Black Holes: New Horizons (11w5099)

Valeri Frolov (University of Alberta, Edmonton, Canada),
Sang Pyo Kim, (APCTP and Kunsan National University, Korea),
Misao Sasaki (Yukawa Institute of Theoretical Physics, Kyoto, Japan),
Don Page (University of Alberta, Edmonton, Canada)

November 20 - November 25, 2011

This meeting was a joint activity of 3 theoretical physics institutes: Institute of Theoretical Physics of the University of Alberta (Canada), Asia Pacific Center for Theoretical Physics (Korea), and Yukawa Institute of Theoretical Physics (Kyoto, Japan). The meeting attracted leading experts in the field from 10 countries. As a result we had very fruitful and informative discussions of the exciting and intriguing problems of the physics and mathematics of black holes.

1 Overview of the Field

Black holes are one of the most amusing predictions of the Einstein's General Relativity. A first solution of the Einstein equations describing a black hole was obtained by Schwarzschild in 1916, soon after these equations were formulated. A prediction that the gravitational collapse of a massive star may produce a black hole was first made by Oppenheimer and Snyder (1939). However till the beginning of seventies of the past century there were no astrophysical confirmation of the existence of black holes. During this long period a lot of work was done by theoretical physicists and mathematicians which results in very deep understanding of properties of black holes.

A mathematical model of a physical spacetime adopted in the General Relativity is a differential manifold. Points of this manifold are identified with events. The (pseudo-) Riemannian metric on the spacetime manifold is used to determine an interval between a pair of near-by events. Directions generated by null vectors at a given point form a local null cone. Particle worldlines are represented by timelike smooth curves, with a tangent vectors (velocity) inside future directed local null cones, while light propagates along null geodesics. A spacetime is asymptotically flat if it has a distant region similar to the Minkowski space where the curvature becomes small. A black hole is a region in an asymptotically flat spacetime from where no information carrying signals propagating along causal (timelike and/or null) curves can reach infinity. Existence of a black hole indicated that the global causal structure of the spacetime is non-trivial. The boundary separating 'visible' and 'invisible' regions is called the event horizon. From mathematical point of view study of black holes and their properties is study of global and causal properties of spacetime manifolds with metrics obeying the Einstein equations. The Einstein equations is a set of coupled non-linear second order partial differential equations for 10 functions of four variables, the metric components. The global analysis of this system is quite complicated. However, now there exist quite good understanding of its solutions describing black holes.

An important breakthrough at the end of sixties – beginning of seventies was application of a global geometrical analysis to the black hole theory, which not only allowed one to give a covariant definition of a black hole but also to prove several fundamental theorems. The now-classical theorems stating that black holes have no hair (that is, no external individual attributes except mass, angular momentum, and charge), that a black hole contains a singularity inside it, and that the black hole area cannot decrease were proved during this period. It was demonstrated that black hole solutions are classically stable. However, in 1974 it was shown by Hawking that black holes are unstable with respect to quantum process of particle creation. Black holes emit particles with (practically) thermal spectrum and black holes obey the laws similar to the laws of thermodynamics. In particular they have entropy proportional to their surface area. All these and other results made it possible to construct a qualitative picture of the formation of black holes, to describe their possible further evolution and interaction with matter and other classical physical fields.

During next 30 years the status of black holes changed dramatically. Many stellar mass black holes were discovered in the X-ray binaries. Moreover, it is now believed that centers of many galaxies (including our) contain supermassive black holes with the mass of millions and billions of the solar mass. Gamma-ray bursts, the most powerful sources of high frequency electromagnetic radiation in the Universe, are explained by a model, where a center engine producing the energy is a black hole.

More recently another aspect of black hole physics became very important for astrophysical applications. The collision of a black hole with a neutron star or coalescence of a pair of black holes in binary systems is a powerful source of gravitational radiation which might be strong enough to reach the Earth and be observed in a new generation of gravitational wave experiments (LIGO and others). The detection of gravitational waves from these sources requires a detailed description of the gravitational field of a black hole during the collision. In principle, gravitational astronomy opens remarkable opportunities to test gravitational field theory in the limit of very strong gravitational fields. In order to be able to do this, besides the construction of the gravitational antennas, it is also necessary to obtain the solution of the gravitational equations describing this type of situation. Until now there exist no analytical tools which allow this to be done. Under these conditions one of the important tasks is to study colliding black holes numerically.

