Cluster-related interests of Hugh Thomas

Atomic bases (=canonically positive bases). An element of a cluster algebra is called *positive* if its expansion in terms of any cluster has non-negative coefficients. The positive elements of a cluster algebra form a cone in the cluster algebra. An atomic basis for a cluster algebra is a basis whose non-negative linear combinations generate the positive cone. (Such a basis need not exist, but if it exists, it is unique.)

Until recently, only few examples of cluster algebras with atomic bases were known. Giovanni Cerulli recently showed that for finite type cluster algebras, the cluster monomials form an atomic basis. Subsequently, building on his approach, Grégoire Dupont and I found the atomic basis for coefficient-free cluster algebras of type $\widetilde{A}_n$ (confirming a conjecture of Dupont). We hope that it may be possible to extend this approach to all cluster algebras arising from surfaces (using bases provided by Musiker-Schiffler-Williams).

Higher dimensional analogues of cluster algebra theory. Two of the best-understood classes of cluster algebras have a "two" associated to them: for cluster algebras with an additive categorification, the "two" is the 2-Calabi-Yau property, while for cluster algebras arising from surfaces, the "two" is the two-dimensionality of the surface. It is natural to ask if these twos can be (simultaneously) increased.

In recent work with Steffen Oppermann, we showed that the link between the combinatorics of the A_n cluster algebra (triangulations of a polygon) and the type A_n cluster category, can be generalized by replacing the polygon by an even-dimensional cyclic polytope, and replacing the cluster category by a similar construction starting from the higher Auslander algebra of an A_n quiver. Various elements of cluster algebra theory persist; in particular, there is an interpretation of a tropical version of exchange relations.

However, many elements of cluster algebra theory are still missing (including, most obviously, an analogue of cluster algebras themselves). We are also curious if there is any connection to (higher-dimensional) hyperbolic geometry, Poisson geometry, or any of the other flavours of representation theory which are associated to cluster algebras.

Preprojective algebras and total positivity. Fix Q a quiver (Dynkin or not). Let Λ_Q be the associated preprojective algebra, and W the corresponding Weyl group. For $w \in W$, there is an ideal I_w in Λ_Q. Write $(I_w)_{kQ}$ for I_w viewed as a kQ-module. Define $\mathcal{C}_w$ to consist of the finitely-generated kQ-modules contained in $\mathrm{add}(I_w)_{kQ}$.

In ongoing work with Steffen Oppermann and Idun Reiten, we show that the indecomposable modules in $\mathcal{C}_w$ encode the *positive distinguished* subexpression for w inside the word $(s_1 s_2 \dots s_n)^\infty$ if Q is non-Dynkin, or inside the $s_1 \dots s_n$-sorting word for w_0 if Q is Dynkin. The positive distinguished subexpression for w in closely tied to total positivity. Given the link between total positivity and preprojective algebras, one is led to expect a conceptual explanation for our result, which should also be related to the Geiss-Leclerc-Schröer approach to categorification of cluster algebras associated to Kac-Moody groups.