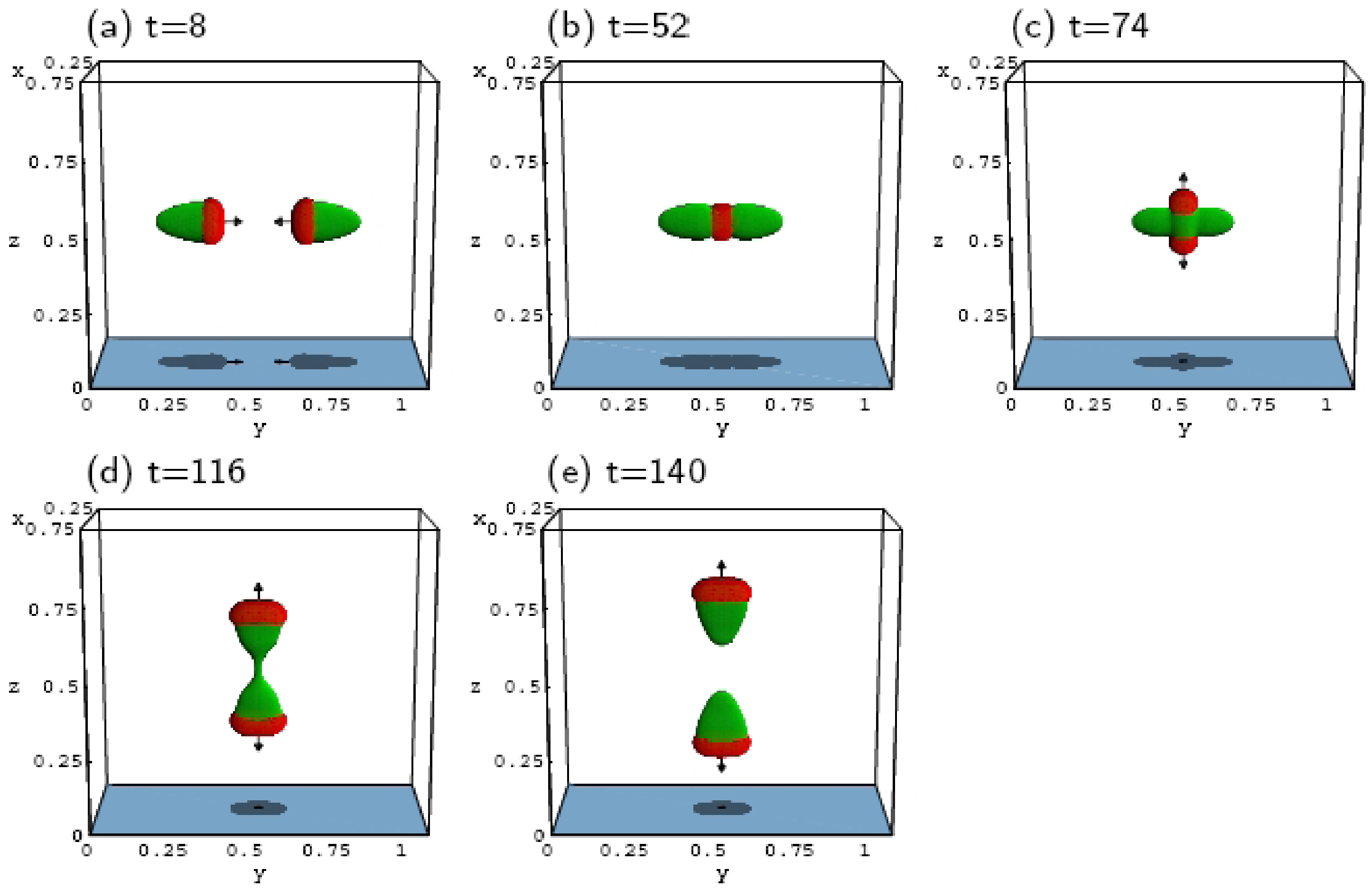
Numerical Solution of the Generalized FN Equation in 3-Dimensional Space: Weak Scattering of 2 DSs II (Movie)