For many years, black holes have been considered as interesting solutions of the theory of General Relativity with a number of amusing mathematical properties. Now, after the discovery of astrophysical black holes, the Einstein gravity has become an important tool for their study [1]. Black holes are considered now as the most powerful sources of the gravitational radiation in the Universe. Black holes play also an important role of probes of new physical concepts, such as the modern string theory and recent models with large extra dimensions. Study of different aspects of black hole physics requires developed mathematical tools. (A more detailed discussion of the modern status of black holes can be found in a recent book [2]).

2 Recent Developments and Open Problems

(1) LIGO and other gravitational observatories search now for gravitational waves. Black holes in binary systems are considered as the most probable powerful sources of the strong gravitational radiation. Theoretical modeling of the black hole coalescence requires solving Einstein equations in the regime of extremely strong field and fast evolution. Numerical relativity is the only available ‘final resource’ for obtaining detailed description of such phenomena. The computational aspects of black holes, such as study of black hole coalescence and gravitational radiation, is important and rapidly developing area of research (see e.g. [3]).

(2) To identify black holes as astrophysical objects one uses theoretical results concerning light and particle propagation in the close vicinity of black holes. From mathematical point of view this is a problem of study highly non-linear ODEs for geodesic motion and solutions of partial differential equations for fields in the given metric. A lot of work has been done in this area. Now, when the General Relativity provides tools in the modern astrophysics, different aspects of these ‘old’ problems require new more detailed consideration. For example, for study polarized light propagation near rotating black holes one needs to develop WKB methods for multi-component fields (such as electromagnetic and gravitational ones) in a curved spacetime, which gives correct long-time asymptotics of the solution of the wave equations.

(3) One of the most important new development of the black hole theory is connected with higher dimensional gravity. The idea that spacetime may have more that 4 dimensions is rather old. The existence of extra dimensions in the string theory is required for consistency this theory. Models with large extra dimensions became popular in the brane world theories. In such theories the matter is confined to 4-dimensional

spacetime submanifold embedded in the higher dimensional bulk space, while gravity can propagate in extra dimensions. Black holes, which are solitonic type solutions of the Einstein equations, play the role of natural probes of extra dimensions in these theories.

(4) A lot of work was done recently in study of black hole solutions in asymptotically anti-deSitter spacetime in connection with AdS/CFT correspondence. In this approach it is possible to relate conformal quantum field, 'living in the AdS 4D boundary, with the properties of the classical solutions of the gravitational equations in the bulk spacetime. In this approach bulk black hole solutions correspond to the quantum field at finite temperature.

(5) Recently it was demonstrated that there exists a large variety of black hole solutions in higher dimensional Einstein theory of gravity. These solutions differ by the topology of their event horizons. Explicit stationary black hole solutions were obtained in 5 dimensions. There are indications that similar and more complicated black hole solutions exist in 6 and higher dimensions [4]. Important problems, which require developed mathematical tools are: to prove the existence of such solutions and to find and classify them. Another open problem is dynamical stability of higher dimensional black holes.

(6) It has been known for a long time that geodesic equations in the Kerr metric, describing a 4-dimensional rotating black hole, are completely integrable. Recently it was discovered that the complete integrability is a characteristic common property of ALL higher dimensional black holes with the spherical topology of the horizon. The geodesic motion in the spacetime of higher dimensional rotating black holes is a new physically interesting case of completely integrable systems. There exists a number of interesting mathematical problems, such as construction of Lax pairs for such systems and application of KAM method for study general properties of such dynamical systems. It was also shown that the same hidden symmetries, which are responsible for the complete integrability of geodesic equations, also imply complete separation of variables in the physically interesting field equations. ODE obtained by such a separation are second order linear equations with polynomial coefficients. The power of these polynomials grows with increasing the number of the spacetime dimensions. Study of the Sturm-Liouville problem for 'angular' eigen-modes and general properties of the 'radial equations' is practically open problems, interesting for the physical applications (see e.g. a review [5]).

(7) Another interesting problem is study of 'different phases' in a space of higher dimensional black-hole solutions in a spacetime with compact extra dimensions, and transition between these phases (see e.g. a review [6]).

