Collective Behavior in Excitable Media: Interacting Particle-Like Waves

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West Virginia University

Localized Multi-dimensional Patterns in Dissipative Systems
Banff International Research Station
Banff, Canada, July 25-29

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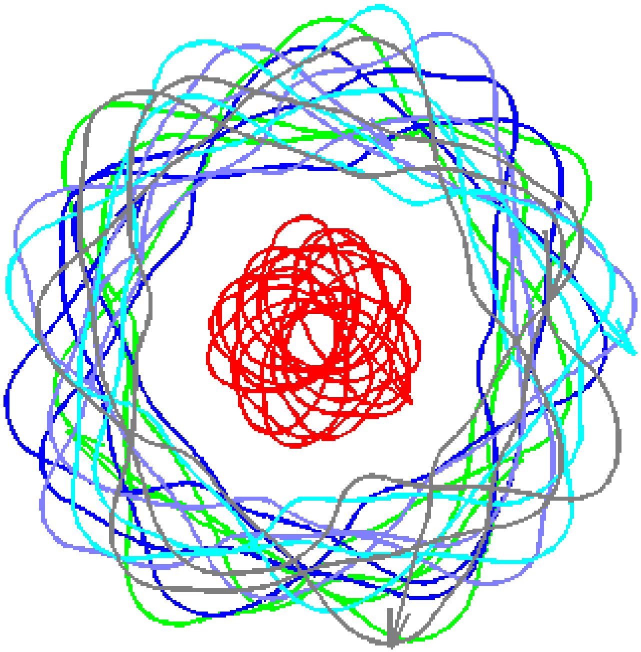
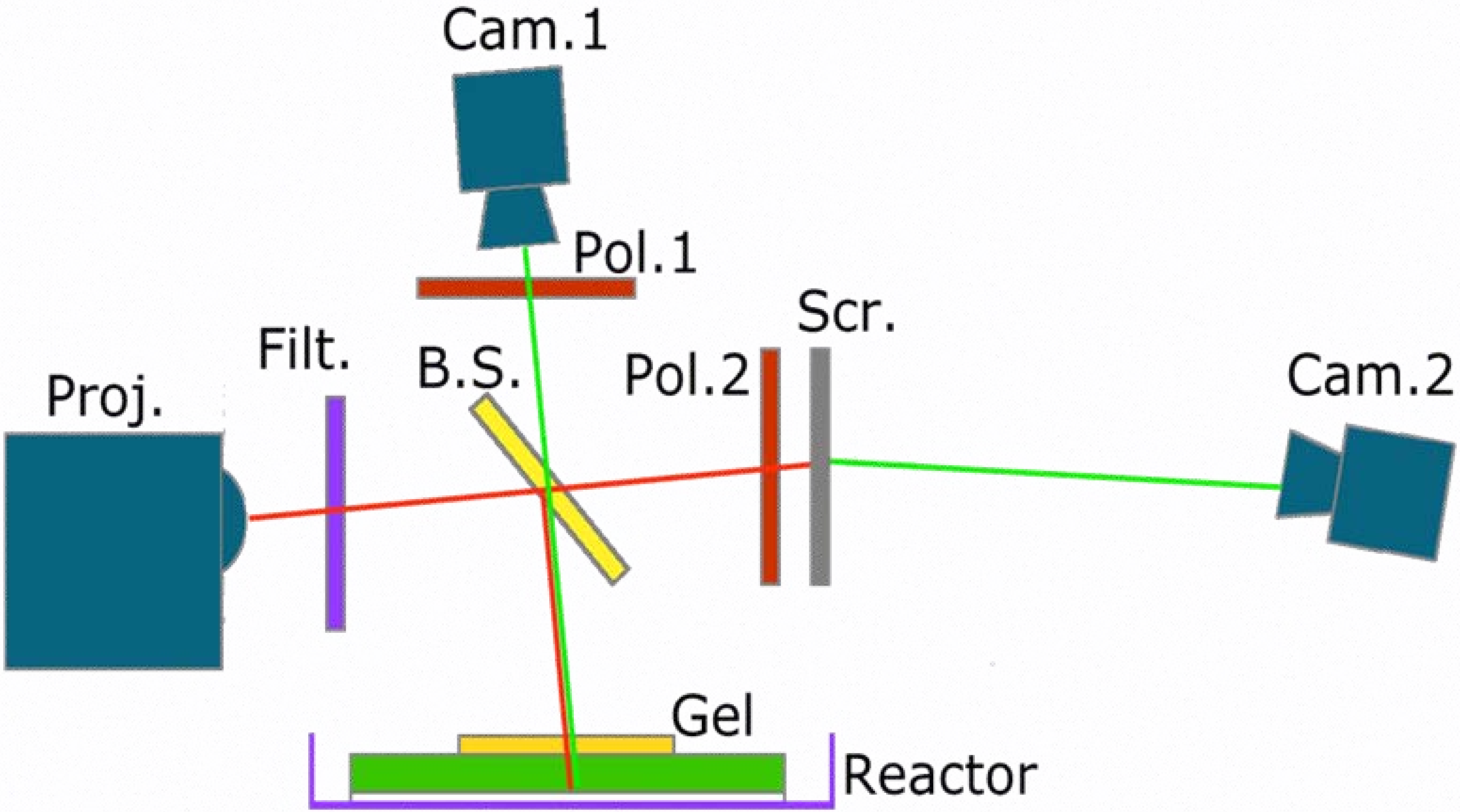


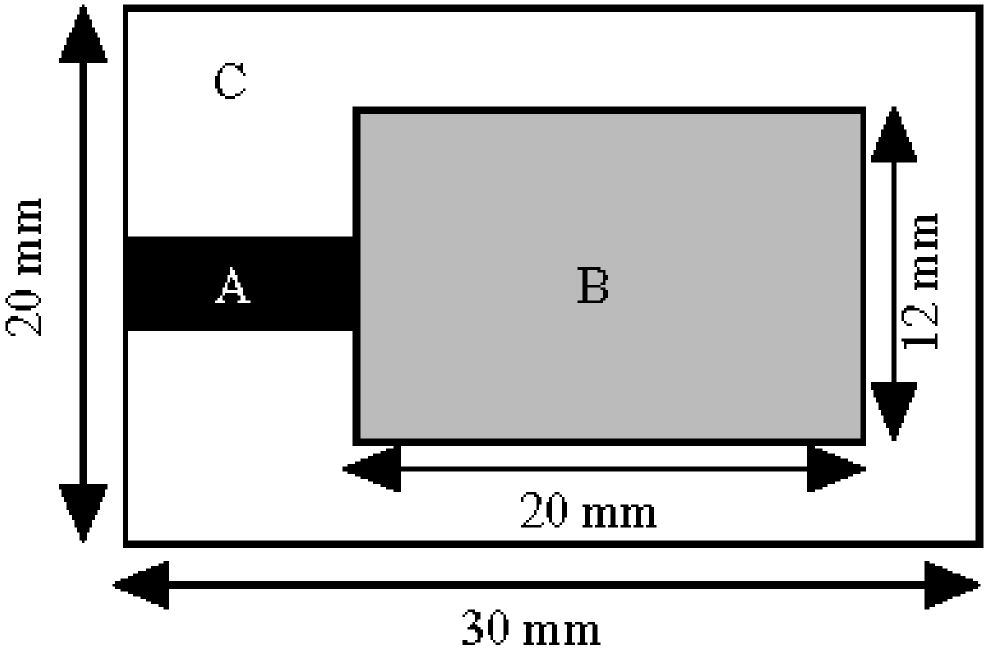
<u>Funding:</u>
NSF



$$Ru(bipy)_3^{2+} + BrMA + h\nu \rightarrow Br^-$$

Control of Medium Excitability

High intensity → Low excitability
Low intensity → High excitability



Zone A: excitable
Zone B: excitability controlled
Zone C: nonexcitable

Wave Propagation in Subexcitable Media

15 mm

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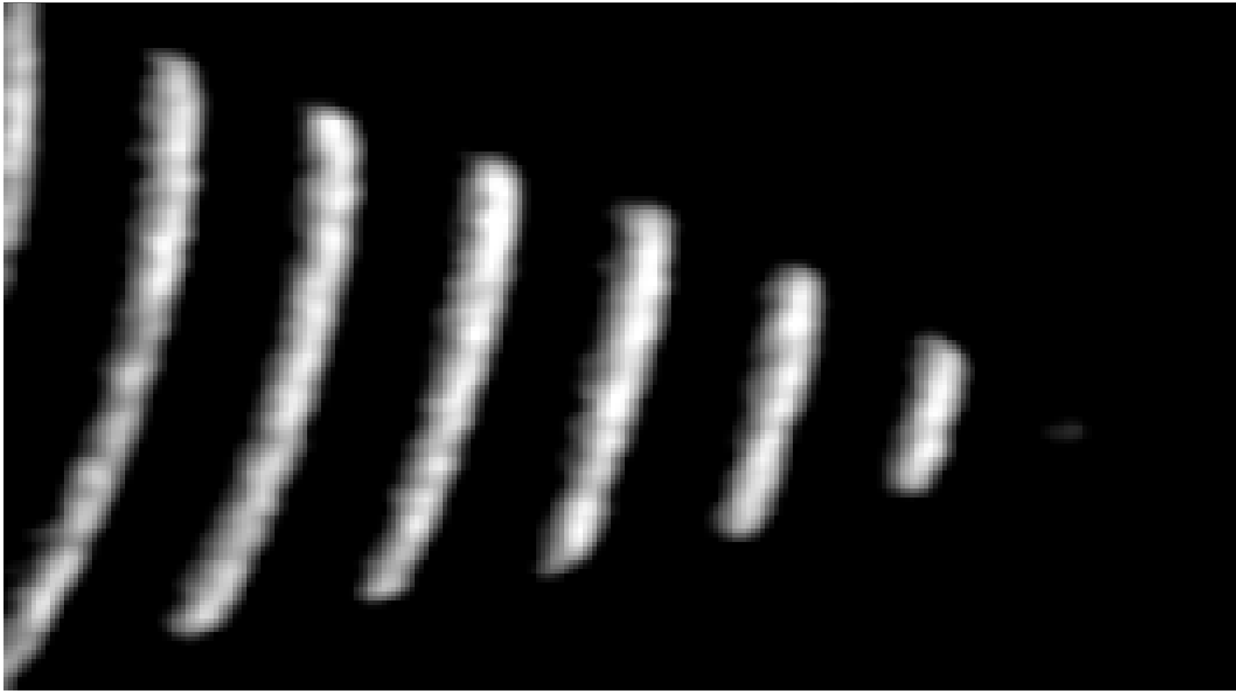
5 mm



At low excitability waves with free ends contract tangentially.

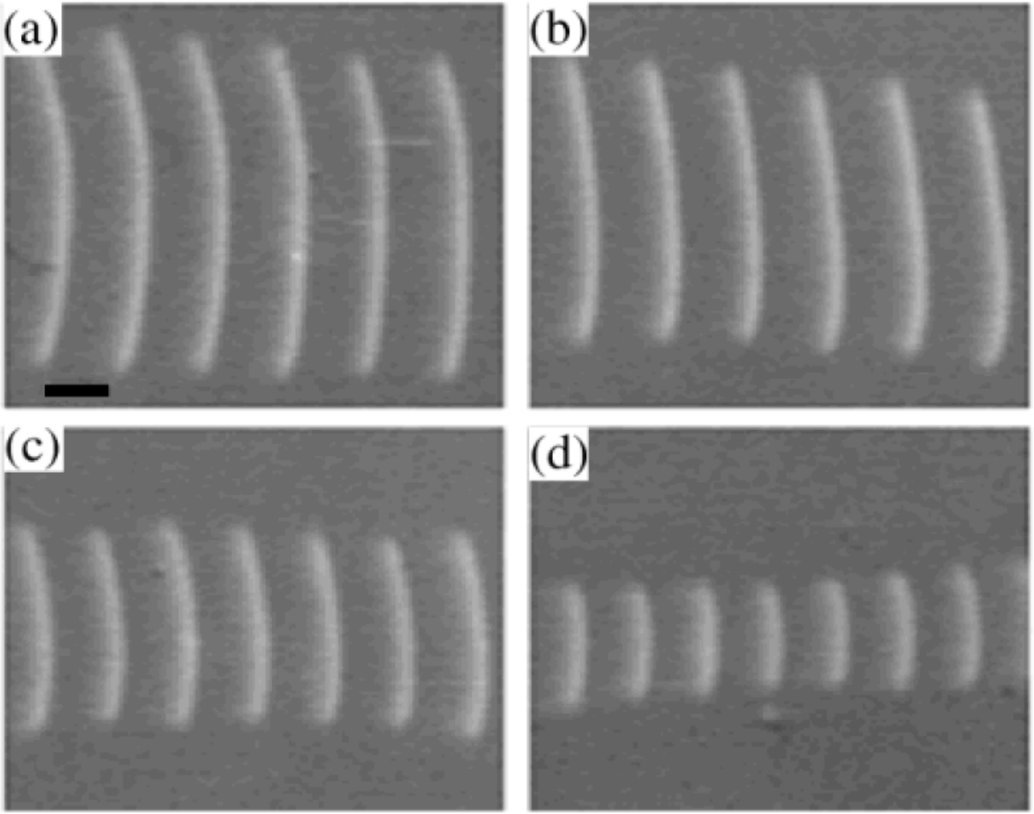
Medium does not support sustained wave propagation; hence, subexcitable.