if not linked: start movie "sqt250199a4_ elastischerStoß.avi" in the folder

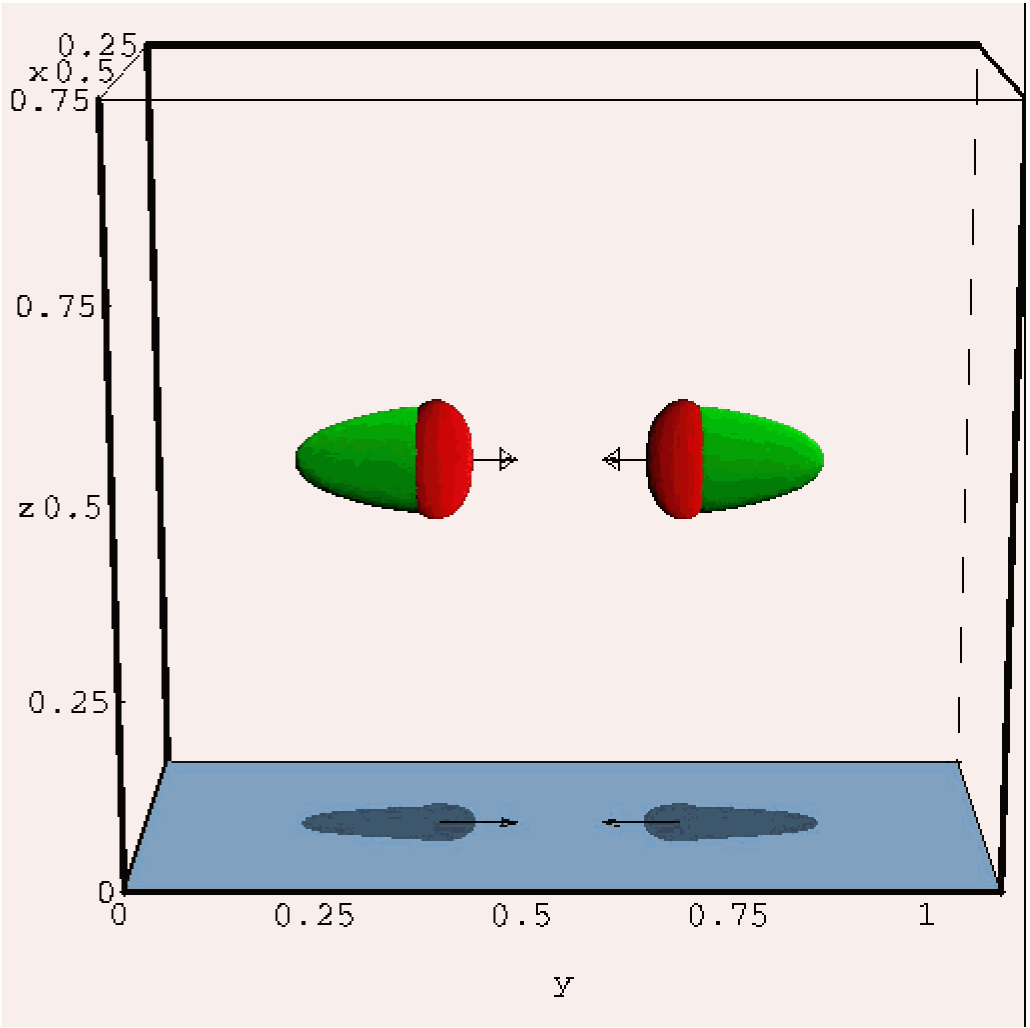
See also: http://www.uni-muenster.de/Physik.AP/Purwins/Research-Summary.

Numerical Solution of the Generalized FN Equation in 3-Dimensional Space: Scattering of 2 DSs with Strong Interaction I



