CURRENT RESEARCH

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My recent research is in the following subjects:

(1) Cluster algebras coming from representation theory of affine algebras and quantum affine algebras: Q-systems, T-systems and their quantization. These are quite universal structures, and appear in various other contexts, although their origin is as a relation between characters of special representations of quantum affine algebras introduced in the context of the generalized Heisenberg spin chain. For example, the Q-system describes a certain (double Coxeter) subset of the double Bruhat cells of GL_n or, alternatively, the totally positive factorization. It also describes certain elements in the dual canonical basis (shown by the recent work by Nakajima).

(2) Application of results from cluster algebras to solve certain conjectures about fermionic formulas in representation theory, originating in integrable statistical mechanical models on the lattice and conformal field theory. I'm interested in the quantized version of these identities and its relation to the quantum cluster algebras related to the Q-systems.

(3) Discrete and quantum integrability manifested by such cluster algebras: Integrability survives quantization, and manifests itself in different aspects of the cluster algebra evolutions. For example, in the existence of conserved quantities and linear recursion relations which allow a solution of the system. The Toda flows of Gekhtman, Shapiro and Vainshtein are compatible with certain cluster algebra mutations which appear from the Q-system equations mentioned above. These are integrable in the Poisson sense, and Q-system mutations are coordinate transformations in the double Bruhat cells where the Toda flows take place. The quantized T-system is related to the quantum discrete Liouville equations of Faddeev and Volkov.

(4) Positivity proofs using techniques of statistical mechanics: The use of weighted statistical models gives explicit expressions for the cluster variables. Having a statistical model therefore gives more than positivity. Moreover, it may carry over to the non-commutative version with some work, as we have shown in the special case of Q-system. In cases where quasi-statistical models can be used, one may hope to lift the situation to the non-commutative setting. Our recent work on the non-commutative rank 2 models suggested by Kontsevich was recently generalized using such ideas to the symmetric non-integrable setting by Kyungyong Lee.

Most of this research is in collaboration with Philippe Di Francesco.

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