On the Stability of Swarms with Second Order Agent Dynamics

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- ✔ Agent model
- ✔ Aggregation
- ✔ Formation control
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- ✔ Simulations
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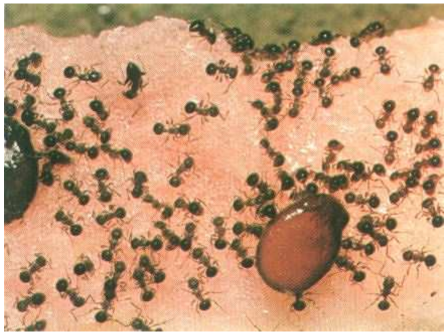
Introduction

- ✔ Individuals perform better as a group than individually
- ✔ Relatively simple individuals can collectively perform complex tasks
- ✔ Benefits: cheaper, more robust, and more flexible systems
- ✔ Engineering applications: formation ctrl, search/rescue/surveillance/military operations, demining, pollution clean-up, deep space/undersea exploration, air traffic management, etc.

➜

Can we learn from biological swarms to develop engineering multi-agent systems?

Agent Model

Consider a swarm of agents with dynamics based on Newton's law of motion

$$\dot{x}_i = v_i, \qquad \dot{v}_i = u_i \tag{1}$$

- ✔ $x_i \in \mathbb{R}^n$ is the position of agent i
- ✔ $v_i \in \mathbb{R}^n$ is its velocity
- ✔ $u_i \in \mathbb{R}^n$ is its control (force) input
- ✔ $x^\top = [x_1^\top, x_2^\top, ..., x_N^\top] \in \mathbb{R}^{Nn}$ is the vector of agent positions
- ✔ $v^\top = [v_1^\top, v_2^\top, ..., v_N^\top] \in \mathbb{R}^{Nn}$ is the vector of agent velocities
- ✔ (x, v) is the state of the swarm
- ✔ $(x_i, v_i), i = 1, ..., N$ are the states of agents
- ✔ Unity mass agents.

Assumption: The agents move simultaneously and know the relative positions of the other agents in the swarm.

- ✔ Inter-agent interactions in the swarm are represented by a potential function $J : \mathbb{R}^{Nn} \rightarrow \mathbb{R}$.
- ✔ Interactions of the agents with the environment are represented by an environment potential $\sigma : \mathbb{R}^n \rightarrow \mathbb{R}$.
- ✔ The environment potential $\sigma(\cdot)$ is sometimes called the resource profile.
- ✔ We will consider
  - ➔ Aggregation
  - ➔ Formation control
  - ➔ Social foraging

  within the same framework.
- ✔ Design the agent control inputs u_i and the potential function $J(x)$ such that desired behavior is achieved.

Aggregation

- ✔ Fundamental behavior seen in swarms in nature

Problem: Design the agent control inputs u_i such that aggregation is achieved.

- ✔ Design the potential $J(x)$ such that it is minimized by aggregation
- ✔ Use a damping term to prevent oscillations
- ✔ Choose the control input u_i of agent i in the form

$$u_i = -kv_i - \nabla_{x_i} J(x) \tag{2}$$

Aggregation Potential

Choose the potential of the form

$$J(x) = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \left[J_a\Big(\|x_i - x_j\|\Big) - J_r\Big(\|x_i - x_j\|\Big) \right] \tag{3}$$

- ✔ $J_a : \mathbb{R}^+ \rightarrow \mathbb{R}$ - attraction component, $J_r : \mathbb{R}^+ \rightarrow \mathbb{R}$ - repulsion component

Representing the gradients $\nabla_y J_a(\|y\|)$ and $\nabla_y J_r(\|y\|)$ as

$$\nabla_y J_a(\|y\|) = y g_a(\|y\|) \quad \text{and} \quad \nabla_y J_r(\|y\|) = y g_r(\|y\|)$$

and defining the attraction/repulsion function $g(\cdot)$ as

$$g(y) = -y\Big[g_a(\|y\|) - g_r(\|y\|)\Big] \tag{4}$$

the control input of agent i becomes

$$u_i = -kv_i - \sum_{j=1, j\neq i}^{N} \Big[g_a(\|x_i - x_j\|) - g_r(\|x_i - x_j\|)\Big](x_i - x_j) \tag{5}$$

- ✔ $g_a : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ - long range attraction, $g_r : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ - short range repulsion
- ✔ $g(\cdot)$ is odd and symmetric with respect to the origin

Assumption 1: The potential function $J(x)$ is such that the level sets $\Omega_c = \{x | J(x) \leq c\}$ are compact, the corresponding attraction/repulsion function $g(\cdot)$ is odd, and $g_a(\cdot)$ and $g_r(\cdot)$ are such that there exists a unique distance δ at which we have $g_a(\delta) = g_r(\delta)$. Moreover, we have $g_a(\|y\|) > g_r(\|y\|)$ for $\|y\| > \delta$ and $g_r(\|y\|) > g_a(\|y\|)$ for $\|y\| < \delta$.

- ✔ The minimum of $J_a(\|x_i - x_j\|)$ occurs on or around $\|x_i - x_j\| = 0$
- ✔ The minimum of $-J_r(\|x_i - x_j\|)$ occurs when $\|x_i - x_j\| \to \infty$
- ✔ The minimum of $J_a(\|x_i - x_j\|) - J_r(\|x_i - x_j\|)$ occurs at $\|x_i - x_j\| = \delta$ where attraction and repulsion balance

Example potential function which satisfies above is

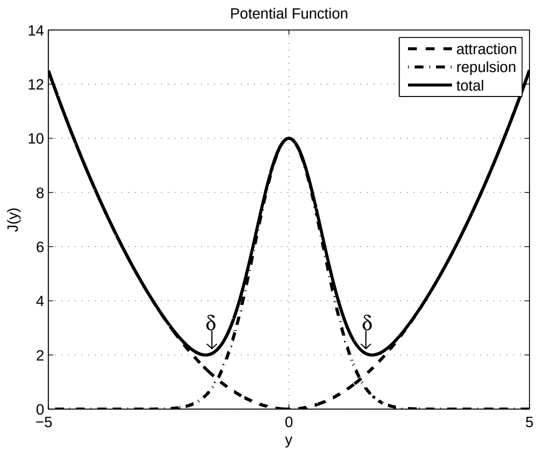
$$J(x) = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \left[\frac{a}{2} \|x_i - x_j\|^2 + \frac{bc}{2} \exp\left(-\frac{\|x_i - x_j\|^2}{c} \right) \right] \quad (6)$$

where a, b, and c, $b > a$, are parameters to be chosen by the designer.

