Banff International Research Station
for Mathematical Innovation and Discovery

STOCHASTIC ANALYSIS & STOCHASTIC PARTIAL DIFFERENTIAL EQUATIONS
(5-day workshop 12w5023)
Sunday April 1 to Friday April 6, 2012

MEALS

**Breakfast* (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
**Lunch* (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
**Dinner* (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
**Coffee Break*s: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)
***Remember to scan your meal card at the host station in the dining room for every meal**

MEETING ROOMS

All lectures will be held in the new lecture theater in the TransCanada Pipelines Pavilion (TCPL). LCD projector and blackboards are available for presentations.

SCHEDULE

Sunday

16:00	Check-in begins (Front Desk—Professional Development Centre—open 24 hours)
17:30–19:30	Buffet *Dinner*, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall (if desired)
	Beverages and a small assortment of snacks are available on a cash honor system

Monday

7:00–8:45	*Breakfast*
8:45–9:00	Introduction and Welcome by BIRS Station Manager, TCPL
9:00–10:00	Carl Mueller
10:05–10:25	*Coffee Break*, TCPL
10:25–10:55	Wenbo Li
11:00–11:30	Daniel Conus
11:35–12:55	*Lunch*
13:00–14:00	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:00	**Group Photo; meet in foyer of TCPL**
	(you might need warm clothing, as the photograph will be taken outdoors)
14:05–14:35	Sandra Cerrai
14:40–15:10	Breakout problem sessions
15:15–15:30	*Coffee Break*, TCPL
15:30–16:00	Raluca Balan
16:05–16:35	John Walsh
16:40–17:30	Informal problem sessions
17:30–19:30	*Dinner*

Tuesday

7:00–9:00	*Breakfast*
9:00–10:00	Michael Röckner
10:05–10:25	*Coffee Break*, TCPL
10:30–11:00	Yaozhong Hu
11:05–11:35	Le Chen
11:40–13:30	*Lunch*
13:30–14:00	Leonid Mytnik
14:05–14:35	Martina Zähle
14:40–15:10	*Coffee Break*, TCPL
15:10–15:50	Francesco Russo
15:55–16:25	Martina Hoffmanová
16:30–17:00	Xia Chen
17:05–17:30	Informal problem sessions
17:30–19:30	*Dinner*

Wednesday

7:00–9:00	*Breakfast*
9:00–10:00	David Nualart
10:05–10:25	Coffee Break, TCPL
10:25–10:55	Frederi Viens
11:00–11:30	Samy Tindel
11:35–13:30	*Lunch*
Free afternoon	
17:30–19:30	*Dinner*

Thursday

7:00–9:00	*Breakfast*
9:00–10:00	Robert Dalang
10:05–10:25	Coffee Break, TCPL
10:25–10:55	Martin Grothaus
11:00–11:30	Leif Döring
11:35–13:30	*Lunch*
13:30–14:00	Peter Imkeller
14:05–14:35	Jan van Neerven
14:40–15:20	*Coffee Break*, TCPL
15:20–15:50	Lluís Quer–Sardanyons
15:55–16:25	Tusheng Zhang
16:30–17:00	Annie Millet
17:05–17:30	Informal problem sessions
17:30–19:30	*Dinner*

Friday

7:00–9:00	*Breakfast*
9:00–10:00	Informal problem sessions
10:00–10:15	Coffee Break, TCPL
10:15–11:30	Informal problem sessions
11:30–13:30	*Lunch*

Checkout by 12 noon.

