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Deformation-induced spot dynamics in reaction-diffusion systems

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This is a joint work with Professor Y. Nishiura

Traveling spots in three-component systems

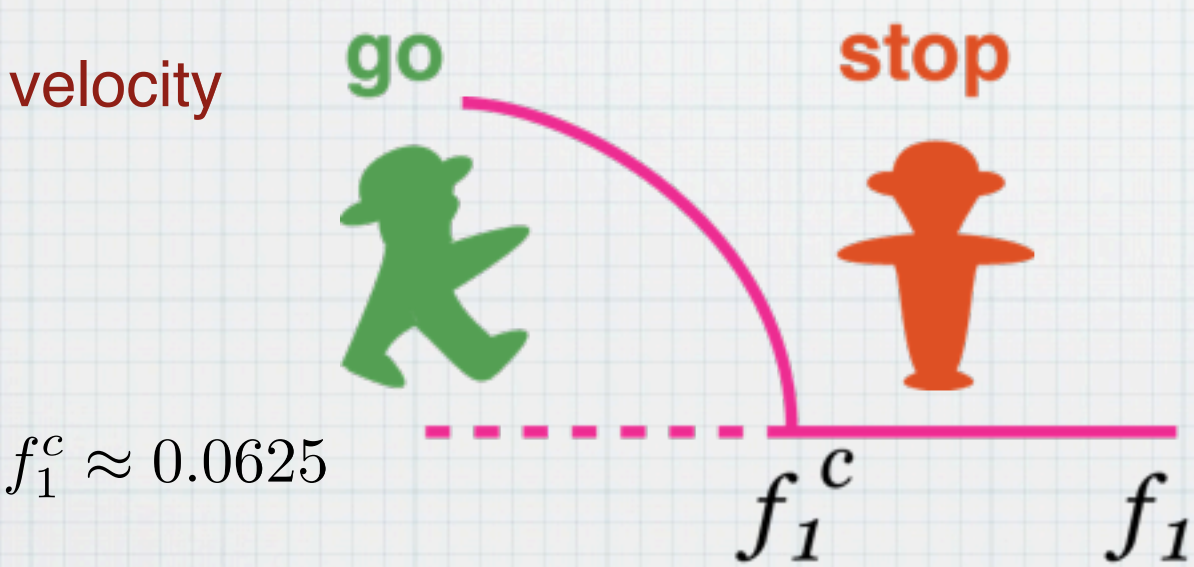
substrate **U**; activator **V**; inhibitor **W**; (see Vanag and Epstein)

$$\begin{cases} u_t = & D_u \triangle u - \dfrac{uv^2}{1+f_2 w} + f_0(1-u), \\ v_t = & D_v \triangle v + \dfrac{uv^2}{1+f_2 w} - (f_0+f_1)v, \\ \tau w_t = & D_w \triangle w + f_3(v-w), \end{cases}$$

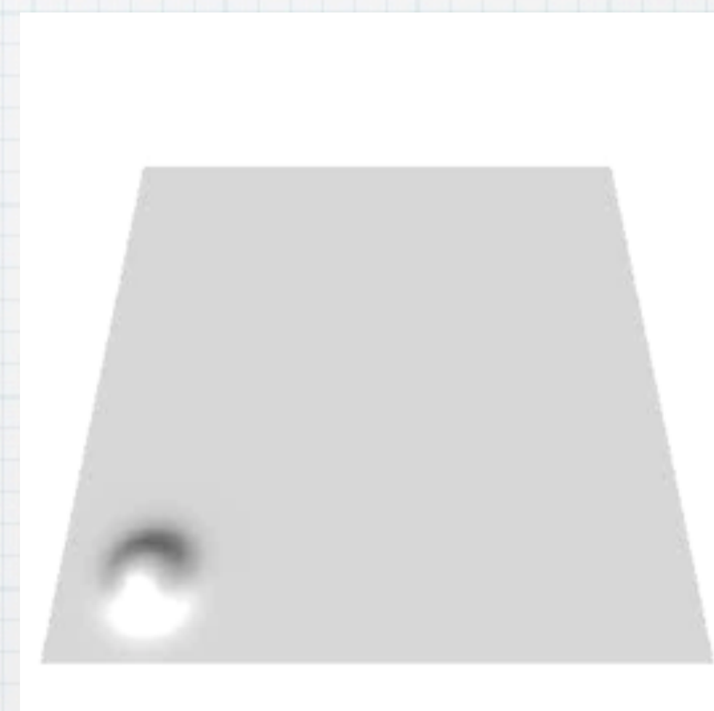
0

The third inhibitor **W** is a necessary condition for traveling spot to avoid a decay of pattern.

(see Purwins)



symmetry-breaking bifurcation



Traveling spots keep the shape firmly and propagate in a straight way with constant speed.

Strategy for analyzing spot behaviors

Blended methodology between computers and mathematics

* Phase 1: Computers

  Numerical simulations

  → careful observation of change of pattern dynamics

  Newton method and spectral analysis

  → characterization of instabilities

  unstable patterns and local dynamics around them

  Continuation and bifurcation analysis

  → global bifurcation diagram and higher singularity search

* Network of unstable patterns is a key to understand the large deformation during collision dynamics.

Scattering of traveling spots in dissipative systems,
Chaos 15 (2005) 047509.

Strategy for analyzing spot behaviors

- Phase 2: Mathematical analyses

  Extraction of essential dynamics based on center manifold theory

  Weak interaction

  $\rightarrow$ reduction to motion of particle (ODE) near bifur. pt.

  Investigation of underlying mechanism from a view of dynamical systems theory

  $\rightarrow$ standard dynamics classification

  prototypical bifurcation diagram

  unfolding of global bifurcations ($T \nearrow \infty$)

  degeneration of singularities

  Rigorous analysis by using singular perturbation theory

Detection and characterization of instabilities

Application of the dynamical systems theory

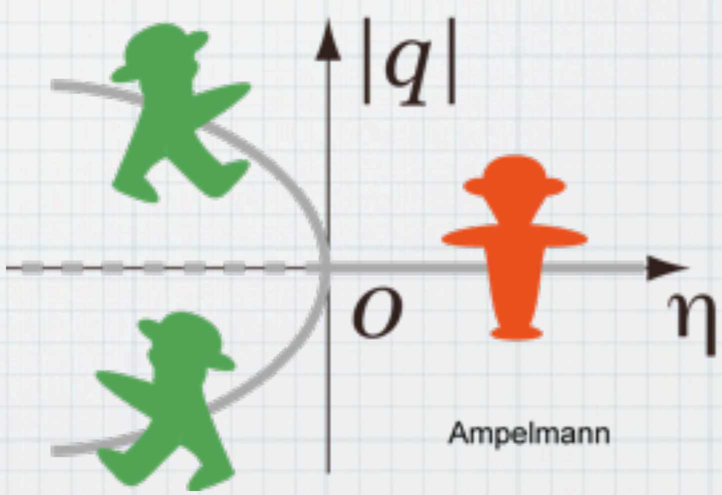
Dynamics of spot solution in the neighborhood of codimension 1 bifurcation point

Interacting spots in reaction diffusion systems,
Ei, Mimura, Nagayama, Disc. Cont. Dyn. Syst. 14 (2006) 31-62.

A general setup for the PDE system in a neighborhood of drift-bifurcation point reads, with small parameter η as $\lambda = \lambda_c + \eta,$

$$u_t = D\triangle u + F(u;\lambda) \equiv \mathcal{L}(u;\lambda^c) + \eta g(u),$$

We assume that the nontrivial standing spot solution $S(\mathbf{r};\lambda)$ exists at $\lambda = \lambda_c,$ i. e., $\mathcal{L}(S;\lambda^c) = 0.$

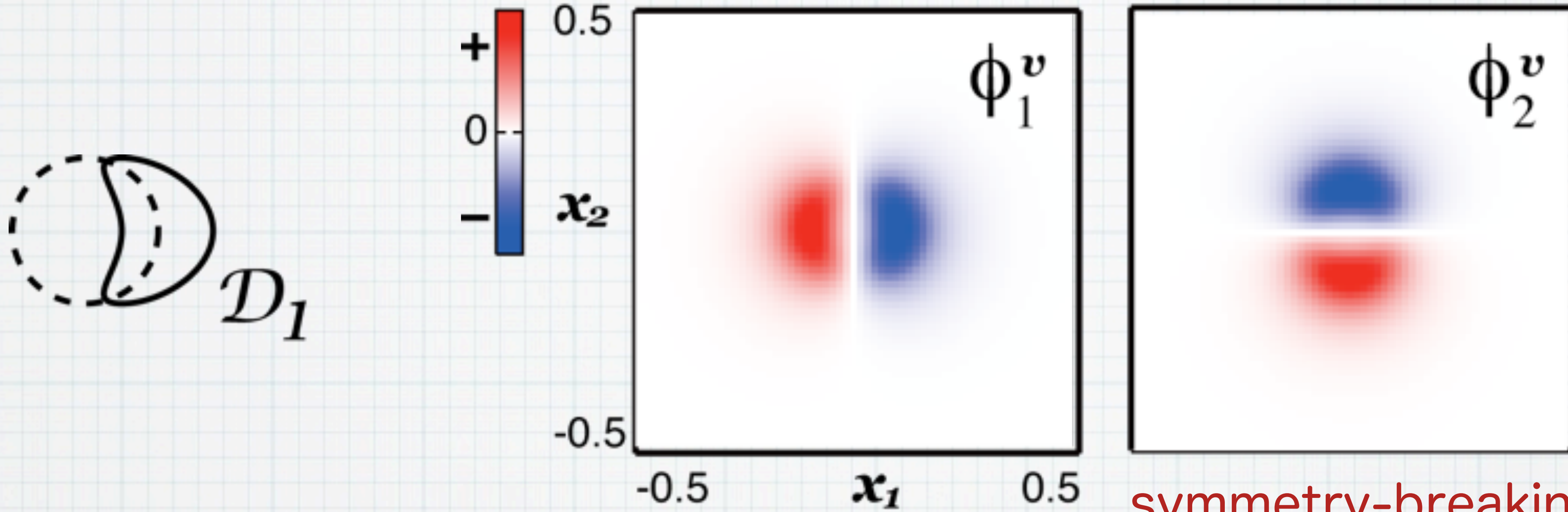


Linearized operator; $L = \mathcal{L}'(S(\mathbf{r},\lambda^c))$

$$L\phi_i = 0, \quad L\psi_i = -\phi_i,$$

where $\phi_i = \partial S/\partial x_i$ and ψ_i represents the deformation vector with Jordan from for the drift bifurcation.

Similar properties also holds for L^*, $L^*\phi_i^* = 0, \quad L^*\psi_i^* = -\phi_i^*.$



symmetry-breaking
(drift mode)

Let $E = \mathrm{span}\{\phi_i, \psi_i\}$.

Normalization; $$\langle \phi_i, \psi_i^* \rangle_{L^2} = \langle \psi_i, \phi_i^* \rangle_{L^2} = \begin{cases} \pi & i = j, \\ 0 & i \neq j. \end{cases}$$

The motion of a spot solution **u** is essentially described by two-dimensional vector functions of time **t** ;

Location of the spot; $\mathbf{p} = (p_1, p_2)$

Velocity of the spot; $\mathbf{q} = (q_1, q_2)$

For small η, we can approximate a solution u by

$$U = \tau(\mathbf{p}) \left\{ S(\mathbf{r}) + \sum_{i=1}^{2} q_i \psi_i(\mathbf{r}) + \zeta^\dagger \right\},$$

2

where $(\tau(\mathbf{p})u)(\mathbf{r}) = u(\mathbf{r} - \mathbf{p})$.

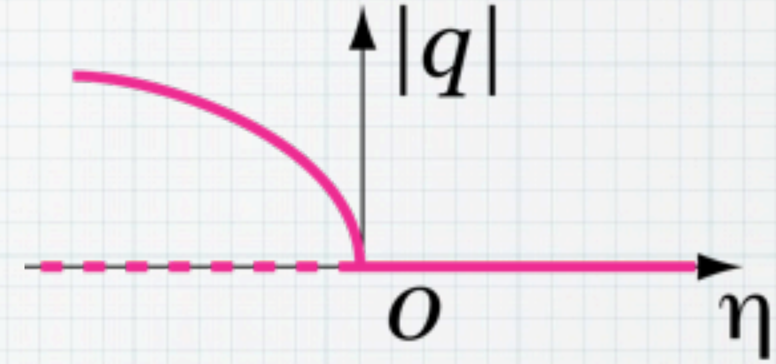
The remaining term, $\zeta^\dagger = {q_1}^2 \zeta_1 + {q_2}^2 \zeta_2 + q_1 q_2 \zeta_3 + \eta \zeta_4,$

with $\zeta_k (k = 1, \cdots 4) \in E^\perp$ are defined by solutions of

$$\begin{aligned}
-L\zeta_1 &= \frac{1}{2} F''(S) {\psi_1}^2 + \psi_{1x_1}, \\
-L\zeta_2 &= \frac{1}{2} F''(S) {\psi_2}^2 + \psi_{2x_2}, \\
-L\zeta_3 &= F''(S) \psi_1 \psi_2 + \psi_{1x_2} + \psi_{2x_1}, \\
-L\zeta_4 &= g(S).
\end{aligned}$$

Substituting 2 into 1 and taking inner product with the adjoints, we obtain the principal part by :

$$\begin{cases} \dot{p}_i = q_i, \\ \dot{q}_i = M_1 \sum_{j=1}^{2} {q_j}^2 q_i + M_2 \eta q_i, \end{cases}$$



$$M_1 \approx -246 < 0,$$
$$M_2 \approx -101 < 0.$$

$$\begin{cases} \pi M_1 = \frac{1}{6} \langle F'''(S) {\psi_1}^3, \phi_1^* \rangle_{L^2} \\ \qquad + \langle F''(S) \psi_1 \zeta_1, \phi_1^* \rangle_{L^2} + \langle \zeta_{1x_1}, \phi_1^* \rangle_{L^2}, \\ \pi M_2 = \langle g'(S) \psi_1, \phi_1^* \rangle_{L^2} \\ \qquad + \langle F''(S) \psi_1 \zeta_4, \phi_1^* \rangle_{L^2} + \langle \zeta_{4x_1}, \phi_1^* \rangle_{L^2}. \end{cases}$$

The coefficients M_1, M_2 are crucial for understanding the dynamics of spot.

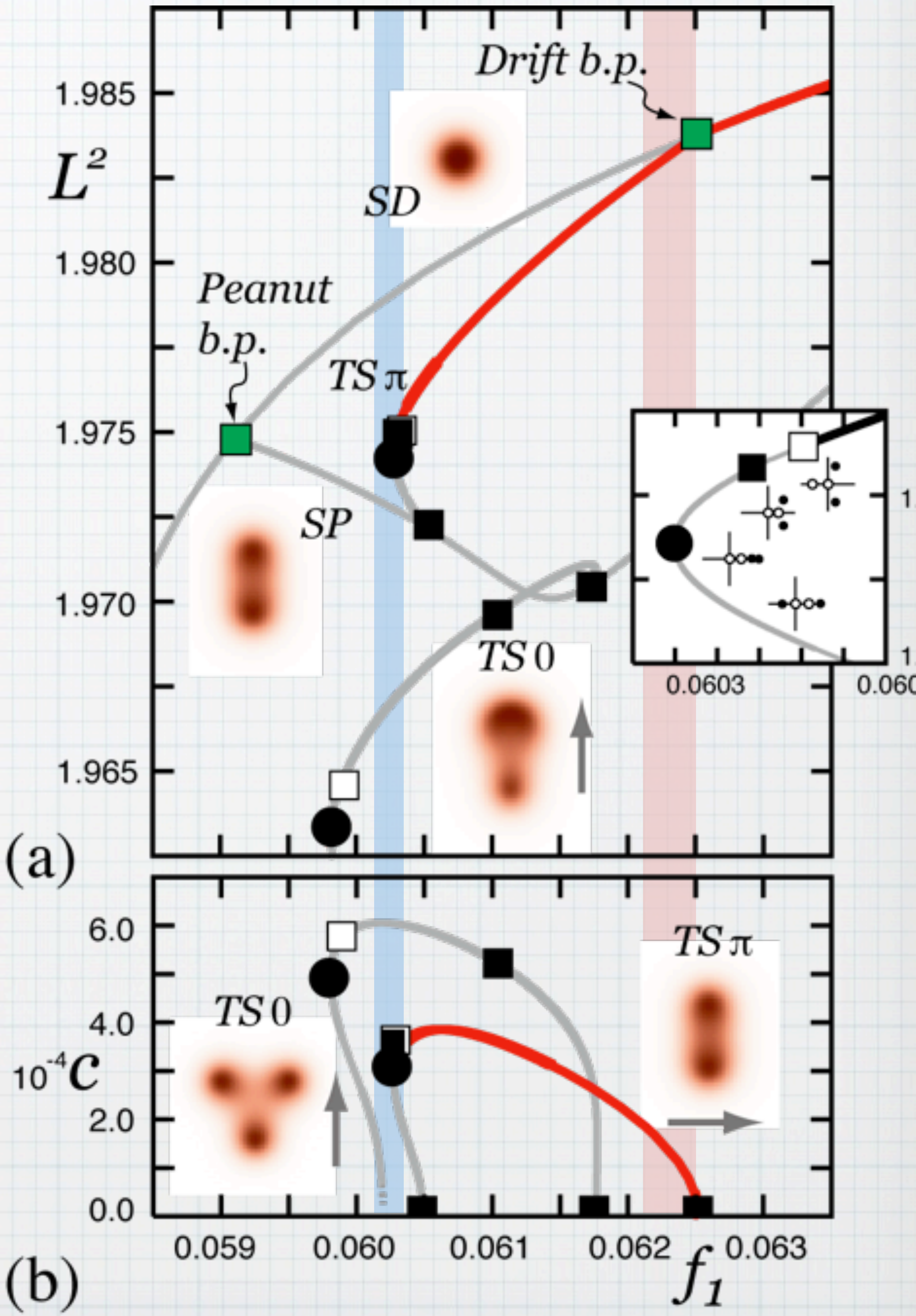
Information specific to the original PDEs is contained in M_1, M_2.

codim 1

Rotational motion of traveling spot

serendipity !