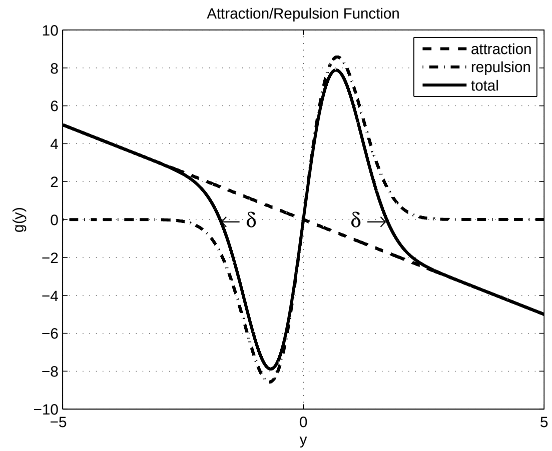
The corresponding attraction/repulsion function is

$$g(y) = -y\left[a - b\exp\left(-\frac{\|y\|^2}{c}\right)\right] \tag{7}$$

✔ Linear attraction and bounded repulsion



(a) Potential function.



(b) Attraction/repulsion function.

Figure 1: Plot of the potential functions in (6) (between two agents only) and the corresponding attraction/repulsion function in (7) for $a = 1$, $b = 10$, and $c = 1$.

Analysis of Swarm Motion

Define the centroid of the swarm as

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i, \qquad \bar{v} = \frac{1}{N}\sum_{i=1}^{N} v_i$$

Lemma 1: *The centroid $\bar{x}$ of the swarm consisting of agents with dynamics in* (1) *and with control input in* (5) *with potential function $J(x)$ which satisfies Assumption 1 converges to a stationary point x_c exponentially fast.*

- ✔ x_c depends only on the initial positions and velocities of the agents.
- ✔ It is possible to show that $x_c = \bar{x}(0) + \frac{1}{k}\bar{v}(0)$

Denote the invariant set of equilibrium (or stationary) points for the swarm with

$$\Omega_e = \{(x, v) : v_i = 0 \text{ and } \nabla_{x_i} J(x) = 0 \;\; \forall i\} \tag{8}$$

- ✔ $(x, v) \in \Omega_e$ implies that all agents are stationary.

Theorem 1: *Consider a swarm consisting of agents with dynamics in* (1) *and with control input in* (5) *with a potential function* $J(x)$ *which satisfies Assumption 1. For any* $(x(0), v(0)) \in \mathbb{R}^{2Nn}$*, as* $t \to \infty$ *we have* $(x(t), v(t)) \to \Omega_e$.

- ✔ The agents will eventually stop.
- ✔ $\nabla_{x_i} J(x) = 0$ is achieved for all agents.

☆ For flocking replace the velocity damping term with a velocity matching term

$$u_i = -k \sum_{j=1, j \neq i}^{N} (v_i - v_j) - \nabla_{x_i} J(x) \tag{9}$$

☆ For tracking a reference trajectory $\{\ddot{x}_r, \dot{x}_r, x_r\}$ use controllers

$$u_i = \ddot{x}_r - k_v(v_i - \dot{x}_r) - k_p(x_i - x_r) - \nabla_{x_i} J(x) \tag{10}$$

Swarm Cohesion Analysis

- ✔ Will the swarm be cohesive?
- ✔ Is there a bound on the swarm size?
- ✔ From Theorem 1 we know that as $t \to \infty$ we have

$$\nabla_{x_i} J(x) = \sum_{j=1, j\neq i}^{N} \Big[g_a\big(\|x_i - x_j\|\big) - g_r\big(\|x_i - x_j\|\big)\Big](x_i - x_j) = 0 \tag{11}$$

for all agents i.

- ✔ For the potential function in (6), equation (11) becomes

$$\nabla_{x_i} J(x) = \sum_{j=1, j\neq i}^{N} \left[a - b\exp\left(-\frac{\|x_i - x_j\|^2}{c}\right)\right](x_i - x_j) = 0 \tag{12}$$

✔ Using $\sum_{j=1,j\neq i}^{N}(x_i - x_j) = Ne_i$ and rearranging (12) can be written as

$$e_i = \frac{1}{aN}\sum_{j=1,j\neq i}^{N} b\exp\left(-\frac{\|x_i - x_j\|^2}{c}\right)(x_i - x_j)$$

from which one obtains

$$\|e_i\| \leq \frac{b}{a}\sqrt{\frac{c}{2}}\exp\left(-\frac{1}{2}\right)$$

Theorem 2: *Consider a swarm consisting of agents with dynamics in* (1) *and with control input in* (5) *with a potential function $J(x)$ given in* (6) *(which satisfies Assumption 1, and has linear attraction and bounded repulsion). As time progresses all agents in the swarm will converge to a hyperball*

$$B_\epsilon(\bar{x}) = \left\{x : \|x - \bar{x}\| \leq \epsilon = \frac{b}{a}\sqrt{\frac{c}{2}}\exp\left(-\frac{1}{2}\right)\right\}$$

Formation Control

Formation Control Problem: Design the agent control inputs u_i such that the swarm achieves a predefined geometrical shape.

☆ Pair specific inter-agent interactions.

✔ Choose the potential function $J(x)$ as

$$J(x) = J_{formation}(x) = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \left[J_{ija}\Big(\|x_i - x_j\| \Big) - J_{ijr}\Big(\|x_i - x_j\| \Big) \right] \tag{13}$$

✔ The corresponding control input in (5) is given by

$$u_i \quad = \quad -kv_i - \sum_{j=1, j\neq i}^{N} \left[g_{ija}\Big(\|x_i - x_j\| \Big) - g_{ijr}\Big(\|x_i - x_j\| \Big) \right] (x_i - x_j) \tag{14}$$

✔ $J(x)$ satisfies Assumption 1.