** Participants are welcome to use BIRS facilities (BIRS Coffee Lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

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ABSTRACTS

Speaker: **Balan, Raluca** (University of Ottawa, *Canada*)
Title: *Recent advances related to some linear SPDEs with fractional noise*
Abstract: In a seminal article in 1944, Itô introduced the stochastic integral with respect to the Brownian motion, which turned out to be one of the most fruitful ideas in mathematics in the 20th century. This lead to the development of stochastic analysis, a field which includes the study of stochastic partial differential equations (SPDEs). One of the approaches for the study of SPDEs was initiated by Walsh (1986) and relies on the concept of random-field solution. This concept allows us to investigate the dynamical changes in the probabilistic behavior of the solution, simultaneously in time and space. In this talk, we examine the question of existence of a random-field solution for a parabolic equation of the form $u_t = \mathcal{L}u + \dot{W}$ and a hyperbolic equation of the form $u_{tt} = \mathcal{L}u + \dot{W}$, where $\mathcal{L}$ is the generator of a d-dimensional Lévy process $X = (X_t)_{t\geq 0}$. The noise $\dot{W}$ perturbing this equation has the spatial homogeneous structure introduced by Dalang and Frangos (1998), a structure which lead to many interesting investigations in the case when the noise is white in time. Motivated by the recent interest in random fractals, in the present talk, we assume in addition that the noise has the temporal covariance structure of the fractional Brownian motion of index $H > 1/2$. In the case of the parabolic equation, we show that the question of existence of a random-field solution is closely related to the potential theory of the Lévy process X (when viewed as a Markov process) and also to the question of existence of a weighted intersection local time of two independent copies of X. Our study was motivated by a recent article of Foondun and Khoshnevisan (2011) in which a similar problem was examined in the case of temporal white noise.

Speaker: **Chen, Le** (École Polytechnique Fédérale de Lausanne, *Switzerland*)
Title: *Intermittency and exponential growth indices for some parabolic and hyperbolic Anderson models*
Abstract: This talk will focus mainly on the parabolic Anderson model

$$\frac{\partial}{\partial t}u(t,x) = \frac{\nu}{2}\frac{\partial^2}{\partial x^2}u(t,x) + \lambda\, u(t,x)\dot{W}(t,x), \quad t>0, x\in\mathbf{R}, \lambda\in\mathbf{R}, \nu>0\ .$$

I will show that this equation has a random field solution for general initial data (certain Radon measures), and moreover its second moment has an explicit form. This moment formula allows us to derive a sharp lower bound to the exponential growth indices proposed by Conus and Khoshnevisan (2010), which then answers their first open problem. If enough time remains, then I will briefly highlight some related results for the stochastic wave equation

$$\frac{\partial^2}{\partial t^2}u(t,x) = \kappa\frac{\partial^2}{\partial x^2}u(t,x) + \lambda\, u(t,x)\dot{W}(t,x), \quad t>0, x\in\mathbf{R}, \lambda\in\mathbf{R}, \kappa>0\ .$$

The full intermittency for the three-dimensional stochastic wave equation is proved by Dalang and Meuller (2009). For the one-dimensional case, thanks to another explicit formula of the second moment, we can again derive the full intermittency for this model.

Speaker: **Chen, Xia** (University of Tennessee, *United States*)
Title: *Quenched asymptotics for Brownian motion in generalized Gaussian potential*
Abstract: Recall that the notion of generalized function is introduced for the functions that cannot be defined pointwise, and is given as a linear functional over test functions. A similar idea applies to random fields. In this talk, we study the long-term asymptotic behavior of the quenched exponential moment

$$\mathrm{E}_0 \exp\left\{\int_0^t V(B_s)ds\right\},$$

where $\{B(s);\ s \geq 0\}$ denotes a d-dimensional Brownian motion and $V(\cdot)$ is a generalized Gaussian random field. We present: (i) The solution to an open problem of Carmona and Molchanov, with an answer that is different from what was conjectured; and (ii) The quenched laws for Brownian motions in Newtonian-type potentials, and in potentials driven by white noise or by fractional white noise.

Speaker: **Conus, Daniel** (Lehigh University, *United States*)
Title: *Chaotic character and intermittent islands for the stochastic heat equation*
Abstract: We study the supremum of the solution to a family of non-linear stochastic heat equations under different assumptions on the noise, the non-linearity and the initial condition. Our purpose is to show that the supremum (and, hence the solution to the equation) exhibits strongly different behavior for different initial conditions and non-linearities, thereby illustrating the *chaotic* behavior of the equation. Quantitative estimates are given which will be of particular interest in the case of the parabolic Anderson model via its connection to the KPZ equation. Inspired by the techniques used, we are also able to give some information on the size of the intermittent islands for the solution. Time permitting, we will say a few words on the techniques behind the results. This is based on joint works with M. Joseph (Utah), D. Khoshnevisan (Utah) and S.-Y. Shiu (Academica Sinica).

