Current interests

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1 About semicanonical basis

Let G be a connected and simply connected simple Lie group, $\mathfrak{g}$ its Lie algebra and $\mathfrak{n}$ a maximal nilpotent subalgebra of $\mathfrak{g}$. Lusztig introduced the semicanonical basis of the enveloping algebra $U(\mathfrak{n})$ in type A, D and E. This basis is indexed by the irreducible components of the varieties of representations of the corresponding preprojective algebra. This construction leads to the categorifications of Geiß, Leclerc and Schröer of the cluster structure on the coordinate rings of unipotent subgroups of G. So, the cluster monomials correspond to the rigid representations of the preprojective algebra. In my thesis, I described a method which allows to see, for the non simply-laced types (B, C, F, G), the cluster monomials as coming from Γ-stable rigid irreducible components of the varieties of representations of a preprojective algebra Λ endowed with an action of a finite group Γ. So, one morally obtains a part of the semicanonical basis in this framework.

Problem 1. *Construct a semicanonical basis of $U(\mathfrak{n})$ in non simply-laced types.*

The original method for simply-laced case works indeed also for Kac-Moody case. I think now that it is difficult in such a full generality. So, I tried during last times to understand it at least for specific cases (B_n for example). Thus, the problem could be attacked by explicit computations on crystals. For example, it is known that, in Dynkin case, there is an easy to define bijection between isoclasses of representations of a quiver and the irreducible components of the varieties of representations of the corresponding preprojective algebra. Thus, this leeds to the structure of a crystal on the set of isoclasses of representations of the quiver. If we are able to compute explicitely this structure (it is far from being obvious), then, we can expect to get some much more explicit and computational description (as representations of a Dynkin quiver are easy to compute with). Thus, it could be a starting point for trying to give an analogue in non simply-laced case.

2 More general categorifications of skew-symmetrizable cluster algebras

I tried recently to adapt the work of Derksen, Weyman and Zelevinsky to cluster algebras whose exchange matrices are not skew-symmetric. Up to now, I got only a (quite small) subclass of these cluster algebras (containing for example acyclic ones), which seems to be reachable also by purely combinatorial methods. The natural idea which consists in taking a quiver with potential and an action of a group seems to be too restrictive. In other words, there should exist a generalization of such a situation.

To be more explicit, the problem in this case comes from the fact, if one takes the natural (or easy) candidate, is that the space of potentials becomes reducible (for Zariski topology). Thus, the natural argument for the existence of a non-degenerate potential vanishes (intersection of countably many open conditions). So, the problem could be to extend the notion of potentials in such a way that the space of potential becomes irreducible.

In the same spirit, we could probably construct in such a case generalized cluster categories if we had a good notion of potential (indeed, we can already, but, we often do not have generic potential, and therefore, these categories do not categorify the corresponding cluster algebras).