✔ For all (i, j) the pair dependent

$$g_{ij}(y) = -y\Big[g_{ija}(y) - g_{ijr}(y)\Big] \tag{15}$$

are such that attraction and repulsion balance at pair dependent equilibrium distances δ_{ij}.

✔ In the formation control framework the potential function in (6) becomes

$$J(x) = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \left[\frac{a_{ij}}{2}\|x_i - x_j\|^2 + \frac{b_{ij}c_{ij}}{2} \exp\left(-\frac{\|x_i - x_j\|^2}{c_{ij}}\right)\right] \tag{16}$$

where a_{ij}, b_{ij}, and c_{ij}, $b_{ij} > a_{ij}$ are pair dependent.

✔ The control input for agent i is of the form

$$u_i = -kv_i - \sum_{j=1, j\neq i}^{N} \left[a_{ij} - b_{ij} \exp\left(-\frac{\|x_i - x_j\|^2}{c_{ij}}\right)\right](x_i - x_j) \tag{17}$$

- ✔ The desired formation is specified a priori by formation constraints of the form

$$\|x_i - x_j\| = d_{ij}$$

  for all $(i,j), j \neq i$.

- ✔ Choose $J(x)$ (the corresponding $g_{ij}(\cdot)$) such that $\delta_{ij} = d_{ij}$ for all (i,j).
- ✔ For $J(x)$ in (16) one can choose $b_{ij} = b$, $c_{ij} = c$ for all (i,j) and for some constants b and c and calculate a_{ij} as $a_{ij} = b\exp\left(-\frac{d_{ij}^2}{c}\right)$.

Corollary 1: *Consider a swarm consisting of agents with dynamics in* (1) *and with control input in* (14) *with potential function $J(x)$ which satisfies Assumption 1 and has pair dependent interactions. Assume that the pair dependent inter-agent attraction/repulsion functions $g_{ij}(\cdot)$ are chosen such that the distances δ_{ij} (at which the inter-agent attractions and repulsions between pairs (i,j) balance) satisfy $\delta_{ij} = d_{ij}$, where d_{ij} are the desired formation distances. Then, the equilibrium at the desired formation is locally asymptotically stable.*

Social Foraging

Design agent controllers u_i such that the swarm moves towards favorable regions and avoids favorable regions of the environment while preserving cohesiveness.

- ✔ The environment is represented by a resource profile $\sigma : \mathbb{R}^n \rightarrow \mathbb{R}$.
- ✔ The overall potential $\bar{J}(x)$ becomes

$$\bar{J}(x) = \sum_{i=1}^{N} \sigma(x_i) + J(x) \tag{18}$$

where $J(x)$ is the aggregation potential.

The corresponding controller is given by

$$u_i = -kv_i - \nabla_{x_i}\sigma(x_i) - \sum_{j=1, j\neq i}^{N} \left[g_a\Big(\|x_i - x_j\|\Big) - g_r\Big(\|x_i - x_j\|\Big) \right](x_i - x_j) \tag{19}$$

- ✔ The term $-\nabla_{x_i}\sigma(x_i)$ guides agent i towards minima and away from maxima of the resource profile $\sigma(x_i)$.

Theorem 3: *Consider a foraging swarm consisting of agents with dynamics in* (1) *and with control input in* (19) *with inter-agent interaction potential function* $J(x)$ *which satisfies Assumption 1. Assume that the resource profile* $\sigma(\cdot)$ *of the environment is one of the following*

- *A valley-type quadratic profile (i.e., the profile in* (22) *below with* $A_\sigma > 0$*), or*
- *A valley-type Gaussian profile (i.e., the profile in* (23) *below with* $A_\sigma > 0$*)*

Then, as $t \to \infty$ *we have* $(x(t), v(t)) \to \Omega_e$*.*

✔ The motion of the centroid is given by

$$\dot{\bar{x}} = \bar{v}, \qquad \dot{\bar{v}} = -k\bar{v} - \frac{1}{N}\sum_{i=1}^{N} \nabla_{x_i}\sigma(x_i) \tag{20}$$

✔ The swarm centroid is “guided” by the average of the gradient of the resource profile evaluated at the agent locations.

Plane Resource Profile

- ✔ Plane resource profile described by an equation of the form

$$\sigma(y) = a_\sigma^\top y + b_\sigma \tag{21}$$

  where $a_\sigma \in \mathbb{R}^n$ and $b_\sigma \in \mathbb{R}$.

- ✔ Its gradient at a point $y \in \mathbb{R}^n$ is given by: $\nabla_y \sigma(y) = a_\sigma$

- ✔ The motion of the centroid of the swarm is described by

$$\dot{\bar{x}} = \bar{v}, \qquad \dot{\bar{v}} = -k\bar{v} - a_\sigma$$

Lemma 2: *Consider a foraging swarm consisting of agents with dynamics in* (1) *and with control input in* (19) *with inter-agent interaction potential function* $J(x)$ *which satisfies Assumption 1. Assume that the resource profile* $\sigma(\cdot)$ *of the environment is given by* (21). *Then, as* $t \to \infty$ *the centroid of the swarm moves along the negative gradient of the profile towards infinity with velocity* $\bar{v}(t) = -\frac{1}{k} a_\sigma$.

✔ To analyze the relative dynamics define $z_i = x_i - \bar{x}$ and $\zeta_i = v_i - \bar{v}$.

✔ The relative agent dynamics can be expressed as

$$\dot{z}_i = \zeta_i, \qquad \dot{\zeta}_i = -k\zeta_i - \sum_{j=1, j\neq i}^{N} \left[a - b \exp\left(-\frac{\|z_i - z_j\|^2}{c} \right) \right] (z_i - z_j)$$

which is exactly the equation for aggregation.