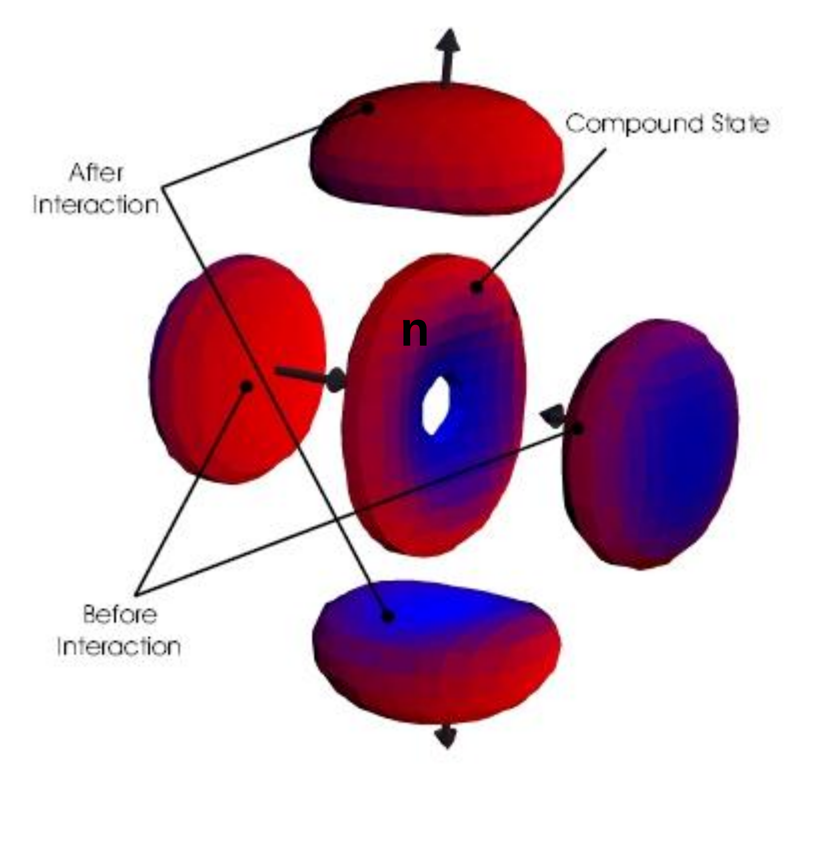
Schenk et al., in: *High Performance Computing in Science and Engineering `99,* E. Krause, W. Jäger (Eds.), p. 354 (2000); Bode et al., Physica D, 161, 45-66 (2002); http://www.uni-muenster.de/Physik.AP/Purwins/Research-Summary.

Numerical Solution of the Generalized FN Equation in 3-Dimensional Space: Scattering of 2 DSs with Strong Interaction II (Movie)



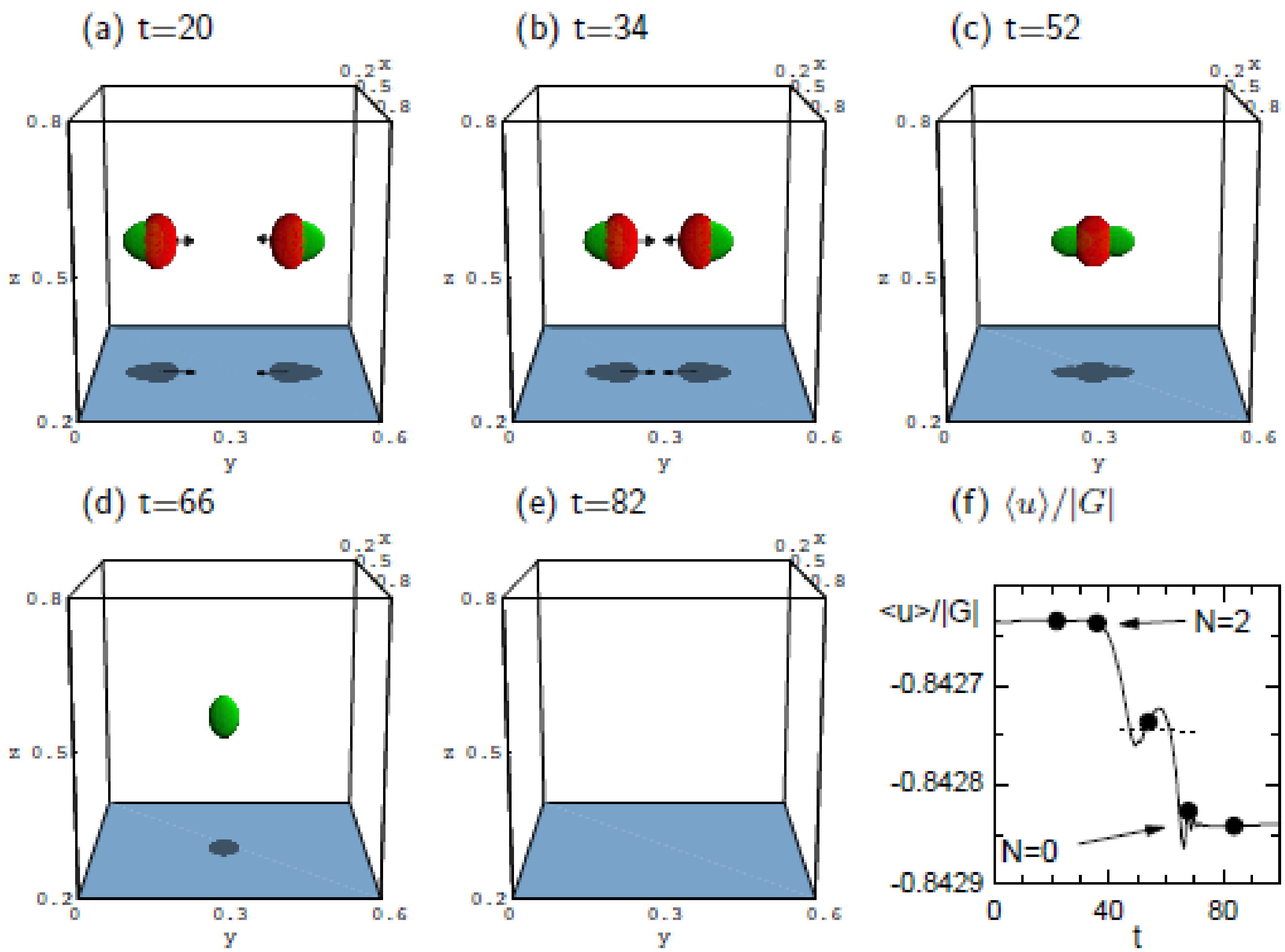
if not linked: start movie "sqt120898a9_kurzzeitigeVerschmelzung.avi" in the folder