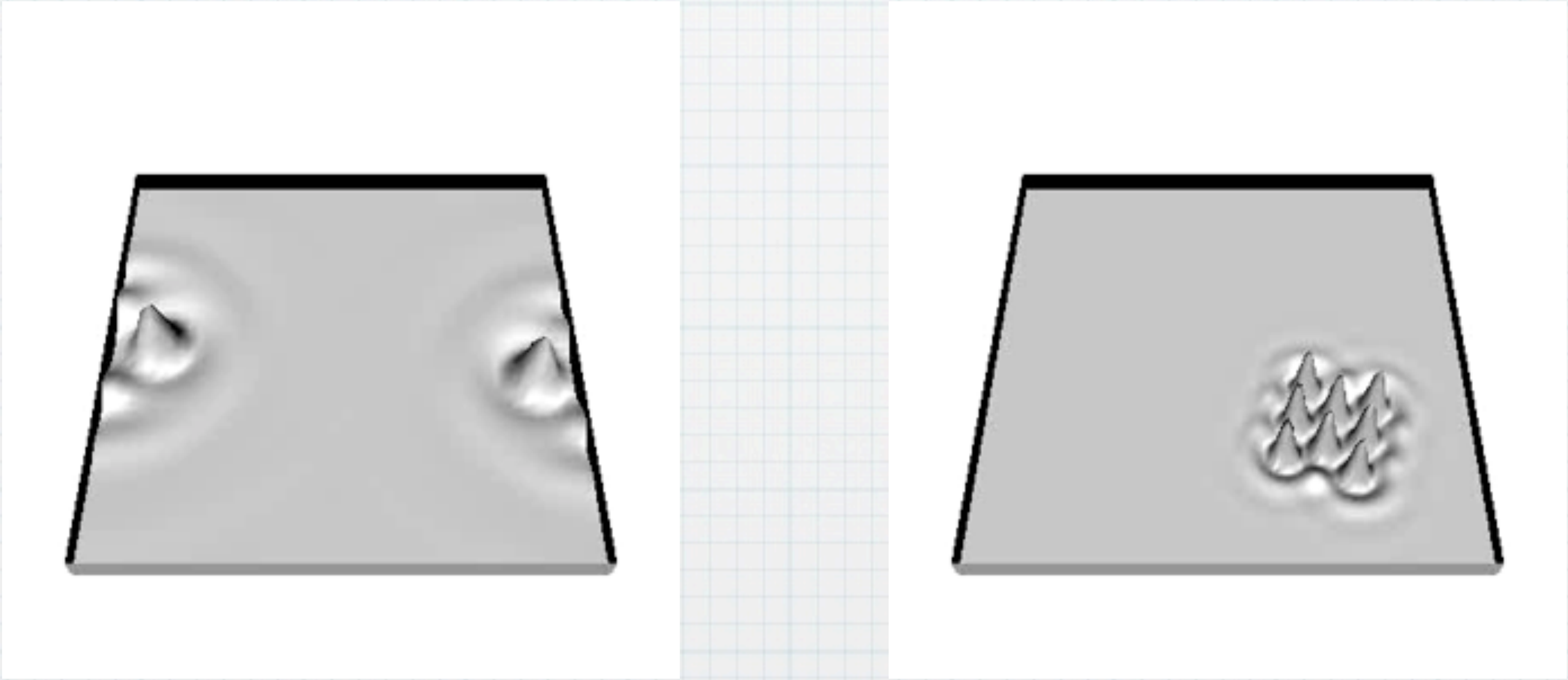
a dog chase its tail and rotate.



Remark: Rotational motion of spots in gas-discharge system

Rotating bound states of dissipative solitons in systems of reaction-diffusion type, Liehr, Moskalenko, Astrov, Bode and Purwins, EPJB 37 (2004) 199-204.

$$\begin{cases} u_t = & D_u \triangle u + k_2 u - u^3 - k_3 v - k_4 w + k_1, \\ \tau v_t = & u - v, \\ 0 = & D_w \triangle w + u - w, \end{cases}$$



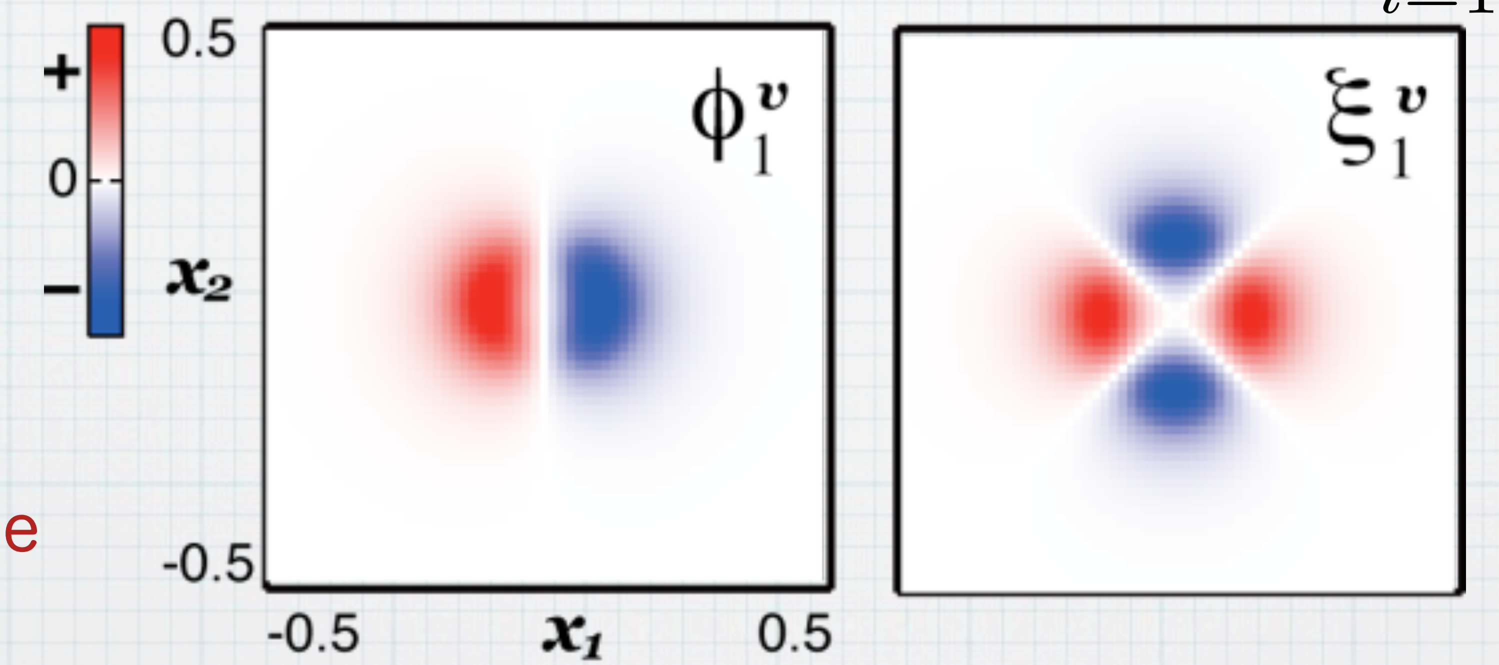
oscillatory tail form --> attractive force --> bound states (cluster)

Dynamics of spot solution in the neighborhood of codimension 2 bifurcation point

The parameter values are located close to the drift and peanut bifurcation points as $(\lambda_1, \lambda_2) = (\lambda_1^c, \lambda_2^c) + (\eta_1, \eta_2).$

$$u_t = \quad D\triangle u + F(u;\lambda) \equiv \mathcal{L}(u;\lambda^c) + \sum_{i=1}^{2} \eta_i g_i(u),$$

drift mode



peanut mode

$$L\phi_i = 0, \quad L\psi_i = -\phi_i, \quad L\xi_i = 0,$$

$$L^*\phi_i^* = 0, L^*\psi_i^* = -\phi_i^*, L^*\xi_i^* = 0,$$

$\mathcal{D}_1$

Drift instability originates in the translation-free mode and the associated deformation vector represents a $\mathcal{D}_1$ symmetry-breaking from a $\mathcal{D}_\infty$ shape.

$\mathcal{D}_2$

Peanut one is by $\mathcal{D}_2$ symmetry-breaking bifurcation.

Normalization;

$$\langle \phi_i, \psi_i^* \rangle_{L^2} = \langle \psi_i, \phi_i^* \rangle_{L^2} = \langle \xi_i, \xi_i^* \rangle_{L^2} = \begin{cases} \pi & i = j, \\ 0 & i \neq j. \end{cases}$$

The motion of a spot solution **u** is essentially described by two-dimensional vector functions of time **t** ;

Location of the spot;	$\mathbf{p} = (p_1, p_2)$
Velocity of the spot;	$\mathbf{q} = (q_1, q_2)$
Deformation of the spot;	$\mathbf{s} = (s_1, s_2)$

Let $E = \mathrm{span}\{\phi_i, \psi_i, \xi_i\}.$

For small η, we can approximate a solution $\mathbf{U}$ by 4

$$U \quad = \tau(\mathbf{p}) \left\{ S(\mathbf{r}) + \textstyle\sum_{i=1}^{2} q_i \psi_i(\mathbf{r}) + \sum_{i=1}^{2} s_i \xi_i(\mathbf{r}) + \zeta^\dagger \right\}.$$

The remaining term, center manifold,

$$\zeta^\dagger = {q_1}^2 \zeta_1 + {q_2}^2 \zeta_2 + q_1 q_2 \zeta_3 + {s_1}^2 \zeta_4 + {s_2}^2 \zeta_5 + s_1 s_2 \zeta_6$$
$$+ q_1 s_1 \zeta_7 + q_2 s_2 \zeta_8 + q_1 s_2 \zeta_9 + q_2 s_1 \zeta_{10} + \eta_1 \zeta_{11} + \eta_2 \zeta_{12}$$

with $\zeta_k (k = 1, \cdots 12) \in E^\perp$

Substituting 4 into 1 and taking inner product with the adjoints,
we obtain :

Here we introduce the complex variables,

$$z_0 = p_1 + ip_2, z_1 = q_1 + iq_2, z_2 = s_1 + is_2.$$

5

location

velocity

deformation

$$\begin{cases} \dot{z_0} = z_1 - \beta' \overline{z_1} z_2, \\ \dot{z_1} = M_1 |z_1|^2 z_1 + M_2 |z_2|^2 z_1 + M_3 z_1 + \beta \overline{z_1} z_2, \\ \dot{z_2} = N_1 |z_2|^2 z_2 + N_2 |z_1|^2 z_2 + N_3 z_2 + \alpha {z_1}^2. \end{cases}$$

M_3, N_3 are used as the new bifurcation parameter set.

Rotational motion of traveling spots in dissipative systems,
Teramoto, Suzuki, Nishiura, Physical Review E 80 (2009) 046208.

The dynamics are essentially governed by the last two equations, exactly the same as the normal form obtained in the study of mode interaction of steady bifurcations in O(2) symmetry.

Through the slave part of equations of motion, richness of dynamics in the master part is converted into that of particle motion.

→ Natural extension to the deformed particle dynamics

The constants are computed as,

$$\begin{aligned}\pi M_1 =& \frac{1}{6}\langle F'''(S){\psi_1}^3, \phi_1^*\rangle_{L^2}\\ &+ \langle F''(S)\psi_1\zeta_1, \phi_1^*\rangle_{L^2} + \langle \zeta_{1x_1}, \phi_1^*\rangle_{L^2},\\ \pi M_2 =& \frac{1}{2}\langle F'''(S){\xi_1}^2\psi_1, \phi_1^*\rangle_{L^2}\\ &+ \langle F''(S)\psi_1\zeta_4, \phi_1^*\rangle_{L^2} + \langle F''(S)\xi_1\zeta_7, \phi_1^*\rangle_{L^2}\\ &+ \langle \zeta_{4x_1}, \phi_1^*\rangle_{L^2} - \beta'\langle \xi_{1x_1}, \phi_1^*\rangle_{L^2},\\ \pi M_3 =& \eta_1\left(\langle F''(S)\psi_1\zeta_{11}, \phi_1^*\rangle_{L^2}\right.\\ &\left.+\langle g_1'(S)\psi_1, \phi_1^*\rangle_{L^2} + \langle \zeta_{11x_1}, \phi_1^*\rangle_{L^2}\right)\\ &+ \eta_2\left(\langle F''(S)\psi_1\zeta_{12}, \phi_1^*\rangle_{L^2}\right.\\ &\left.+\langle g_2'(S)\psi_1, \phi_1^*\rangle_{L^2} + \langle \zeta_{12x_1}, \phi_1^*\rangle_{L^2}\right).\end{aligned}$$

$$\begin{aligned}\pi N_1 =& \frac{1}{6}\langle F'''(S){\xi_1}^3, \xi_1^*\rangle_{L^2} + \langle F''(S)\xi_1\zeta_4, \xi_1^*\rangle_{L^2},\\ \pi N_2 =& \frac{1}{2}\langle F'''(S){\psi_1}^2\xi_1, \xi_1^*\rangle_{L^2}\\ &+ \langle F''(S)\psi_1\zeta_7, \xi_1^*\rangle_{L^2} + \langle F''(S)\xi_1\zeta_1, \xi_1^*\rangle_{L^2}\\ &+ \langle \zeta_{7x_1}, \xi_1^*\rangle_{L^2} - \beta'\langle \psi_{1x_1}, \xi_1^*\rangle_{L^2},\\ \pi N_3 =& \eta_1\left(\langle F''(S)\xi_1\zeta_{11}, \xi_1^*\rangle_{L^2} + \langle g_1'(S)\xi_1, \xi_1^*\rangle_{L^2}\right)\\ &+ \eta_2\left(\langle F''(S)\xi_1\zeta_{12}, \xi_1^*\rangle_{L^2} + \langle g_2'(S)\xi_1, \xi_1^*\rangle_{L^2}\right).\end{aligned}$$

$$M_1 \approx -61.3 < 0,\ M_2 \approx -3.9,$$

$$N_1 \approx -240.0 < 0,\ N_2 \approx -35.6 < 0$$

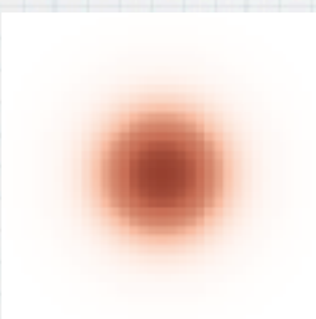
$$\alpha \approx -31.8 < 0, \beta \approx 1.0 > 0, \beta' \approx -326.7 < 0$$

M_1, M_2, N_1, N_2 are all negative. $\beta > 0, \alpha < 0, \beta' < 0.$

Letting $z_1 = Qe^{i\phi}, z_2 = Se^{i\psi}$, we rewrite 5 as

$$\begin{cases} \dot{Q} = (M_1 Q^2 + M_2 S^2 + M_3) Q + \beta Q S \cos\theta, \\ \dot{S} = (N_1 S^2 + N_2 Q^2 + N_3) S + \alpha Q^2 \cos\theta, \\ \dot{\theta} = -\left(2\beta S + \dfrac{\alpha Q^2}{S}\right) \sin\theta, \end{cases} \tag{6}$$

where we set $\theta = \psi - 2\phi$.



trivial fixed points →

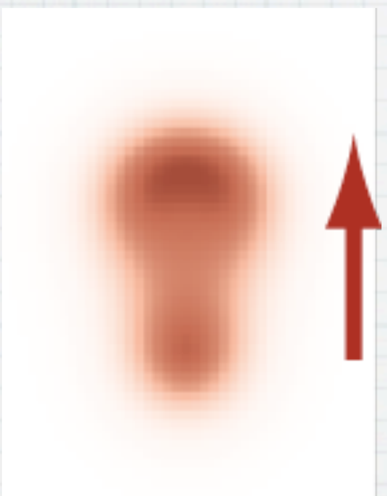
Standing disk (SD) spot solution: $Q = S = 0$

fixed points with $|\cos\theta| = 1$ →

Standing peanut (SP) spot solution: $Q = 0, \quad S^2 = -N_3/N_1$

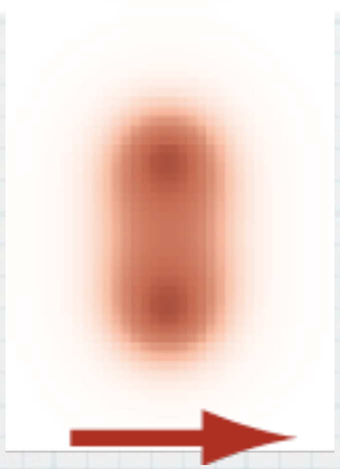
Traveling spot (TS) solution bifurcates from SD spot at $M_3 = 0$
and from SP spot at $M_3 - M_2 N_3/N_1 \pm \beta(-N_3/N_1)^{1/2} = 0.$

$$\begin{cases} M_1 Q^2 + M_2 S^2 + M_3 \pm \beta S = 0, \\ (N_1 S^2 + N_2 Q^2 + N_3) S \pm \alpha Q^2 = 0, \end{cases}$$



Traveling spot TS_0 with $\cos\theta = +1$ corresponds to a propagation direction **parallel** to the long axis of the deformed shape.

Dictyostelid type



Traveling spot TS_π with $\cos\theta = -1$ corresponds to a propagation direction **perpendicular** to the long axis of the deformed shape.

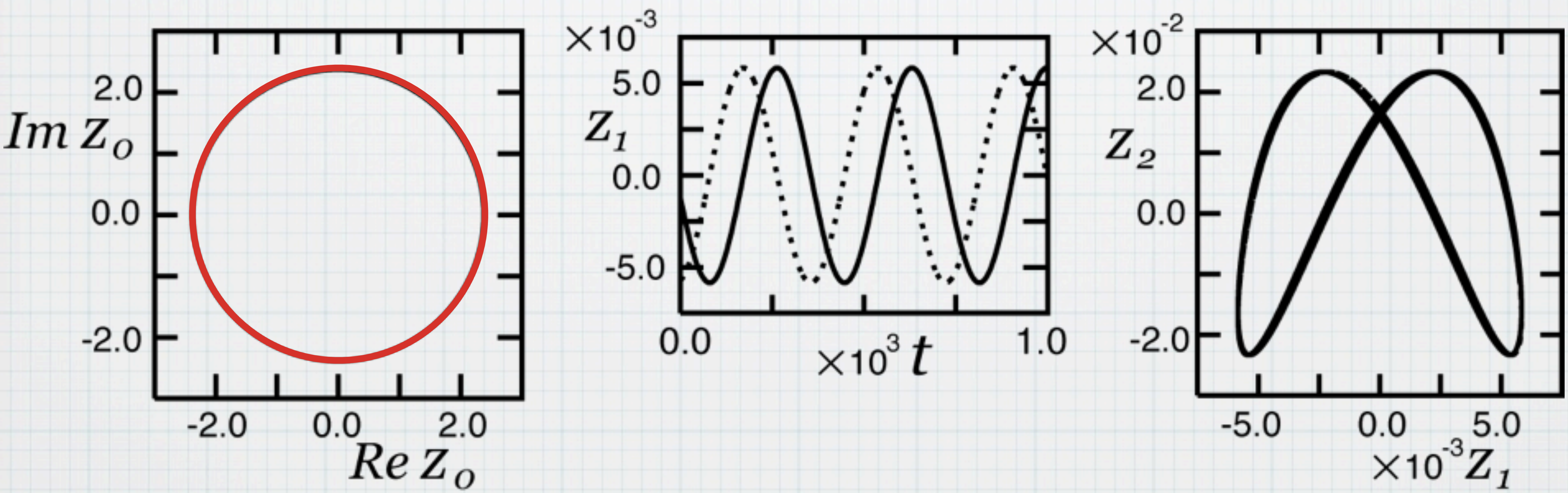
Keratocyte type

Higher codimension singularity includes the lower ones and its dynamics owns the global property.