Corollary 2: *Consider a swarm consisting of agents with dynamics in* (1) *and with control input in* (19) *with inter-agent interaction potential* $J(x)$ *given in* (6) *(which satisfies Assumption 1 with linear attraction and bounded repulsion). Assume that the resource profile* $\sigma(\cdot)$ *of the environment is given by* (21)*. As time progresses all agents in the swarm will converge to a hyperball*

$$B_\epsilon(\bar{x}) = \left\{ x : \|x - \bar{x}\| \leq \epsilon = \frac{b}{a}\sqrt{\frac{c}{2}} \exp\left(-\frac{1}{2} \right) \right\}$$

Quadratic Resource Profile

- ✔ Quadratic resource profile described by an equation of the form

$$\sigma(y) = \frac{A_\sigma}{2}\|y - c_\sigma\|^2 + b_\sigma \tag{22}$$

where $A_\sigma \in \mathbb{R}$, $b_\sigma \in \mathbb{R}$, and $c_\sigma \in \mathbb{R}^n$.

- ✔ It has a global extremum at $y = c_\sigma$.
- ✔ Its gradient at a point $y \in \mathbb{R}^n$ is given by: $\nabla_y \sigma(y) = A_\sigma(y - c_\sigma)$
- ✔ The motion of the centroid $\bar{x}$ can be calculated as

$$\dot{\bar{x}} = \bar{v}, \qquad \dot{\bar{v}} = -k\bar{v} - A_\sigma(\bar{x} - c_\sigma)$$

- ✔ Defining $\bar{x}_c = \bar{x} - c_\sigma$ and $\bar{v}_c = \bar{v}$ these can be expressed as

$$\begin{bmatrix} \dot{\bar{x}}_c \\ \dot{\bar{v}}_c \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -A_\sigma & -k \end{bmatrix} \begin{bmatrix} \bar{x}_c \\ \bar{v}_c \end{bmatrix}$$

✔ $\bar{x} = c_\sigma$ and $\bar{v} = 0$ is an equilibrium point of this system.

✔ The eigenvalues of the system matrix are $\lambda_{1,2} = \frac{-k \pm \sqrt{k^2 - 4A_\sigma}}{2}$.

Lemma 3: *Consider a foraging swarm consisting of agents with dynamics in* (1) *and with control input in* (19) *with inter-agent interaction potential $J(x)$ which satisfies Assumption 1. Assume that the resource profile $\sigma(\cdot)$ of the environment is given by* (22)*. As $t \to \infty$ we have*

- *If $A_\sigma > 0$, then $\bar{x}(t) \to c_\sigma$ (i.e., the centroid of the swarm converges to the global minimum c_σ of the profile), or*
- *If $A_\sigma < 0$ and $\bar{x}(0) \neq c_\sigma$, then $\bar{x}(t) \to \infty$ (i.e., the centroid of the swarm diverges from the global maximum c_σ of the profile).*

✔ For $A_\sigma > 0$, the sign of $k^2 - 4A_\sigma$ will determine the characteristics of the centroid motion.

➔ For $0 < A_\sigma < \frac{k^2}{4}$ its motion will be overdamped

- For $A_\sigma > \frac{k^2}{4}$ its motion will be underdamped.

Theorem 4: *Consider a foraging swarm consisting of agents with dynamics in* (1) *and with control input in* (19) *with inter-agent interaction potential $J(x)$ in* (6) *(which satisfies Assumption 1 and has linear attraction and bounded repulsion). Assume that the resource profile $\sigma(\cdot)$ of the environment is given by* (22) *with $A_\sigma > 0$. Then, as $t \to \infty$ all agents $i = 1, \ldots, N$, will enter*

$$B_\epsilon(c_\sigma) = \left\{ x : \|x - c_\sigma\| \leq \epsilon = \frac{b(N-1)}{aN + A_\sigma} \sqrt{\frac{c}{2}} \exp\left(-\frac{1}{2}\right) \right\}$$

- ✔ The swarm will aggregate around the minimum of the resource profile.

Gaussian Resource Profile

- ✔ Gaussian resource profile described by an equation of the form

$$\sigma(y) = -\frac{A_\sigma}{2}\exp\left(-\frac{\|y-c_\sigma\|^2}{l_\sigma}\right) + b_\sigma \tag{23}$$

where $A_\sigma \in \mathbb{R}$, $b_\sigma \in \mathbb{R}$, $l_\sigma \in \mathbb{R}^+$, and $c_\sigma \in \mathbb{R}^n$.

- ✔ Its gradient at $y \in \mathbb{R}^n$ is: $\nabla_y \sigma(y) = \frac{A_\sigma}{l_\sigma}(y - c_\sigma)\exp\left(-\frac{\|y-c_\sigma\|^2}{l_\sigma}\right)$
- ✔ For all $y \in \mathbb{R}^n$ it is bounded and satisfies

$$\|\nabla_y \sigma(y)\| \leq \bar{\sigma} = \frac{|A_\sigma|}{\sqrt{2 l_\sigma}}\exp\left(-\frac{1}{2}\right) \tag{24}$$

- ✔ The dynamics of the centroid can be obtained as

$$\dot{\bar{x}} = \bar{v}, \qquad \dot{\bar{v}}(t) = -k\bar{v} - \frac{A_\sigma}{N l_\sigma}\sum_{i=1}^{N}(x_i - c_\sigma)\exp\left(-\frac{\|x_i - c_\sigma\|^2}{l_\sigma}\right)$$

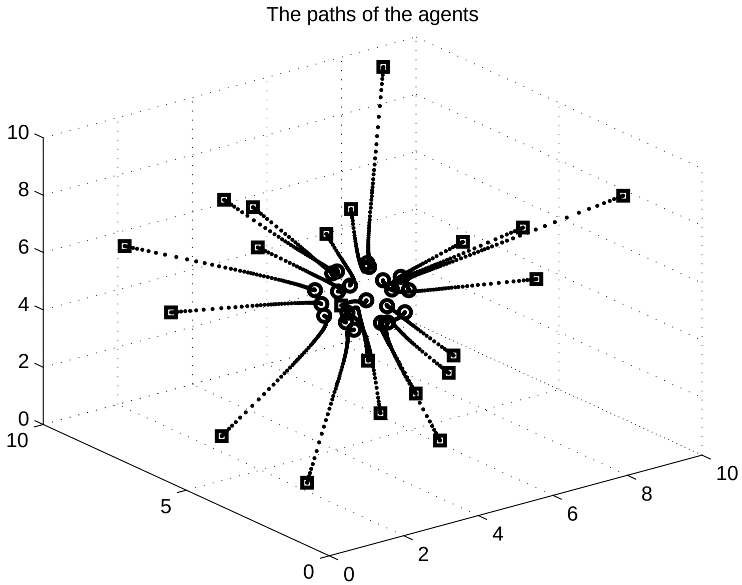
Lemma 4: *Consider a foraging swarm consisting of agents with dynamics in* (1) *and with control input in* (19) *with inter-agent interaction potential* $J(x)$ *which satisfies Assumption 1. Assume that the resource profile* $\sigma(\cdot)$ *of the environment is given by* (23) *with* $A_\sigma > 0$*. Then, as* $t \to \infty$ *we have* $c_\sigma \in conv\{x_1, ..., x_N\}$ *where* $conv$ *stands for the convex hull (i.e., the agents encircle the minimum point of the profile* c_σ*).*