Superimposed images at equal time intervals

J. Maselko, KS, Nature ***391****, 770-772 (1998).*

Feedback Stabilized Waves

$$\phi(x, y) = a \cdot A + b$$



A = area
a = gain
b = offset

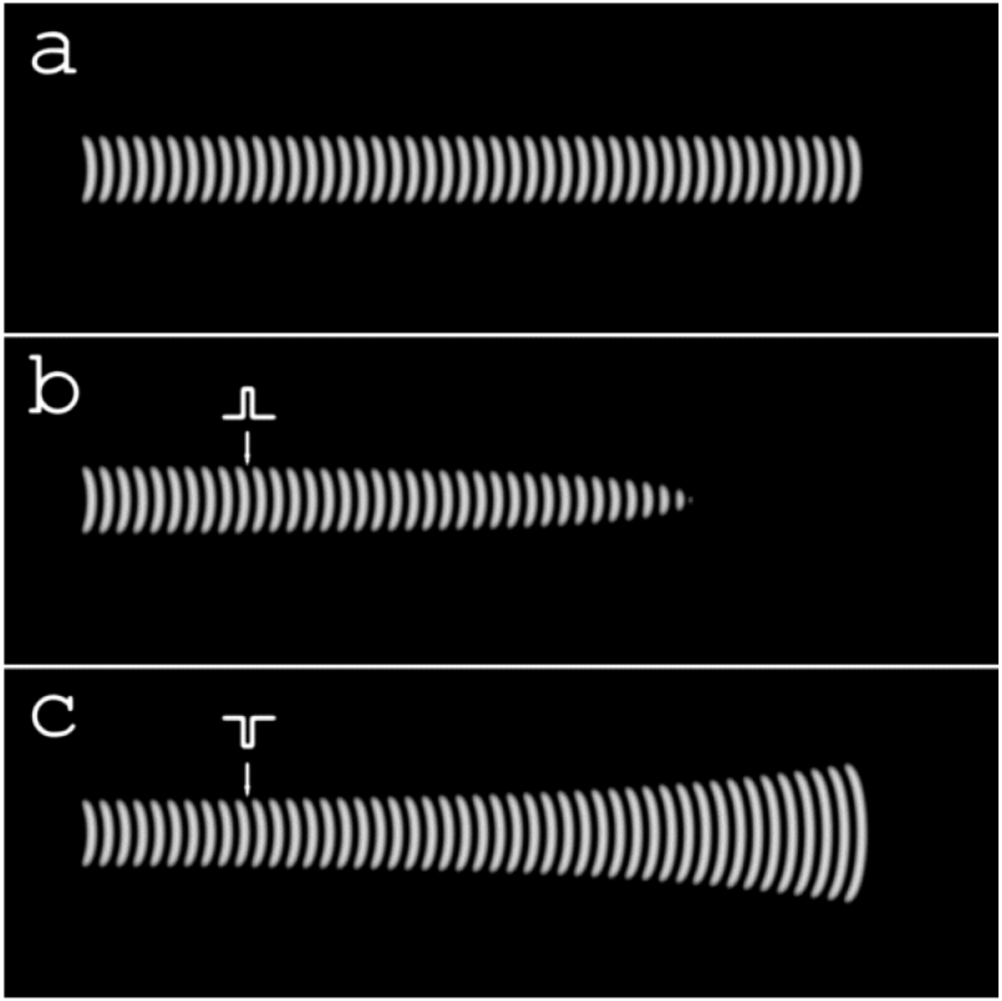
Excitability is determined by offset:

b = -0.0744 (a)
-0.0248 (b)
0.0248 (c)
0.0744 (d)
mW/cm^2

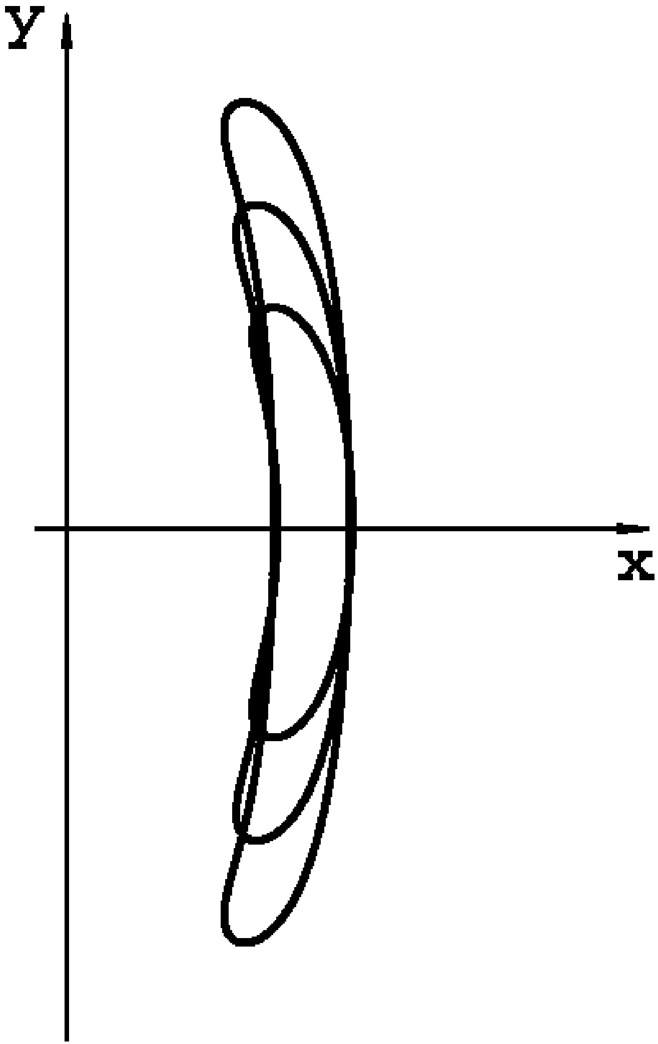
E. Mihaliuk, T. Sakurai, F. Chirila, KS, Phys. Rev. E ***65****, 065602 (2002).*
V.S. Zykov, KS, Phys. Rev. Lett. ***94****, 068302 (2005).*

Saddle Character of Unstable Waves

Perturbation: $\Delta\Phi = \pm\ 1.0 \times 10^{-3}$, $\Delta t = 0.1$



Wave Size Dependence on Medium Excitability



Gain: $a = 2.5 \times 10^{-4}$

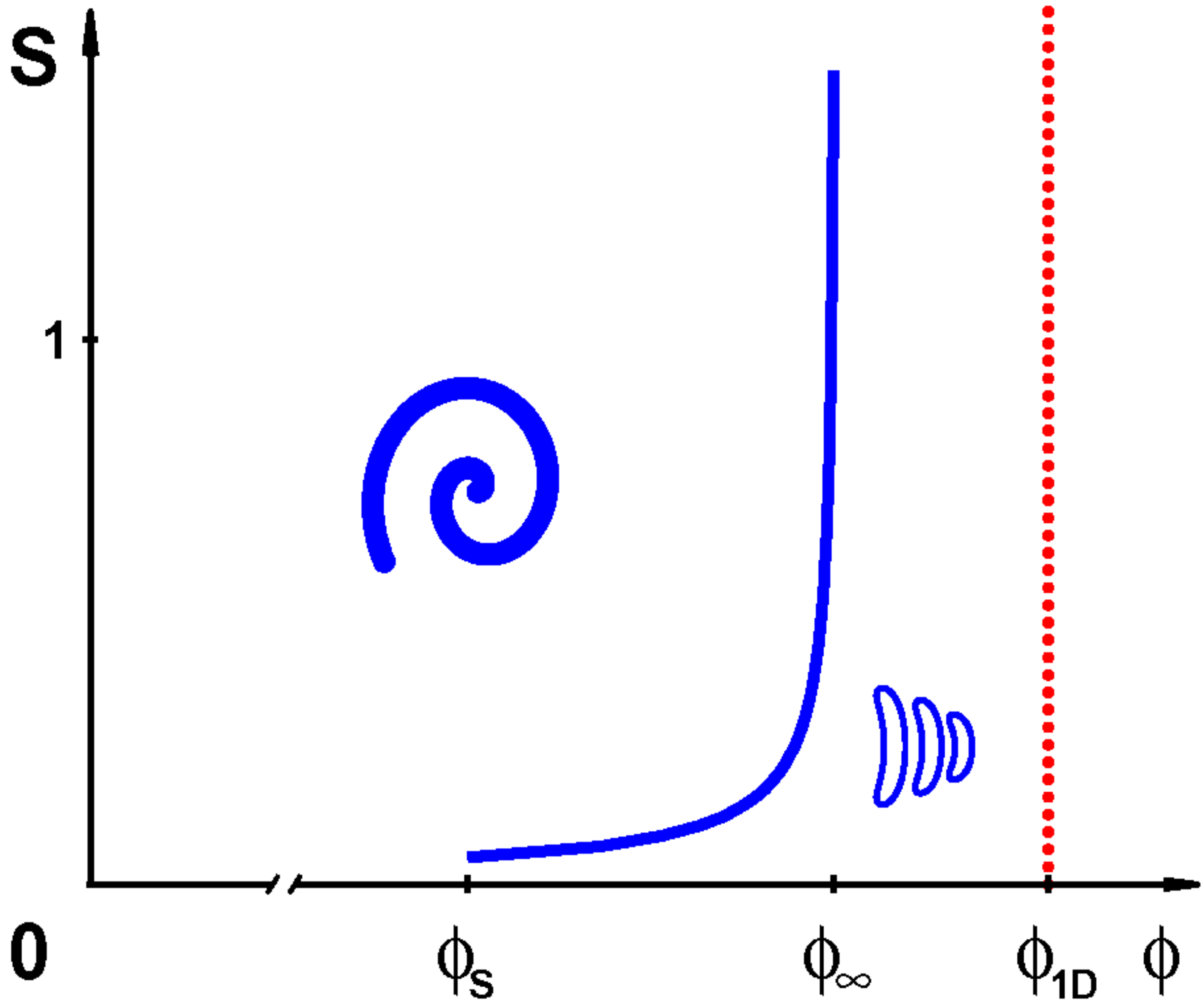
Offset: b = - 0.1 (small wave)

- 0.2 (medium wave)

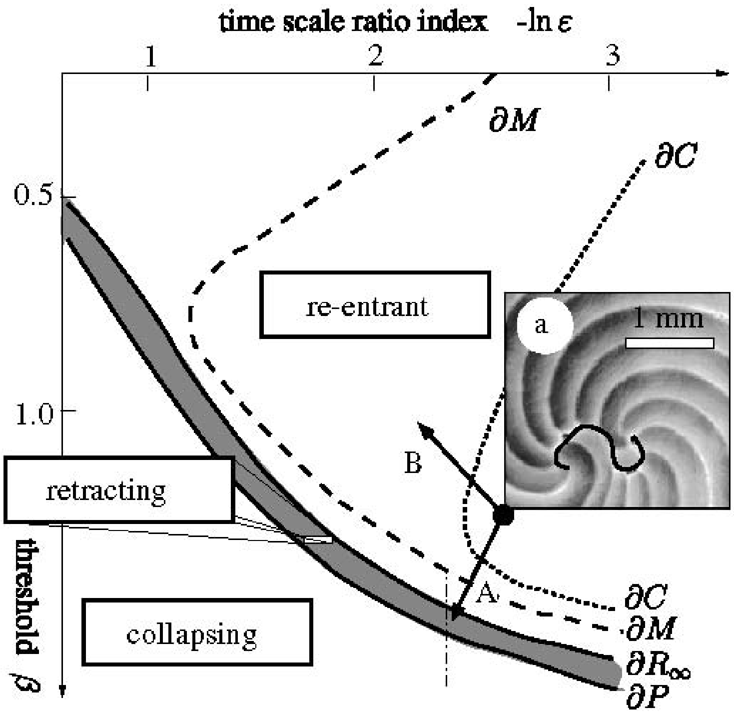
- 0.3 (large wave)

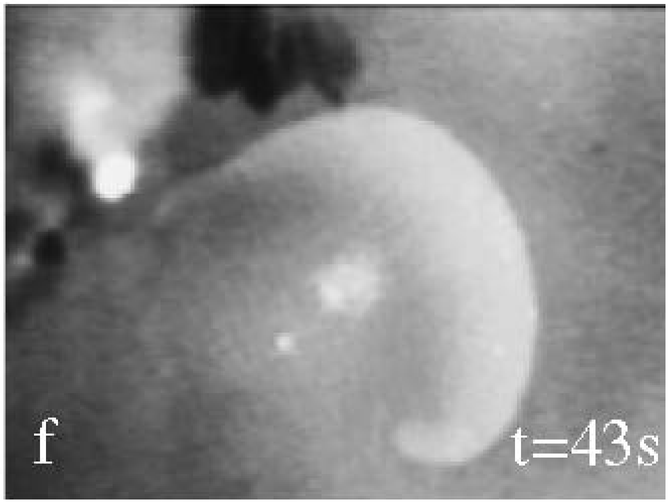
Stabilized wave size increases with decreasing excitability.

Excitability Boundary for Spiral Waves



Spreading Depolarization (SD) Wave Segments



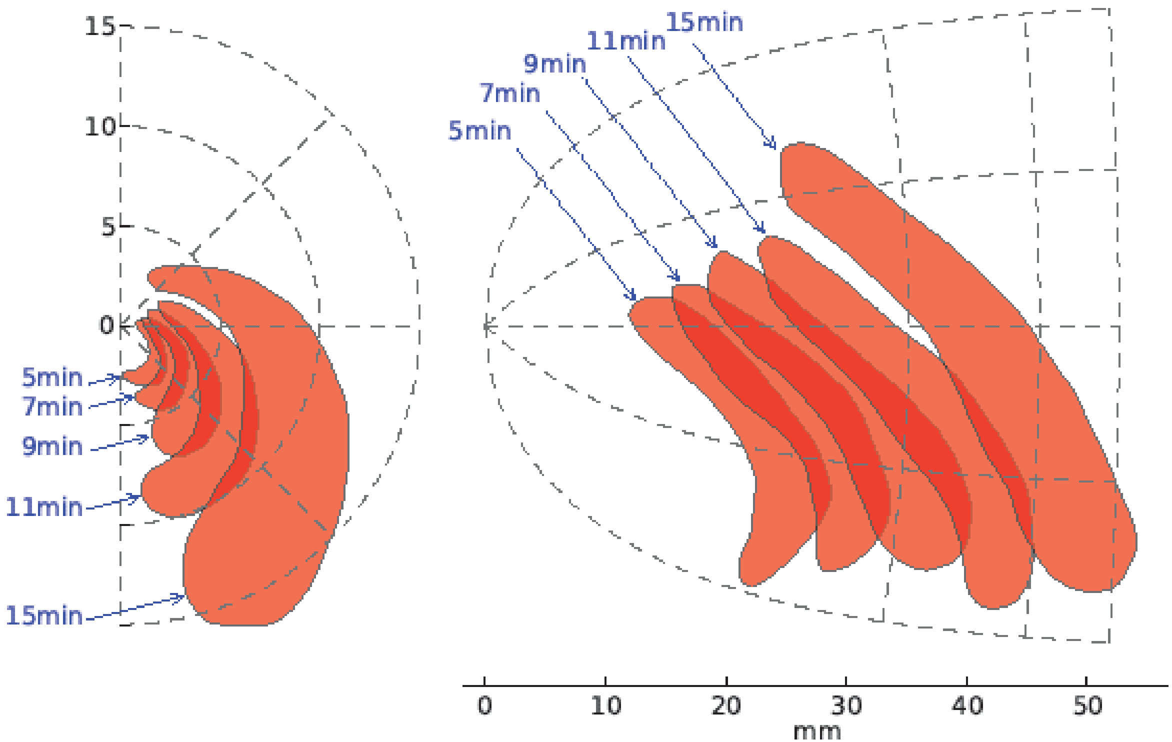


Subexcitable regime in the FitzHugh–Nagumo equations

SD wave segment in chicken retina

M.A. Dahlem et al. Physica D **239**, 889-903 (2010).

Spreading Depolarization (SD) Wave Segments



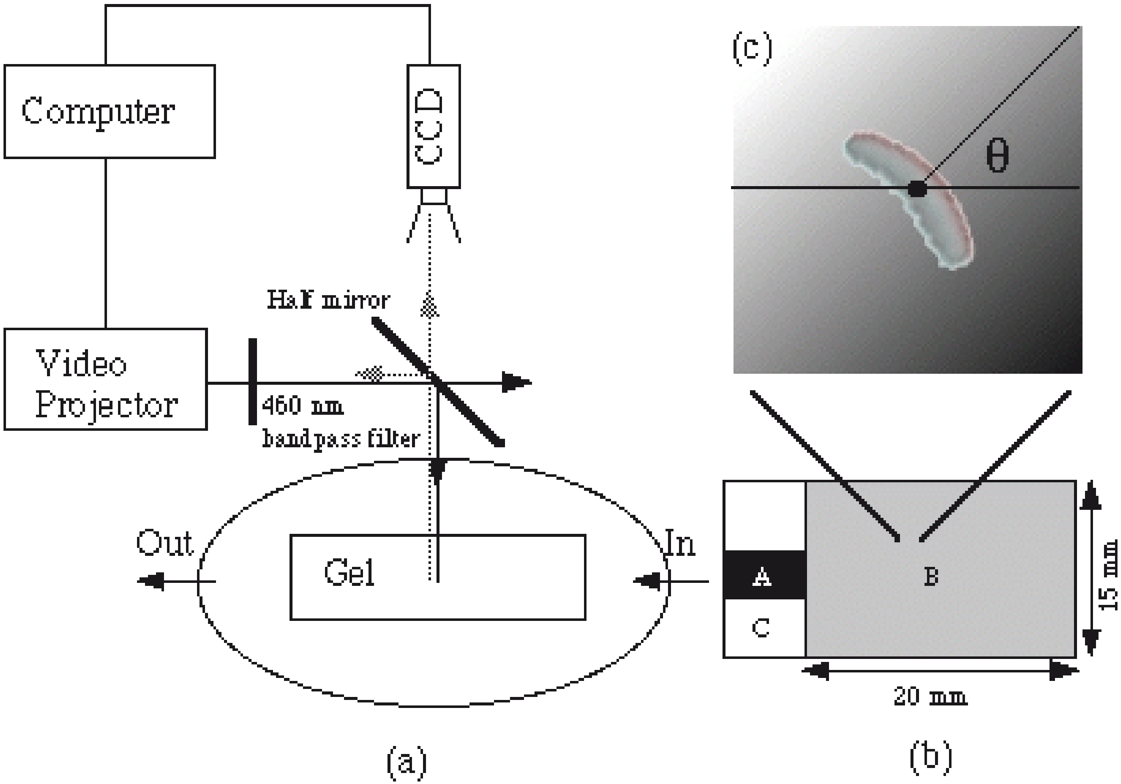
Propagating visual migraine aura (left) and (presumed) corresponding spreading depression wave segments (right) according to reversed retinotopic mapping onto a flat model of the primary visual cortex.