Rotating spot solutions with $|\cos\theta| \neq 1$ emanate via pitchfork bifurcation,

$$\begin{cases} Q^2 = \left(-\dfrac{2\beta}{\alpha}\right) S^2 = \left(-\dfrac{2\beta}{\alpha}\right) \dfrac{N_3 + 2M_3}{K}, \\ \cos^2\theta = \dfrac{(N_3(M_2 - 2\beta M_1/\alpha) - M_3(N_1 - 2\beta N_2/\alpha))^2}{\beta^2 (N_3 + 2M_3) K}, \end{cases}$$

where $K = 4\beta M_1/\alpha - 2M_2 - N_1 + 2\beta N_2/\alpha.$



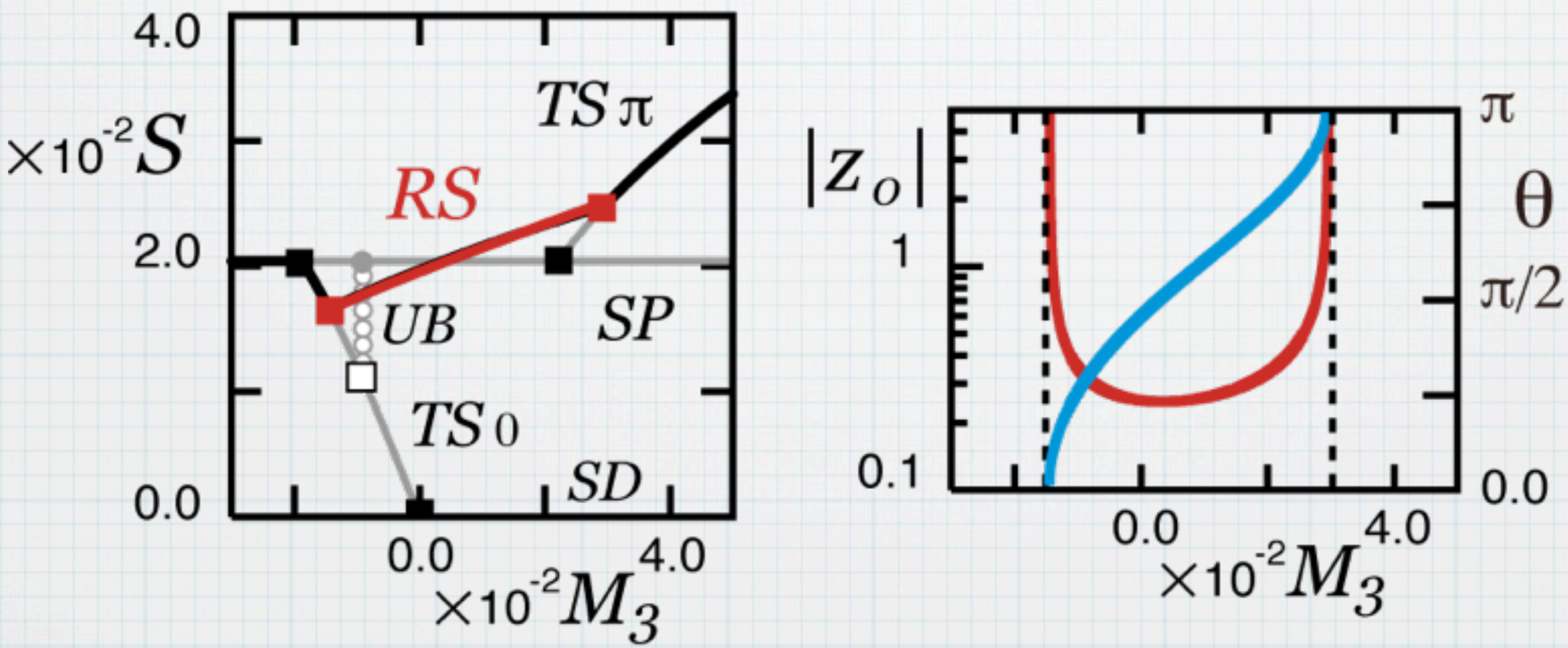
We solve the slave part in 5 as

$$z_0 = (2/\alpha\beta)^{1/2}(\beta' S e^{i\theta_0} - 1)e^{i\beta S \sin\theta t}/\sin\theta.$$

This allows the occurrence of RS motion with radius

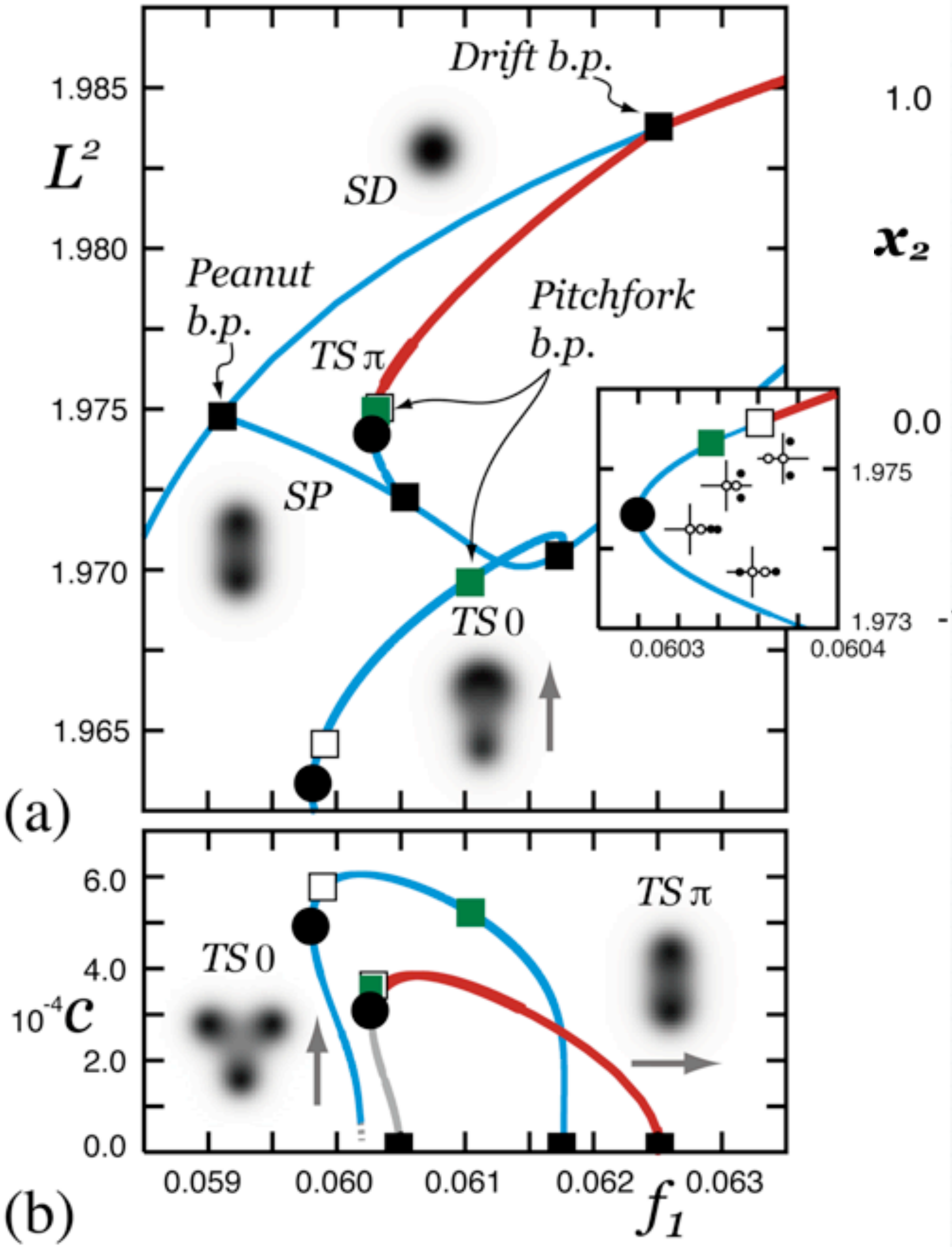
$$|z_0|^2 = 2((\beta' S)^2 - 1)/(\alpha\beta\sin^2\theta)$$

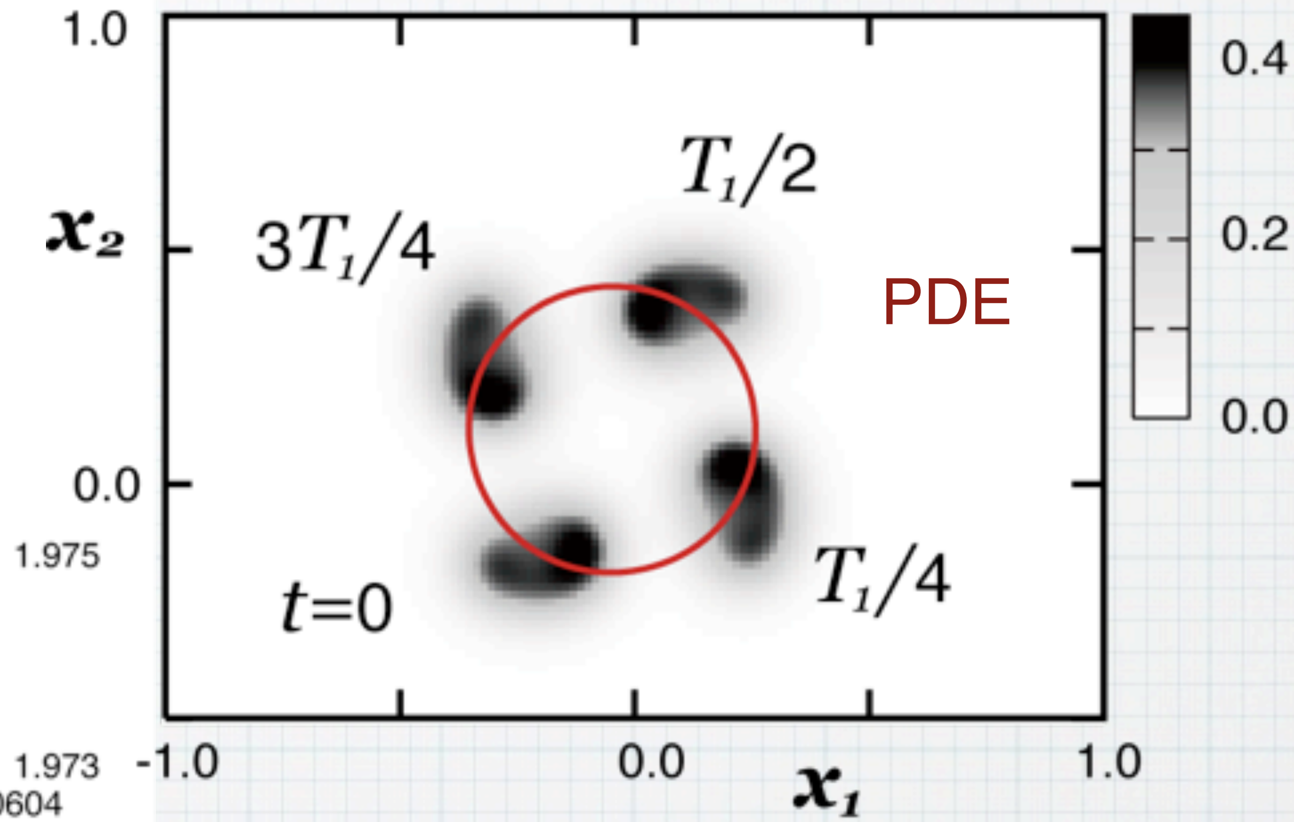
and the phase speed $\dot{\psi} = 2\dot{\phi} = 2\beta S\sin\theta.$

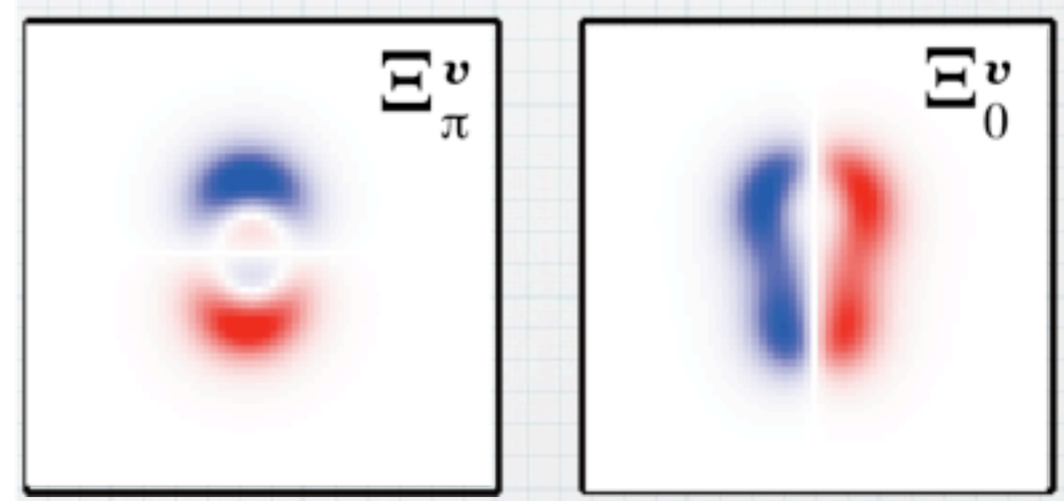


Clockwise and counter-clockwise rotational motions bifurcate from a straight motion via the pitchfork bifurcation.

Rotational motion of traveling spots

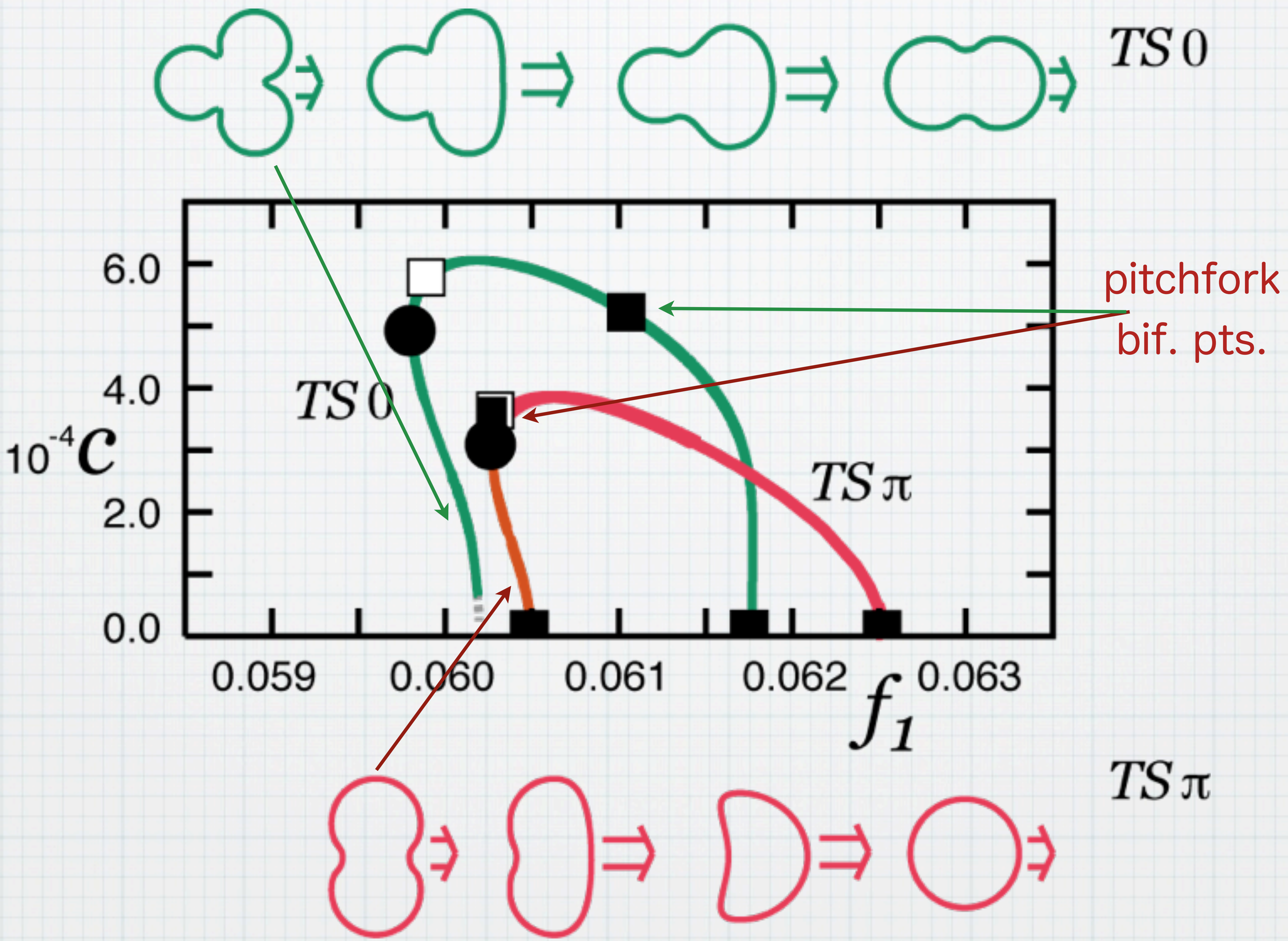






symmetry-breaking modes for TSs

Bifurcation branches for traveling spot solutions



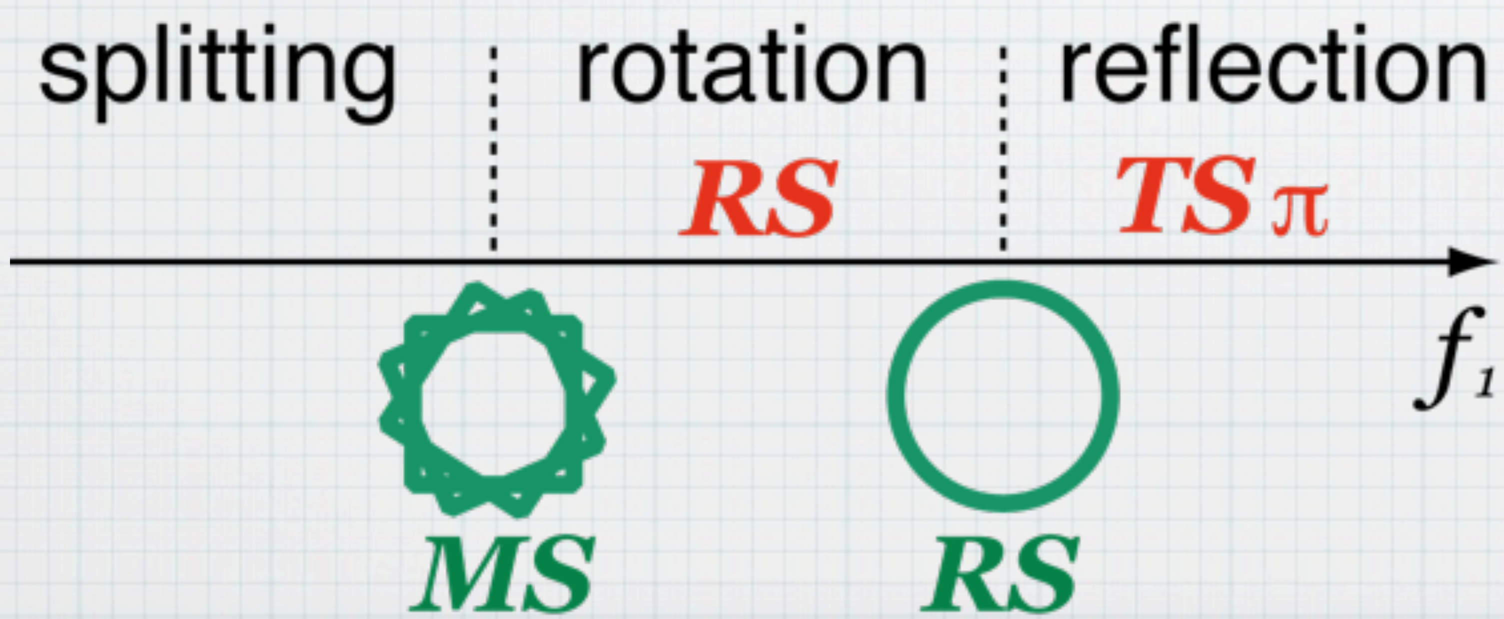
How does RS destabilize ?

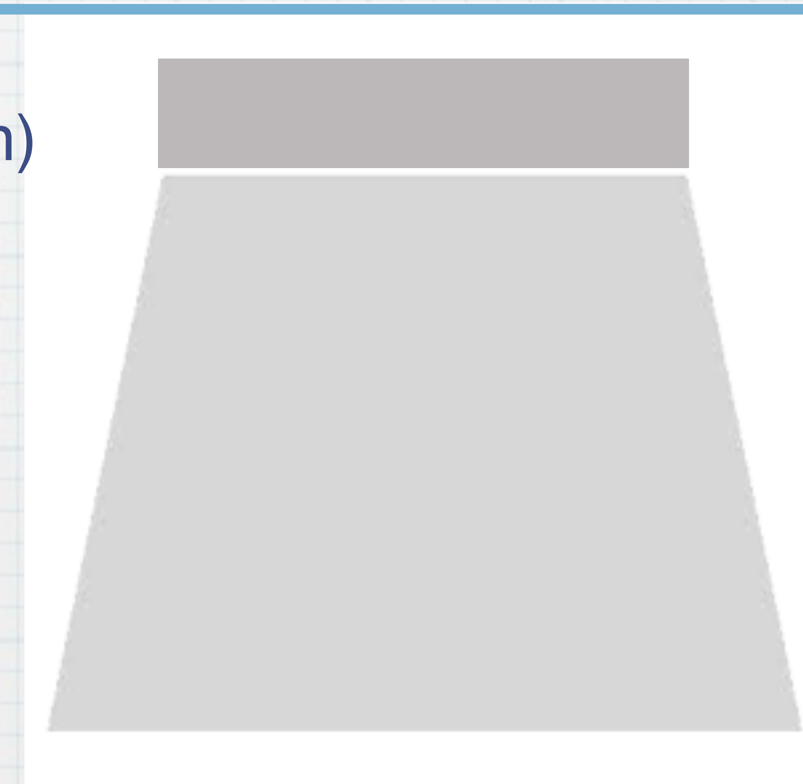
Spots are asymptotically stable, but ...