(8) An interesting subject is study of black holes in non-Einstein gravity.

The purpose of the workshop was to discuss physical and mathematical aspects of black holes, recent progress in this area, and its open problems.

3 Presentation Highlights

3.1 Black string instability

Several quite interesting talks were devoted to the problem of higher dimensional black holes. Some of new results were obtained in this area by combination of numerical and analytical results.

A simplest example of higher dimensional object with non-trivial topology of a horizon is a black string (brane). Such solutions exist in a spacetime with large compact extra dimensions. Any space which is a direct sum of a Ricci flat 4D black hole metric and flat torus is again Ricci flat and, hence, it is a higher dimensional black hole solution of vacuum Eintein equations. It has been known for a long time that such solutions become unstable when the size of flat extra dimensions become sufficiently large as compared with the gravitational radius. This so called Gregory-Laflamme instability discovered in 1993 [7] later was studied in details in the linearized regime. But there remained an open question: what is the final state of such instability. At the present meeting there were demonstrated results of recent numeric simulations, performed by Luis Lehner and Frank Pretorius [8], which finally allow one to make definite conclusions concerning the dynamics of unstable black string solutions. There simulations show that at first stage the long black string looses its homogeneity in the flat z-direction and a single black hole is formed, with a black string attached to it. Later the string becomes thinner, its energy is partly absorbed by the black hole. When thickness of the string reaches the critical value, new smaller size black holes are formed. This process continues in time with formation of a self-similar discrete structure, containing smaller and smaller size black holes.

3.2 Merger transitions

Another interesting subject discussed at the meeting is the phase transitions of black objects in a space with large compact extra dimensions. In this approach one studies condition of coexistence of black hole solutions with different topology in the parameter space and focuses on stationary critical (maybe unstable) solutions when the topology of the black object changes. Barak Kol [9] proposed a conjecture that horizons during the phase of reconstruction has the structure of conefolds. Roberto Empharan at his talk demonstrated a simple analytical model which describes merges of the horizons. Namely, he considered a higher dimensional rotating black hole in a spacetime with the cosmological constant. Such black holes allow arbitrary large rotation parameters. When this parameter is large enough the black hole horizon crosses the cosmological horizon. At this point the global horizon changes its topology. The results presented in the talk show that the Kol's conefold conjecture is valid in this case. This result opens an interesting possibility for search of the analytical proof of this conjecture in more general cases.

3.3 Black holes in Randall-Sundrum model

Randall-Sundrum model is another approach with large extra dimensions. In this model the bulk space is a higher-dimensional Universe described by warped geometry, while four dimensional submanifold (brane) describes our 'visible' world. Finding black hole solutions in such spacetimes, and even proof of their existence is a rather complicated problem. Recently the progress was achieved in the work of the group of Wiseman [10]. Using the numerical methods based on the Ricci-flow approach, they succeeded to obtain 5 dimensional regular black hole solutions attached to the brane. Don Page and collaborators presented at the meeting a new independent proof of the existence of the large black holes in the Randall-Sundrum model. Instead of solving numerically the 5D Einstein equations, they used numerical tools to find a minimum of the positive definite integral, which vanishes when such equations are satisfied. Since the problem is quite complicated this independent result obtained by a different method is quite important. However, there still exists an unsolved puzzle connected with black holes solutions in the Randall-Sundrum model. Sometime ago, using arguments based on the ideas of AdS/CFT correspondence, several scientists arrived to a conclusion that such static solutions are impossible because of the emission by the black hole quantum radiation of conformal fields. The obtained numerical solutions (at least for large black holes) do not have indication on the existence of such radiation. Possible resolutions of this paradox were discussed in the talk by Takahiro Tanaka.

3.4 Black hole solutions with reduced symmetry

Most of the known higher dimensional black hole solutions possess rather high symmetry. Robert Mann demonstrated the existence of higher dimensional black hole solutions in the presence of scalar complex fields with reduced symmetry [11]. This is a generalization of recent 4 dimensional results by Gary Horowitz and collaborators [12]. Numerical simulations of black hole solutions with axion hairs and gravitational collapse in such systems were presented by Hirotaka Yoshino.