Speaker: **Dalang, Robert C.** (École Polytechnique Fédérale de Lausanne, *Switzerland*)
Title: *Hitting probabilities for systems of non-linear stochastic heat equations in spatial dimension $k \geq 1$*
Abstract: We consider a system of d non-linear stochastic heat equations in spatial dimension $k \geq 1$. The d-dimensional driving noise is white in time and with a spatially homogeneous covariance defined as a Riesz kernel with exponent β, where $0 < \beta < (2 \wedge k)$. The non-linearities appear both as additive drift terms and as multipliers of the noise. Using techniques of Malliavin calculus, we establish an upper bound on the two-point density of the $\mathbf{R}^{2d}$-valued random vector $(u(s, y), u(t, x))$, that, in particular, quantifies how this density degenerates as $(s, y) \to (t, x)$. From this result, we deduce a lower bound on hitting probabilities of the process $\{u(t\,, x)\}_{t\in\mathbf{R}_+, x\in\mathbf{R}^k}$, in terms of Newtonian capacity. We also establish an upper bound on hitting probabilities of the process in terms of Hausdorff measure. These estimates make it possible to show that points are polar when $d > \frac{4+2k}{2-\beta}$ and are not polar when $d < \frac{4+2k}{2-\beta}$ (if $\frac{4+2k}{2-\beta}$ is an integer, then the case $d = \frac{4+2k}{2-\beta}$ is open). We also show that the Hausdorff dimension of the range of the process is $\frac{4+2k}{2-\beta} \wedge d$ a.s. This is joint work with D. Khoshnevisan and E. Nualart.

Speaker: **Döring, Leif** (Université de Paris, *France*)
Title: *Self-similar Markov processes and generalized voter processes*
Abstract: In a seminal article, John Lamperti has established in 1972 a characterization of all positive self-similar Markov processes as time-changes of exponentials of Lévy Processes. This representation breaks down for self-similar Markov processes that hit zero or are started from zero. Several authors have studied the problem at zero in recent years and it is now completely solved. In the talk I will present a new approach developed via jump-type SDEs. The approach seems to be robust in the sense that it can be directly extended to real-valued self-similar Markov processes but has the challenging drawback that weak or strong uniqueness has to be established. I will link the appearing one-dimensional processes to a class of generalized voter processes found by Klenke and Mytnik that can be interpreted as weak solutions to jump-type SPDEs which arise as limits of mutually catalytic branching processes when the branching rate is sent to infinity.

Speaker: **Grothaus, Martin** (Technische Universität Kaiserlauten, *Germany*)
Title: *On a non-linear stochastic partial differential algebraic equation arising in industrial mathematics*
Abstract: An existence and uniqueness result for stochastic partial differential equations via non-time-homogeneous evolution systems is presented. The studies are motivated by a non-linear stochastic partial differential algebraic equation arising in industrial mathematics.

Speaker: **Hoffmanová, Martina** (Ecole Normale Supérieure de Cachan, *France*)
Title: *Degenerate parabolic stochastic partial differential equations*
Abstract: We study the Cauchy problem for a scalar semilinear degenerate parabolic partial differential equation with stochastic forcing. In particular, we are concerned with the well-posedness in any space dimension. We adapt the notion of kinetic solution which is well suited for degenerate parabolic problems and supplies a good technical framework to prove the comparison principle. The proof of existence is based on the vanishing viscosity method: the solution is obtained by a compactness argument as the limit of solutions of nondegenerate approximations.

Speaker: **Hu, Yaozhong** (University of Kansas, *United States*)
Title: *Feynman–Kac formula for stochastic partial differential equations*
Abstract: I will talk about joint works with David Nualart, Jian Song and Fei Lu on Feynman–Kac formulas to represent the solutions of stochastic partial differential equations driven by fractional Brownian fields. Some applications to the Hölder continuous of the solution, the smoothness of the density will also presented. Some recent work will also mentioned.