Theorem 5: *Consider a foraging swarm consisting of agents with dynamics in* (1) *and with control input in* (19) *with inter-agent interaction potential* $J(x)$ *in* (6) *(which satisfies Assumption 1 and has linear attraction and bounded repulsion). Assume that the resource profile* $\sigma(\cdot)$ *of the environment is given by* (23) *with* $A_\sigma > 0$ *(and whose gradient is bounded by* $\bar{\sigma}$ *in* (24)*). Then, as* $t \to \infty$ *we have* $x_i(t) \to B_\epsilon(\bar{x}(t))$ *for all* i*, where*

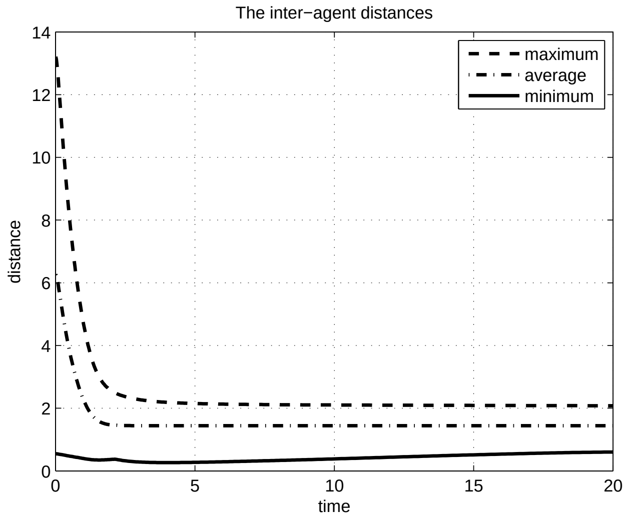
$$B_\epsilon(\bar{x}(t)) = \left\{ y(t) : \|y(t) - \bar{x}(t)\| \leq \epsilon = \frac{\bar{\sigma}}{Na} + \frac{b}{a}\sqrt{\frac{c}{2}} \exp\left(-\frac{1}{2}\right) \right\}$$

Simulation Examples

Aggregation



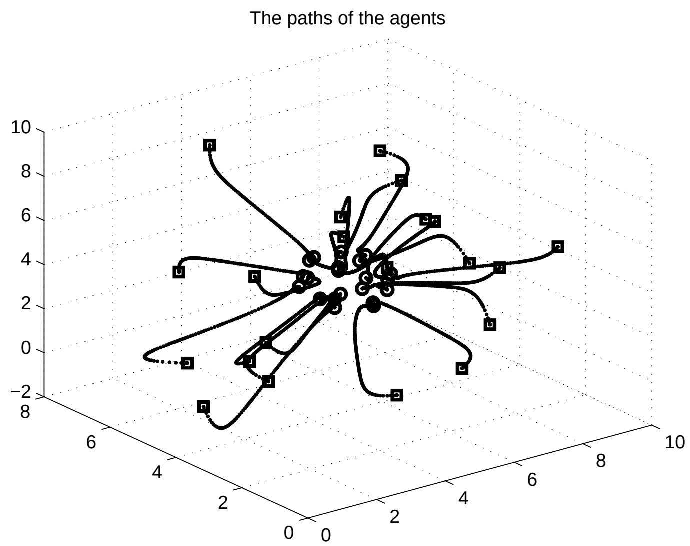
(a) Agent paths.



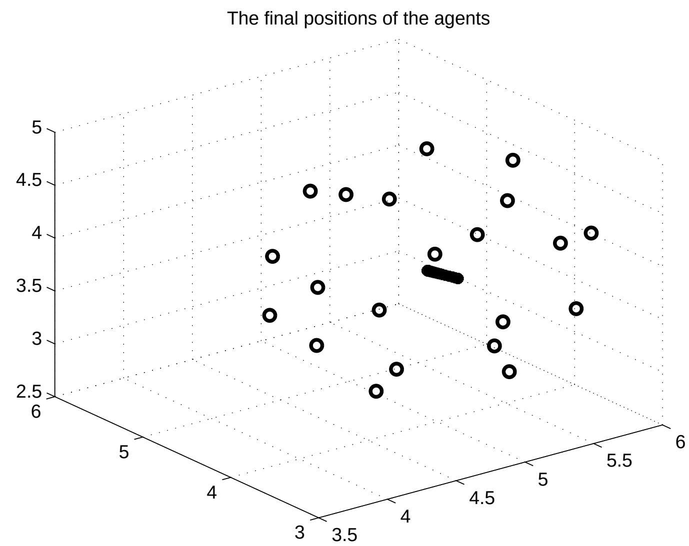
(b) Min, max, and average distances.

Figure 2: Aggregating swarm (zero initial velocities).