Numerical Solution of the Generalized FN Equation in 3-Dimensional Space: Scattering of 2 DSs with Strong Interaction III



Schenk et al., in: *High Performance Computing in Science and Engineering `99,* E. Krause, W. Jäger (Eds.), (2001)

Numerical Solution of the Generalized FN Equation in 3-Dimensional Space: Annihilation in the Course of Collision of 2 DSs

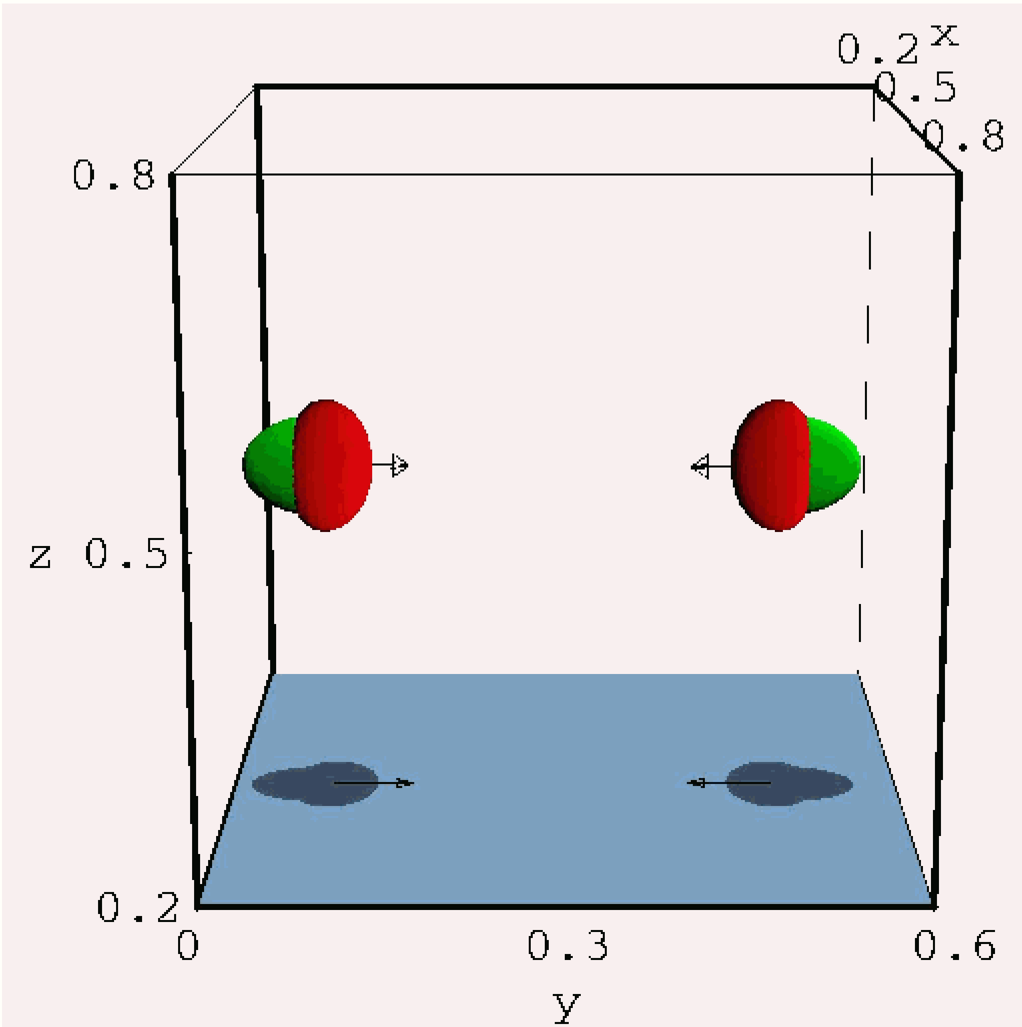


Bode et al. 2002

Pu 26.07.2011

http://www.uni-muenster.de/Physik.AP/Purwins/Research-Summary

Numerical Solution of the Generalized FN Equation in 3-Dimensional Space: Annihilation in the Course of Collision of 2 DSs (Movie)