* Intrinsic instability
    destruction, drift, splitting, Hopf ...
  → Spot have a potential ability that display a variety of dynamics

  External interaction (perturbation)
    head-on and oblique collision, heterogeneous media ...
  → Hidden instability emerges through interactions

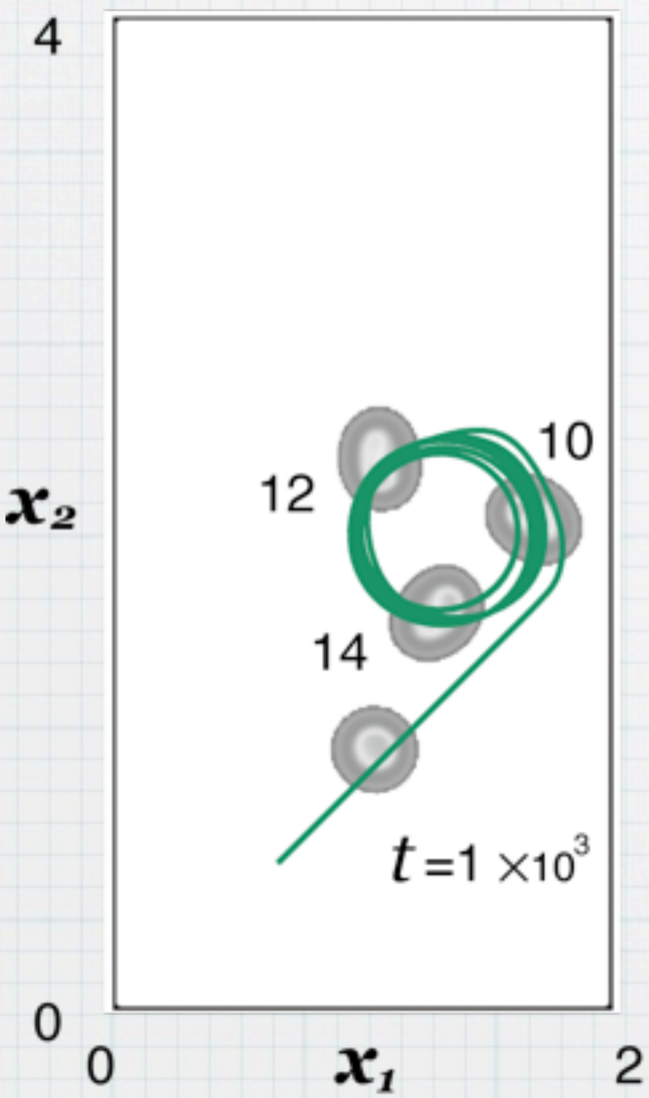
Oblique collision with Neumann wall (external perturbation)



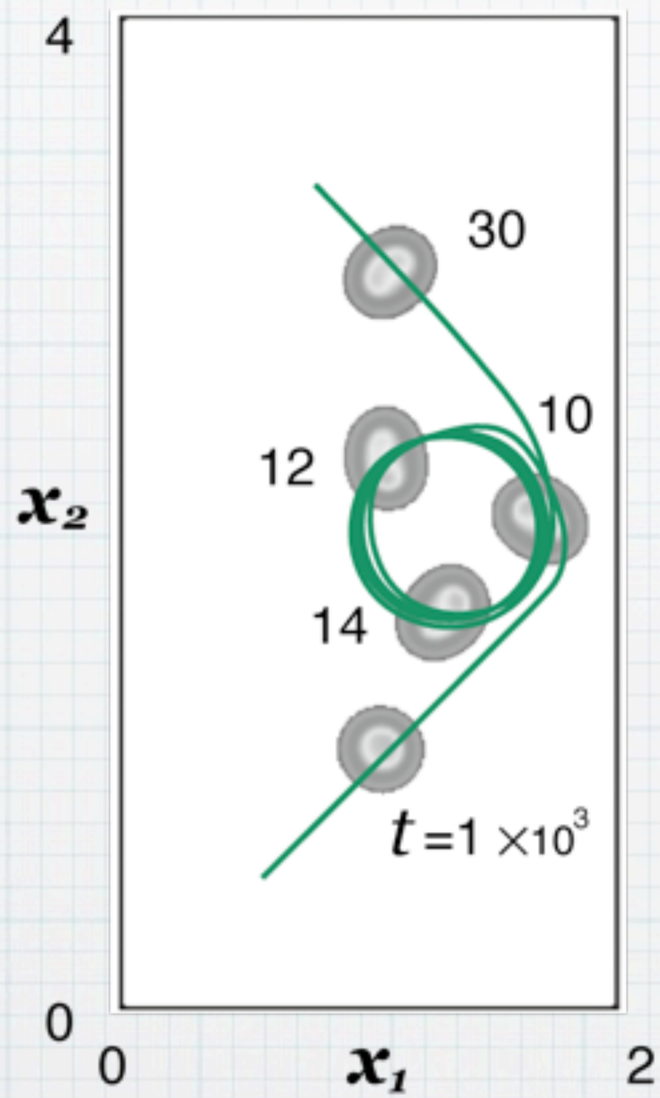


RS loose its stability via saddle-node bifurcation ?

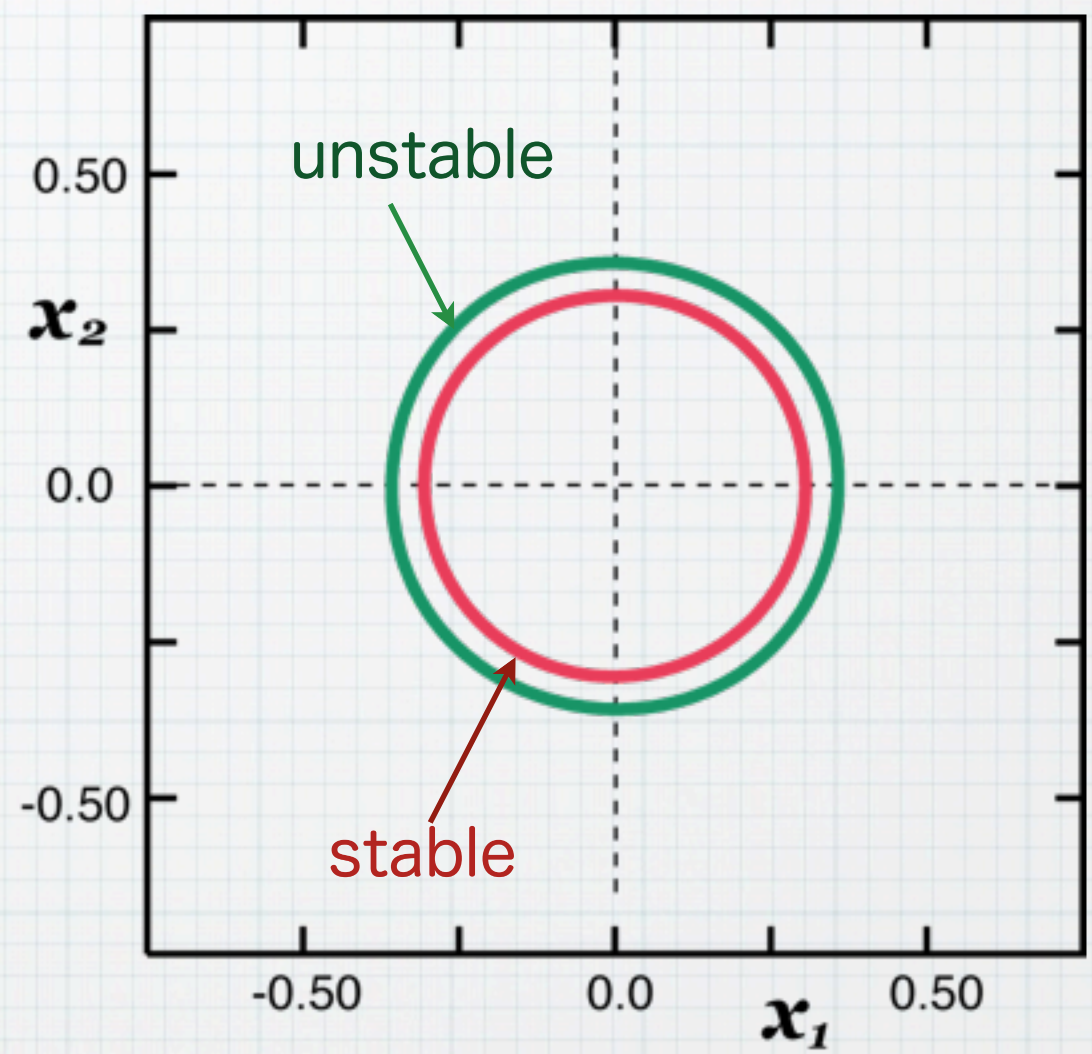
Unstable rotational spot appears as as scattor between the rotation and reflection behaviors after collision.



rotation



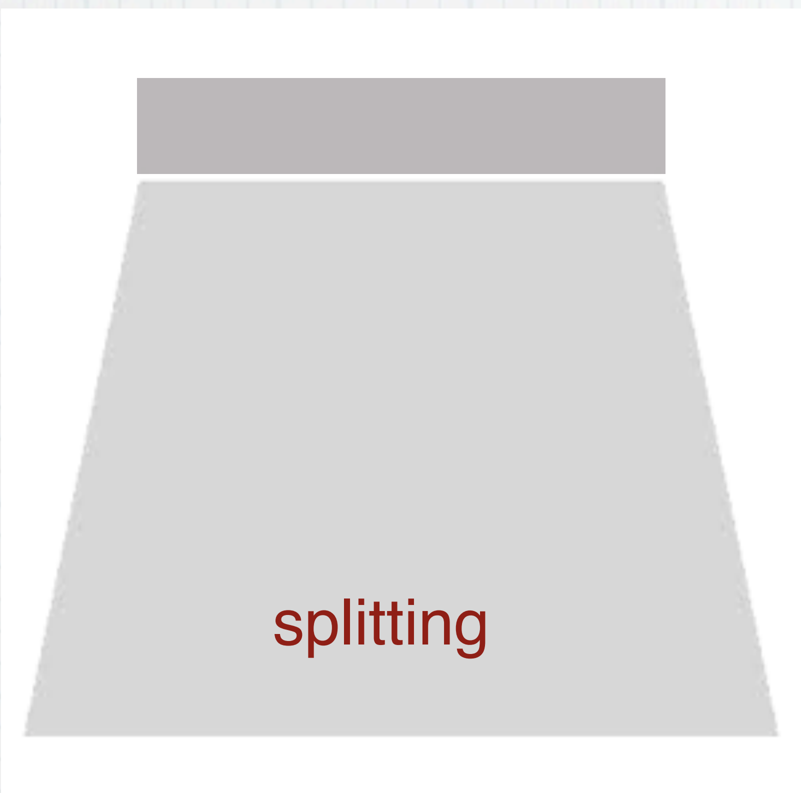
reflection

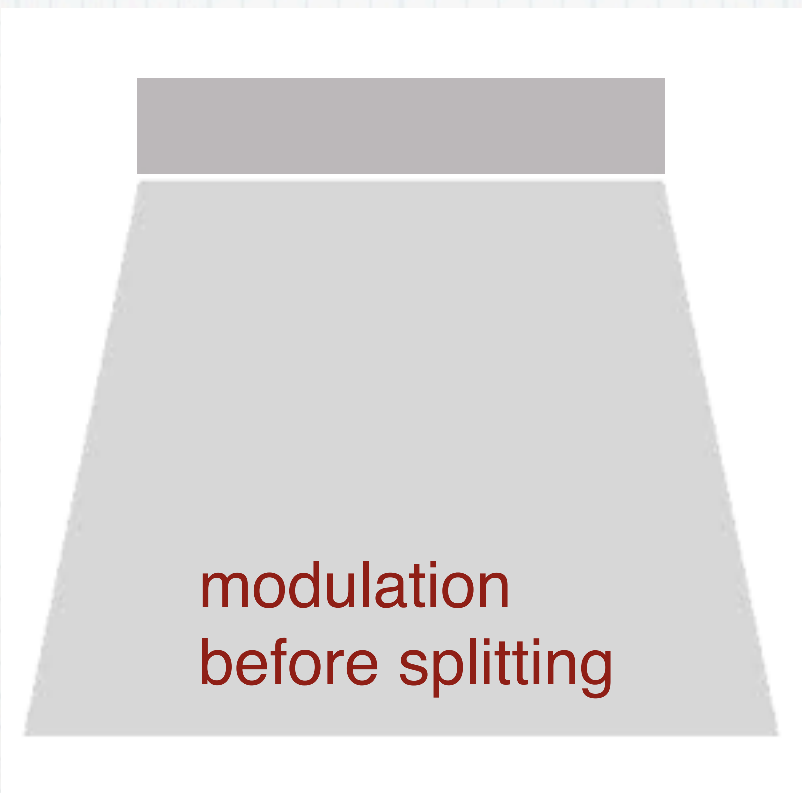


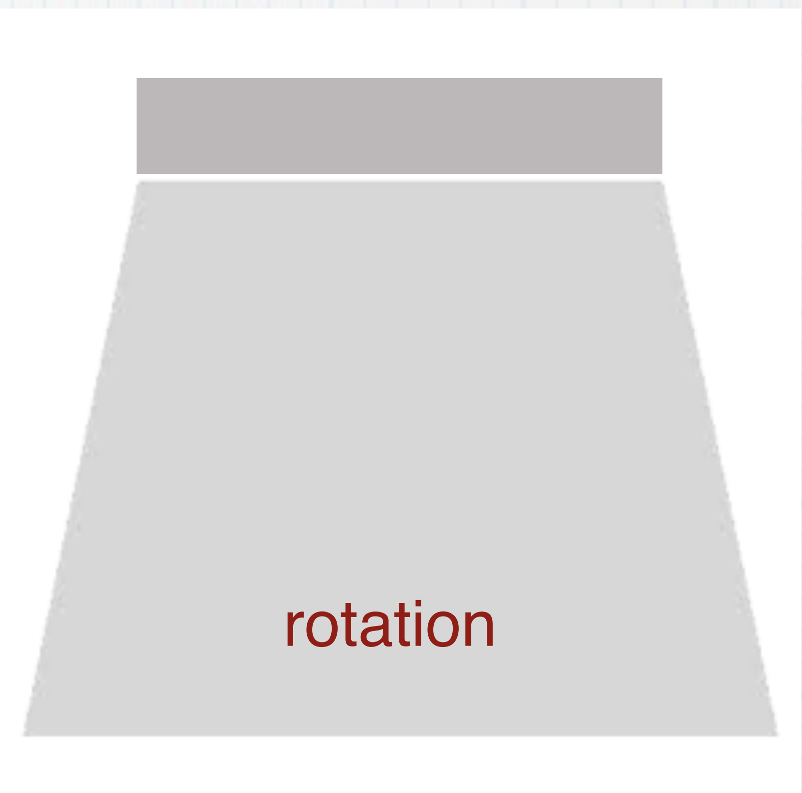
Unstable rotational motion detected by shooting Newton method

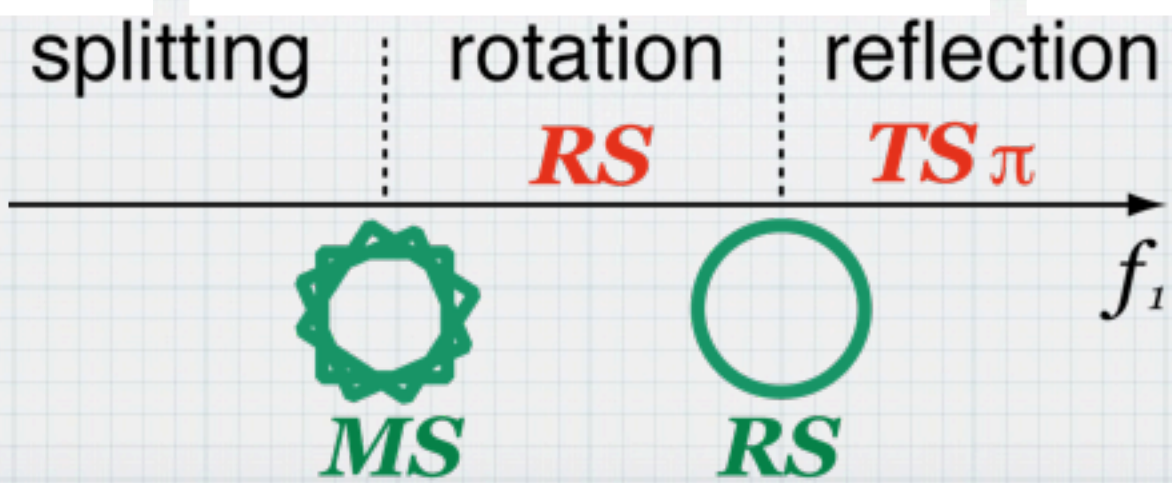
RS loose its stability via Torus bifurcation ?

Modulation of rotational motion occurs after collision for the phase boundary between splitting, rotation, and reflection behaviors.
→ Modulated spot (MS) motion plays a role of scattor for the complex behaviors.