Speaker: **Peter Imkeller** (Humbolt–Universität zu Berlin, *Germany*)
Title: *A Fourier approach of pathwise integration*
Abstract: In 1961, Ciesielski established a remarkable isomorphism of spaces of Hölder continuous functions and Banach spaces of real valued sequences. This isomorphism leads to wavelet decompositions of Gaussian processes giving access for instance to a precise study of their large deviations, as shown by Baldi and Roynette. We will use Schauder representations for a pathwise approach of Young integrals, using Ciesielski's isomorphism. This talk is based on work in progress with N. Perkowski (HU Berlin).

Speaker: **Li, Wenbo** (University of Delaware, *United States*)
Title: *Small value probabilities and applications*
Abstract: Small value probabilities or small deviations study the decay probability that positive random variables behave near zero. In particular, small ball probabilities provide the asymptotic behavior of the probability measure inside a ball as the radius of the ball tends to zero. In this talk, we will provide an overview on developments related to stochastic analysis and stochastic partial differential equations, including one or two sided boundary crossing for Brownian sheets, smoothness of densities for SPDEs, and branching related processes.

Speaker: **Millet, Annie** (Université de Paris, *France*)
Title: *On the stochastic Cahn–Hilliard/Allen–Cahn equation*
Abstract: We prove existence and uniqueness of the solution to a Cahn–Hilliard/Allen–Cahn equation in dimension 1 to 3 subject to a multiplicative stochastic perturbation driven by space time white noise. This models absorption/desorption mechanisms in surface interface or on cluster interface morphology. Some regularity properties of the solution will also be described. This is joint work with D. Antonopoulou, G. Karali and Y. Nagase.

Speaker: **Mueller, Carl** (University of Rochester, *United States*)
Title: *Recent nonuniqueness for some stochastic PDE*
Abstract: The uniqueness theories for PDE and stochastic PDE (SPDE) are quite different. Recently there has been substantial progress in clarifying uniqueness questions for SPDE. Proofs use a broad range

of techniques, such as superprocesses, backward equations, and others. I will review some of these results and discuss the latest progress. First I will discuss parabolic SPDE of the following type,

$$\begin{aligned} \partial_t u &= u_{xx} + |u|^\gamma \dot{W}(t,x) \\ u(0,x) &= u_0(x) \end{aligned} \tag{1}$$

Then I will move on to more general equations which require additional techniques, such as backward stochastic equations.

Speaker: **Mytnik, Leonid** (Technion, *Israel*)
Title: *Multifractal analysis of superprocesses with stable branching in dimension one*
Abstract: It has been well-known for a long time that the super-Brownian motion with $1+\beta$-stable branching mechanism has densities at any fixed time, provided that the spatial dimension d is small enough ($d < 2/\beta$). Then, in 2003, it was shown that in the case of $\beta < 1$ there is a dichotomy for the corresponding density functions: there are either continuous if $d = 1$, or locally unbounded in dimensions $d \in (1, 2/\beta)$. Recently we determined the spectrum of singularities of the continuous densities in dimension $d = 1$.

Speaker: **van Neerven, Jan** (Delft Institute of Applied Mathematics, *Netherlands*)
Title: *Global existence for stochastic reaction diffusion equations*
Abstract: We prove global existence for stochastic reaction diffusion equations with multiplicative noise and a polynomially bounded reaction term satisfying suitable dissipativity conditions. Generalising previous work of Brzezniak-Gatarek and Cerrai, the operator governing the linear part of the equation can be an arbitrary uniformly elliptic second order elliptic operator. This is joint work with Markus Kunze.