3.5 Black hole numerics

Review of recent progress in numerical study of black hole merger and collisions was given by Matt Choptuik and Masaru Shibata. Namely, Choptuik presented recent results of the numerical simulation of higher dimension black hole collisions and new results on the gravitational critical collapse of the matter. Shibata and his collaborators focused of 'real' processes of a 4D black hole formation in the neutron-star–neutron-star and neutron-star–black-hole binaries and in the stellar core collapse. They presented an updated estimation of the rate of the gravitational waves emission and its dependence on the equation of state of the matter in the neutron stars.

3.6 Black holes in non-Einstein gravity

Quite large number of talks were devoted to black hole solutions in different generalizations of the Einstein gravity. There exist at least two reasons why such solutions are important:

- In order to apply the methods similar to AdS/CFT correspondence for the description of usual quantum systems in the strong coupling regime the modification of bulk gravity equations are required.
- Low energy gravity equations in the string theory contain higher in curvature corrections and other fields.

The popular now modifications of the first type form a class of so called Horava-Lifshitz gravity theories. Recent results concerning existence of black hole and cosmological solutions in these theories were presented and summarized in the talks by Ruth Gregory and Shinji Mukohyama.

An example of the second kind of non-Einstein theory is a so called Gauss-Bonet gravity. In 4 dimensions there exists a special linear combination of quadratic in curvature invariants which is a total derivative and the corresponding action is a topological invariant. Adding such a term to the Einstein action in 4 dimensions does not affect the Einstein equations. However, in 5 and higher dimensions adding of the Gauss-Bonet term to the action modifies the Einstein equation. A similar modifications also occurs in 4 dimensions if there exists an additional scalar (dilaton) field and the Gauss-Bonet contribution to the action contains a prefactor depending on this field. A special property of these theories is that the dynamical equations do not contain metric derivatives higher than the second order.

Asymptotically AdS solutions of the Gauss-Bonet gravity in 5 dimensional spacetime were discussed in the talk by Rong-Gen Cai, who demonstrated how these solutions might be used for description of holographic superconductors in the related AdS/CFT description. The talk of Jutta Kunz summarized recent numerical results of study black holes in the dilaton Gauss-Bonet gravity in 4 dimensions. A new unexpected result obtained in this work is the existence of static spherically symmetric wormhole solutions [13]. It was argued that these solutions are stable.

3.7 Hidden symmetries of black holes

During recent years a lot of work was done in study of hidden symmetries of higher dimensional black hole. The main result obtained in this area is the proof of the complete integrability of the geodesic particle and light motion in the background of arbitrary higher dimensional rotating black hole with spherical topology of the horizon. This result and similar results on the complete separability of the Hamilton-Jacobi, Klein-Gordan and some other relativistic field equations is based on the existence of the closed conformal rank 2 Killing-Yano tensors in such spaces. Moreover, it was also demonstrated that the most general solutions describing higher dimensional rotating black holes in asymptotically AdS spacetime do possess this property. New development presented at the meeting was demonstration that the complete integrability property is also valid for the motion of particles with internal (spin) degrees of freedom (talk by Pavel Krtous, see also [14, 15]). Another important development was presented by Claude Warnick. He demonstrated how the notion of the closed conformal rank 2 Killing-Yano tensors can be generalized to the case of the connections with torsion and described interesting applications to Kerr-Sen black hole solutions.

3.8 Spinoptical effects in rotating black holes

Application of the geometrical optics approximation for the light propagation in a curved spacetime has quite long story. The well known statement is that in the high-frequency approximation light propagates along null geodesics and if it is linearly polarized, its polarization vector is parallel transported (see e.g. [16]). In the talks by Valeri Frolov and Andrey Shoom it was demonstrated that this conclusion seams to be oversimplified. It should be emphasized that the geometrical optics approximation (as well as similar WKB approximation) are well defined locally. To reduce solution of the wave equations in the high frequency limit one constructs a Lagrangian submanifold of the phase space satisfying the eikonal equation. However, even small change of this equation can modify long time behavior of the Lagrangian submanifold, and, hence, the asymptotic form of the solution. In the talk it was proposed to improve the standard geometrical optics approach in order to take care of this problem. For this purpose from the very beginning the Maxwell equations are written as two independent sets of the equations: one for right-polarized and the other for left-polarized light. Geometric optics is constructed as a high frequency approximation in each of the independent sectors and the lowest order polarization dependent corrections are included in the eikonal equations. As a result, right and left polarized beams of light have slightly different trajectories, while a beam with initial linear polarization splits in two spatially separated circular polarized beams. Possible application of this effects to the polarized light propagation near a rotating black hole was discussed.