Hidden dynamics in 1:2 mode interaction systems

location $\dot{z}_0 = z_1 - \beta' \overline{z}_1 z_2$

velocity / deformation

$$\begin{cases} \dot{z}_1 = M_1|z_1|^2 z_1 + M_2|z_2|^2 z_1 + M_3 z_1 + \beta \overline{z_1} z_2 \\ \dot{z}_2 = N_1|z_2|^2 z_2 + N_2|z_1|^2 z_2 + N_3 z_2 + \alpha {z_1}^2 \end{cases}$$

Heteroclinic cycles and modulated traveling waves in systems with O(2) symmetry, Armbruster, Guckenheimer, Holmes, Physica D 29 (1988) 257-282

$\mathcal{D}_1$

Remark: derivation using symmetry: translation and reflection invariance
(the symmetry of rotations and reflections of a circle)

$\mathcal{D}_2$

$$(z_1, z_2) \to (e^{i\theta} z_1, r^{2i\theta} z_2), \; (z_1, z_2) \to (\overline{z_1}, \overline{z_2})$$

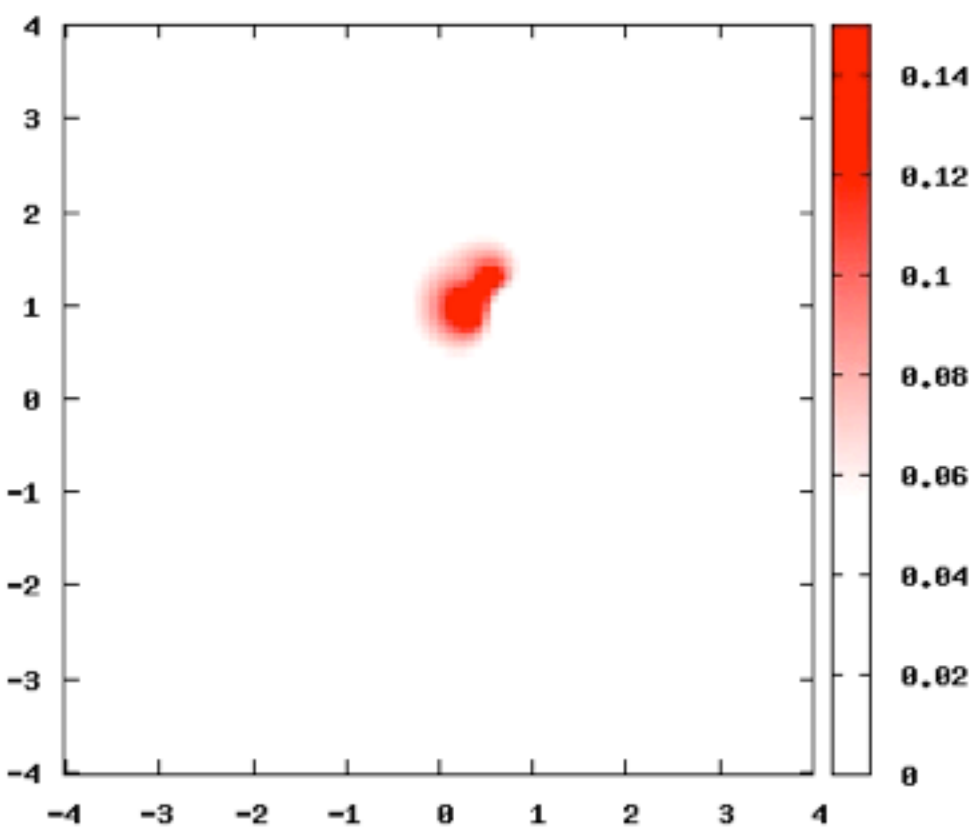
Armbruster et al shows that traveling wave (TW) solution exist only when $\alpha < 0$, and it emerge from mixed mode solutions (MMs) in pitchfork bifurcations for

$$((2M_1 + M_2)N_3 - (2M_2 + N_1)M_3)^2 \leq -(4M_1 + 2M_2 + 2N_2 + N_1)(2M_3 + N_3).$$

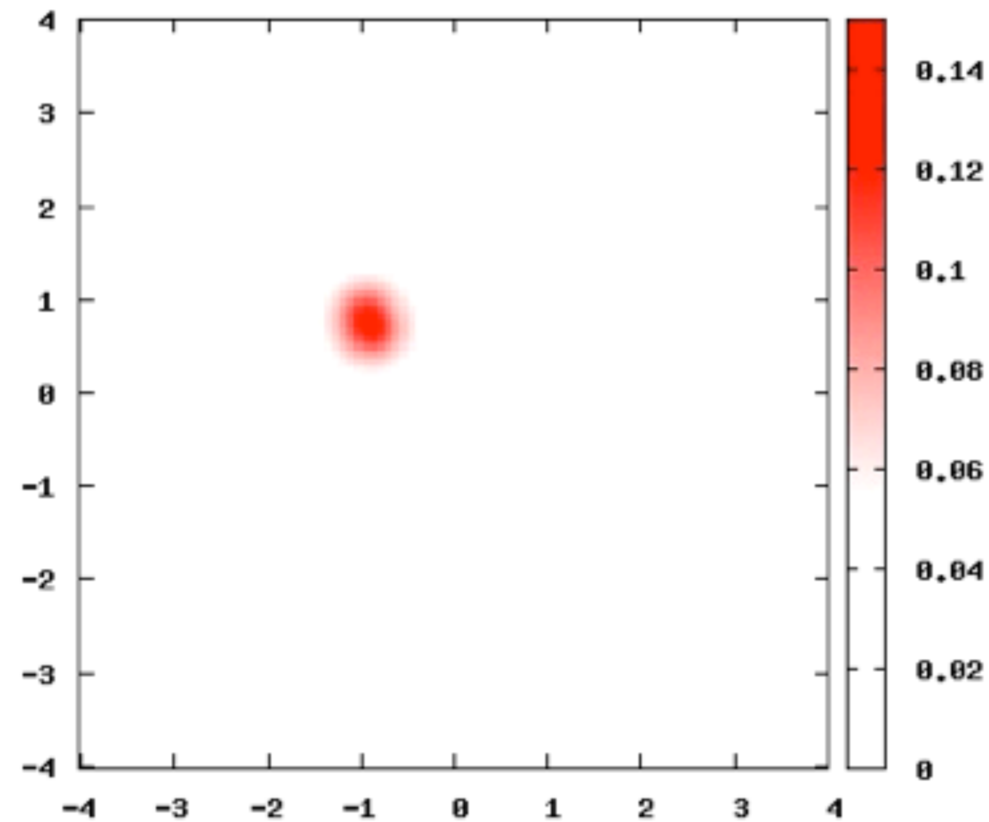
Here is the correspondence table.

O (trivial)	SS (Standing Spot)
P (Pure mode)	SP (Standing Peanut)
MM (Mixed mode)	TS (Traveling Spot)
TW (Traveling wave)	RS (Rotating Spot)
SW (Standing wave)	TB (Traveling Breather)
MTW (Modulated Traveling Wave)	MS (Modulated Spot)
AGH notations	notations for spot dynamics

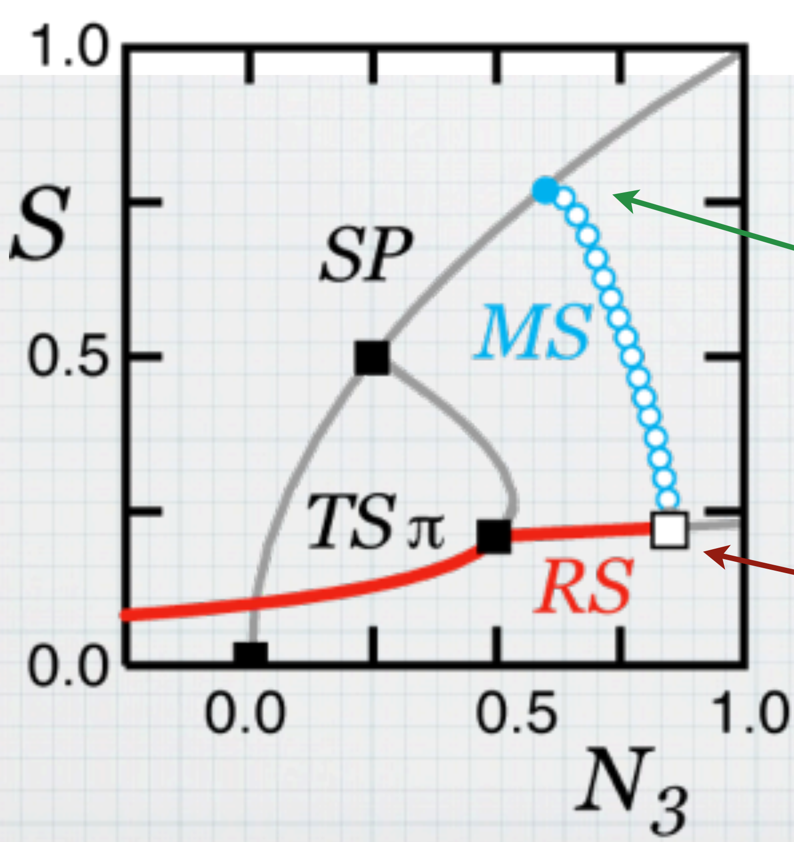
Modulated Spot motion



Heteroclinic cycle motion



(movie from folder)

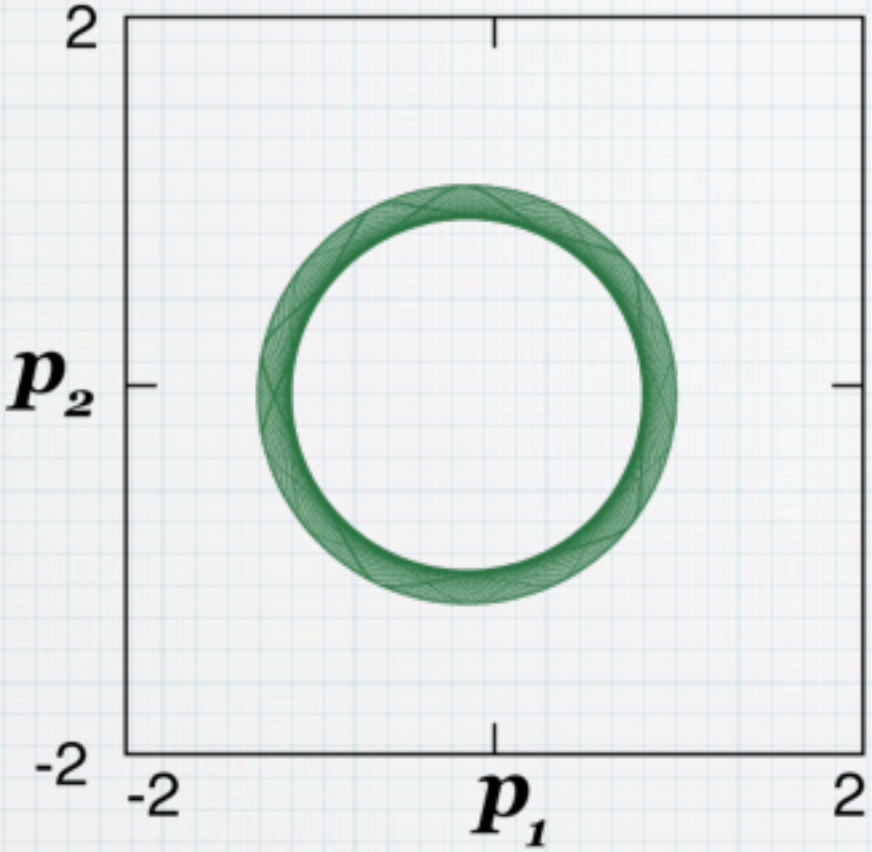


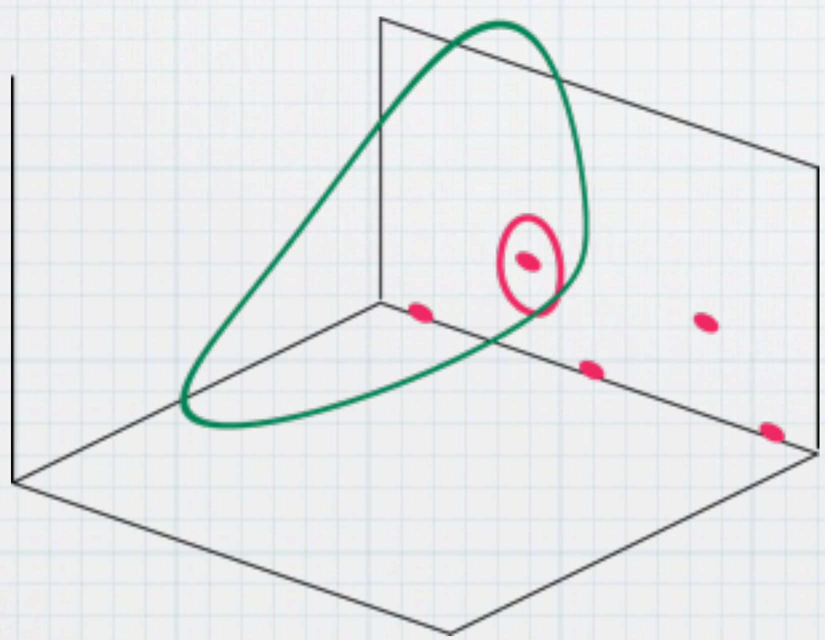
Numerical diagrams obtained by AUTO

MS disappears at Heteroclinic connection between $\mathbf{SP}_0$ and $\mathbf{SP}_\pi$.

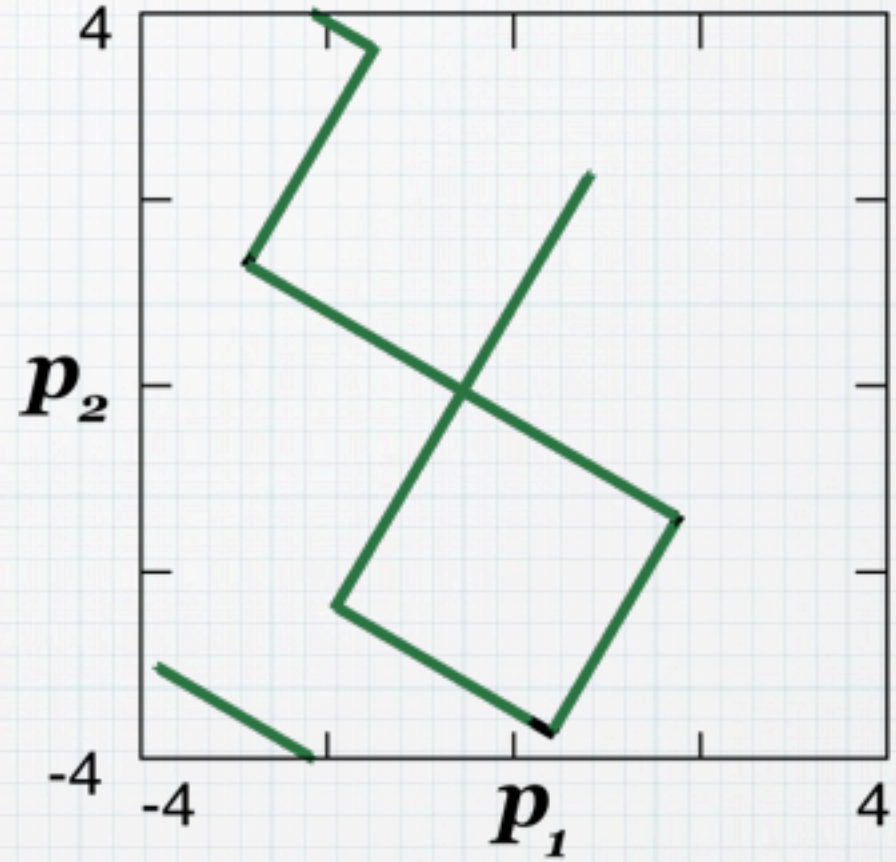
MS emanates from RS via Torus bifurcation.

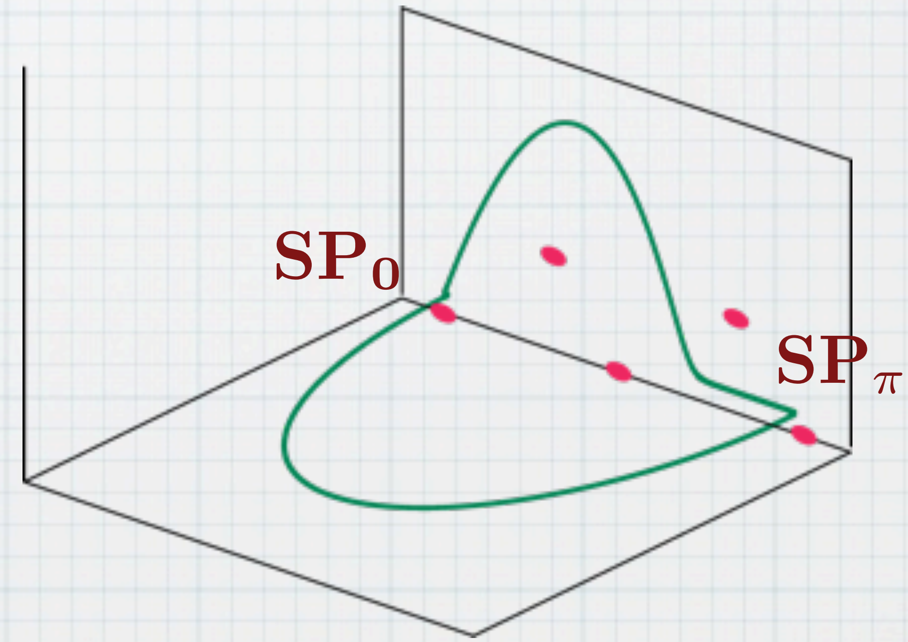
Trail of MS forms a torus.



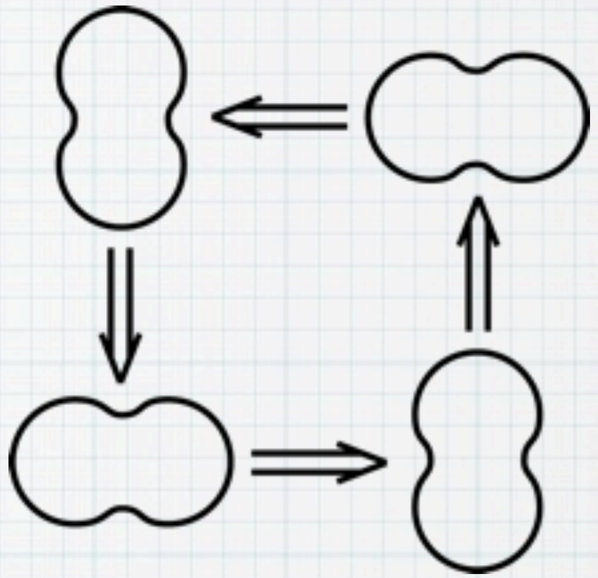


Trail of AGH cycle





Armbruster-Guckenheimer-Holmes cycle:



$\mathbf{SP_0} \rightarrow \mathbf{SP_\pi} \rightarrow \mathbf{SP_0}$

linear stability of SPs (four eigenvalues):

$$0, -2N_3, \sigma_\pm = M_3 - \frac{N_3 M_2 \pm \sqrt{-N_3 N_1}}{N_1}$$

Stability with respect to 1-mode direction is assumed as $\sigma_- < 0 < \sigma_+$.

$\mathbf{SP_0}$ is unstable in the direction associated with σ_-.

$\mathbf{SP_\pi}$ is unstable in the direction associated with σ_+.

Armbruster et al proved that

if $M_1 < 0, N_1 < 0, M_2 + N_2 < 2\sqrt{M_1 N_1}, N_3 > 0, M_3 > 0,$

there is a heteroclinic cycle of connecting SPs.
The cycle is locally asymptotically stable, if $\min\{2N_3, -\sigma_-\} > \sigma_+$.