M.A. Dahlem and N. Hadjikhani, PLoS One **4**(4), E5007 (2009).

Directing Stabilized Waves with Excitability Gradients

$$\phi(x,y) = a \cdot A + b + c \cdot G(x,y)$$

$$A = \sum_{x,y} \Theta(p(x,y) - p_{th})$$

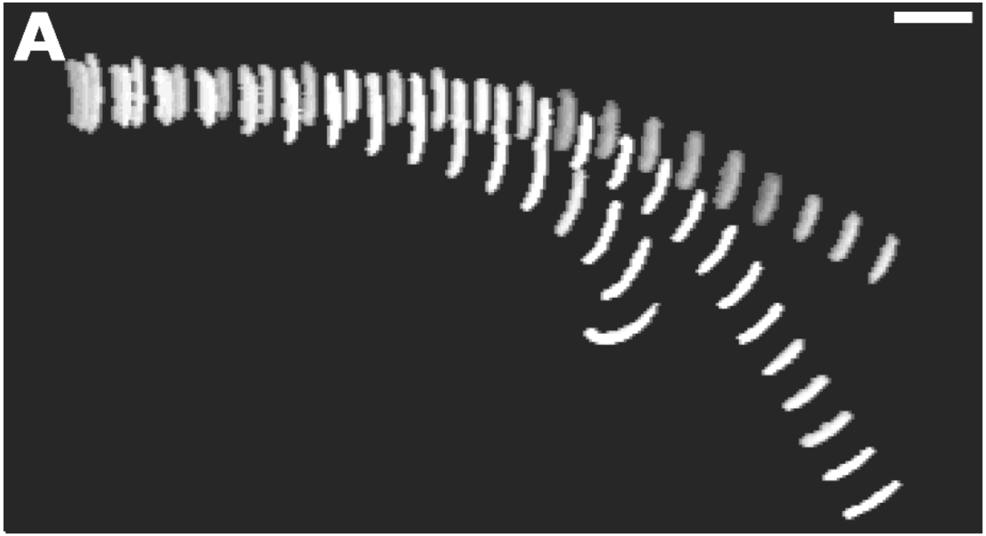


Wave trajectories for radially symmetrical excitability distribution:

$G(r) = ln(r) - ln(r_m)$

Superposition of snapshots of experimental (A) and simulated (B) wave behavior.

Scale bar in (A): 2.0 mm.



Hypotrochoid trajectories

$$X = (\alpha - \beta)\cos(\theta) + \gamma\cos((\tfrac{\alpha}{\beta} - 1)(\theta))$$

$$Y = (\alpha - \beta)\sin(\theta) - \gamma\sin((\tfrac{\alpha}{\beta} - 1)(\theta))$$

Circular trajectories:

$\alpha = \gamma = 0, \beta = 100$

Three-lobed trajectories:

$\alpha = 60, \beta = 20, \gamma = 80$

Four-lobed trajectories:

$\alpha = 60, \beta = 15, \gamma = 80$

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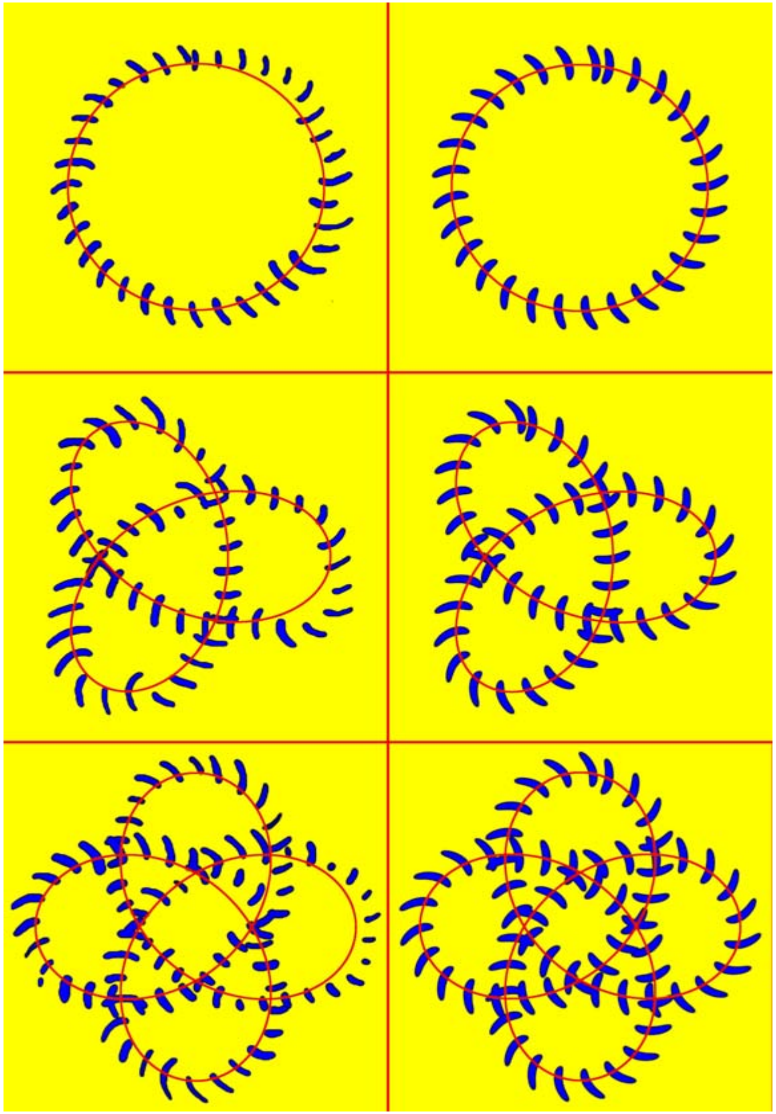


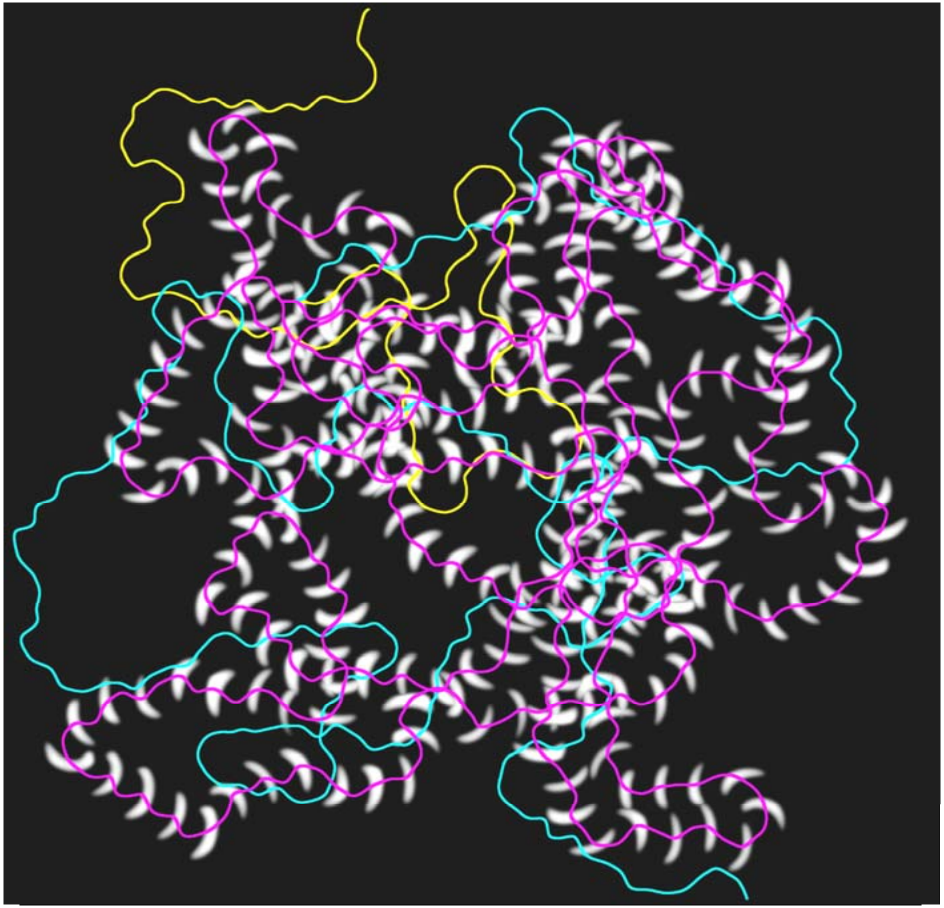
T. Sakurai, E. Mihaliuk, F. Chirila, KS, Science ***296****, 2009 (2002).*

Waves Undergoing Random Walks

The slope of G randomly selected from a uniform probability distribution between -0.002 and 0.002.

CLICK PICTURE TO PLAY MOVIE

Three different random walk trajectories, all starting at the same point, shown in red, blue and yellow.

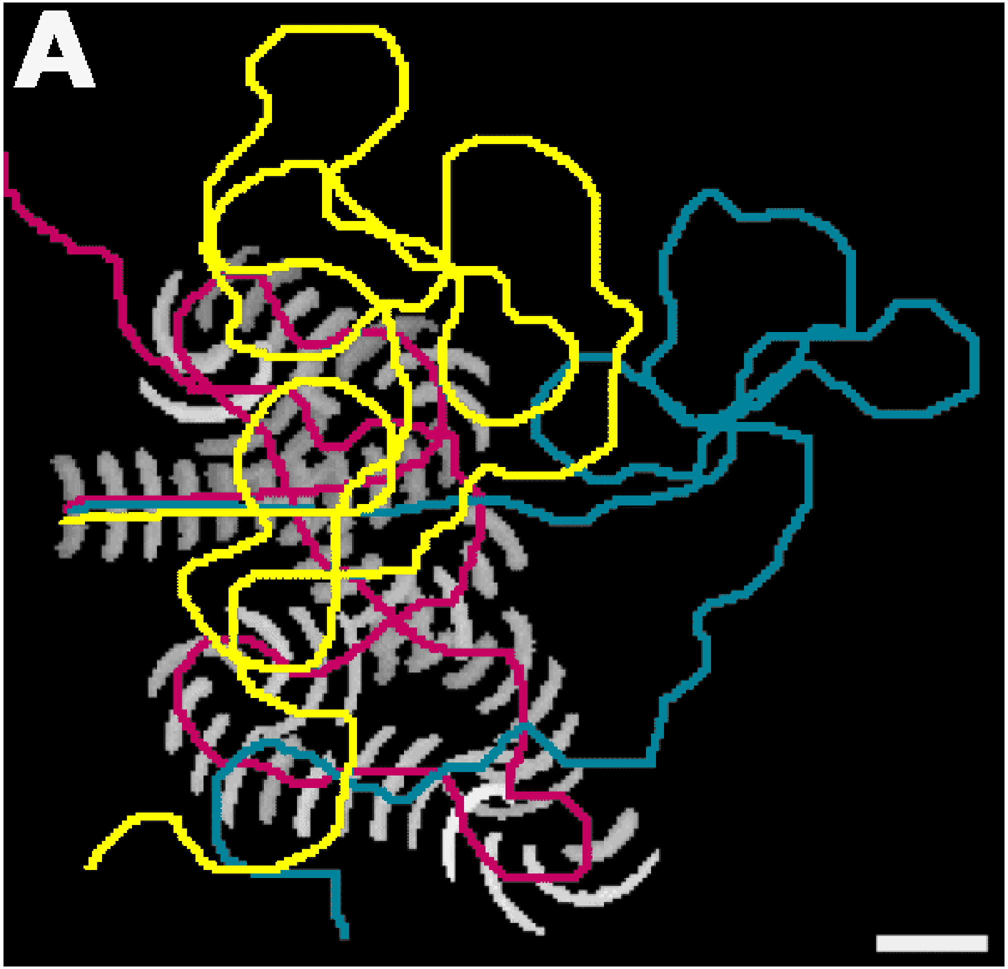


Experimental Waves Undergoing Random Walks

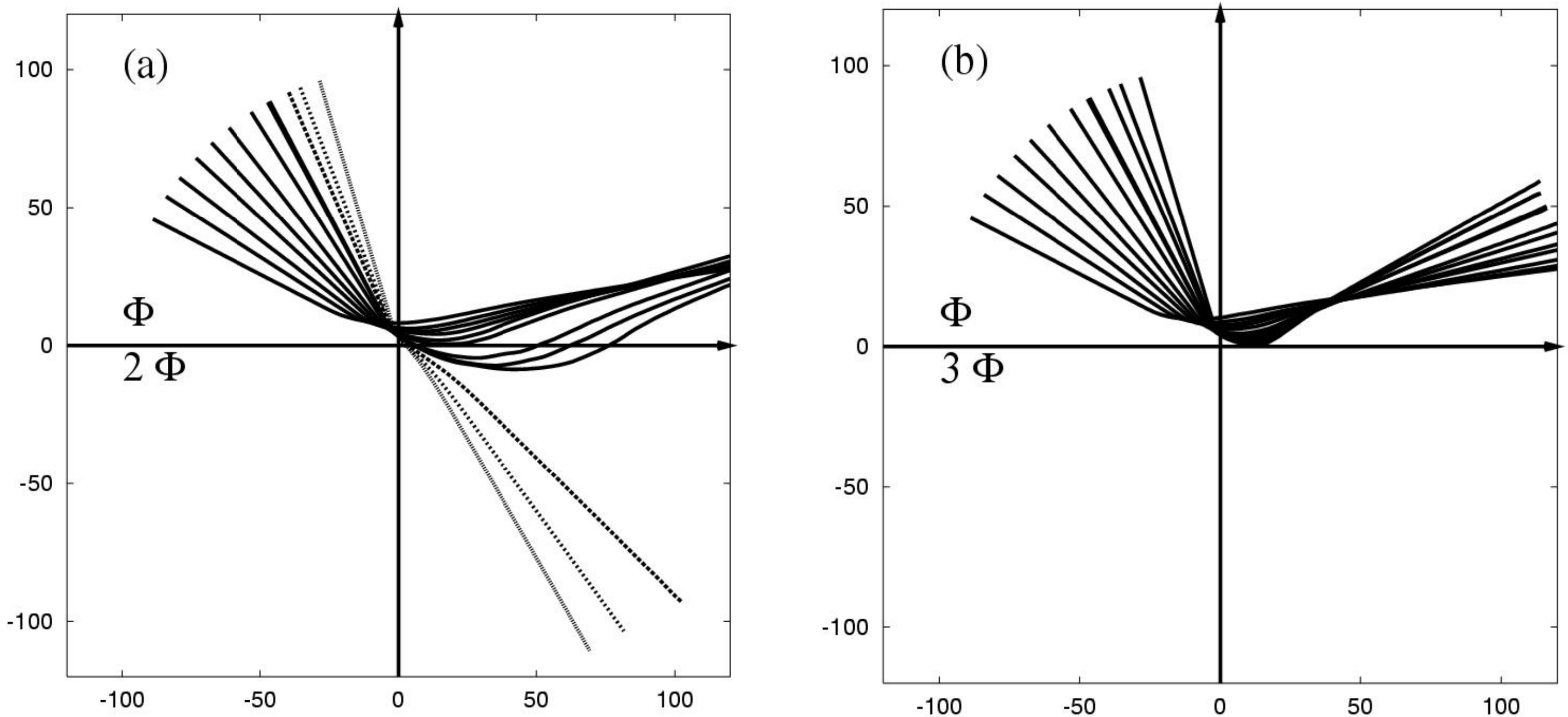
Three different random walk trajectories, each starting at the same point.