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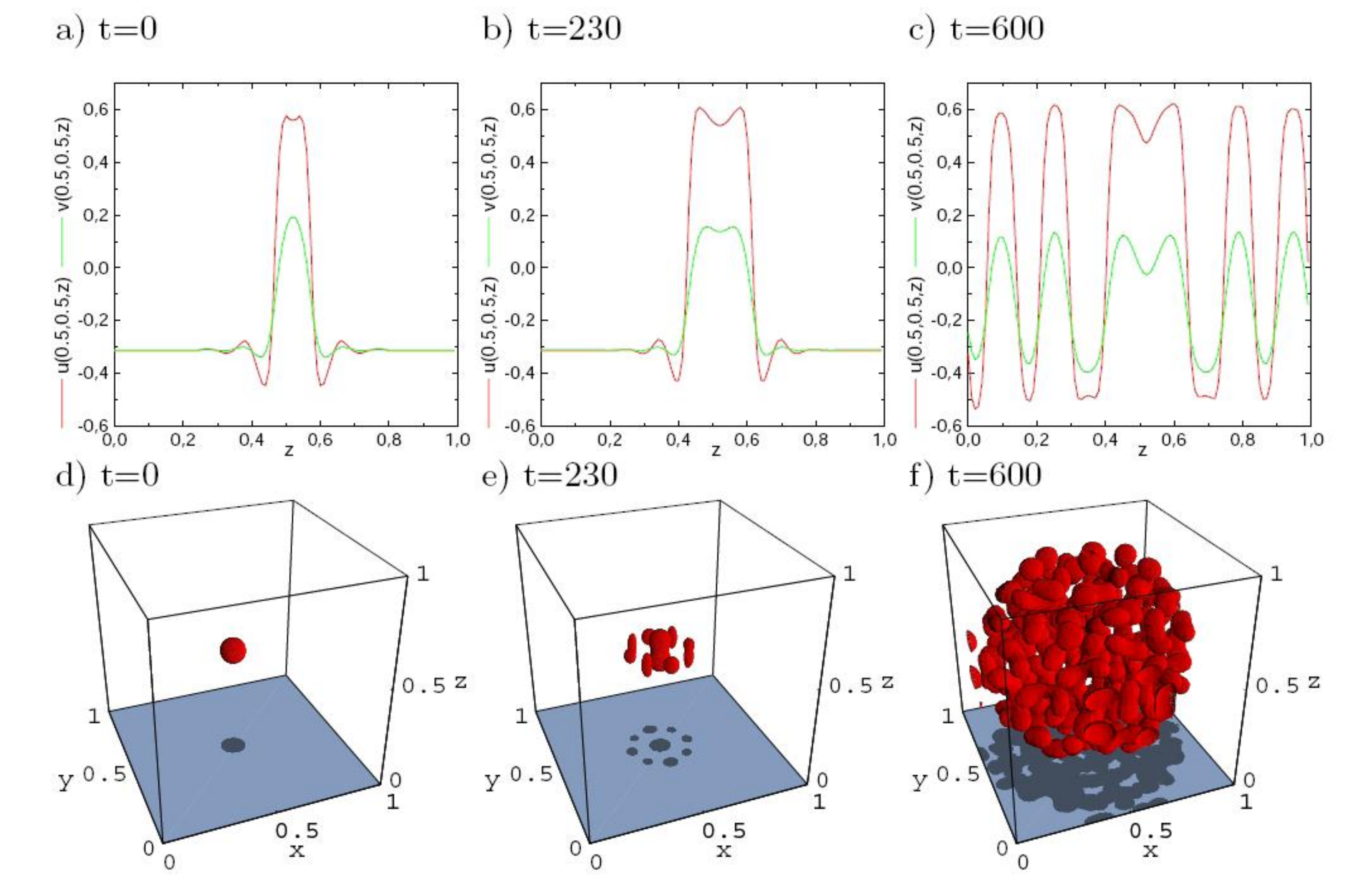