Attracting structurally stable heteroclinic cycle !
(Robust heteroclinic cycles: see Krupa, and Sandstede and Scheel)

Equation for inhibitor: linear PDE

$$\tau w_t = D_w \Delta w + f_3(v - w)$$

rescale by $\tilde{r} = \dfrac{r}{\sqrt{D_w}}, \ f_3 \sim O(1),$

and consider the radial spot solution with m-mode deformation

$$w(r, \theta, t) = \overline{w}(r) + \hat{w}(r) e^{im\theta + \lambda t}$$

stationary problem: $\overline{w}_{rr} + \dfrac{1}{r}\overline{w}_r - \left(\dfrac{m^2}{r^2} + 1\right)\overline{w} = 0$

Stationary radial spot solution:

$$\overline{w}(r) = \begin{cases} 1 - 2RK_1(R)I_0(r) & 0 < r < R \\ 2RI_1(R)K_0(R) & r = R \\ 2RI_1(R)K_0(r) & r > R \end{cases}$$

K_m, I_m : modified Bessel function

$\mathcal{D}_1$

$\mathcal{D}_2$

$\mathcal{D}_3$

Eigenvalue problem:

$$\hat{w}_{rr} + \frac{1}{r}\hat{w}_r - \frac{m^2}{r^2}\hat{w} = (1+\tau\lambda)\hat{w}$$

Leading order eigenfunction: (van Heijster's Talk)

$$\hat{w}_m(r) = \begin{cases} -2CRK_m(R)I_m(r) & 0 < r < R \\ -2CRI_m(R)K_m(R) & r = R \\ 2CRI_m(R)K_m(r) & r > R \end{cases}$$

location	$\dot{z}_0$	$=$	$z_1 - \beta' \overline{z_1} z_2$
velocity	$\dot{z}_1$	$=$	$M_1 \lvert z_1\rvert^2 z_1 + M_2 \lvert z_2\rvert^2 z_1 + M_3 z_1 + \beta \overline{z_1} z_2$
deformation	$\dot{z}_2$	$=$	$N_1 \lvert z_2\rvert^2 z_2 + N_2 \lvert z_1\rvert^2 z_2 + N_3 z_2 + \alpha {z_1}^2$

$$z_0 = p_1 + ip_2,\ z_1 = q_1 + iq_2,\ z_2 = s_1 + is_2$$

$$W = \tau(\mathbf{p}) \left\{ \overline{w}(\mathbf{r}) + \sum_{i=1}^2 q_i \hat{w}_{1i}(\mathbf{r}) + \sum_{i=1}^2 s_i \hat{w}_{2i}(\mathbf{r}) \right\}.$$

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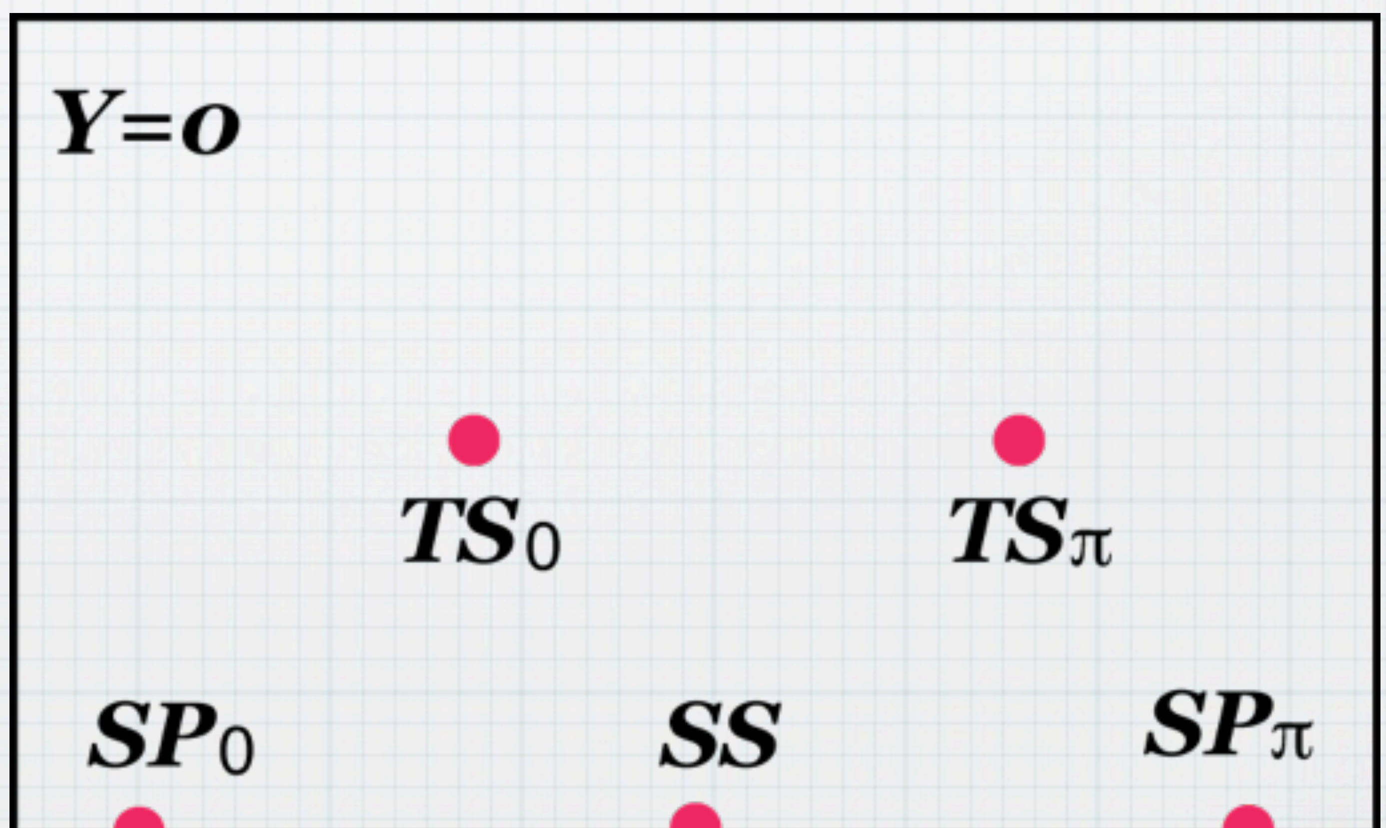
Letting $X = S\cos\theta,\ Y = S\sin\theta$, we rewrite 6 as

the following autonomous systems in $\mathbb{R}^3$

$$\begin{cases} \dot{Q} = (M_1 Q^2 + M_2(X^2+Y^2) + M_3)Q + \beta Q X, \\ \dot{X} = (N_1(X^2+Y^2) + N_2 Q^2 + N_3)X + \alpha Q^2, \\ \dot{Y} = (N_1(X^2+Y^2) + N_2 Q^2 + N_3)Y - 2\beta XY, \end{cases} \tag{8}$$

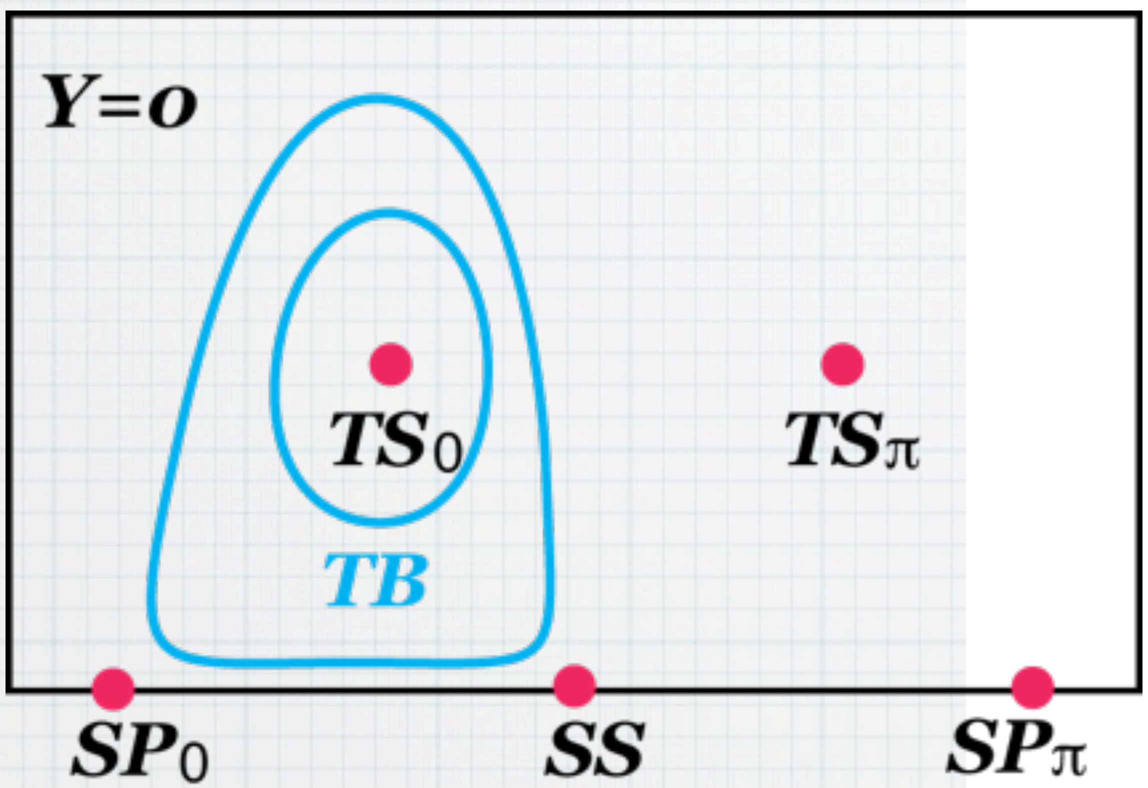
Stationary solutions exist on the plane of Y=0.

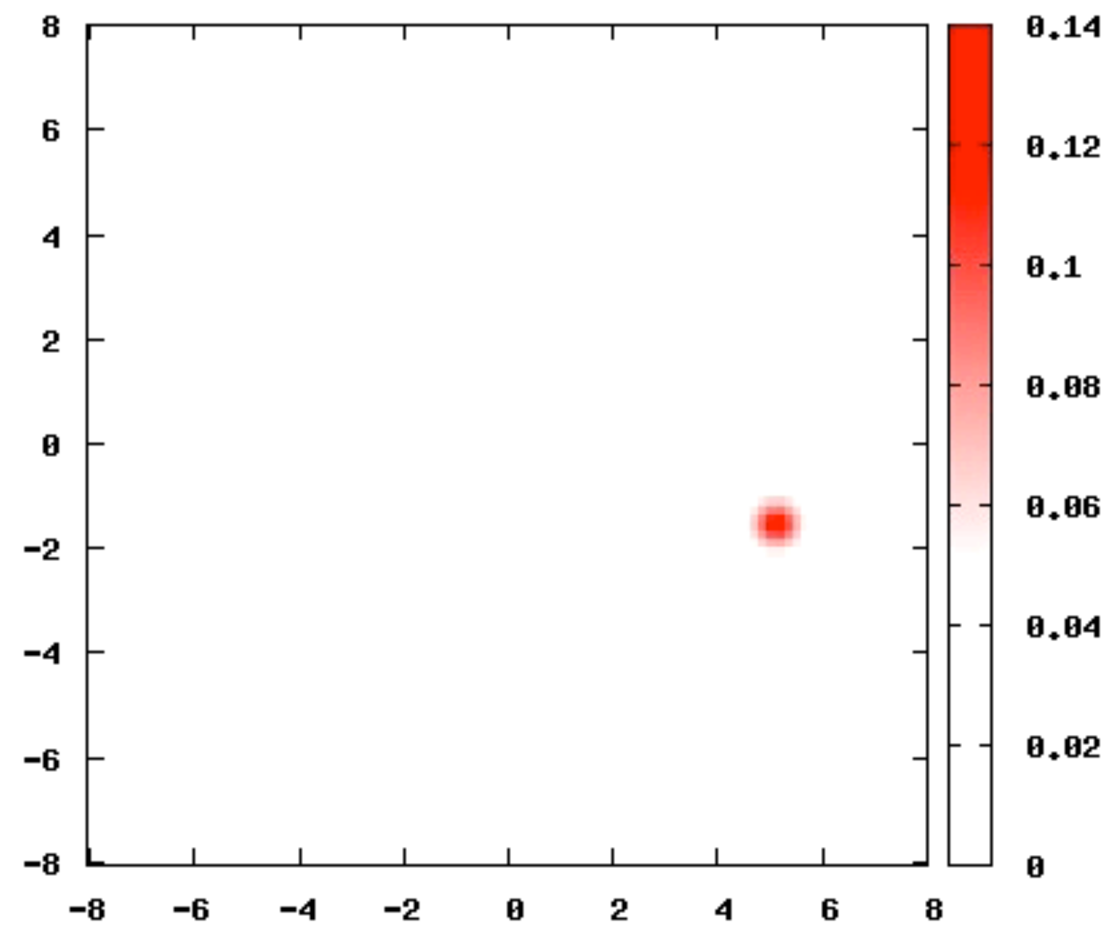
O	SS
P	SP
MM	TS
TW	RS
SW	TB
MTW	MS



Armbruster et al. formulate the conditions on the coefficients for the Hopf bifurcation on MMs, implying the appearance of standing wave (SW) solution.

if $M_1 < 0, N_1 < 0$ and $\alpha < 0,$ Hopf bifurcations can occur only $\mathbf{TS_0}$.





$$\alpha Q^2 - 2M_1Q^2x - 2N_1x^3 = 0$$

$$(2N_1x^3 - \alpha Q^2)M_1 - (2M_2x + 1)(2N_2x + \alpha)x > 0$$

Traveling Breather (TB) appears from $\mathbf{TS_0}$ via Hopf bifurcation and

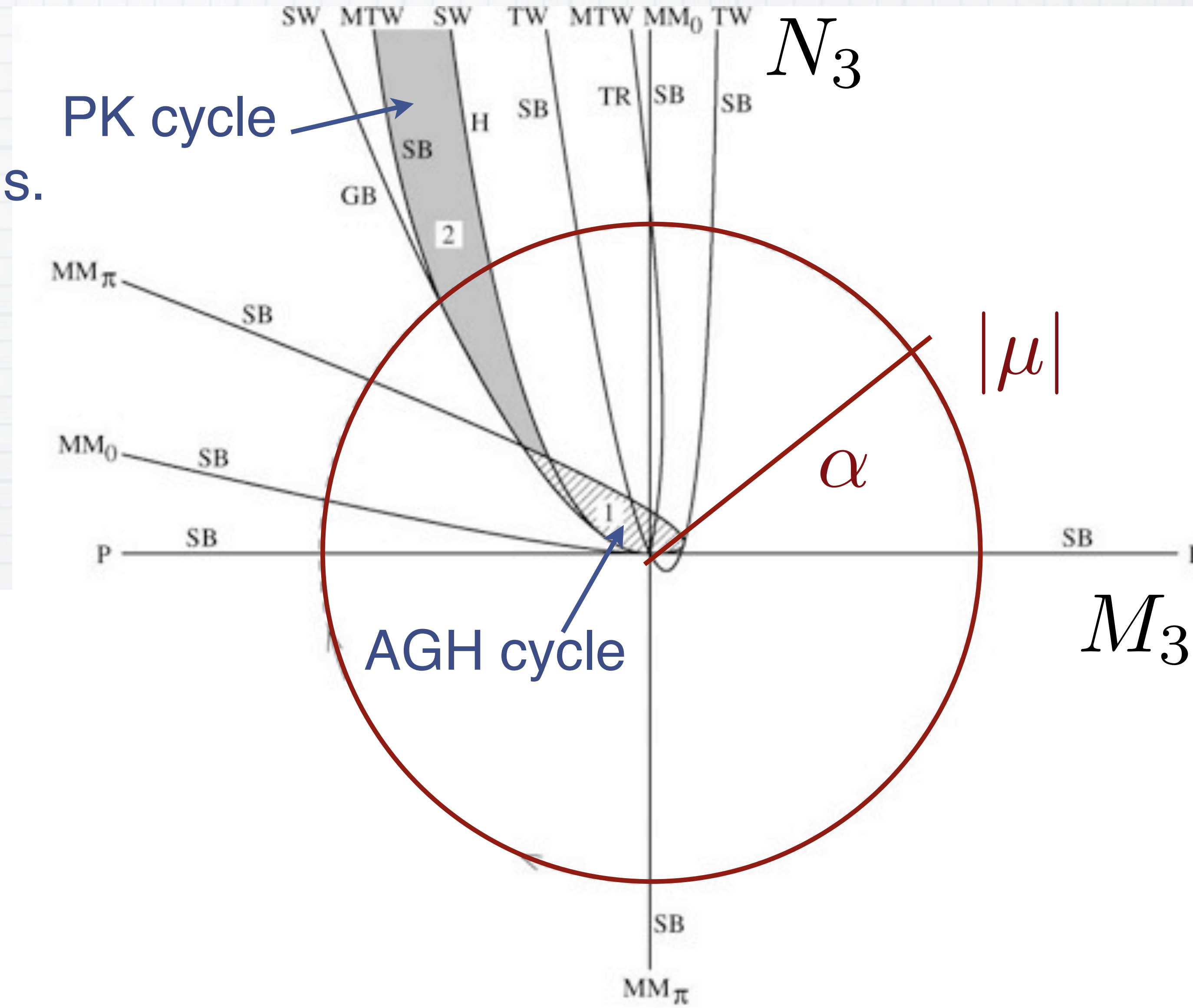
its orbit grows up to Heteroclinic cycle of $\mathbf{SS} \rightarrow \mathbf{SP_0} \rightarrow \mathbf{SS}$.

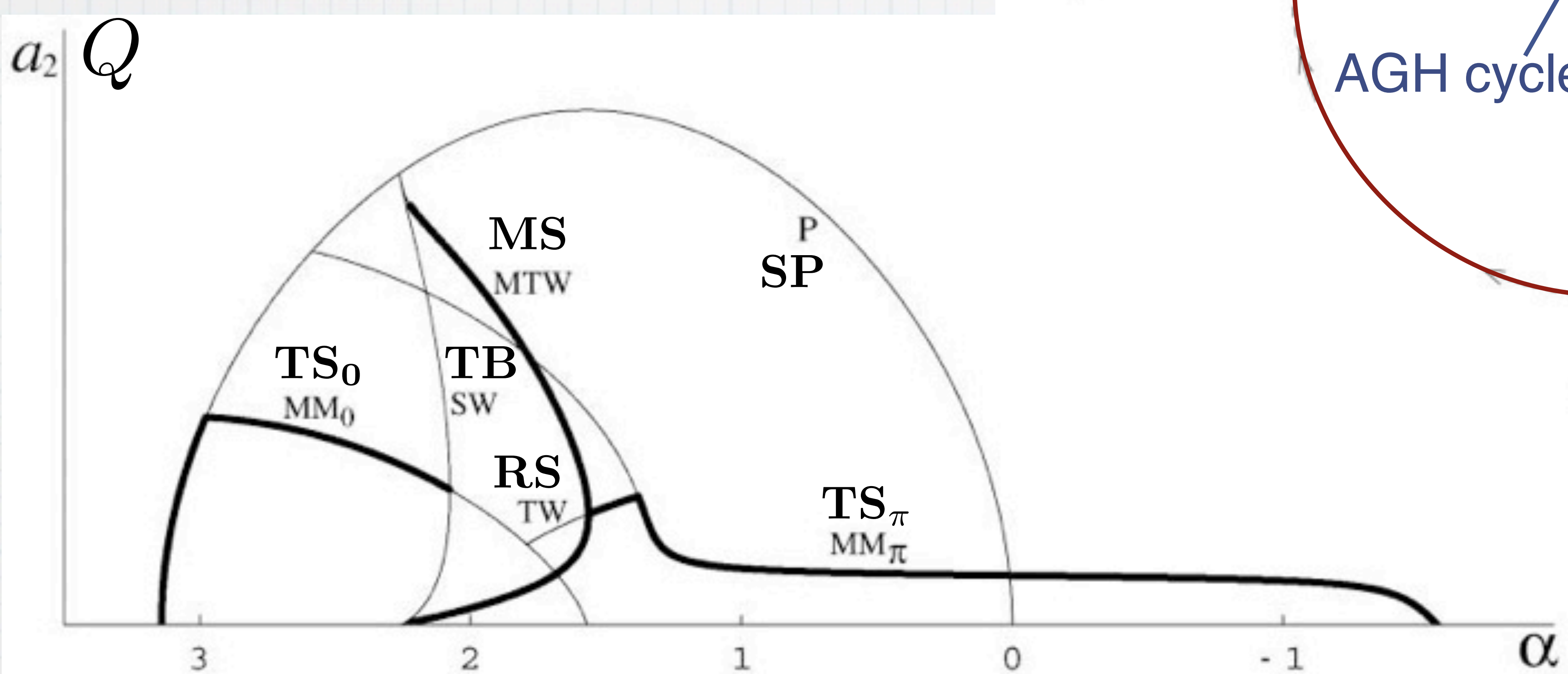
Phase diagrams by Porter and Knobloch

New type of complex dynamics in the 1:2 spatial resonance,
Porter and Knobloch, Physica D 159 (2001) 125-154.

The polar angle is chosen as a bifurcation parameter by taking clockwise circular paths.

