September 20, 2012

Banff International Research Station

Integrable Systems, Growth Processes and KPZ Universality

September 23–28, 2012

Titles & Abstracts of Talks

SPEAKER: **Mark Adler** (Brandeis University)
TITLE: *Nonlinear PDEs for Fredholm determinants arising from string equations*
ABSTRACT: The unitary random matrix model considered by Claeys and Vanlessen ($k = 1$) and more generally Claeys, Its and Krasovsky, where the density of state dies at the endpoint of an interval like $x^{(4k+1)/2}$ gives rise to kernels involving string relations. This leads to PDEs for the associated Fredholm determinants describing gap probabilities which explicitly depend on solutions to Painlevé hierarchies arising from the string relations. Other non-RMT cases will be indicated, involving the critical and tricritical Ising model as time permits. This is joint work with M. Cafasso and P. van Moerbeke.

SPEAKER: **Jinho Baik** (University of Michigan)
TITLE: *Toeplitz determinant of discrete measure*
ABSTRACT: We discuss a simple identity which reduces the asymptotic study of a Toeplitz or Hankel determinant of a discrete measure to that of the orthogonal polynomials of a continuous measure (instead of a discrete measure). We then illustrate the use of the identity in a few examples in non-intersecting processes and other probability models. This is a joint work with Zhipeng Liu (graduate student at U. Michigan).

SPEAKER: **Pavel Bleher** (Indiana University-Purdue University Indianapolis)
TITLE: *Riemann-Hilbert approach to orthogonal polynomials associated with the normal matrix model with a cubic potential*
ABSTRACT: We consider the normal matrix model with a cubic potential. The model is ill-defined, and in order to regularize it, Elbau and Felder introduced a model with a cut-off and corresponding system of orthogonal polynomials with respect to a varying exponential weight on the cut-off region on the complex plane. In the present work we show how to define orthogonal polynomials on a specially chosen system of infinite contours on the complex plane, without any cut-off, which satisfy the same recurrence algebraic identity that is asymptotically valid for the orthogonal polynomials of Elbau and Felder.

The main goal of our work is to develop the Riemann-Hilbert (RH) approach to the orthogonal polynomials under consideration and to obtain their asymptotic behavior on the complex plane as the degree n of the polynomial goes to infinity. As the first step in the RH approach, we introduce an auxiliary vector equilibrium problem for a pair of measures (μ_1, μ_2) on the complex plane. We then formulate a 3×3 matrix valued RH problem for the orthogonal polynomials in hand, and we apply the nonlinear steepest descent method of Deift-Zhou to the asymptotic analysis of the RH problem. The central steps in our study are a sequence of transformations of the RH problem, based on the equilibrium vector measure (μ_1, μ_2), and the construction of a global parametrix. The main result of this work is a derivation of the large n asymptotics of the orthogonal polynomials on the whole complex plane. We prove that the distribution of zeros of the orthogonal polynomials converges to the measure μ_1, the first component of the equilibrium measure. We also obtain analytical results for the measure μ_1 relating

it to the distribution of eigenvalues in the normal matrix model which is uniform in a domain bounded by a simple closed curve. This is a joint project with Arno Kuijlaars.

SPEAKER: **Alex Bloemendal** (Harvard University)
TITLE: *Spiking Tracy-Widom* (β)
ABSTRACT: Investigating the Baik–Ben Arous–Péché phase transition using the Ramírez–Rider–Virág soft edge operator limit led to new distributions: deformations of Tracy-Widom (β), generalizing those found near the transition in the original $\beta = 2$ case. I will review how these distributions are characterized in terms of a matrix-valued version of the stochastic Airy operator, and alternatively in terms of an interesting diffusion and related linear PDE (joint work with B. Virág). There is an explicit but somewhat mysterious connection to the Painlevé II structure at $\beta = 2$ and 4 (joint work also with J. Baik and A. Its). Finally, there is promising evidence that the PDE is numerically useful (joint work with B. Sutton).

SPEAKER: **Alexei Borodin** (MIT)
TITLE: *A CLT for Plancherel representations of the infinite-dimensional unitary group*
ABSTRACT: We study asymptotics of traces of (noncommutative) monomials formed by images of certain elements of the universal enveloping algebra of the infinite-dimensional unitary group in its Plancherel representations. We prove that they converge to (commutative) moments of a Gaussian process that can be viewed as a collection of simply yet nontrivially correlated two-dimensional Gaussian Free Fields. The limiting process has previously arisen via the global scaling limit of spectra for submatrices of Wigner Hermitian random matrices. Based on joint work with Alexey Bufetov.