Aggregation



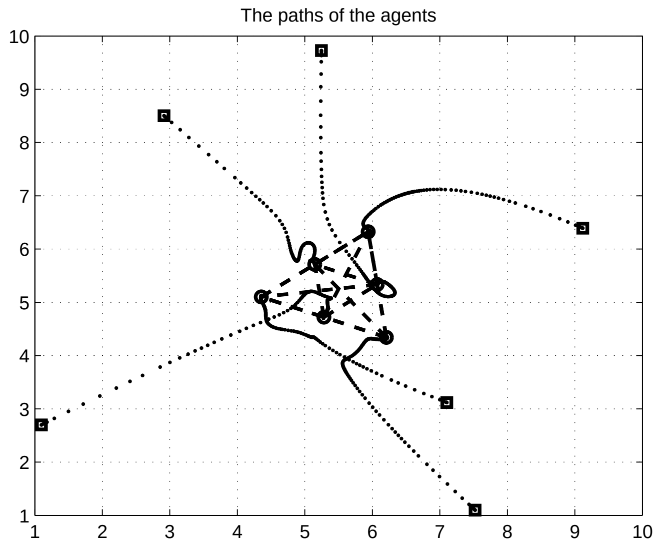
(a) Agent paths.



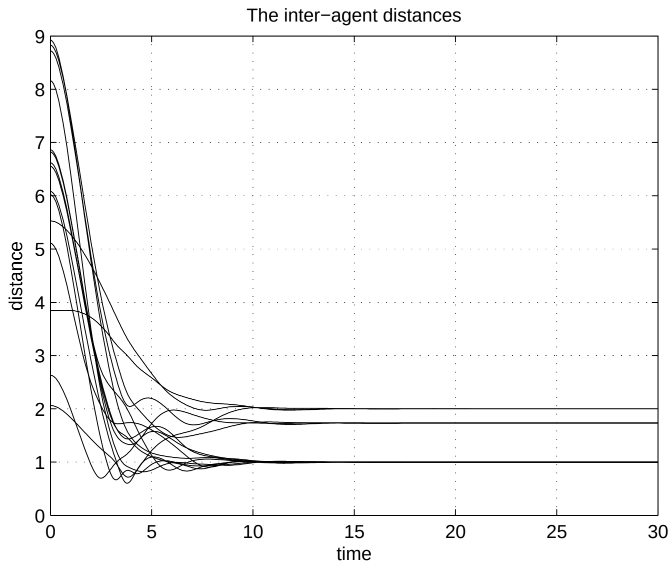
(b) Final agent arrangement.

Figure 3: Aggregating swarm (random initial velocities).

Formation Control



(a) Agent paths.

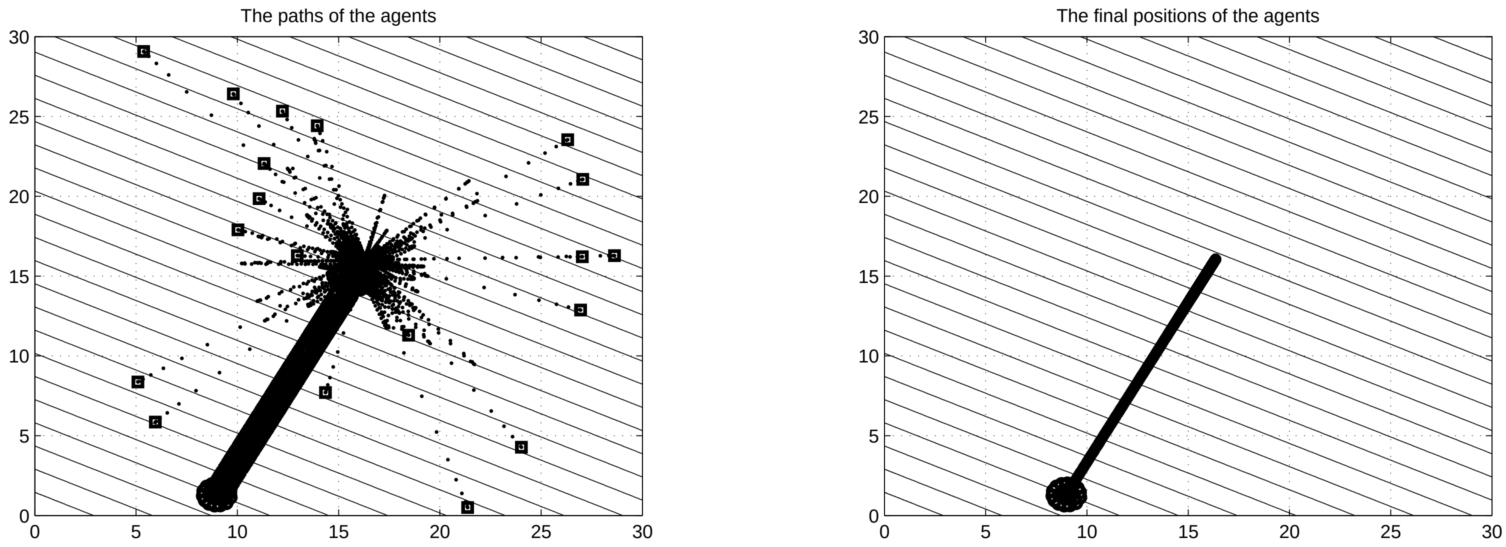


(b) Inter-agent distances.

Figure 4: Equilateral triangle formation by 6 agents.

Social Foraging

Plane Profile

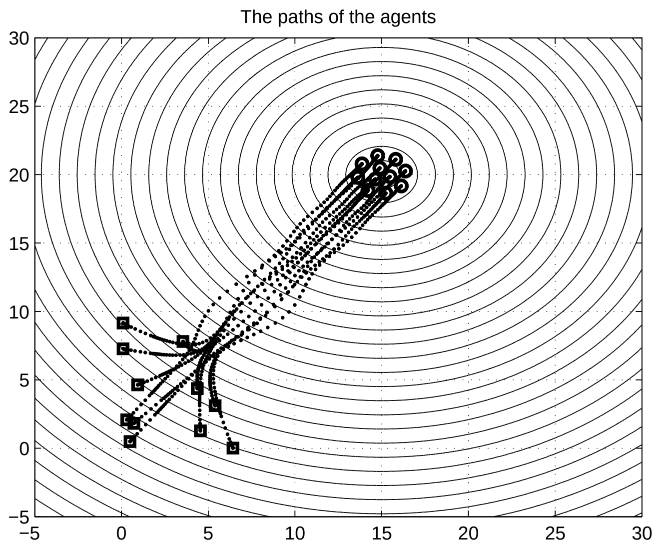


(a) Agent paths. (b) Motion of the centroid.