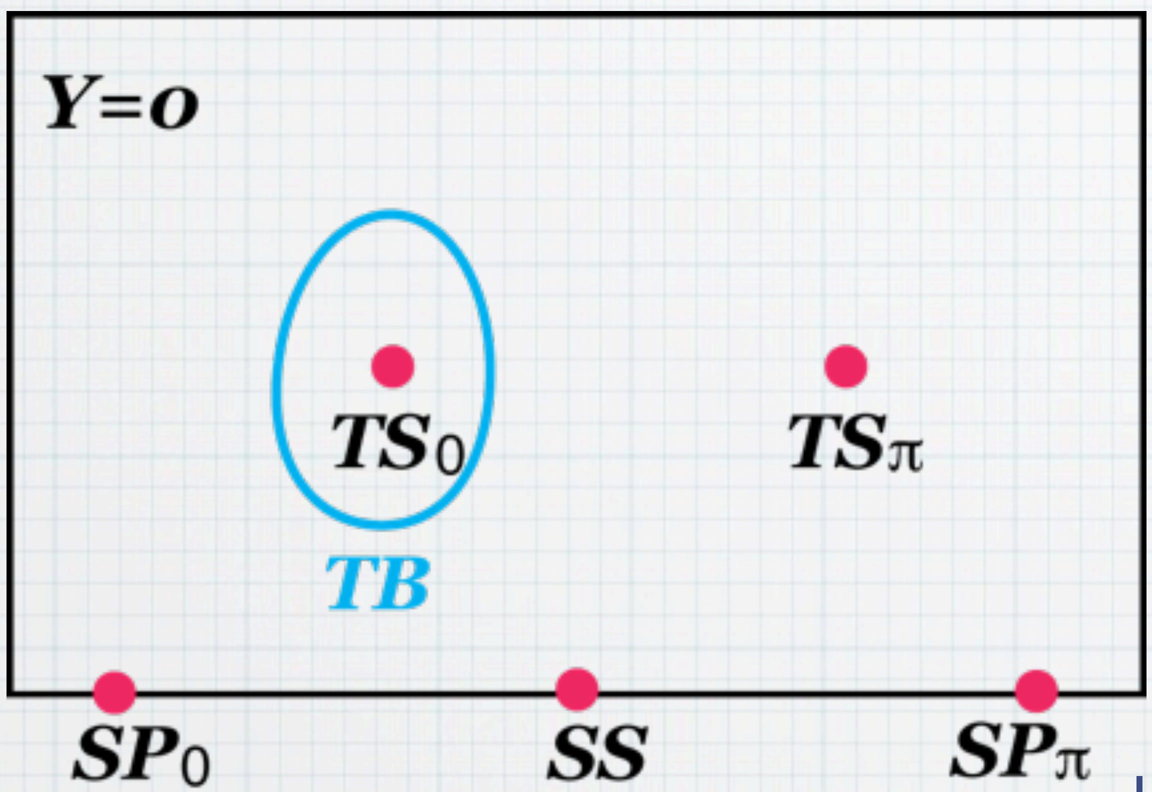
$$(M_3, N_3) = |\mu|(\cos\alpha, \sin\alpha)$$

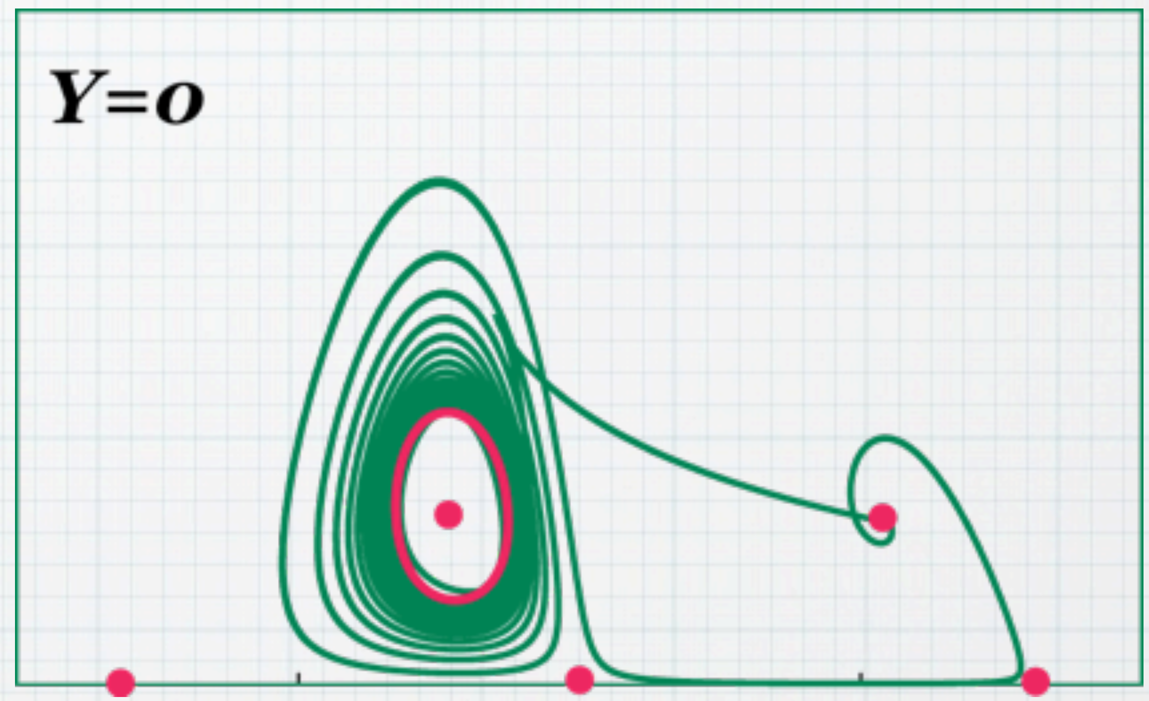




New types of Complex behaviors: Porter-Knobloch cycle

Hopf instability for $\mathbf{TS_0}$, the appearance of unstable $\mathbf{TB}$, brings complex spot behaviors to the system.





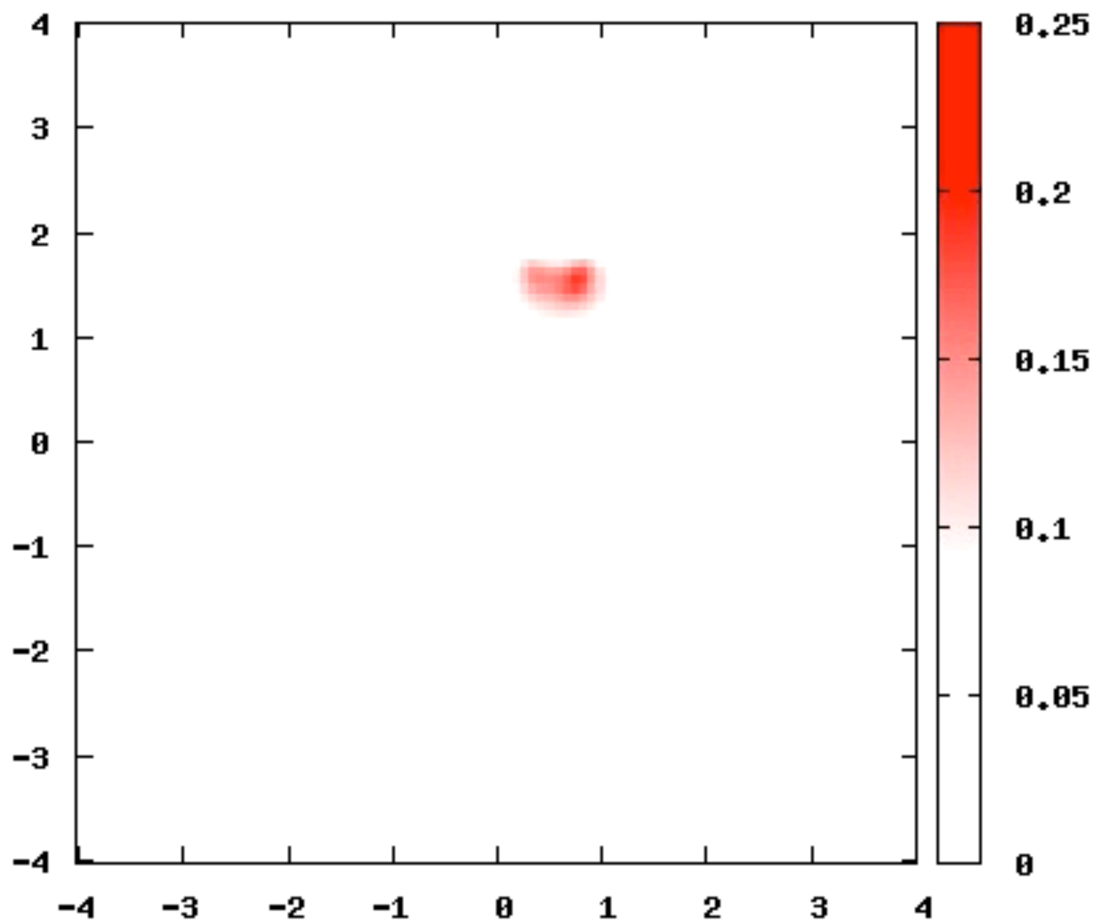
long time periodic motion associated with the heteroclinic cycle of

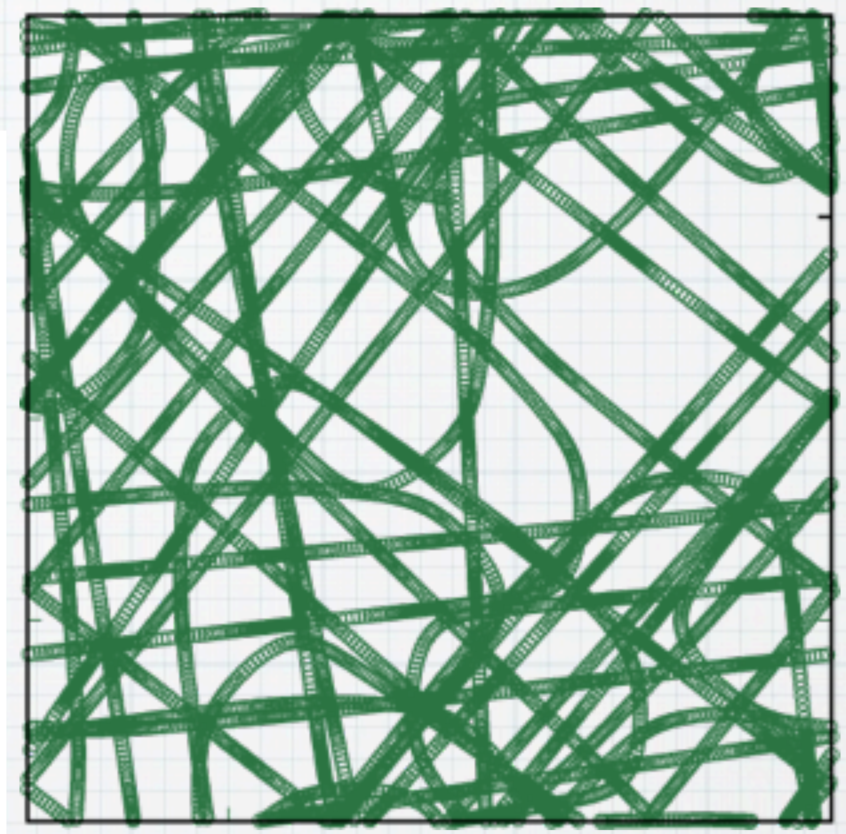
$$\mathbf{SS} \rightarrow \mathbf{SP}_\pi \rightarrow \mathbf{TS}_\pi \rightarrow \mathbf{TB} \rightarrow \mathbf{SS}$$

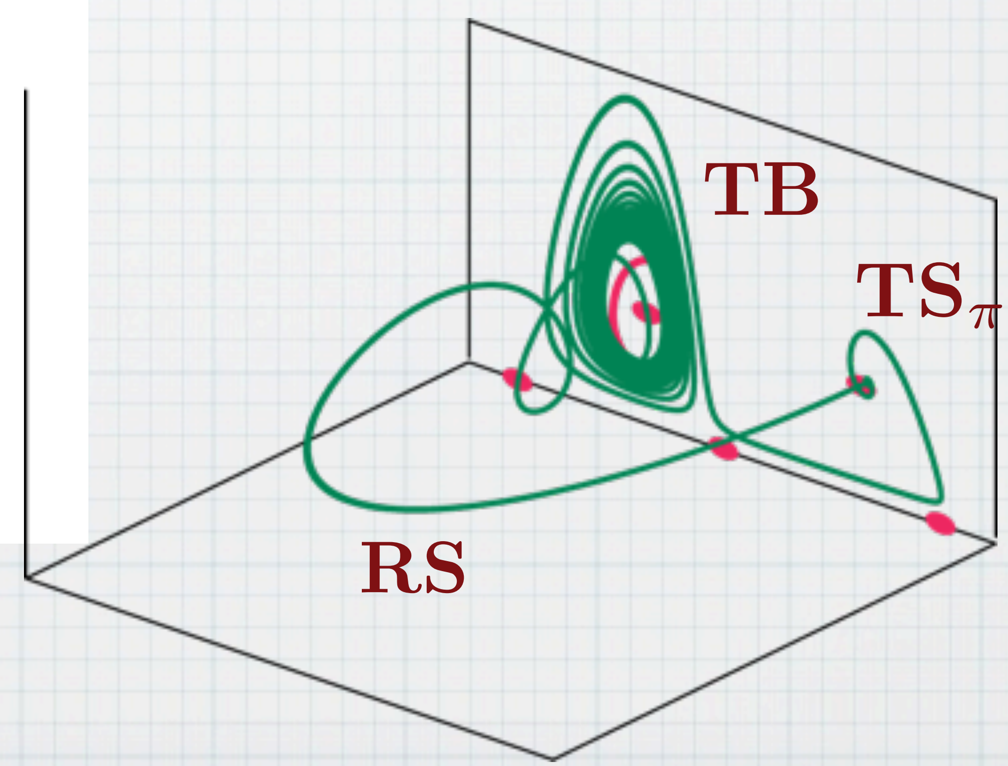
$$Q^2 = -\frac{M_3 + x + M_2 x^2}{M_1}$$

$$0 = \alpha M_3 + (\alpha + M_2 M_3 - M_1 N_3)x + (N_2 + \alpha M_2)x^2 + (M_2 N_2 - M_1 N_1)x^3$$

Long time periodic behavior associated with PK cycle







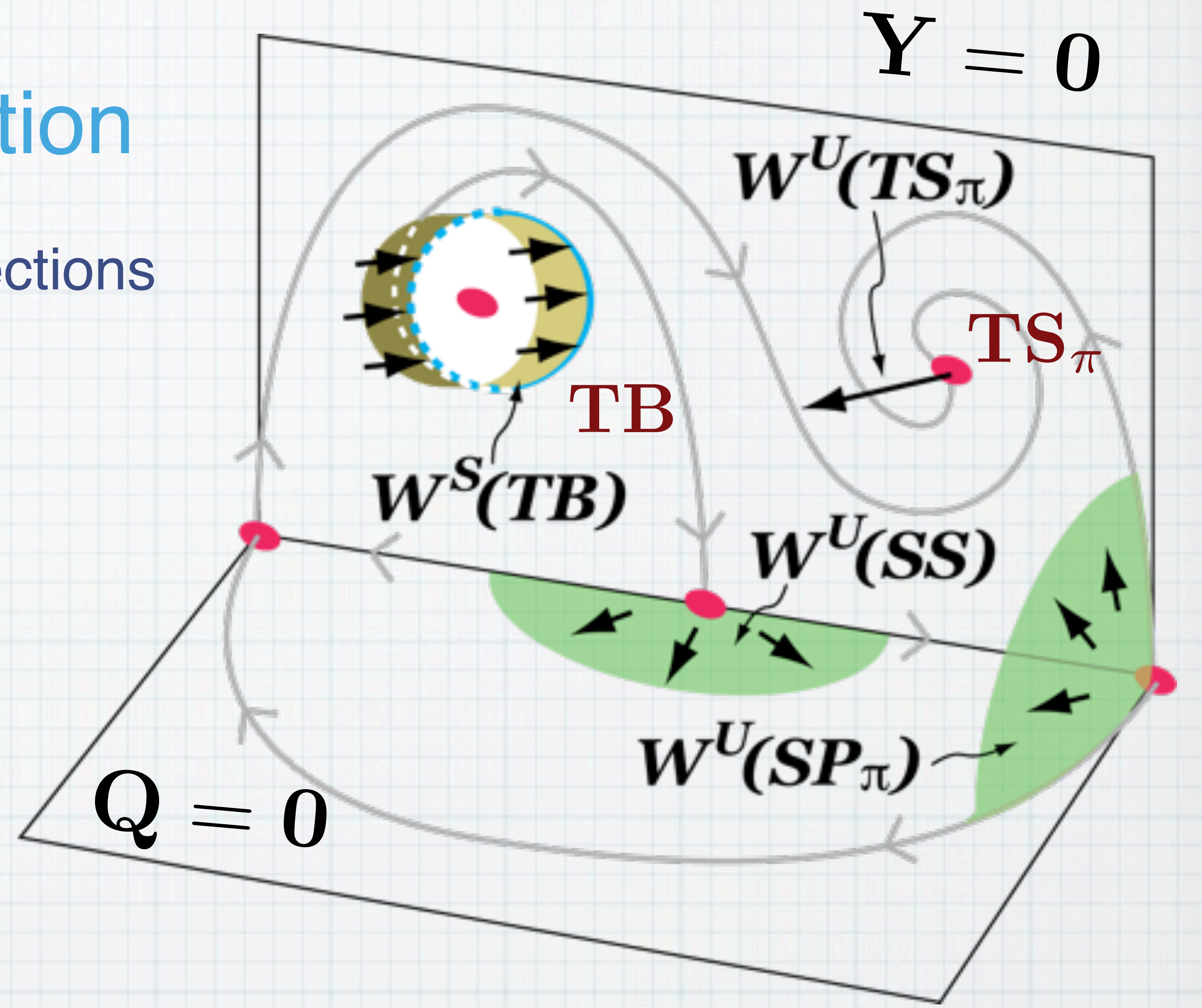
Intuitive Geometric Interpretation

There are several types of heteroclinc connections in the parameter regime:

$$M_1 < 0, N_1 < 0, M_2 + N_2 < 2\sqrt{M_1 N_1}$$

$$\mathbf{SS} \to \mathbf{SP}_{\mathbf{0},\pi} \qquad \mathbf{TB} \to \mathbf{SS}$$

$$\mathbf{SP}_{\mathbf{0},\pi} \to \mathbf{TS}_\pi \qquad \mathbf{TS}_\pi \to \mathbf{TB}$$



There exists the PK cycle of

$$\mathbf{SS} \to \mathbf{SP}_\pi \to \boxed{\mathbf{TS}_\pi \to \mathbf{TB}} \to \mathbf{SS}$$