SPEAKER: **Yang Chen** (University of Macau)
TITLE: *Towards a unified characterization of MIMO mutual information distribution: A Painlevé based approach*
ABSTRACT: We make use of the recently derived Painlevé V characterization of the mutual information for single user MIMO system to study the left and right tails of the PDF of the associated distribution.

SPEAKER: **Sunil Chhita** (KTH Royal Institute of Technology)
TITLE: *Some coupling functions of domino tilings of Aztec diamonds*
ABSTRACT: The inverse Kasteleyn matrix of a bipartite graph holds much information about the perfect matchings of the graph such as its local statistics. These statistics can be used to find global and local asymptotic behavior. In this talk, we present three different weightings (one-periodic, two-periodic and q^{vol} weightings) for domino tilings of the Aztec diamond each with different but striking limiting behavior. We show that it is possible to find recurrence relations which allows a derivation of the inverse Kasteleyn matrix. This is joint work with Benjamin Young (University of Oregon).

SPEAKER: **Ivan Corwin** (Clay Mathematics Institute and MIT)
TITLE: *From duality to determinants*
ABSTRACT: I will explain duality for ASEP and then solve the associated PDEs for special initial conditions via a contour integral ansatz. Using the resulting formulas I develop generating series which identify particle location distributions and which can be expressed as Fredholm determinants (one of which coincides with the famous Tracy-Widom ASEP formula). Asymptotics of such expressions are readily computable and lead to KPZ universality class fluctuations. This is based on joint work with Alexei Borodin and Tomohiro Sasamoto.

SPEAKER: **Alexander Gamburd** (The Graduate Center, CUNY)
TITLE: TBA
ABSTRACT:

SPEAKER: **Friedrich Götze** (Bielefeld University)
TITLE: *Local universality of repulsive particle systems and random matrices*
ABSTRACT: We show that local correlation universality holds for deformations of the GUE ensemble which don't allow a spectral interpretation, but have a local repulsion exponent equal to that of GUE eigenvalues. This is joint work with M. Venker. Extensions of these results for β-ensembles will be discussed.

SPEAKER: **John Harnad** (CRM, University of Montreal & Concordia University)
TITLE: *Convolution symmetries and flows*
ABSTRACT: We show how an infinite abelian group of "additional symmetries" can be used to generate new KP tau functions from old. In particular, this can be applied to obtain "externally coupled" matrix model partition functions (like Brezin-Hikami) from the usual ones (e.g. hermitian matrix ensembles with linear exponential deformation classes of $U(N)$ invariant measures). This can also be applied dynamically, to generate new forms of tau functions, expressed as fermionic expectation values, which represent flows of an infinite abelian group acting by convolutions; (i.e., diagonally in the Fourier series representation). This explains certain non-standard fermionic tau function formulae; e.g., those of Okounkov for tau functions generating crystal growth and those of Bettelhem, Wiegmann et al. for electronic correlators. This is joint work with Alexander Orlov.

SPEAKER: **Alexander Its** (Indiana University-Purdue University Indianapolis)
TITLE: *Quantum entanglement and the block Toeplitz determinants*
ABSTRACT: We calculate the von Neumann and Rényi entropies of a large block of consecutive spins in the ground state of the XY spin chain on an infinite lattice. We also evaluate the spectrum of the corresponding reduced density matrix. Analytically, the problem is reduced to the asymptotical evaluation of certain Toeplitz determinants with matrix valued symbols—the "block" Toeplitz determinants. The key feature of our approach is the use of Riemann-Hilbert techniques in conjunction with Widom's limit theorem for the block Toeplitz determinants. The talk is based on the joint works with F. Franchini, B.-Q. Jin, V. E. Korepin, F. Mezzadri, M. Mo, and L. A. Takhtajan. If time permits, we shall outline the work in progress devoted to the entanglement of the subsystem in the XY chain consisting of two separated blocks of consecutive spins (joint project with L. Brightmore, A. B. J. Kuijlaars, V. E. Korepin and F. Mezzadri).