Slope of G randomly selected from a uniform probability distribution between -2.5 and 2.5.

Scale bar: 2.0 mm.

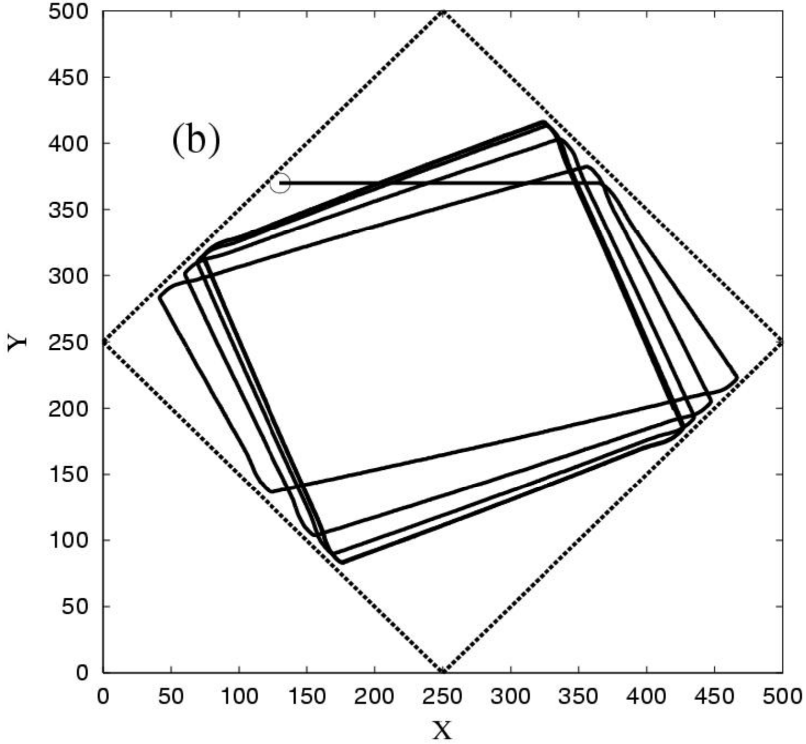


Reflection and Refraction of Wave Segments



Chemical wave "Snell-like" Law: $\frac{\sin\theta_1}{\sin\theta_2} = 0.669\,\Delta\phi$

Wave Segment Confined in a Square Box



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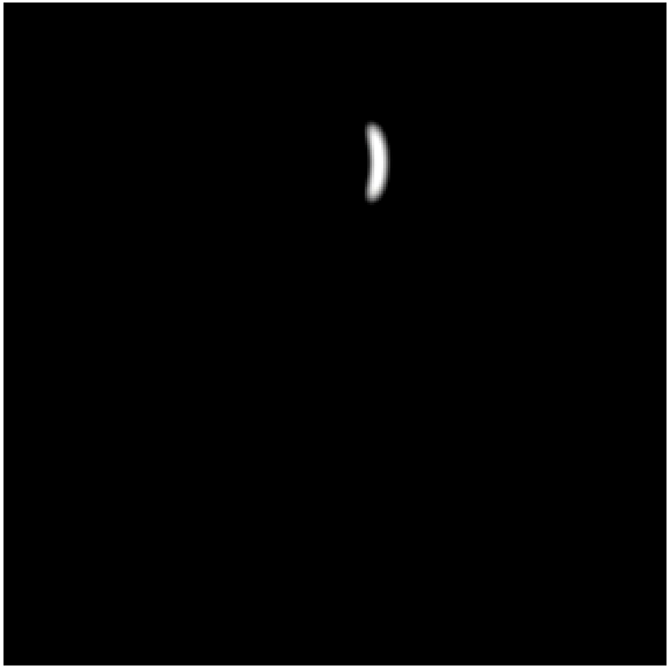
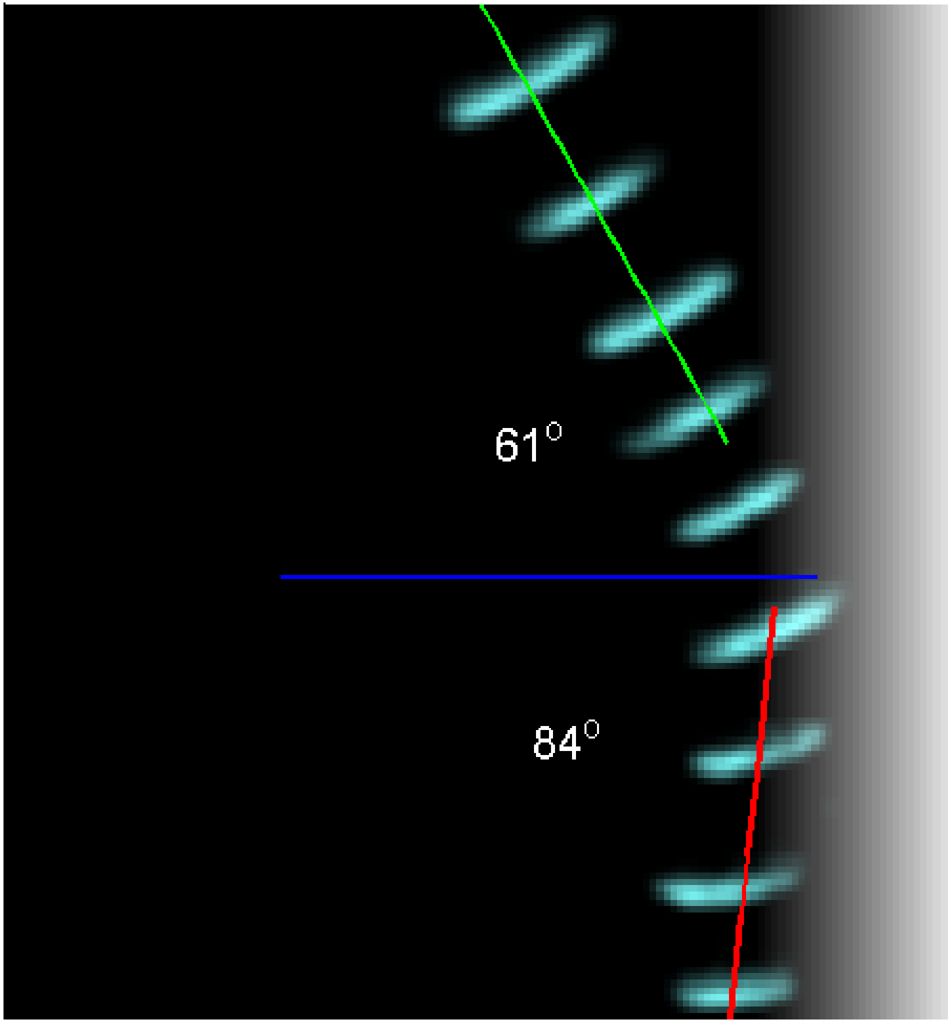


Light intensity outside box ten times larger than inside.

Reflection of Wave Segments: Experiments



Linear gradient reflection boundary.

More effective than step function.

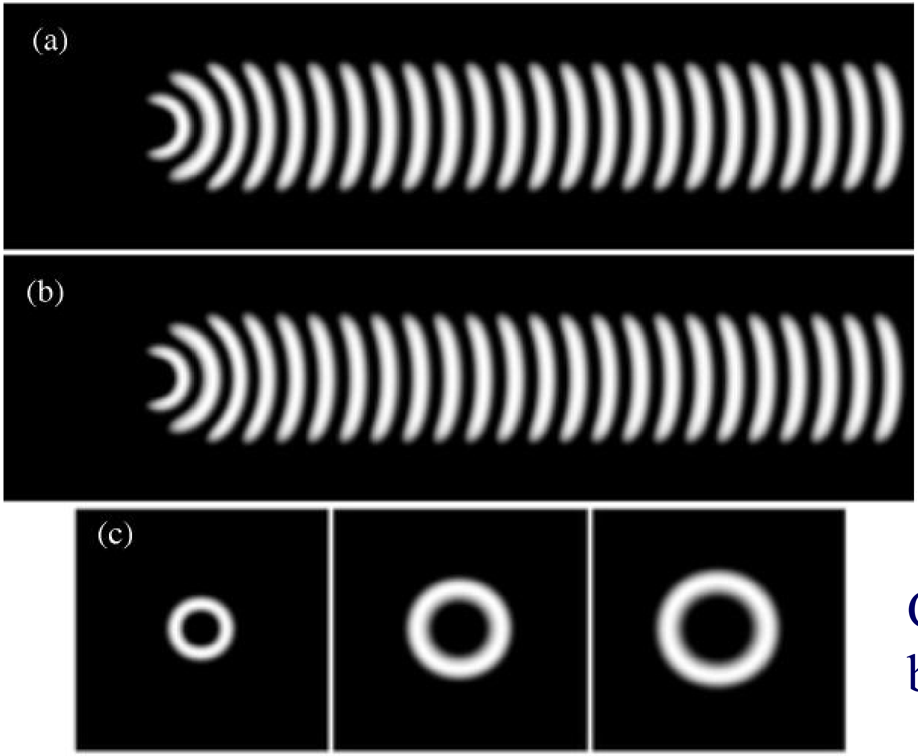
Also used parabolic and cubic functions.

Stabilizing and Controlling 3D Waves

x-z

x-y

y-z



$$\begin{cases} \phi(t) = a + b(V(t) - V_d) \\ V(t) = \sum_{x,y,z} \Theta(p(x,y,z) - p_{th}) \end{cases}$$

Cross section 3 pixels behind "center of mass."

Wave Trajectories in 3D

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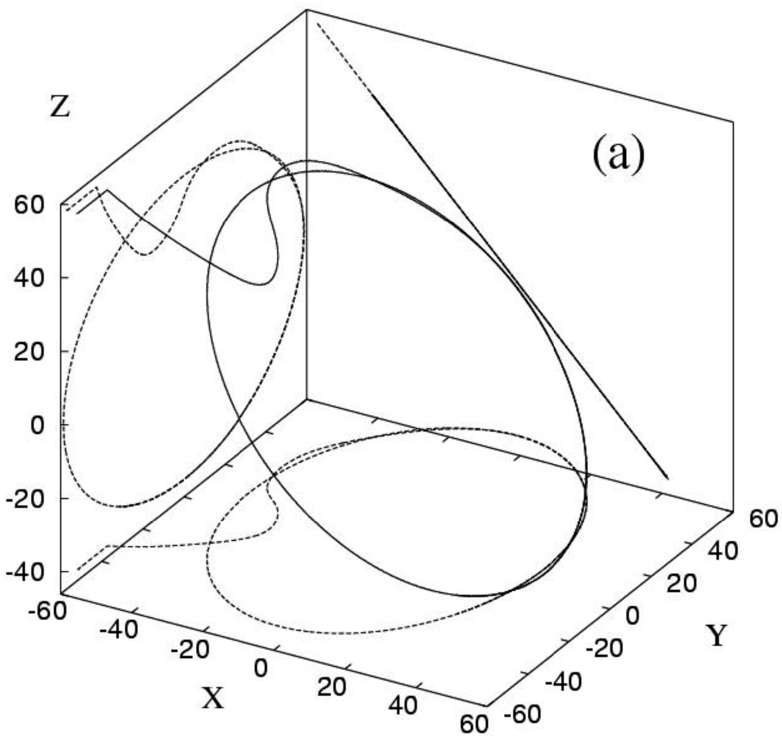
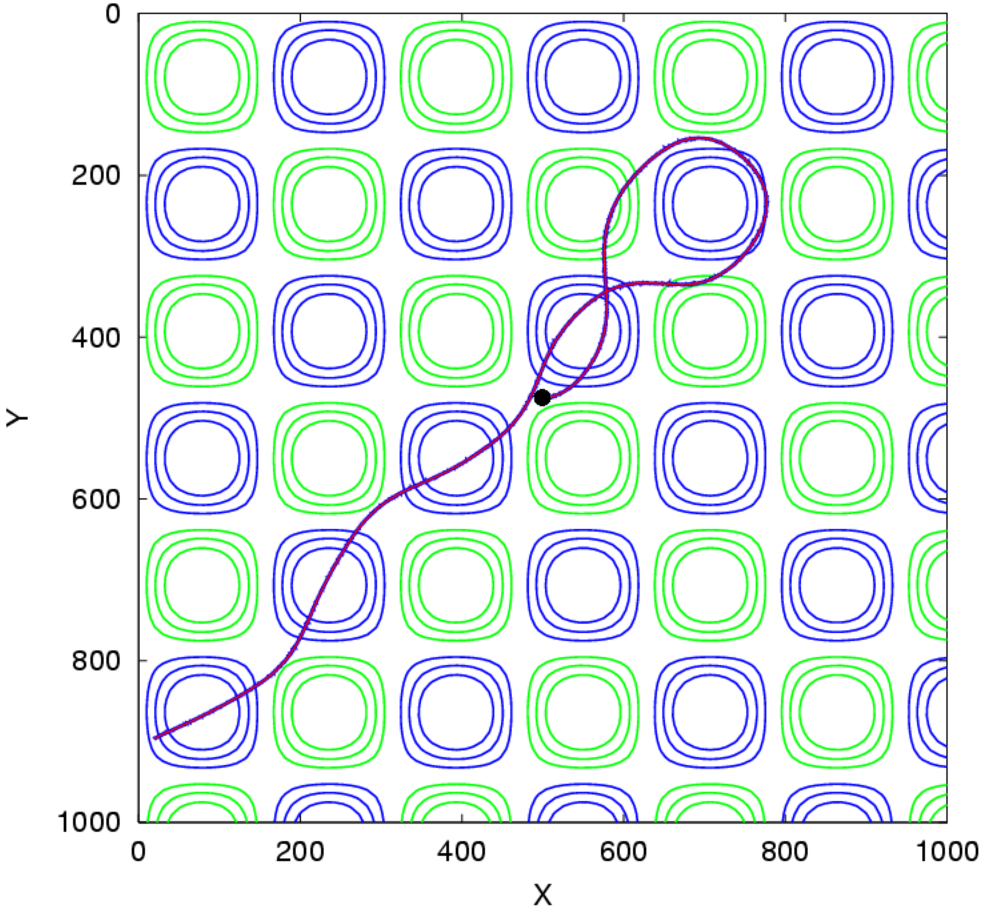


XY plane

XZ plane

Navigating Excitability Landscapes



Contour Plot:

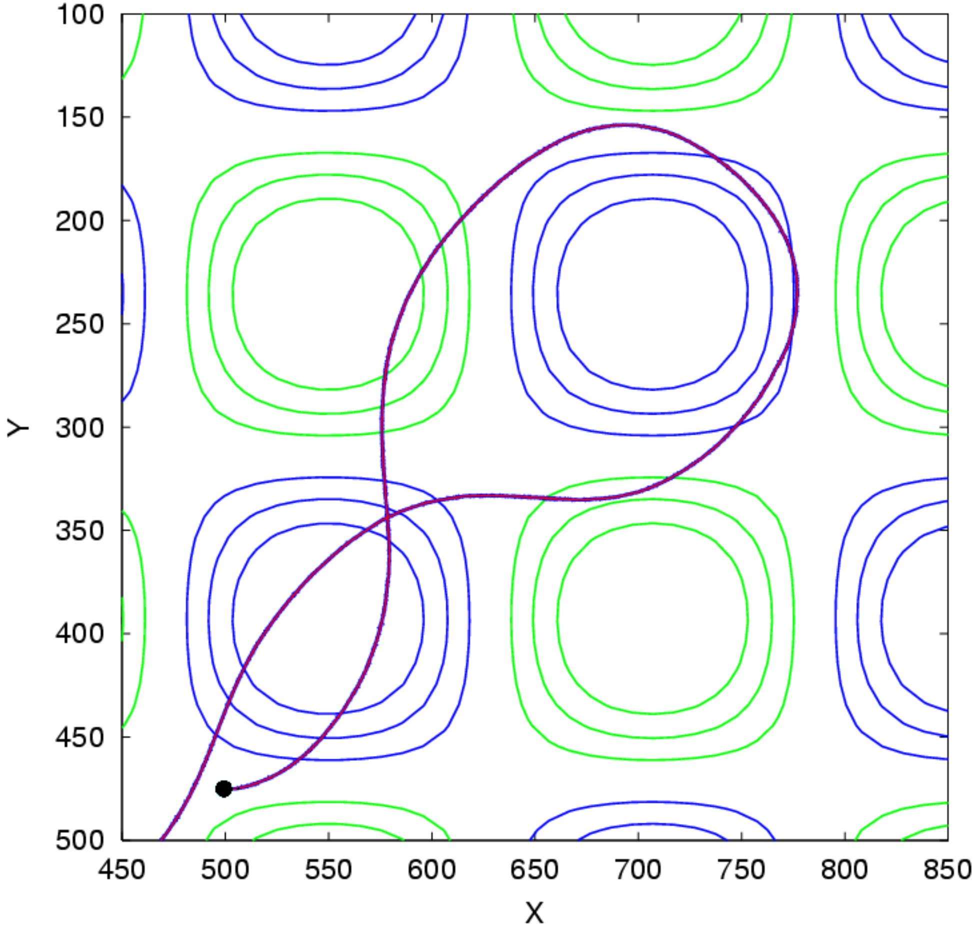
Maxima and Minima above and below average excitability.

$$\phi(x,y) = \phi_{max} \sin(k_x x) \sin(k_y y)$$

Trajectory:

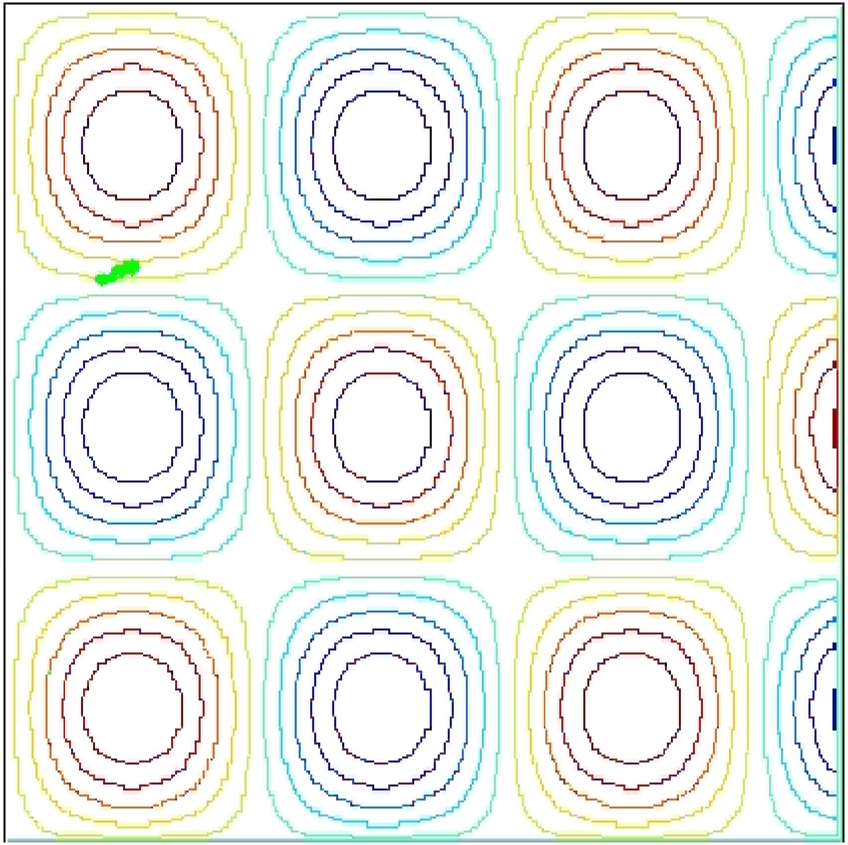
Propagating wave center of mass from starting point ● .

Navigating Excitability Landscapes



Wave stabilization by global feedback. Hence, wave direction is determined by the local excitability gradient.

Navigating Excitability Landscapes: Experiments!

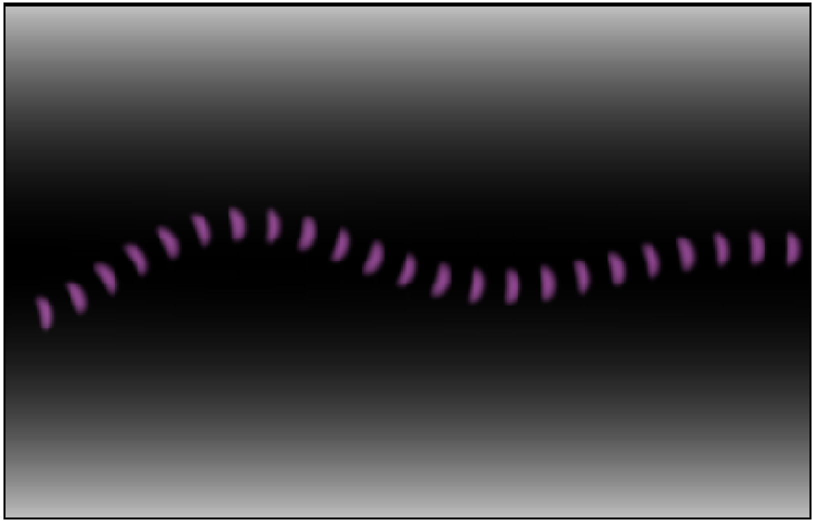


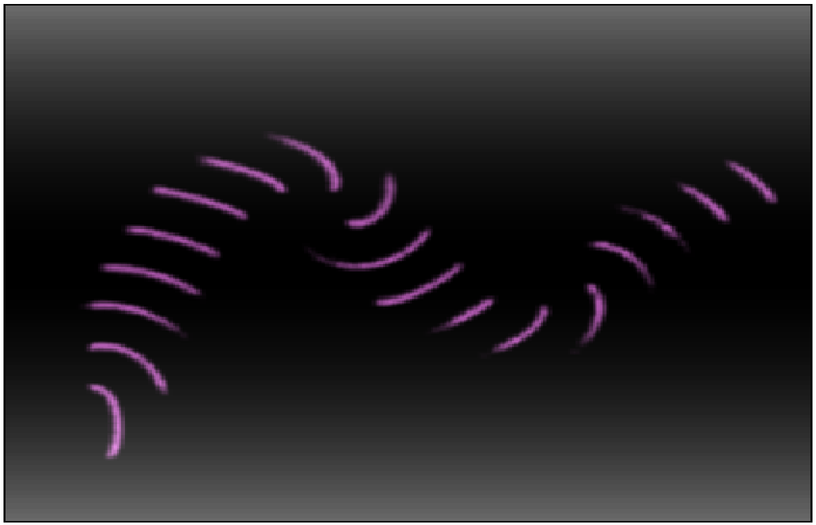
CLICK PICTURE TO PLAY MOVIE

Contour Plot:

Maxima and Minima above and below average excitability.

Navigating Excitability Landscapes





Parabolic valley excitability potential

Simulations (top) and experiments (bottom).

$$U(r) = \alpha_{sim,\exp} y^2$$

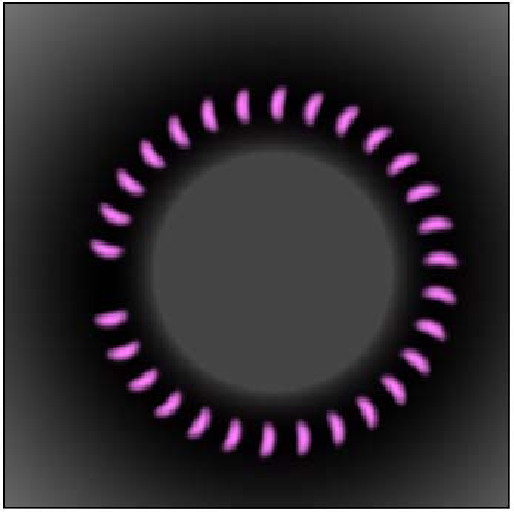
$$\alpha_{sim} = 1.1 \times 10^{-7}$$

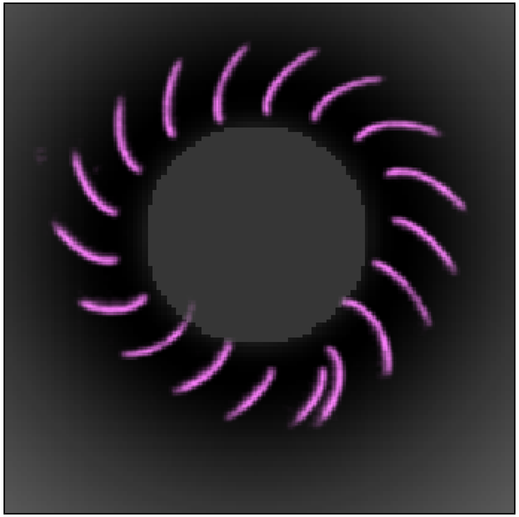
$$\alpha_{\exp} = 0.29$$

Chaos ***18****, 026108-1-8 (2008).*

Eur. Phys. J. ST ***165****, 161–167 (2008).*

Navigating Excitability Landscapes





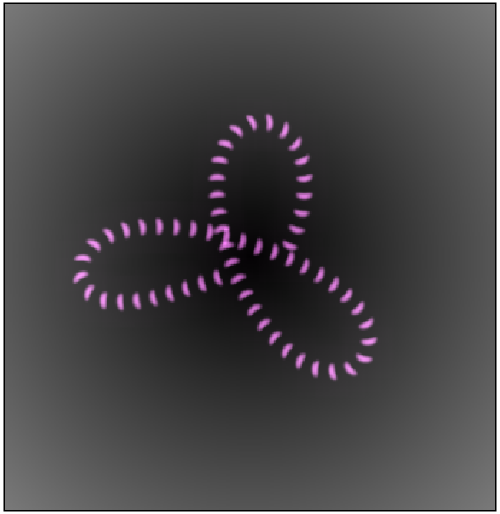
Radial Lennard-Jones

Simulations (left) and experiments (right).

$a_{sim} = 1.13$ $\quad a_{exp} = 0.11$

$b_{sim} = 0.15$ $\quad b_{exp} = 8.4 \times 10^{-2}$

$c_{sim} = 5$ $\quad c_{exp} = 108$



Radial harmonic potential

Simulations (left) and experiments (right).

$$U(r) = \alpha_{sim,exp} r^2$$

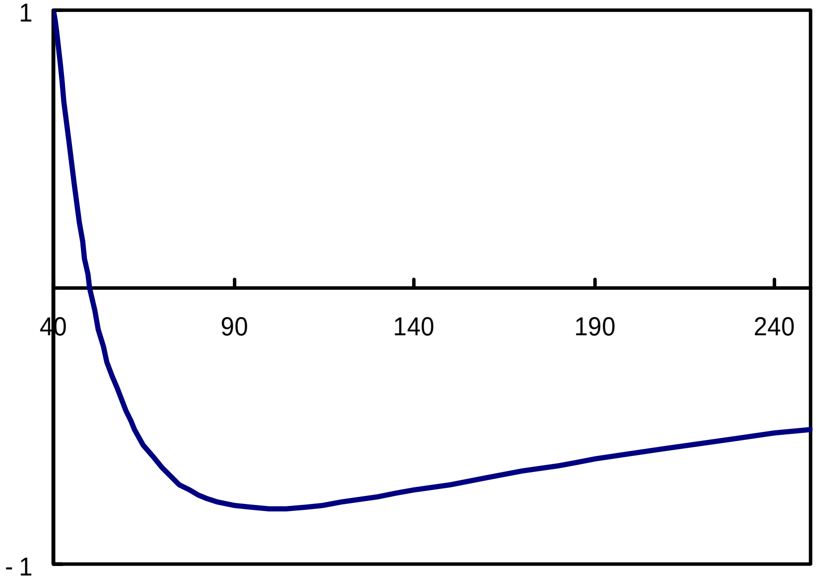
$$\alpha_{sim} = 1.1 \times 10^{-7}$$

$$\alpha_{exp} = 0.29$$

Wave-Wave Interactions via Potentials

Waves experience attractive and repulsive forces according to a Lennard-Jones type potential. Light intensity in control grid is determined by the distance between the centers of mass, and the light distribution is the gradient of the potential applied perpendicular to the velocity:

$$\phi = c\left(\frac{a}{\left|r_1 - r_2\right|^2} - \frac{1}{\left|r_1 - r_2\right|}\right)$$



$$\text{Light intensity gradient} = \hat{v}_{perp} \bullet \nabla \phi$$

where r_1, r_2 = the centers of mass of waves 1,2

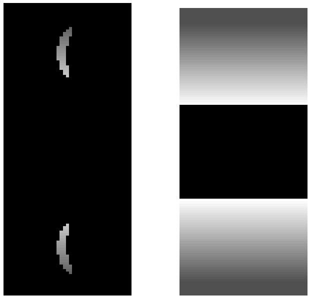
a determines the minimum in the potential

c gives the strength of the interaction:

$c < 50$: noncohesive behavior

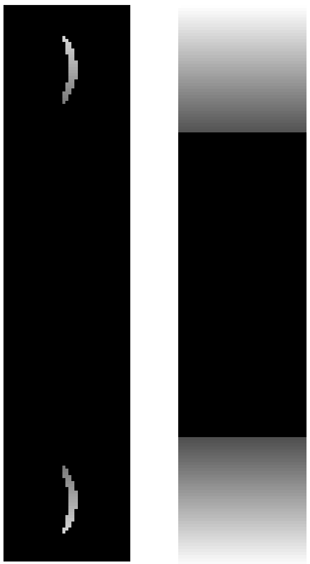
$50 < c < 150$: processional behavior

$150 < c < 350$: rotational & processional behavior



Repulsive Part of Potential

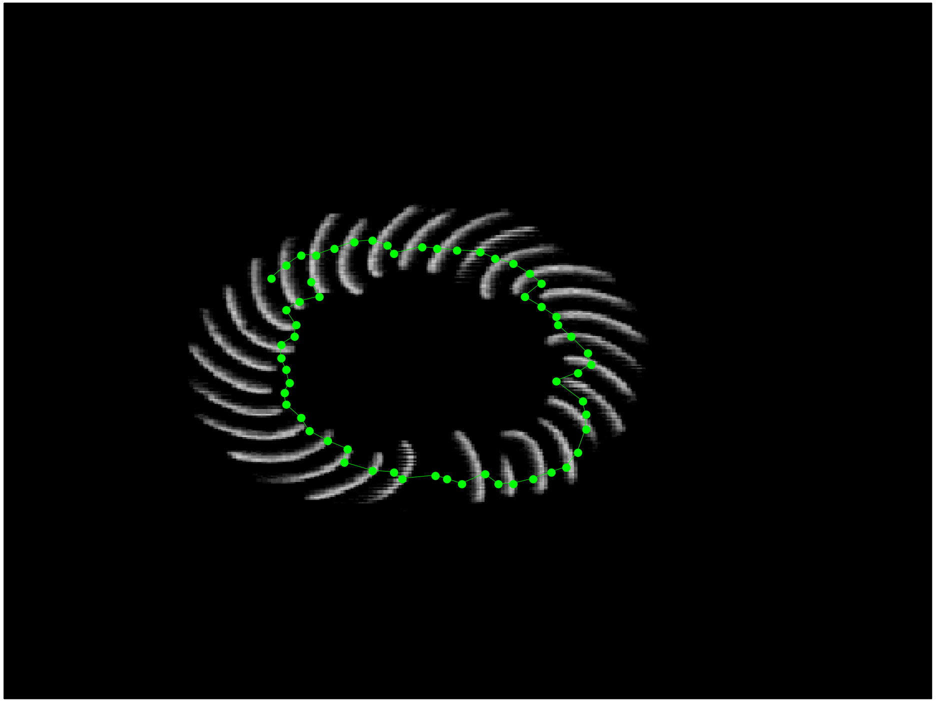
Waves experience repulsive forces:
Separation of center of masses = 80.
Equilibrium distance, $r_{eq} = 100$.