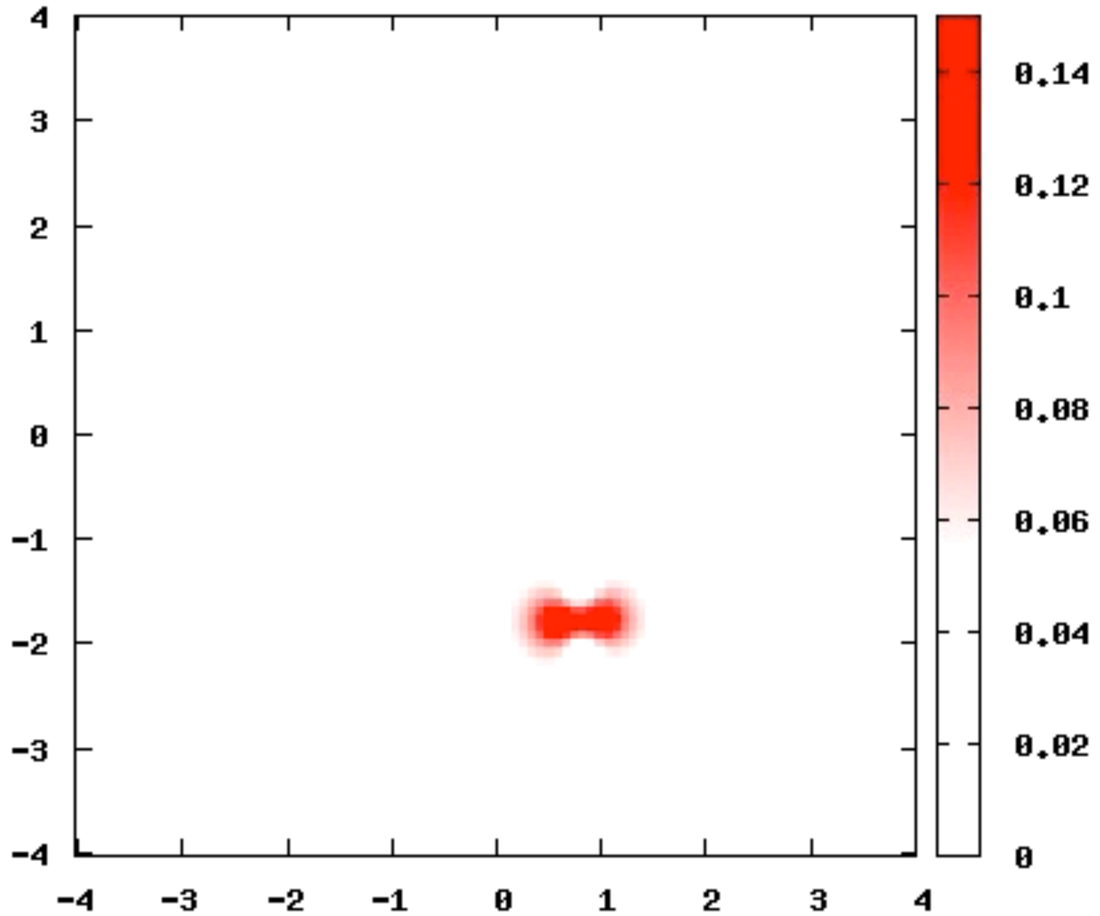
$$\dim T_p W^u(TS_\pi) = 1 \qquad \dim T_p W^s(TB) = 2$$

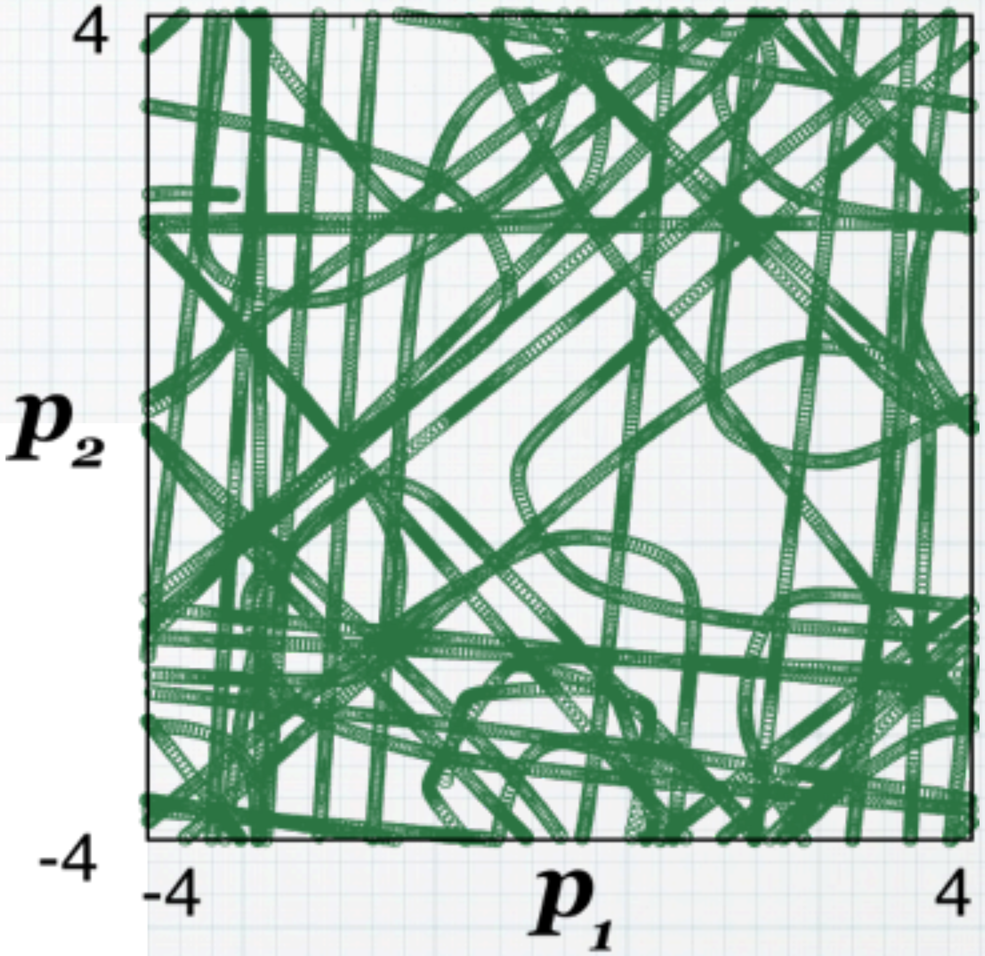
$$\dim(T_p W^u(TS_\pi) + T_p W^s(TB)) \neq 3$$

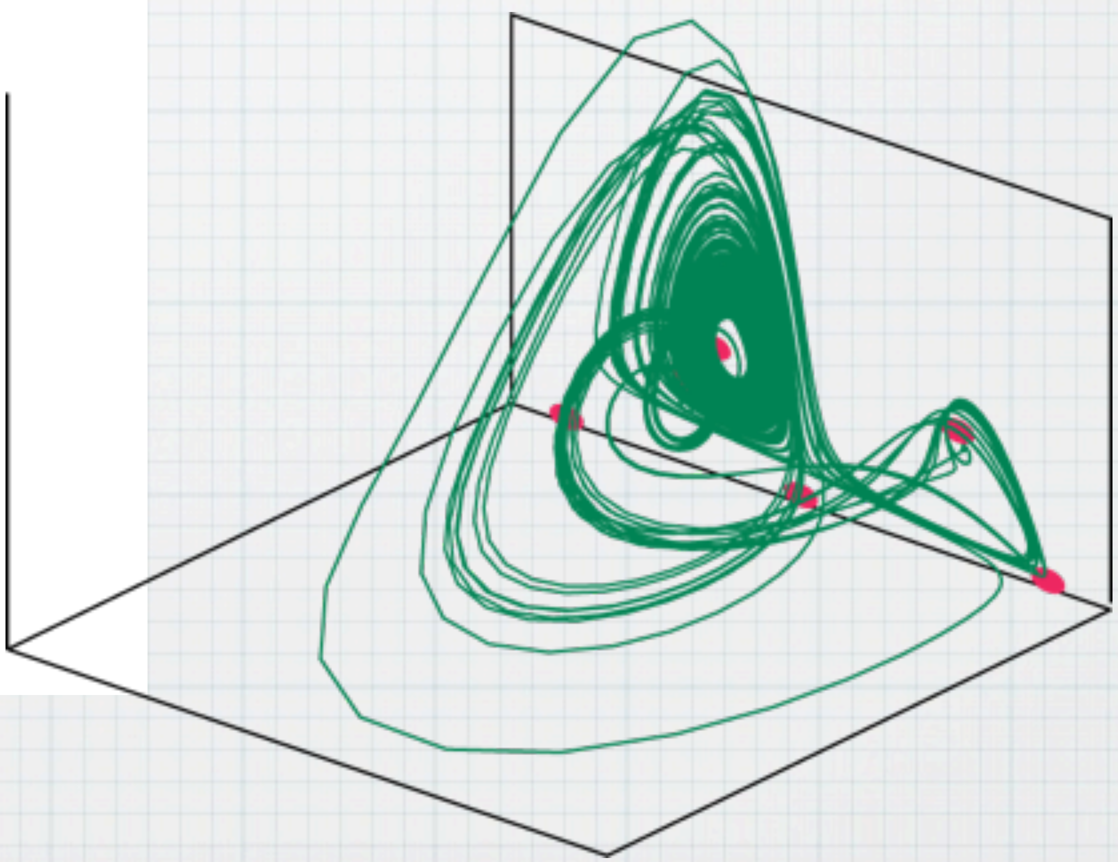
structurally unstable

Variation of PK cycle

long long long time periodic motion ?

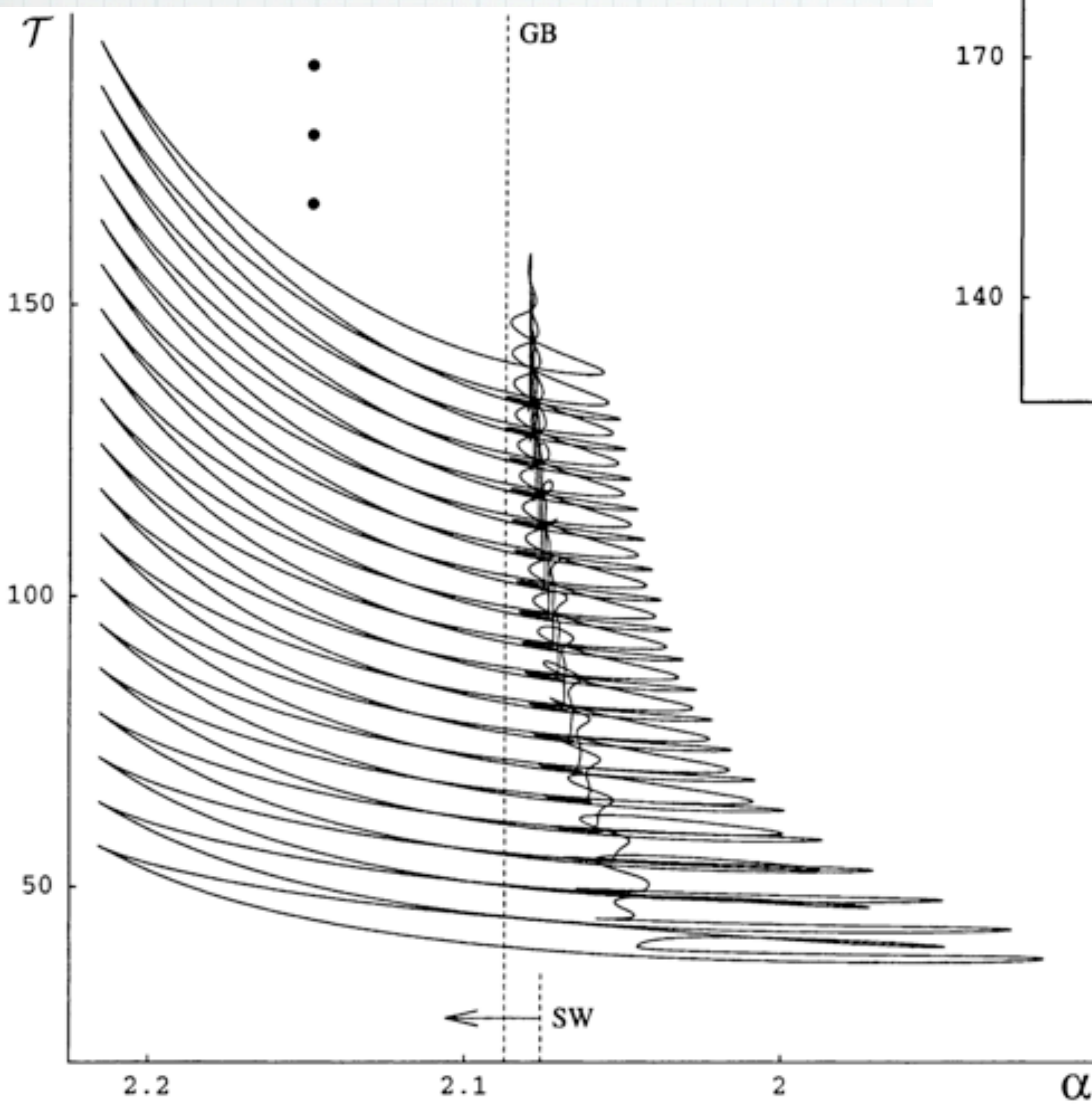




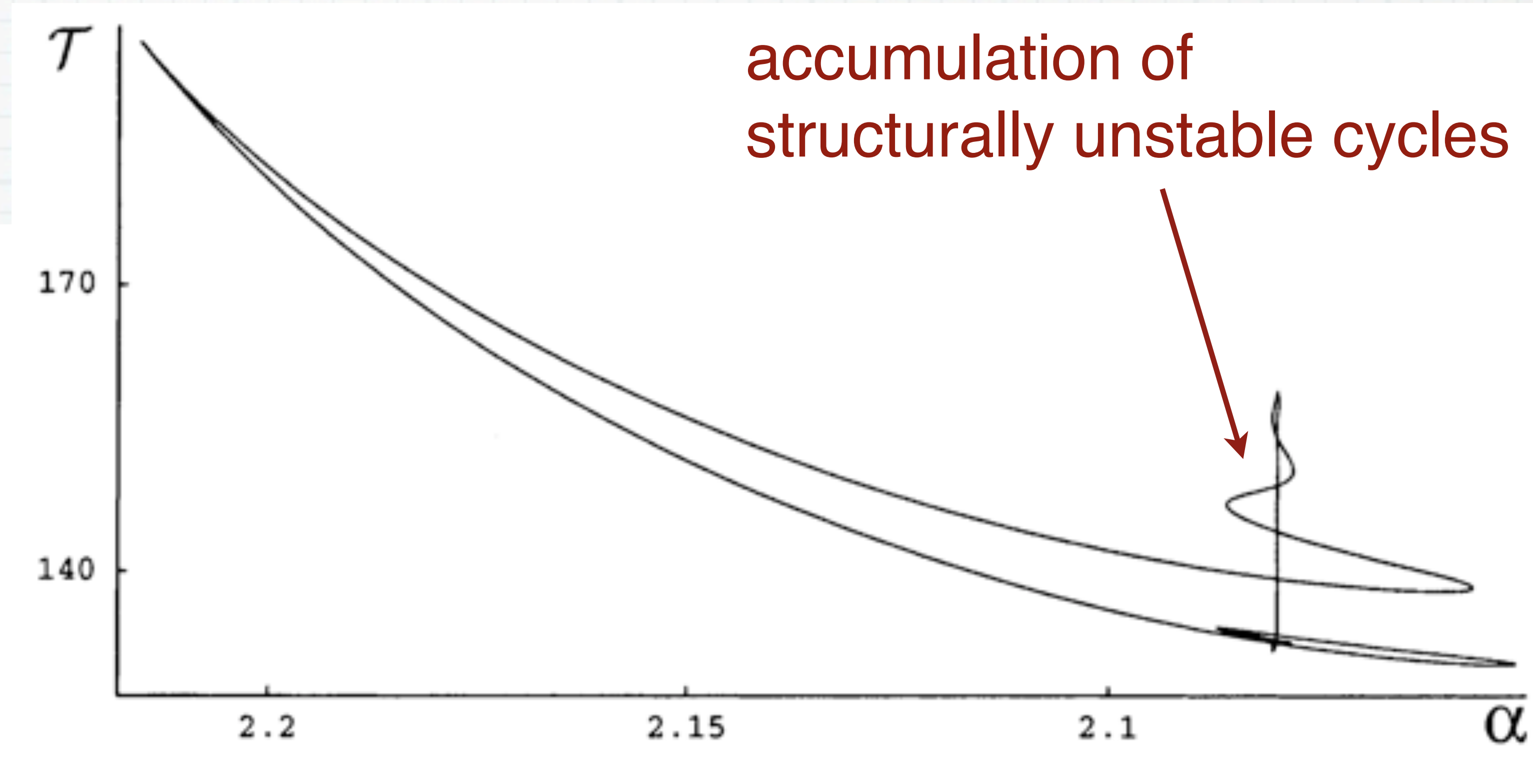


Cascades of isolas and period doubling bifurcations

winding number of TB oscillation

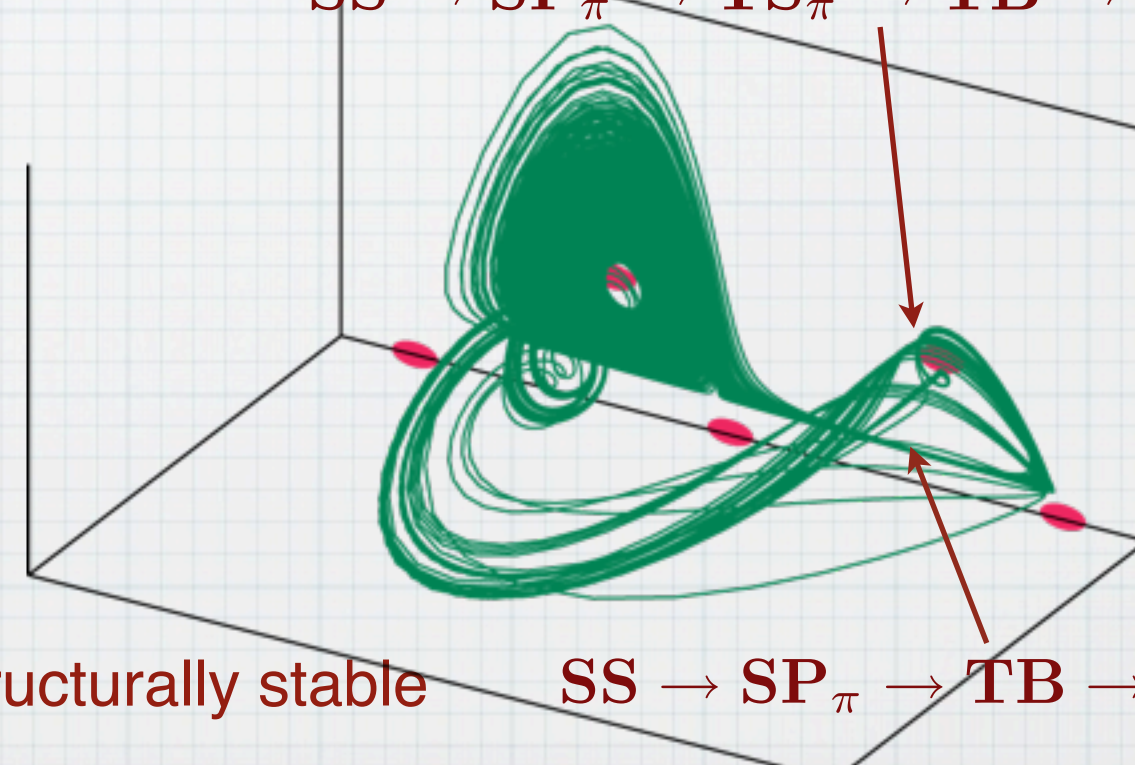


by Porter and Knobloch



accumulation of
structurally unstable cycles

$\mathbf{SS \rightarrow SP_\pi \rightarrow TS_\pi \rightarrow TB \rightarrow SS}$



structurally stable

$\mathbf{SS \rightarrow SP_\pi \rightarrow TB \rightarrow SS}$

Spot dynamics in heterogeneous environments

* Blended methodology between computers and mathematics
  → detection and characterization of instabilities
  application of the dynamical systems theory
  Spot dynamics near codimension 2 singularity
  → Rotational and its modulated motion
  Heteroclinic cycle and its associated long time periodic motion

  References:
  "Rotational motion of traveling spots in dissipative systems",
  Phys.Rev.E 80, 046208 (2009).

* Hidden potential ability (instability) emerges through interactions.
  We have to go through unstable state to catch a new life (dynamics)
  after large deformation.
  Degeneracy of abilities brings us the global properties in life.
  Remark: My first child was born a couple of weeks ago !