SPEAKER: **Kurt Johansson** (KTH Royal Institute of Technology)
TITLE: *Particle and dimer asymptotics in the Aztec diamond*
ABSTRACT: I will discuss the relationship between the particle approach via non-intersecting paths and the dimer approach, using the Kasteleyn method, to determinantal processes in the Aztec and the double Aztec diamond. I will also discuss some asymptotic dimer statistics in the Aztec diamond. The double Aztec diamond will be further explored in the talk by Pierre van Moerbeke. Joint work with Mark Adler, Sunil Chhita, Pierre van Moerbeke and Ben Young.

SPEAKER: **Kenneth D. T-R McLaughlin** (University of Arizona)
TITLE: *Semiclassical analysis of the focusing nonlinear Schrödinger equation with square barrier initial data*
ABSTRACT: The semiclassical scaling of the focusing nonlinear Schrödinger equation is a singular limit of a nonlinear partial differential equation; the underlying model is an elliptic system which is ill-posed as an initial value problem. The full partial differential equation has a reasonable global existence theory and so the puzzle

is to figure out if/how the ill-posed system emerges from the full equation. For real analytic initial data the dichotomy is not present, and this situation has been studied in a small collection of examples. But when the initial data is not analytic, all bets are off. The work to be presented is the first analytical result in the absence of analyticity, and is joint work with Bob Jenkins.

SPEAKER: **Motohico Mulase** (UC Davis)
TITLE: *The Eynard-Orantin recursion and quantum invariants in geometry*
ABSTRACT: In geometry and topology, the Tracy-Widom distribution functions appear as quantum topological invariants. One of the bridges between the two worlds is made by the recent work of Eynard, Orantin, and many others, including string theorists and geometers. In this talk, I will present a few mathematically rigorous examples of this evolving theory. They illustrate the interplay between random matrices, algebraic geometry of moduli spaces, symmetric functions, integrable systems, and the Schrödinger equations satisfied by various quantum invariants. The talk is based on my collaborations with Bouchard, Eynard, Shadrin, Sulkowski, and others.

SPEAKER: **Eric Nordenstam** (University of Vienna)
TITLE: *Tilings of half a hexagon*
ABSTRACT: We study tilings of a certain region in the plane which we call the Novak half-hexagon. It turns out that the number of tilings with rhombuses of a size n Novak half-hexagon is exactly the number of tilings with dominoes of a size n Aztec diamond, which is a tiling model that is extremely well studied in the random matrix community. In our failed attempt to find a combinatorial bijection between these objects, we nevertheless found many interesting similarities between these two models. There is an algorithm to sample tilings of the Novak half-hexagon which is very reminiscent to the shuffling algorithm for the Aztec diamond. The Arctic Circle Theorem for the Aztec diamond can be used to prove an Arctic Parabola Theorem for the Novak half-hexagon. There is also a determinantal point process associated to this model for which we derive the kernel.

SPEAKER: **Neil O'Connell** (University of Warwick)
TITLE: *Tropical RSK correspondence, Whittaker functions and random polymers*
ABSTRACT: The Robinson-Schensted-Knuth (RSK) correspondence is a combinatorial mapping which plays a fundamental role in the theory of Young tableaux, symmetric functions and representation theory. It is deeply connected with Schur functions and provides a combinatorial framework for understanding the Cauchy-Littlewood identity and Schur measures on integer partitions. It is also the basic structure which lies behind the solvability of a particular family of combinatorial models in probability and statistical physics including longest increasing subsequence problems, directed last passage percolation in $1 + 1$ dimensions and the totally asymmetric simple exclusion process. The RSK correspondence can be defined by expressions in the max-plus semi-ring. Replacing these expressions by their analogues in the usual algebra, A.N. Kirillov (2000) introduced a geometric lifting of the RSK correspondence which he called the 'tropical RSK correspondence'. We establish a fundamental connection between the tropical RSK correspondence and $GL(N, R)$-Whittaker functions, analogous to the connection between the RSK correspondence and Schur functions. This gives rise to a natural family of measures associated with $GL(N, R)$-Whittaker functions which are the analogues in this setting of the Schur measures on integer partitions. The corresponding analogue of the Cauchy-Littlewood identity can be seen as a generalisation of an integral identity for $GL(N, R)$-Whittaker functions which was conjectured by Bump (1989) and proved by Stade (2002). As an application, we obtain an explicit integral formula for the Laplace transform of the law of the partition function associated with a one-dimensional directed polymer model with log-gamma weights introduced by Timo Seppäläinen. This is based on joint work with Ivan Corwin, Timo Seppäläinen and Nikos Zygouras. I will also briefly describe some more recent work with Timo Seppäläinen and Nikos Zygouras which clarifies the

connection to Whittaker functions, yields a new proof of Stade's identity and extends naturally to the setting of random polymers with symmetry constraints on the weights.