Attractive Part of Potential

Waves experience attractive forces:
Separation of center of masses = 140.
Equilibrium distance, $r_{eq} = 100$.

Rotational Behavior
Pairwise Interactions



Area = 3.1 cm × 3.1 cm
(160 pixels × 160 pixels)

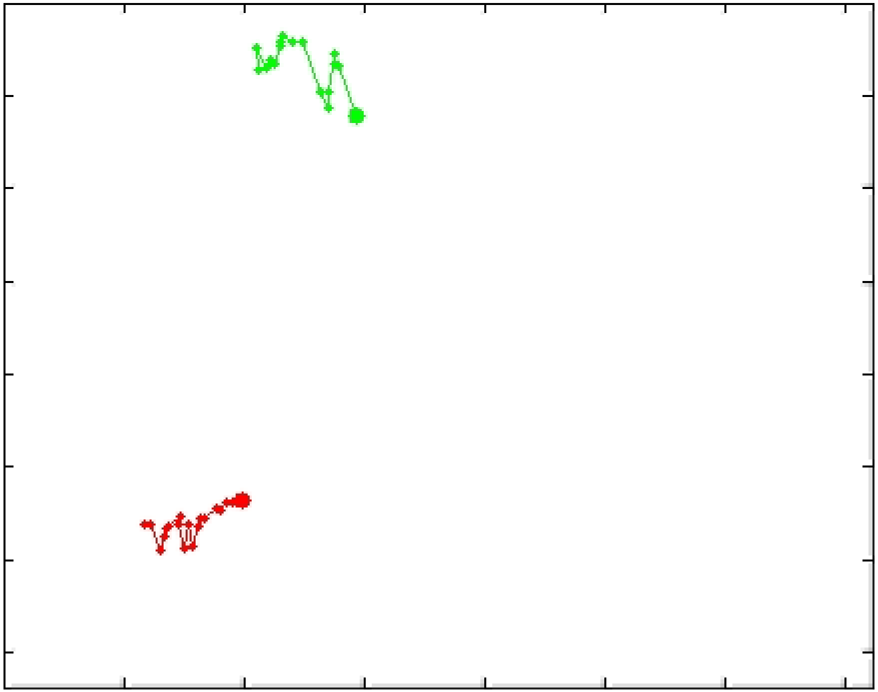
Controller box – 32 pixels by 32 pixels

Parameter a is chosen such that minimum in potential is at 50 pixels.

Rotational behavior is observed for high c values ($c > 7000$ mW/cm^2).

Rotational Behavior

Pairwise Interactions



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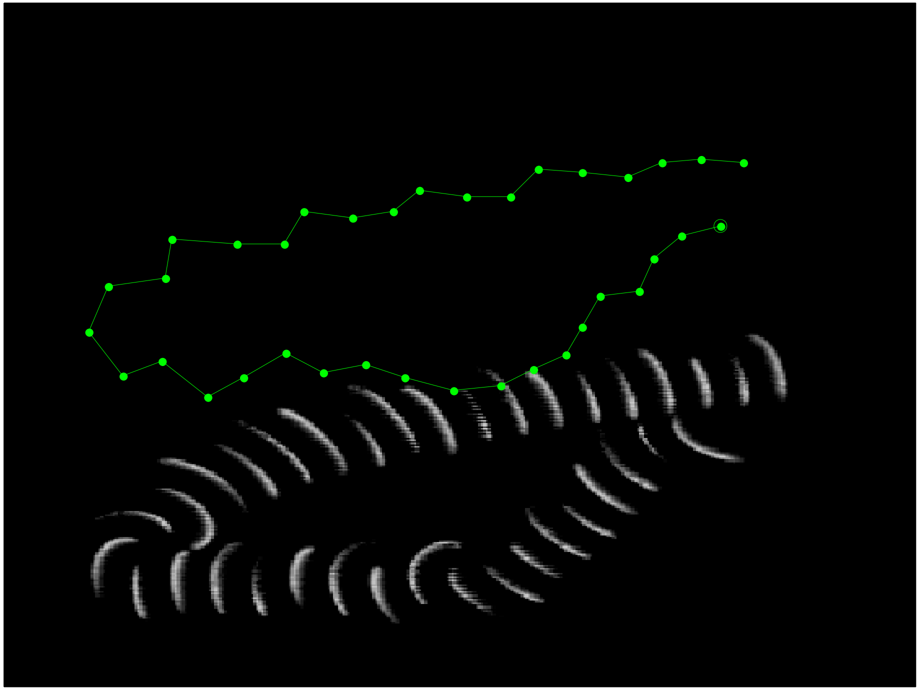
Area = 3.1 cm × 3.1 cm
(160 pixels × 160 pixels)

Controller box – 32 pixels by 32 pixels

Parameter a is chosen such that minimum in potential is at 50 pixels.

Rotational behavior is observed for high c values (c > 7000 mW/cm^2).

Processional Behavior
Pairwise Interaction



Area = 3.1 cm × 3.1 cm
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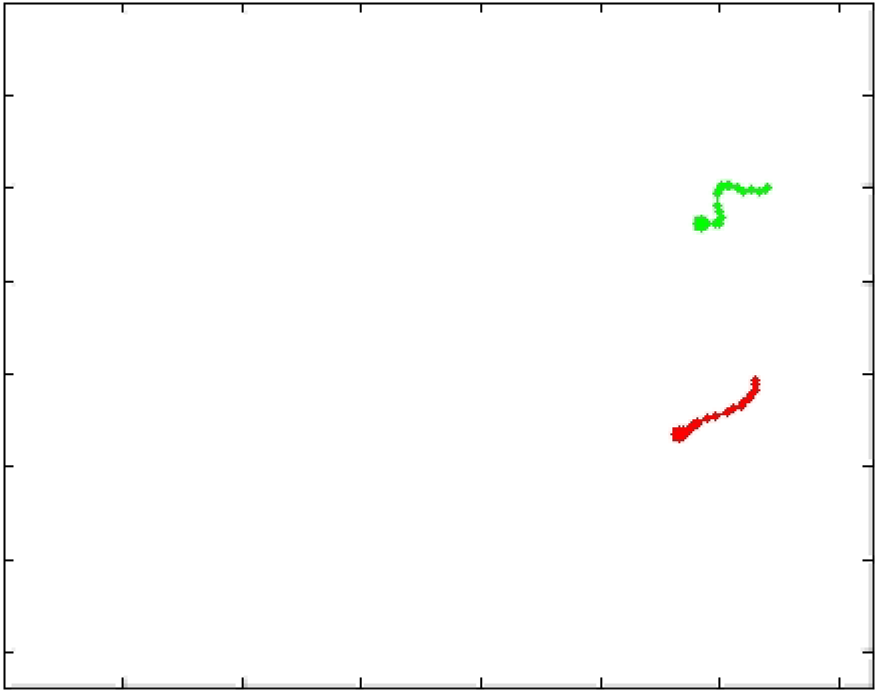
Controller box – 32 pixels by 32 pixels

Parameter a is chosen such that minimum in potential is at 50 pixels.

Processional behavior is observed for medium c values ($35 < c < 7000$ mW/cm^2).

Processional Behavior
Pairwise Interaction



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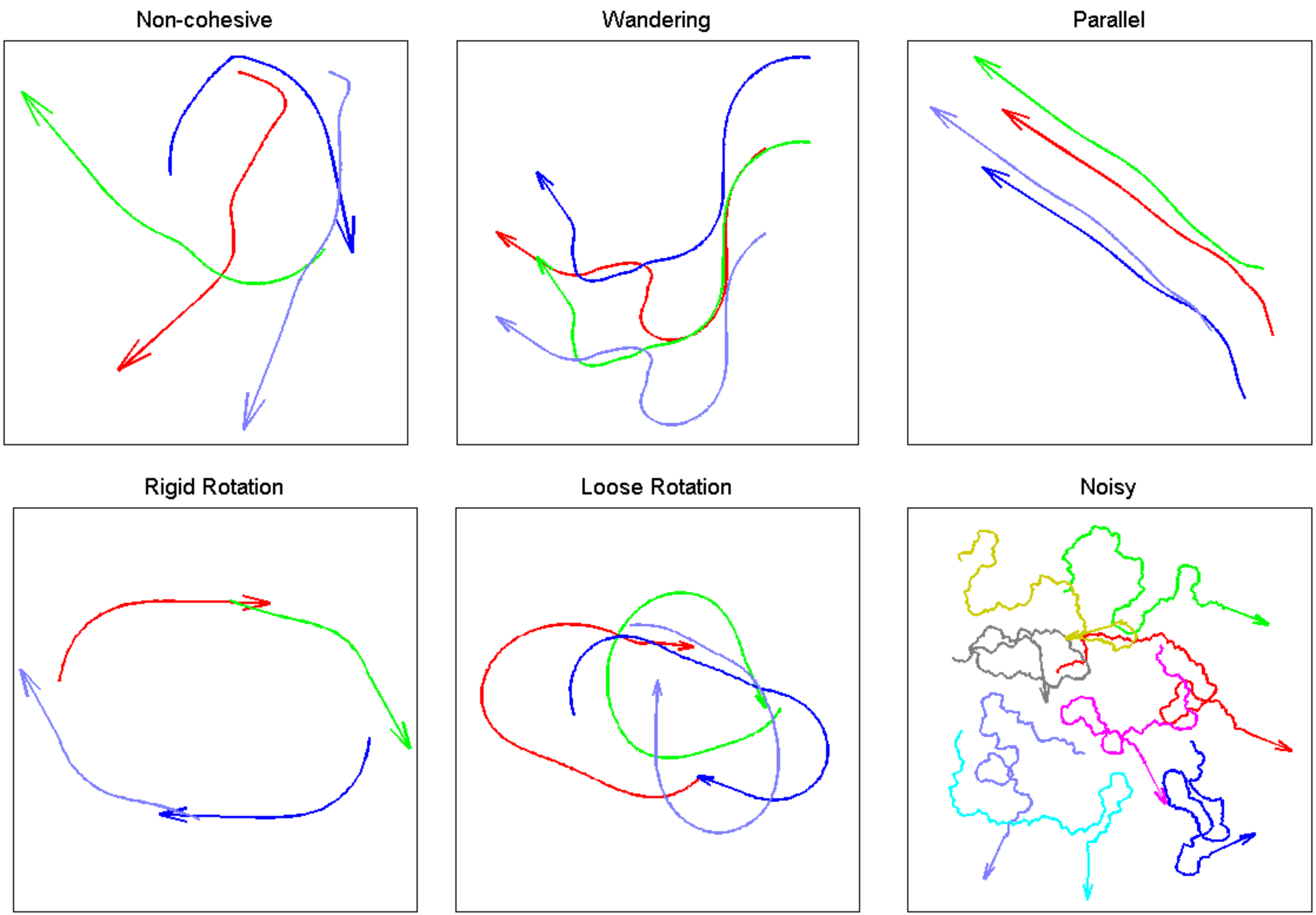
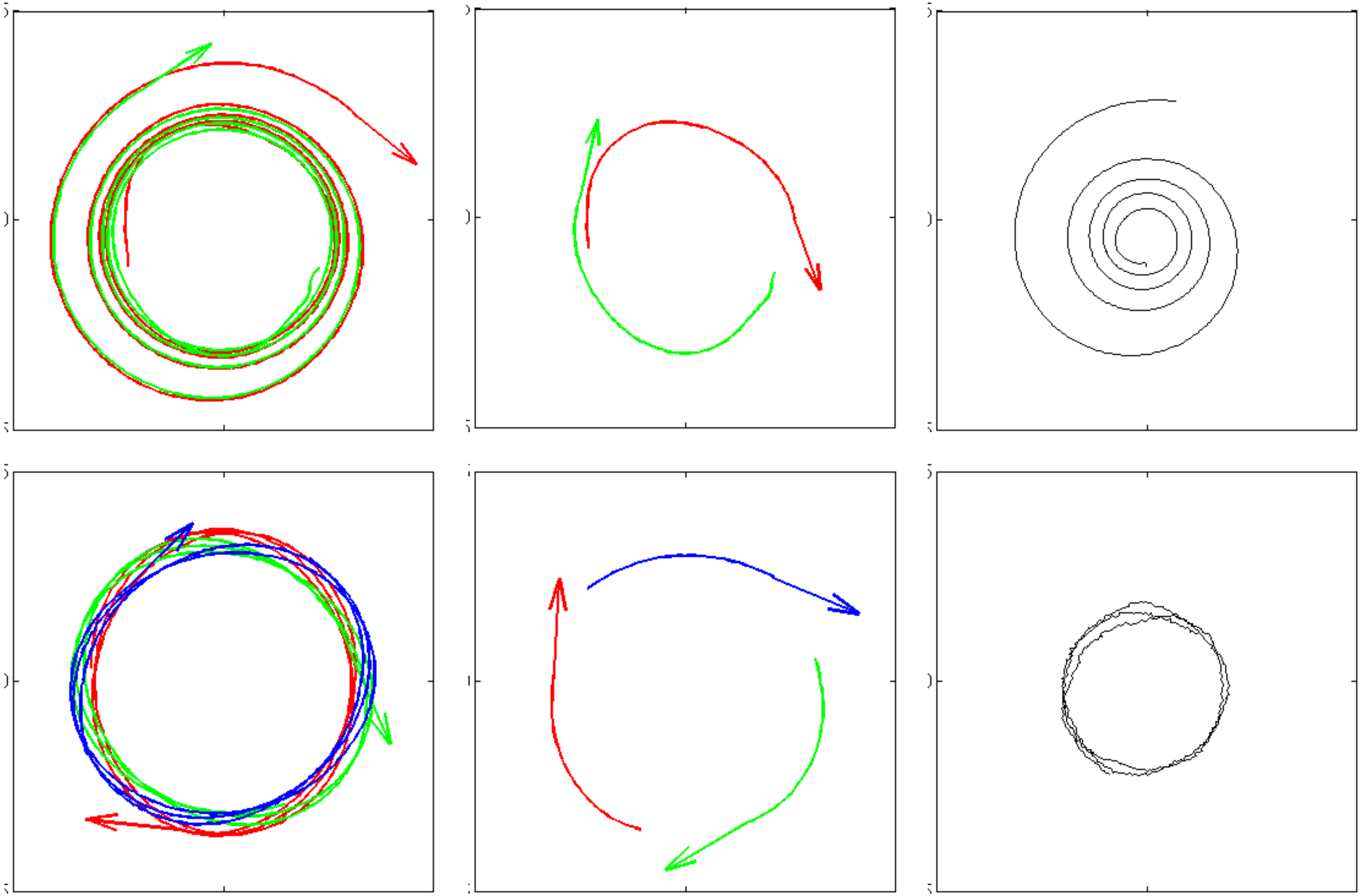