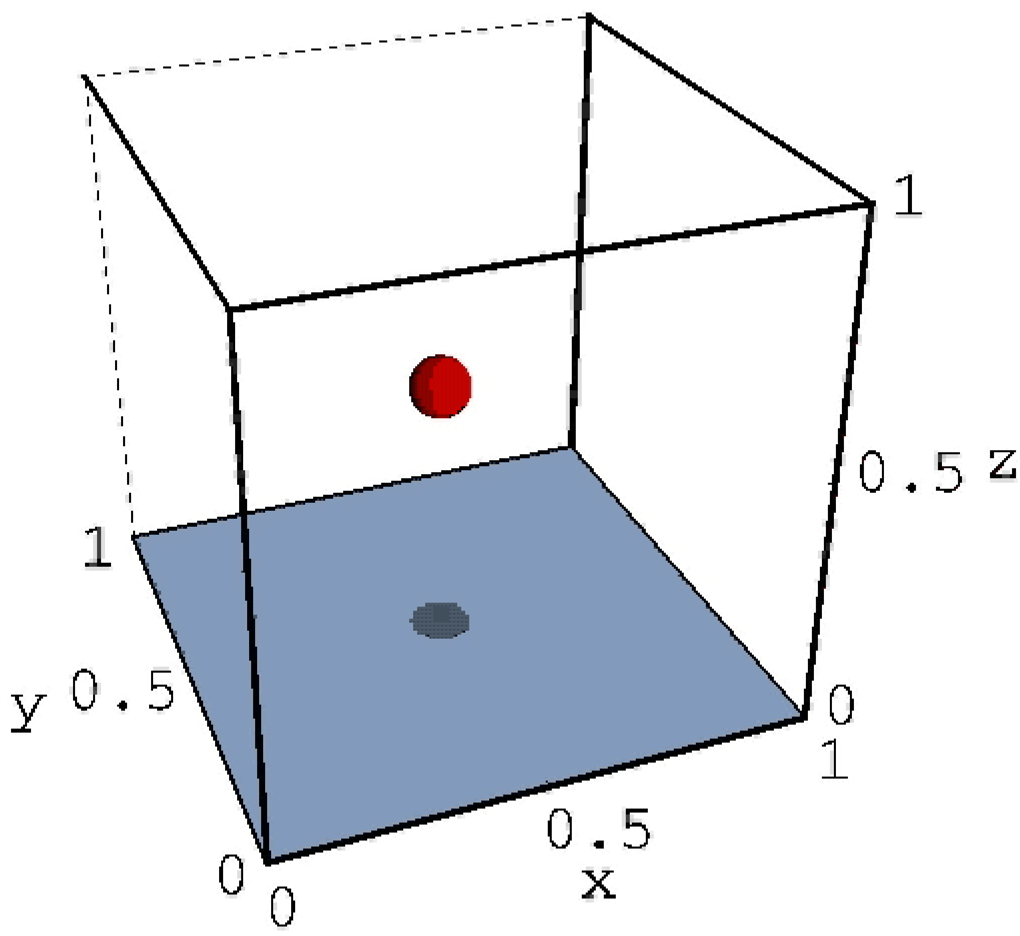
Numerical Solution of the FN-Equation in 3-Dimensional Space: Self-Complition I