Speaker: **Nualart, David** (University of Kansas, *United States*)
Title: *Hölder continuity for the solutions to a class of nonlinear SPDEs*
Abstract: The purpose of this talk is to present a recent joint work with Yaozhong Hu and Lu Fei on the Hölder continuity of the solution to a nonlinear stochastic partial differential equation arising from a one-dimensional superprocess. Considered the stochastic partial differential equation

$$\frac{\partial u}{\partial t}(t,x) = \frac{\partial^2 u}{\partial x^2}(t,x) - \int_{\mathbf{R}} \nabla_x \left(h(y-x)u(t,x)\right) \frac{\partial W}{\partial t}(t,dy) + \sqrt{u(t,x)}\, \frac{\partial^2 V}{\partial t \partial x},$$

where W and V are independent Brownian sheets. This equation describes the density of branching particles in a random environment W. Li, Wang, Xiong and Zhou introduced a notion of mild solution, based on a random heat kernel which coincides with the conditional transition density of a particle whose position ξ_t satisfies

$$\xi_t = \xi_0 + B_t + \int_0^t \int_{\mathbf{R}} h\left(y - \xi_u\right) W\left(du, dy\right),$$

where B is a Brownian motion independent of W. Assuming that $h \in H_2^2(\mathbf{R})$ and $u_0 \in L^2(\mathbf{R})$ is bounded, we have proved that $u(t,x)$ is Hölder continuous of order $\frac{1}{2} - \epsilon$ in x and of order $\frac{1}{4} - \epsilon$ in t, for any $\epsilon > 0$. The proof is based on the techniques of Malliavin calculus which are applied to derive moment estimates for the conditional transition density of ξ_t.

Speaker: **Quer–Sardanyons, Lluís** (Universitat Autònoma de Barcelona, *Spain*)
Title: *The Stratonovich heat equation: a continuity result and weak approximations*
Abstract: We consider a Stratonovich heat equation in $(0,1)$ with a nonlinear multiplicative noise, where this is a trace-class Wiener process and the diffusion coefficient is assumed to be sufficiently smooth. First, we explain how the underlying Stratonovich integral is defined and establish the existence and uniqueness of mild solution for our equation. Secondly, convolutional rough paths techniques are used to provide an almost sure continuity result, in the space of Hölder continuous functions with values in some fractional

Sobolev space, for the solution of our Stratonovich heat equation when compared with the solution of an analogous equation driven by an absolutely continuous general noise. Finally, this continuity result has been used to prove that the solution can be approximated in law, in the above-mentioned spaces, by the solution of the corresponding equation driven by two particular families of absolutely continuous noises: one is constructed in terms of Donsker type approximations and the other one is defined using the Kac-Stroock kernels, the latter being of poissonian nature. tional transition density of ξ_t.

Speaker: **Röckner, Michael** (Universität Bielefeld, *Germany*)
Title: *Regularization of ordinary and partial differential equations by noise*
Abstract: It is well known that there are ordinary differential equations (ODE) which have no or many solutions for a given initial condition, but have a unique solution if one adds a sufficiently large noise. We shall first explain this type of "regularization by noise" on the level of the corresponding Fokker-Planck-Kolmogorov equations both for ODE in finite and infinite dimensional state spaces. Then we shall recall a concrete ODE in finite dimensions given by a merely p-integrable vector field which has a unique strong solution when perturbed by a Brownian noise. Finally, we shall present an analogous new result in infinite dimensions, which is applicable to stochastic partial differential equations.

Speaker: **Russo, Francesco** (ENSTA ParisTech, *France*)
Title: *Probabilistic representation of a generalized porous media type equation: The deterministic and stochastic cases*
Abstract: The object of this talk is a porous media type equation (PME) with monotone irregular possibly discontinuous coefficients and some stochastic perturbation.

- We will recall some recent results about the representation of (PME) via an associated non-linear diffusion describing the microscopic model associated with (PME).
- An important tool for the representation is a uniqueness lemma for partial differential equations for Fokker-Planck type equations with measurable coefficients.
- The probabilistic representation is used for approaching the solutions of (PME) using simulations of a stochastic process.
- Some comments about (PME) perturbed by multiplicative noise is provided. In this case the microscopic model is constituted by a double probabilistic representation. The basic tool is the uniqueness of a stochastic version of Fokker-Planck equation with measurable coefficients.

This is joint work with V. Barbu, M. Röckner and with N. Belaribi and F. Cuvelier.