Non-cohesive
Wandering
Parallel
Rigid Rotation
Loose Rotation
Noisy



Two- and Four-Wave Rotation: All-to-All Interactions

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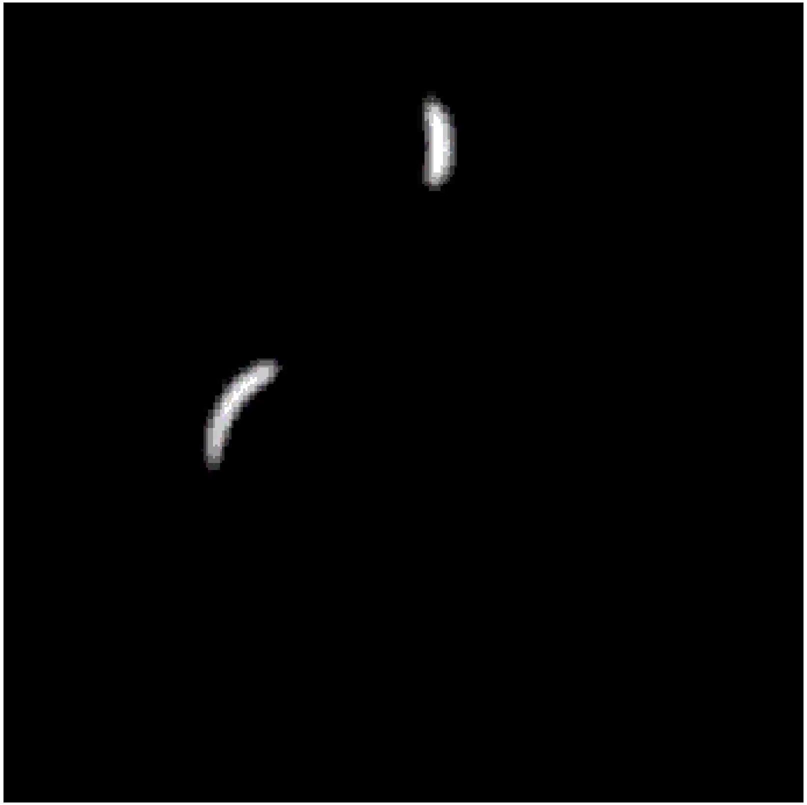
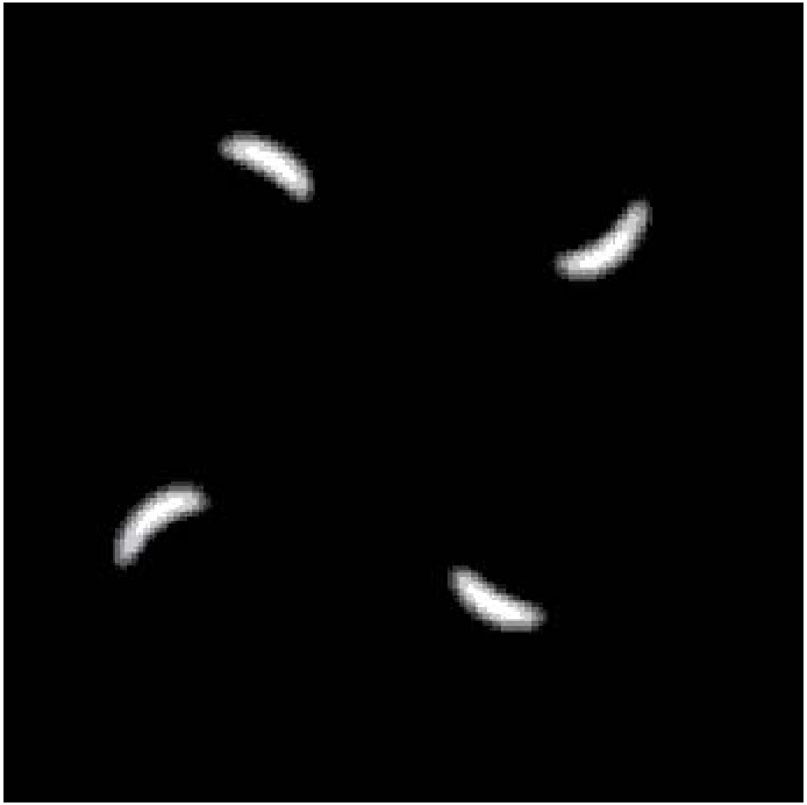
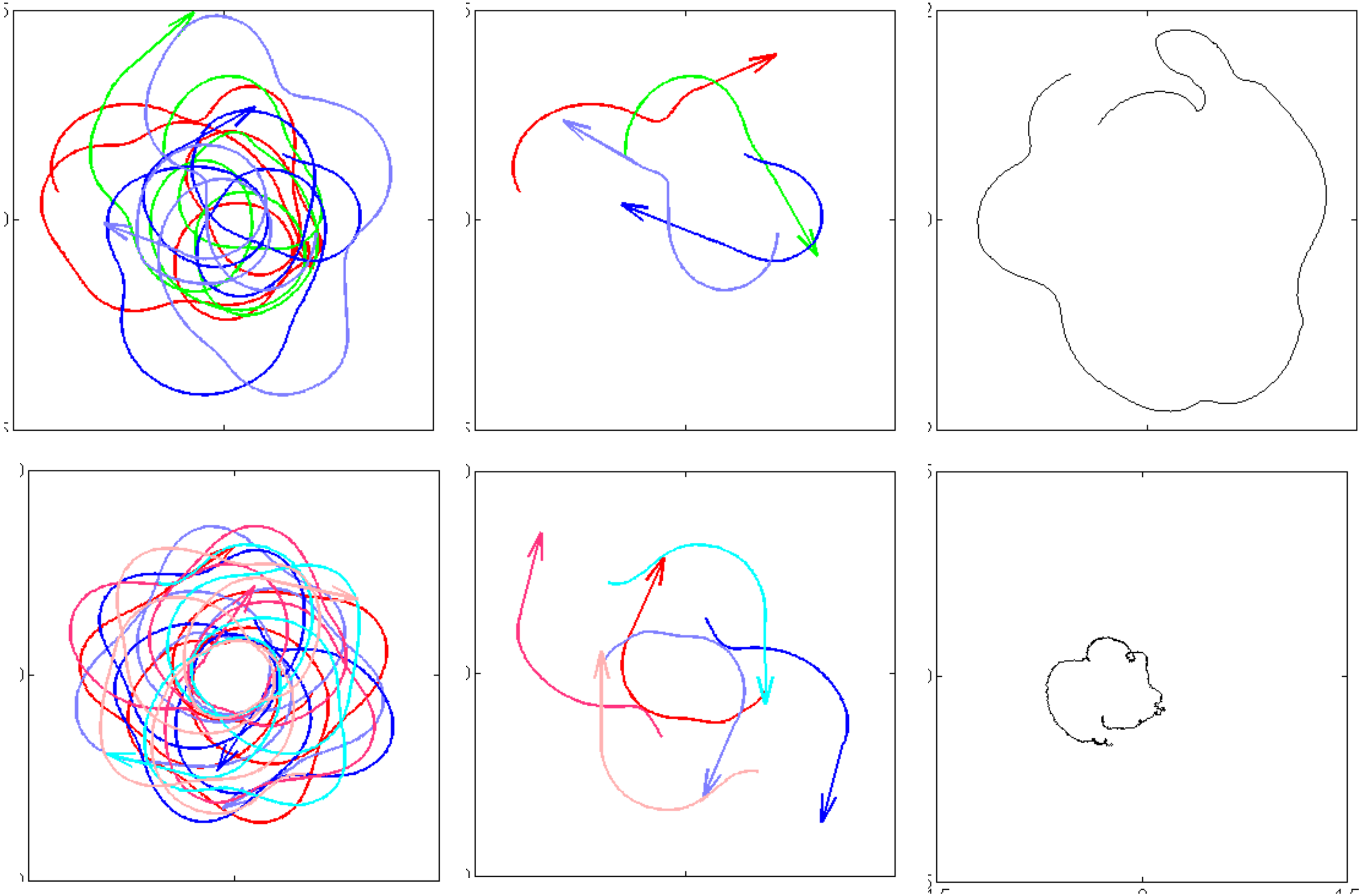
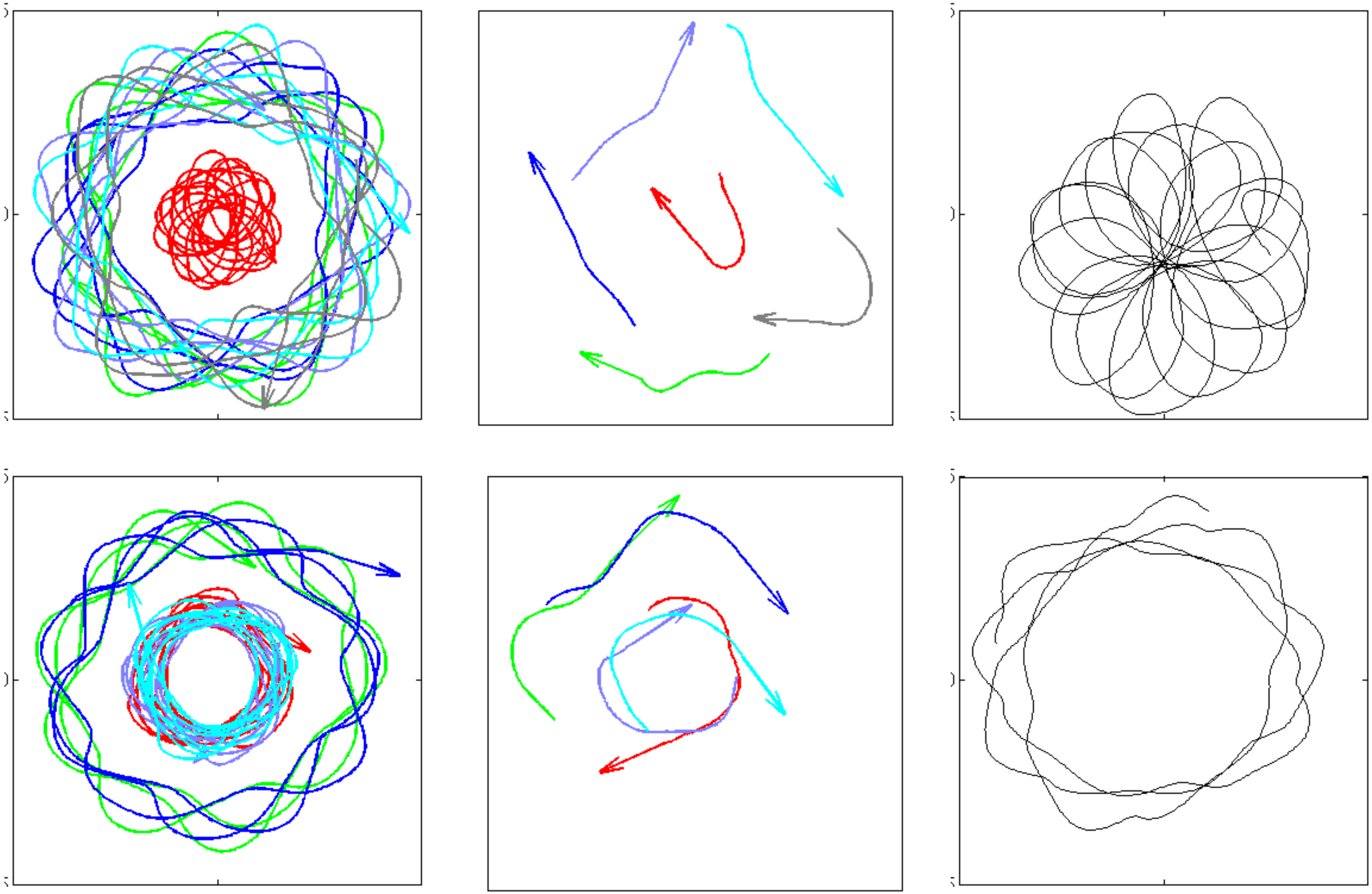


Unstable, spiraling out.



Stable orbit.