SPEAKER: **Janosch Ortmann** (University of Toronto)
TITLE: *Product-form invariant measures for Brownian motion with drift satisfying a skew-symmetry type condition*
ABSTRACT: Motivated by recent developments on positive-temperature polymer models we propose a generalisation of reflected Brownian motion (RBM) in a polyhedral domain. Our process is obtained by replacing the singular drift on the boundary by a continuous one which depends, via a potential U, on the position of the process relative to the domain. We show that our generalised process has an invariant measure in product form, under a certain skew-symmetry condition that is independent of the choice of potential. Applications include TASEP-like particle systems, generalisations of Brownian motion with rank-dependent drift and diffusions connected to the generalised Pitman transform.

SPEAKER: **Eric Rains** (Caltech)
TITLE: *Semiclassical biorthogonal functions, elliptic Painlevé, and discrete isomonodromy deformations*
ABSTRACT: The appearance of Painlevé transcendents (and generalizations) in the asymptotics of growth models is closely connected to their interpretations as parametrizing isomonodromic deformations of differential equations. In addition to the various hierarchies (Garnier, etc.), the Painlevé equations also generalize to various discrete equations, with the most general being the elliptic Painlevé equation of Sakai. Sakai's construction was motivated by the singularity confinement approach, but I'll discuss how a generalization of Magnus' approach to Painlevé via semiclassical orthogonal polynomials leads to an isomonodromy interpretation of Sakai's construction (and generalizations).

SPEAKER: **Brian Rider** (University of Colorado & Temple University)
TITLE: *Spiking the random matrix hard edge*
ABSTRACT: In the present company, the BBP phase transition needs no introduction. Nor does the subsequent extension of the phenomena to the general β ensembles via the "random operator" viewpoint by Bloemendal-Virág (BV). I will show one can perturb the (again general β) hard edge to produce limit laws which contain the BBP-BV family (after a further limiting procedure). This is joint work with José Ramírez. I will also take the opportunity to advertise Painlevé formulas obtained by Igor Rumanov for these perturbed hard edge laws, at $\beta = 2, 4$.

SPEAKER: **Dan Romik** (UC Davis)
TITLE: *The Plancherel-TASEP and limit shapes of bumping routes in the RSK algorithm*
ABSTRACT: In a joint paper with Piotr Sniady from last year, we found a connection between the probabilistic notion of a second class particle in the exclusion process and "jeu de taquin", a combinatorial operation on Young tableaux that plays an important role in algebraic combinatorics and the representation theory of the symmetric group. We used the interplay between these two points of view to prove a theorem on the limiting speed of a second class particle in a variant of the TASEP governed by Plancherel measure dynamics. I will briefly recount some of these ideas and then focus on some new results from a recent follow-up work (joint with Eric Brattain and Piotr Sniady), in which we derive, as a corollary from the results of the Sniady-Romik paper, a limit shape theorem for the so-called "bumping routes" in the RSK algorithm applied to a random permutation.

SPEAKER: **Tomohiro Sasamoto** (Chiba University)

TITLE: *On the replica analysis of the KPZ equation and q-TASEP*
ABSTRACT: We discuss a few aspects of the replica analysis of the KPZ eqatuion and its discrete analogues. Starting from the basics of the replica method, we explain how one can utilize it for the KPZ equation, where are the tricky parts, and how they are regulazied in discrete models like q-TASEP.