Speaker: **Sandra Cerrai** (University of Maryland, *United States*)
Title: Small mass asymptotics for a charged particle in a magnetic field and long-time influence of small perturbations
Abstract: We consider small mass asymptotics of the motion of a charged particle in a potential combined with a magnetic field. After an appropriate regularization, a Smoluchowski–Kramers type approximation is established. This approximation allows to study long-time influence on the motion of various perturbations, deterministic and stochastic. In particular, even in the case of pure deterministic perturbations, the long-time evolution of the perturbed system can be stochastic.

Speaker: **Tindel, Samy** (Université Henri Poincaré, *France*)
Title: *Density of solutions to rough differential equations*
Abstract: This talk will focus on some ongoing works concerning estimates of densities for solutions to differential equations driven by Gaussian rough paths, with a special emphasis on fractional Brownian motion. After recalling what the main problem is, we shall give the main results we aim at and briefly sketch some elements of proof.

Speaker: **Viens, Frederi** (Purdue University, *United States*)
Title: *New Malliavin Calculus techniques in stochastic analysis and stochastic PDEs*
Abstract: The stochastic calculus of variations of Paul Malliavin has found applications in a surprisingly wide array of topics in mathematics, statistics, and other fields. We discuss a recent development in the application of this Malliavin calculus to provide quantitative estimates for random variables on Wiener space. We will see how the notion of Gaussian distribution and of covariance can be generalized by using Malliavin calculus operators, which are then used to extend classical theorems to non-Gaussian fields, including the Fernique–Sudakov comparison for expected suprema, and the Gordon–Slepian conparison of appropriately convex Gaussian functionals. Applications to the stochastic heat equation and the Sherrington–Kirkpatrick model will be given. This is joint work in progress with Ivan Nourdin and Giovanni Peccati, and is based on recent advances by these two authors, and work by the presented, in how to use the Malliavin calculus for Gaussian comparisons.

Speaker: **Walsh, John B.** (University of British Columbia, *Canada*)
Title: *The roughness and smoothness of numerical solutions to the stochastic heat equation*
Abstract: The stochastic heat equation is the heat equation driven by white noise. We consider its numerical solutions using the finite difference method. Its true solutions are Hölder continuous with parameter $(\frac{1}{2} - \epsilon)$ in the space variable, and $(\frac{1}{4} - \epsilon)$ in the time variable. We show that the numerical solutions share this property in the sense that they have non-trivial limiting quadratic variation in x and quartic variation in t. These variations are discontinuous functionals on the space of continuous functions, so it is not automatic that the limiting values exist, and not surprising that they depend on the exact numerical schemes that are used; it requires a very careful choice of scheme to get the correct limiting values. In particular, part of the folklore of the subject says that a numerical scheme with excessively long time-steps makes the solution much smoother. We make this precise by showing exactly how the length of the time-steps affects the quadratic and quartic variations. This is joint work with Yuxiang Chong.

Speaker: **Zähle, Martina** (Friedrich–Schiller–Universität Jena, *Germany*)
Title: *SPDE with fractal noise in metric measure spaces*
Abstract: Pathwise defined stochastic (pseudo) differential equations in σ-finite measure spaces are discussed. We obtain existence, uniqueness and regularity results for a broad class of parabolic problems. The hypotheses are formulated in terms of associated semigroups and regularity is measured by means of abstract potential spaces, which implies Hölder continuity in time. In particular, we consider fractional heat equations driven by fractional Brownian noise in metric measure spaces. The corresponding fractional calculus in Banach spaces is used as a tool. This is joint work with Michael Hinz and with Elena Issoglio.

Speaker: **Zhang, Tusheng** (University of Manchester, *United Kingdom*)
Title: *Existence and uniqueness of invariant measures for SPDEs with two reflecting walls*
Abstract: In this talk is concerned with stochastic partial differential equations with two reflecting walls driven by space-time white noise with non-constant diffusion coefficients under periodic boundary conditions. The existence and uniqueness of invariant measures will be presented. The strong Feller property is also discussed.