Five- and Eight-Wave Rotation: All-to-All Interactions

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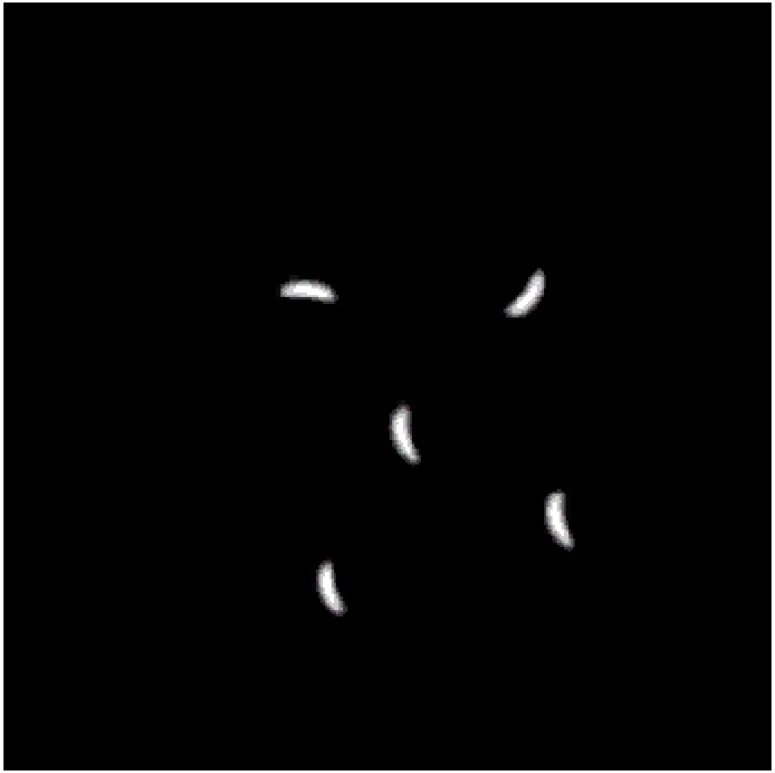
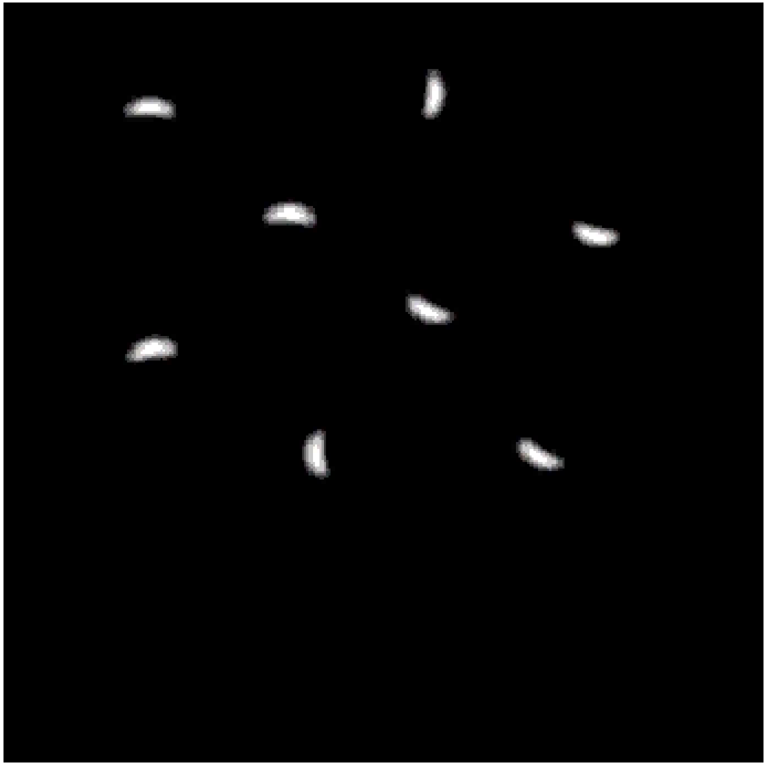
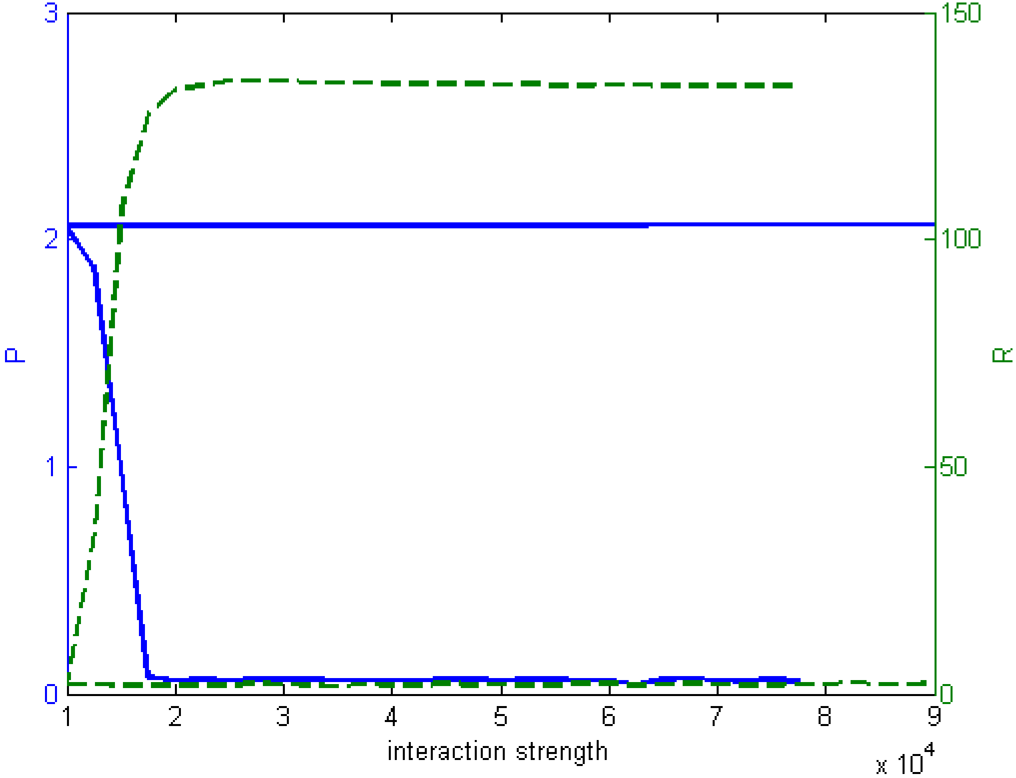


Three-wave center
with two outer waves.



Two-wave center
with six outer waves.

Behavior as a Function of Interaction Strength *c*



Four wave system.

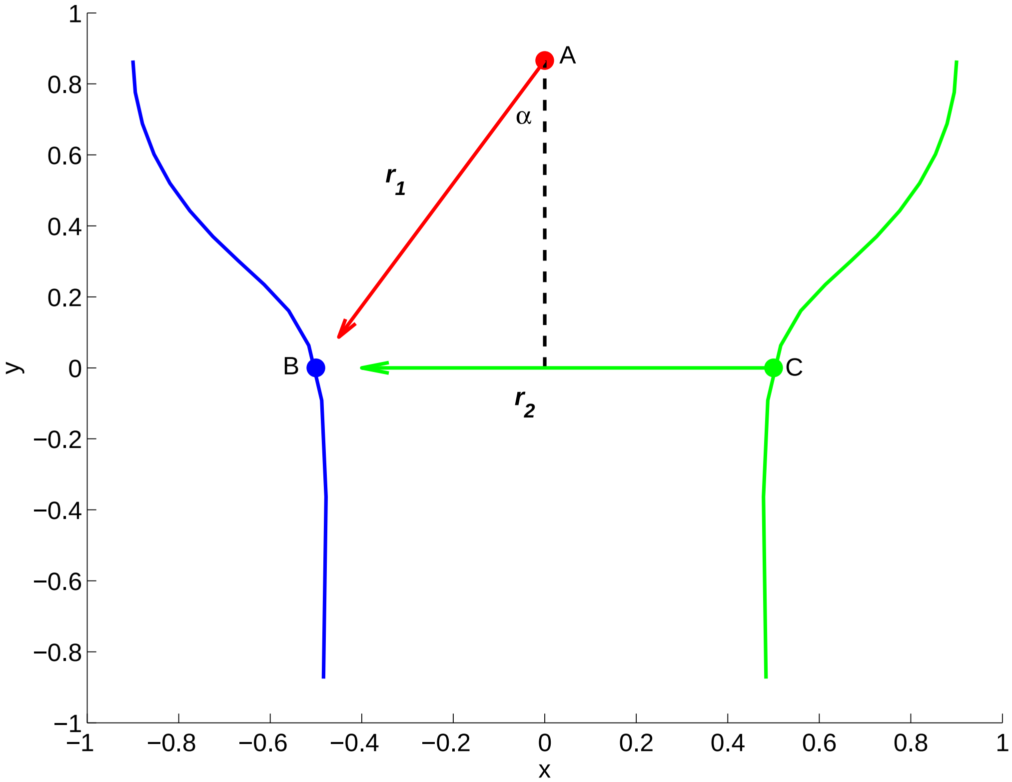
Rotational system at high value of *c*, which is decreased. At low values, system become processional, and stays that way as *c* is increased.

Green dashed line: value of R, giving the rotational character ("angular momentum").

Blue solid line: value of P, giving the processional character (average velocity).

$$R = \left| \sum_{n=1}^{N} \left(\mathbf{r}_{\mathrm{n}} - \mathbf{r}_{gm} \right) \times \mathbf{v}_{n} \right| \qquad P = \left| \sum_{n=1}^{N} \mathbf{v}_{n} \right|$$

Paths of Minimum Potential



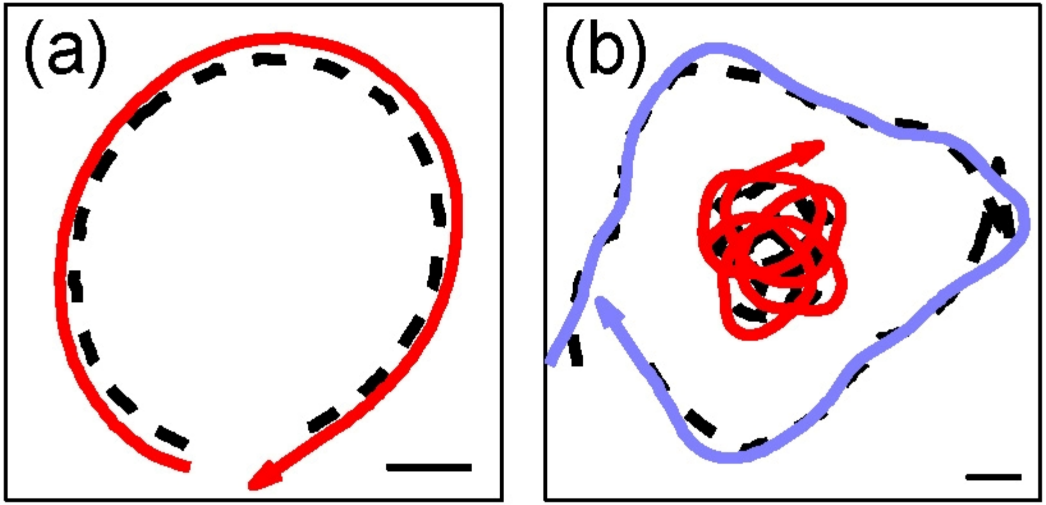
Stable steady state configurations for processional three-wave system.

LJ equilibrium distance is 100.

Wave A experiences equal and opposite gradients from B and C.

$$\hat{\mathbf{v}}_{perp} \cdot \nabla U_{total}^{LJ} = 0$$

Paths of Minimum Potential: Examples

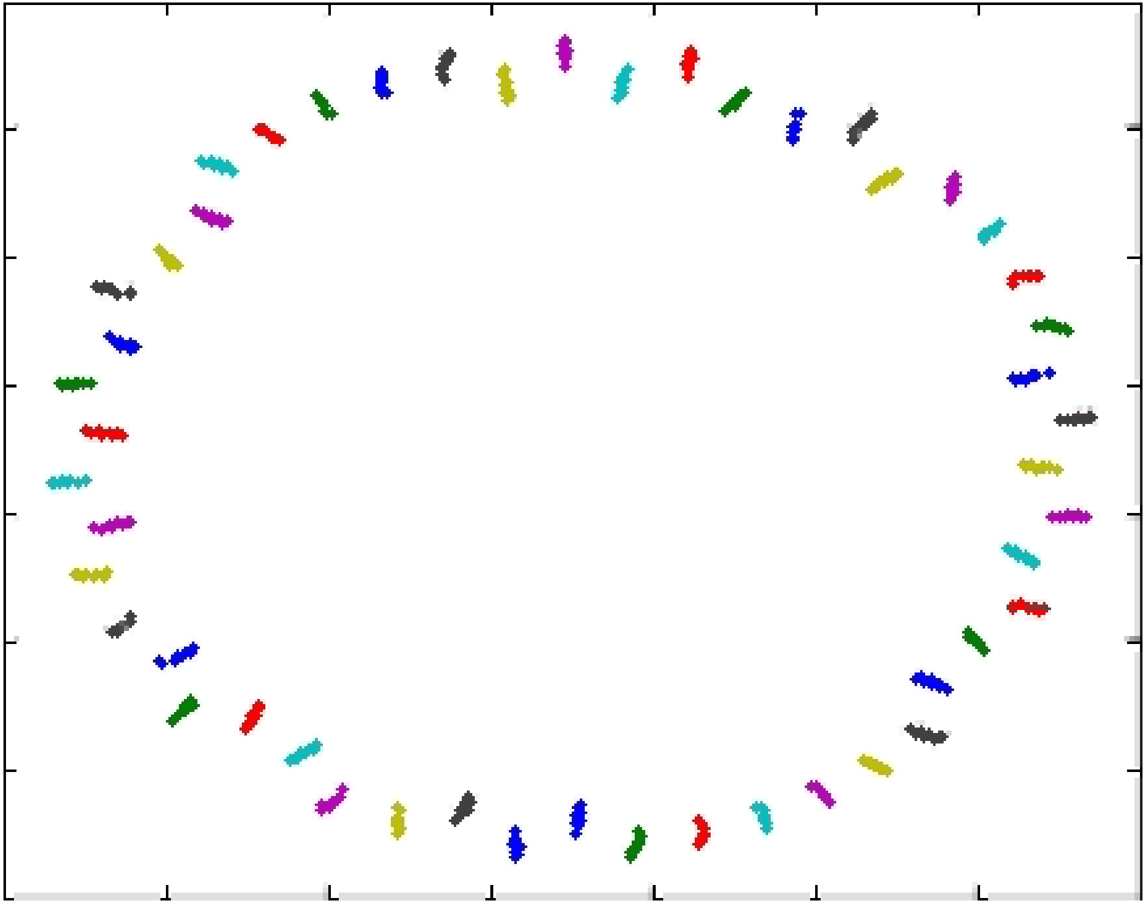


Interacting waves in multiple-wave rotations track paths defined by minimum potential.

(a) The trajectory of one wave in 3 wave rotation following minimum potential path.

(b) Two wave trajectories and the associated minimum potential paths in six-wave system (five waves orbiting around one wave).

Fifty-Wave System



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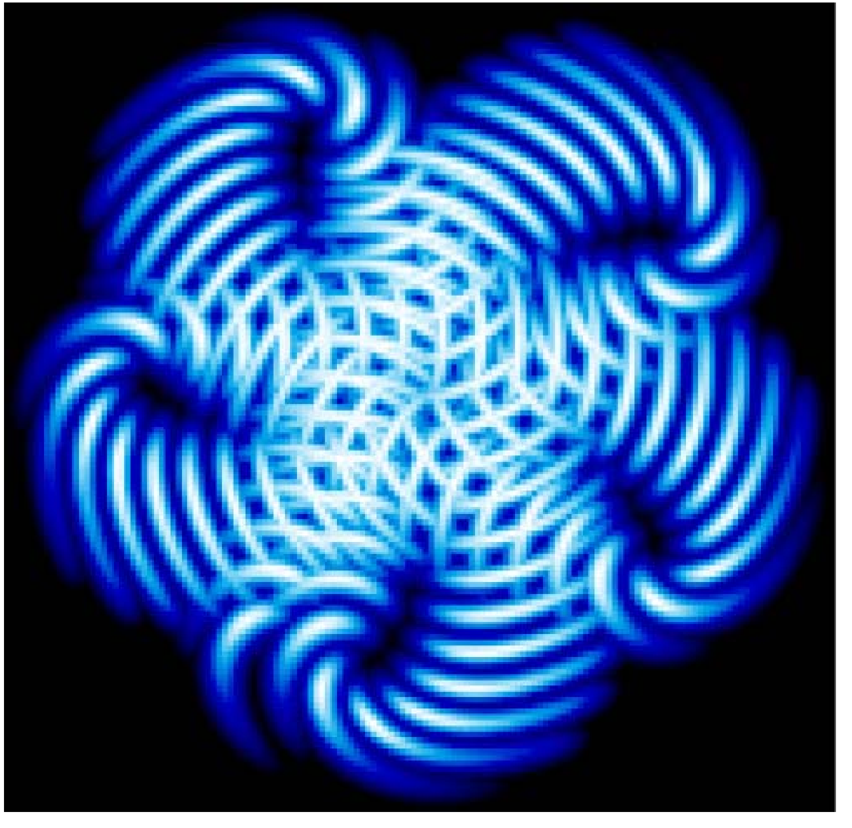
Collective Behavior in Excitable Media: Interacting Particle-Like Waves

Feedback stabilization of waves
Navigating excitability landscapes
Interacting waves via *LJ* potential
Processional and rotational behavior
Paths of minimum potential



http://heracles.chem.wvu.edu