Research Interests
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Since 2008, I have been working on the project of studying the interrelation between *cluster algebras* and the *T- and Y-systems*. The latter arose from the integrable systems based on the conformal field theory (CFT) and the Yang-Baxter equation (or quantum groups) in the 90's. I have done this in collaboration with specialists in various fields, Rei Inoue, Osamu Iyama, Rinat Kashaev, Bernhard Keller, Atsuo Kuniba, Roberto Tateo, Suzuki, and Andrei Zelevinsky. We clarified the integrity of T- and Y-systems, and dilogarithm as well, with cluster algebras; and, as a fruitful outcome we obtained the following results:

— proof of the conjecture on the periodicity of Y-systems in full generality

— proof of the conjecture of the dilogarithm identities in CFT in full generality

— generalization of dilogarithm identities by cluster algebras and their quantum counterpart

— generalization of T- and Y-systems by cluster algebras and their connections to some integrable differential equations

— some basic properties of cluster algebras (duality of C- and G-matrices, extension theorem of periods, etc.)

I continue to study a further development of the basic theory of cluster algebras and its application to various fields, especially, to integrable differential equations.

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