SPEAKER: **Gregory Schehr** (Laboratoire de Physique Theorique d'Orsay)
TITLE: *Extremes of N vicious walkers for large N: Application to the directed polymer and KPZ interfaces*
ABSTRACT: We compute the joint probability density function (jpdf) of the maximal height M and its position τ_M for N non-intersecting Brownian excursions on the unit time interval in the large N limit. For $N \to \infty$, this jpdf is peaked around $M = \sqrt{2N}$ and $\tau_M = 1/2$, while the typical fluctuations behave like $M - \sqrt{2N} \propto \sigma N^{-1/6}$ and $\tau_M - 1/2 \propto \omega N^{-1/3}$. These random variables σ and ω correspond respectively, up to a prefactor, to the maximum of the Airy_2 process minus a parabola and its position. One obtains an explicit expression of the jpdf of σ and ω in terms of the Tracy-Widom distribution for GOE and a ψ-function associated to the Hastings-McLeod solution of the Painlevé-II equation. We discuss the applications of this result to the directed polymer and to interfaces in the Kardar-Parisi-Zhang (KPZ) universality class. Finally we show the equivalence of our formula and the one obtained by Moreno Flores, Quastel and Remenik in terms of the Airy function and an associated operator.

SPEAKER: **Gregory Shinault** (University of Wisconsin)
TITLE: *Inhomogeneous tilings of the Aztec diamond*
ABSTRACT: In this talk we look at random tilings of the Aztec diamond. The tile weights will be determined by their position within the diamond. We will see that the fluctuations of the northern boundary are described by the Tracy-Widom distribution.

SPEAKER: **Alexander Soshnikov** (UC Davis)
TITLE: *Central limit theorem type results in large random matrices with independent entries*
ABSTRACT: In the first part of the talk I will discuss CLT type results for linear statistics in Wigner and band random matrices. The second part of my talk will be devoted to results about the distribution of the outliers in the spectrum.

SPEAKER: **Herbert Spohn** (Technische Universität München)
TITLE: *Interacting diffusions and KPZ universality*
ABSTRACT: We explain a class of diffusions with one-sided interaction, the prime example being the semi-discrete DPRM. Invariant measures can be computed. The map to DPRM is special for the exponential interaction potential. At more depth we discuss Brownian motion in a Weyl chamber with oblique reflection law, which can be considered as a somewhat degenerate limit case.

SPEAKER: **Kazumasa A. Takeuchi** (University of Tokyo)
TITLE: *Experimental realization of KPZ-class interfaces: What were done and need to be done?*
ABSTRACT: We have shown, by a series of real experiments, that KPZ-class universal fluctuations do arise in nature, at least in growing interfaces of liquid-crystal turbulence. In this talk, I will summarize main experimental results and some open problems raised thereby, discussing in particular (1) to what extent existing theoretical results were underpinned by the experiments, (2) which experimental results remain to be explained theoretically, and (3) other open problems from an experimentalist's viewpoint.

SPEAKER: **Benedek Valkó** (University of Wisconsin)
TITLE: *Scaling exponents of lattice gases*
ABSTRACT: We consider a family of lattice gas models (speed change models) where the particles move randomly according to a local rule with the extra condition that there is at most one particle at any site. We study the equilibrium fluctuations at a given particle density, in particular we are interested in the scaling exponents of certain physical quantities. These exponents are predicted to be governed by the local behavior of the macroscopic flux function at the equilibrium density. We will give upper and lower bounds on these exponents, in particular we will confirm the superdiffusive behavior of the models in all the cases where this was predicted by physical arguments. This is based on joint work with Jeremy Quastel.

SPEAKER: **Pierre van Moerbeke** (Université Catholique de Louvain & Brandeis University)
TITLE: *The tacnode GUE-minor kernel*
ABSTRACT: Two different determinantal point process and their kernels can be associated in a natural way to two overlapping Aztec diamonds (double Aztec diamonds). When the size of diamond gets very large, keeping the overlap finite and fixed, the limit of these kernels, for an appropriate scaling, leads to a new kernel, which can be viewed as a coupling two GUE-minor kernels; this happens when the tacnode for the boundary of the frozen region (for the double Aztec diamonds) is macroscopically near the edge of the diamonds. This leads in particular to a finite rank deformation of GUE. This talk follows the one of Kurt Johansson and is joint work with Mark Adler, Sunil Chhita and Kurt Johansson.

SPEAKER: **Bálint Virág** (University of Toronto)
TITLE: *A random operator for the sine kernel process*
ABSTRACT: The Hilbert-Polya conjecture asks for a self-adjoint operator whose eigenvalues are the zeros of $\zeta(1/2 + ix)$; the asymptotic distribution of these zeros are conjectured to be given by the sine kernel. In this talk, I will present an operator whose zeros are distributed according to the sine kernel, and show how it can be obtained as a limit starting from random unitary matrices through the Szego recursion. Joint work with Benedek Valkó.