3.9 Black hole entropy

Two talks devoted to the problem of black hole entropy gave brief review of the problem and recent developments. Steve Carlip focused on the idea of using effective conformal description of black hole entropy [17]. He demonstrated that many of the adopted now approaches are based on the idea that the microscopic constituents responsible for the black hole allows description in terms of 2D conformal fields. He also formulated main generic features of these approaches. An alternative explanation of the black hole entropy in the loop gravity was presented by Hanno Sahlmann.

3.10 Visualization problem

There exist several different methods that are used to visualize properties of special solutions in the General Relativity. Examples are Carter-Penrose conformal diagrams and different embedding diagrams for specially chosen 2D slices of the metric. Kayll Lake presented a new interesting approach which allows one to get better qualitative understanding of global properties of the spacetime. Namely, he proposed to use gradient flows constructed for curvature invariants. He demonstrated that at least in the simplest cases (including interesting black hole solutions) the number and characteristics of the singular points of such flows contain important information concerning global properties of the solution.

3.11 Black hole analogues

In the conclusion, we need also to mention a talk by Bill Unruh. He described a recent experiment performed at UBC. In this experiment amplification of the surface waves in a container with moving water was studied. The profile of the container was chosen so that the water flux in some region moves faster than the speed of waves. Such a system is a liquid analogue of a white hole. It was demonstrated that the amplification coefficient is practically frequency independent in a wide frequency range. This behavior is consistent with theoretical predictions, and might be considered as an experimental evidence for the Hawking effect in the condensed matter analogues of black- and white-hole.

4 Scientific Progress Made and Outcome of the Meeting

It is difficult to expect that during this short period of 5 days of the meeting fundamental problems could be solved. The main result of the workshop is that several of the problems of modern 'mathematical physics of black holes' were identified, formulated and discussed. This subject covers wide spectrum of physical properties of systems containing black holes. Numerical methods developed for simulation of black hole coalescence, are now used for calculation of the cross-section for higher dimensional black hole collision. These results are used in the discussion of possibility of black hole production in colliders. Similar methods allowed one to describe the non-linear stage of the instability of black string in the spacetime with large compact extra dimensions. Combination of numerical and analytical results is important for other black hole problems, such as black holes in non-Einstein gravity and black holes in Randall-Sundrum models. The latter subjects are closely connected with the string theory. Black holes are used in the AdS/CFT correspondence as important component of the construction. At the same time the ideas of this approach play an important role in the explanation of the mechanism of black hole entropy. This close interconnection between different problems of black holes is an important new element of the modern 'state of art'. For this reason an exchange of ideas between experts in different areas of 'mathematical physics of black holes' and using different, both analytical and numerical, tools was very timely, important and productive. In the long term perspective this is the main result of the present event.

There were a number of 'immediate' proposals and 'short term projects', that arose as a result of the talks and discussions. To be more concrete let us give some examples.

The talk of Andrey Zelnikov discussed the self-energy of particles in the vicinity of black holes. At the talk Barak Kol posed a question: Why an additional gravitational force acting on a charged particle is repulsive? This question generated an interesting discussion after the talk. As a result of joint discussions it was proposed a simple mechanism, explaining not only the sign of the effect, but also (at least in the weak field limit) giving a correct numerical factor. The discussion of the question how generic this mechanism is and what might be its applications to other cases continued after the conference by the exchange of e-mails. This may finally results in a joint publication of some participants of the meeting. In any case this discussion clarified a fundamental problem of the self energy of classical charged objects in the external gravitational field.

Another immediate result of the discussion after the talk of Don Page was the following. At the past BIRS Black Hole meeting Hirotaka Yoshino presented arguments, based on his numerical simulations that there do not exist static black hole solutions in the Randal-Sundrum model. Independent arguments in favor of such conclusion based on the AdS/CFT arguments were given earlier. More recently Wiseman and collaborators numerically found such solutions. This result was confirmed by independent calculations of the group of Don Page at the University of Alberta and presented at the meeting. As a result of the discussions during the meeting, Hirotaka Yoshino decided to repeat his old calculations with higher accuracy in order to find a possible solution of this puzzle. Takahiro Tanaka was also stimulated in development of his critical analysis of the AdS/CFT arguments.