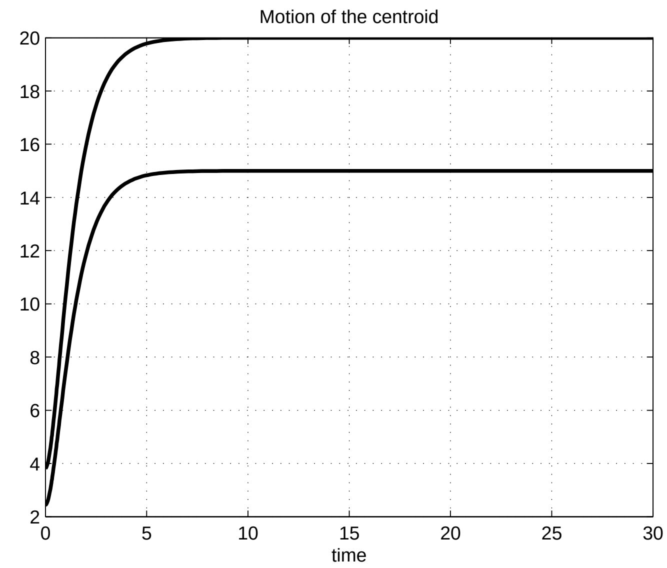
Figure 5: Swarm motion in a plane profile.

Social Foraging

Quadratic Profile



(a) Agent paths.

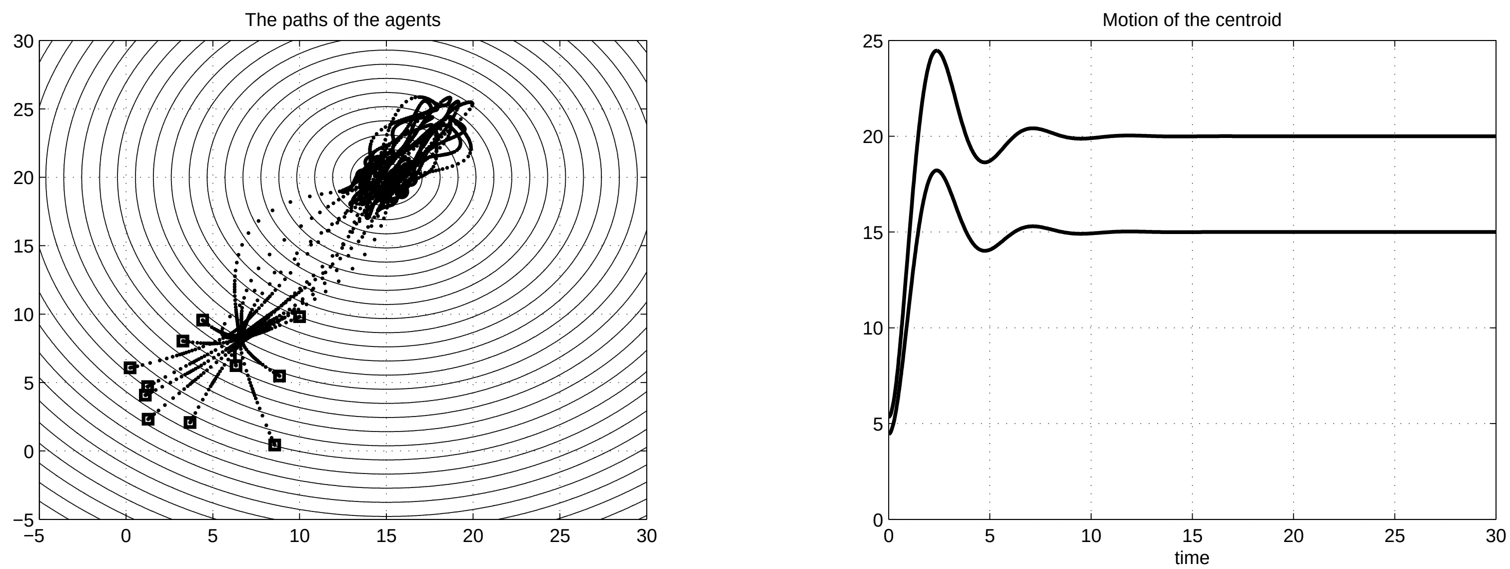


(b) Motion of the centroid.

Figure 6: Swarm motion in a quadratic profile with $k^2 - 4A_\sigma > 0$ (real eigenvalues).

Social Foraging

Quadratic Profile

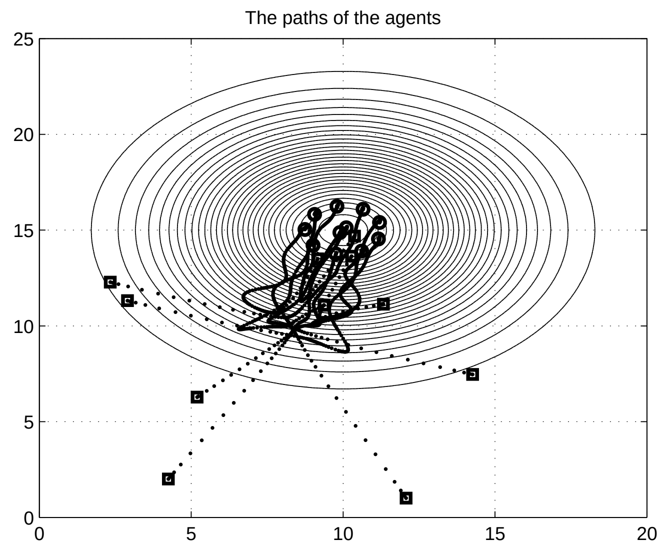


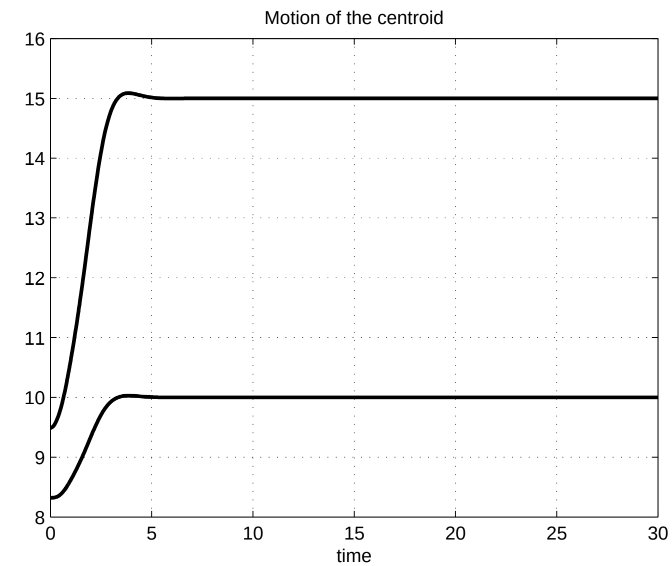
(a) Agent paths. (b) Motion of the centroid.