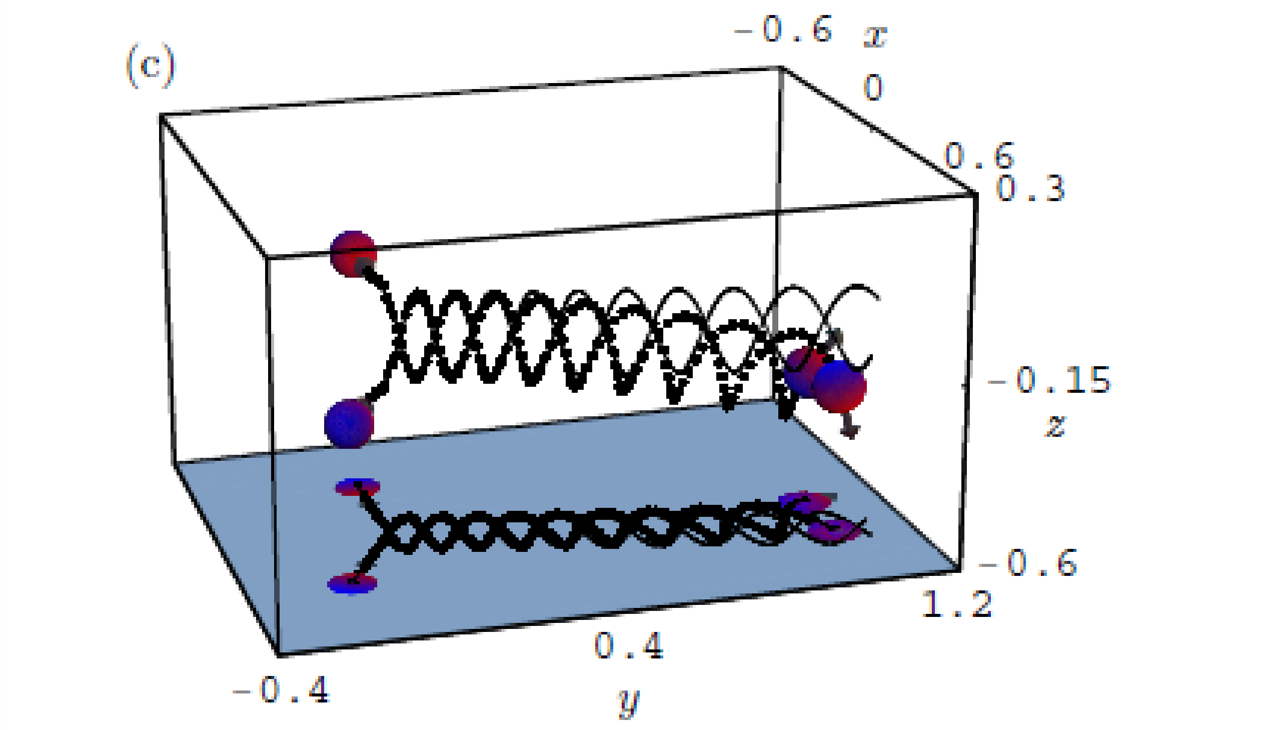
Schenk et al., in: *High Performance Computing in Science and Engineering `99,* E. Krause, W. Jäger (Eds.), p. 354 (2000); **http://www.uni-muenster.de/Physik.AP/Purwins/Research-Summary.**

Numerical Solution of the FN-Equation in 3-Dimensional Space: Self-Complition II (Movie)



if not linked: start movie "Kaskade311298a_3D Destabilisierung.avi" in the folder

Numerical Solution of the FN-Equation in 3-Dimensional Space: Dynamic Cluster of 2 DSs (“Molecule”)



the 2-DS cluster undergoes propagation along the axis that is vertical to the line connecting the centers of the individual DSs, simultaneously the cluster rotates around this axis

11. Summary

The End