One of us (V.F.) may add one, more personal example. Some of our colleagues from Japan expressed high interest in the new results on spinoptics in a curved spacetime presented in our with Andrey Shoom talks. As a results, after my return to Kyoto, where I am staying as a visiting professor, I was invited to give extensive lectures on this subject at Yukawa Institute of Theoretical Physics and at Osaka City University. This illustrates another result: Fresh new ideas presented at the meeting immediately became available to experts from 10 different countries. This demonstrates how effective was a chosen format of the meeting.

To summarize, we would like to say that the BIRS meeting "Black Holes: New Horizon" was very successful. It combined high quality of the talks, and warm friendy atmosphere of discussions. The staff of the BIRS helped us a lot both at the stage of preparation of the meeting, as well as during its work. All the participants enjoyed very much a friendly atmosphere of Banff Center and BIRS. We would like to thank BIRS for the cooperation and help.

(Prepared by Valeri Frolov on the behalf of the organizers.)

References

[1] E. Poisson. *A Relativists Toolkit: The Mathematics of Black-Hole Mechanics*, Cambridge Univ. Press. (2007).

[2] V. P. Frolov and A. Zelnikov, *Introduction to Black Hole Physics*, Oxford University Press, 2011.

[3] U. Sperhake, E. Berti and V. Cardoso. Numerical simulations of black-hole binaries and gravitational wave emission. e-Print: arXiv:1107.2819 [gr-qc] (2011) 46 pp.

[4] R. Emparan, H. S. Reall, Black Holes in Higher Dimensions, Living Rev. Rel. http://www.livingreviews.org/lrr-2008-6, (2008). 11, 6, 76 pp.

[5] V.P.Frolov and D.Kubiznak, D. Higher-dimensional black holes: hidden symmetries and separation of variables, *Class. Quantum Grav.,* **25** (2008) 154005, 22 pp.

[6] T.Harmark, V. Niarchos and N.A. Obers, Instabilities of black strings and branes, *Class. Quant. Grav.,* **24**, (2007) R1-R90.

[7] R. Gregory and R. Laflamme. Black strings and p-branes are unstable. *Phys.Rev.Lett.* **70** (1993) 2837-2840.

[8] L. Lehner and F. Pretorius. Final State of Gregory-Laflamme Instability. e-Print: arXiv:1106.5184 [gr-qc] (2011) 27 pp.

[9] B.Kol, Topology change in general relativity and the black-hole black-string transition,' *JHEP* **0510** (2005), 049.

[10] P. Figueras and T. Wiseman, Gravity and Large Black Holes in Randall-Sundrum II Braneworlds, *Phys. Rev. Lett.* **107** (2011), 081101

[11] S. Stotyn, M. Park, P. McGrath and R. B. Mann. Black Holes and Boson Stars with One Killing Field in Arbitrary Odd Dimensions. e-Print: arXiv:1110.2223 [hep-th] (2011) 29 pp.

[12] O. J.C. Dias, G. T. Horowitz, and J. E. Santos, Black holes with only one Killing field, *JHEP* **1107** (2011), 115

[13] P. Kanti, B. Kleihaus, and J. Kunz. Stable Lorentzian Wormholes in Dilatonic Einstein-Gauss-Bonnet Theory. e-Print: arXiv:1111.4049 [hep-th], 28 pp.

[14] M. Cariglia, P. Krtous and D. Kubiznak. Hidden Symmetries and Integrability in Higher Dimensional Rotating Black Hole Spacetimes. e-Print: arXiv:1112.5446 [hep-th] (2011) 5 pp.

[15] D. Kubiznak, M. Cariglia. On Integrability of spinning particle motion in higher-dimensional black hole spacetimes. e-Print: arXiv:1110.0495 [hep-th] (2011) 5 pp.

[16] C.W.Misner, K.S. Thorne and J.A. Wheeler. Gravitation, W.H. Freeman and company, New York (1973).

[17] S. Carlip. Effective Conformal Descriptions of Black Hole Entropy. e-Print: arXiv:1107.2678 [gr-qc] (2011) 27 pp.