Figure 7: Swarm motion in a quadratic profile with $k^2 - 4A_\sigma < 0$ (complex eigenvalues).

Social Foraging

Gaussian Profile





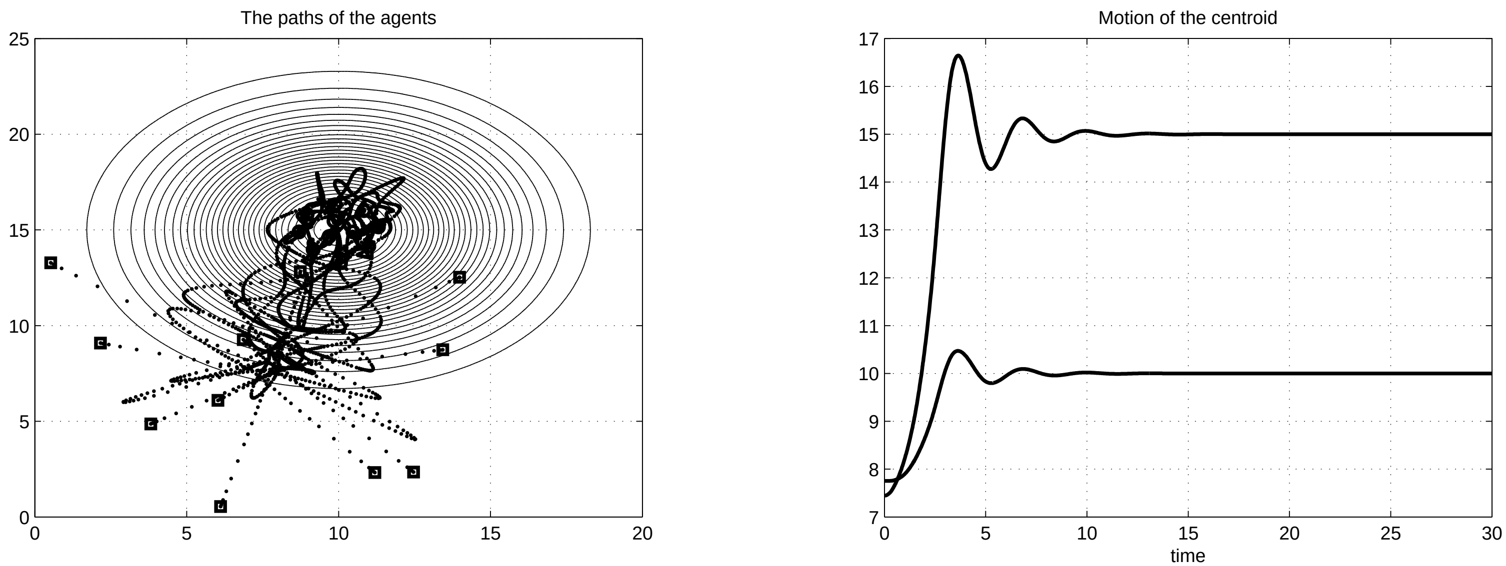
(a) Agent paths.

(b) Motion of the centroid.

Figure 8: Swarm motion in a Gaussian profile for $k = 3$.

Social Foraging

Gaussian Profile



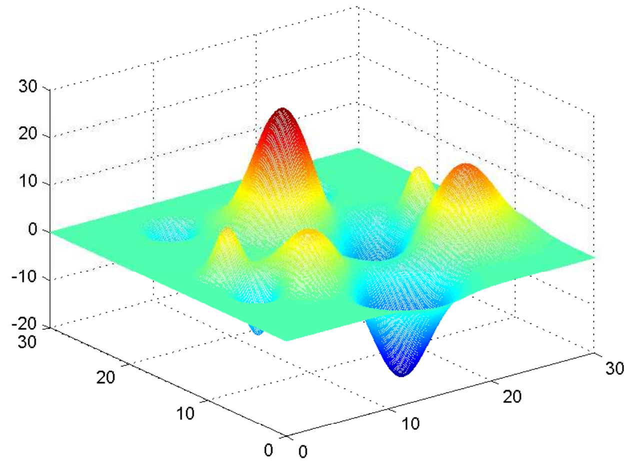
(a) Agent paths. (b) Motion of the centroid.

Figure 9: Swarm motion in a Gaussian profile for $k = 1$.

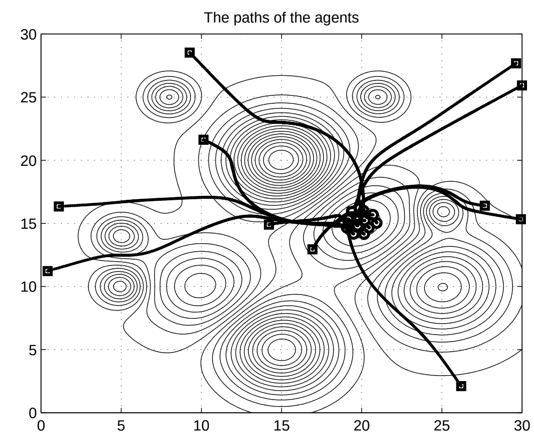
Social Foraging

Multimodal Gaussian Profile

$$\sigma(y) = -\sum_{i=1}^{M} \frac{A_{\sigma i}}{2} \exp\left(-\frac{\|y - c_{\sigma i}\|^2}{l_{\sigma i}}\right) + b_\sigma, \tag{25}$$



(a) The resource profile.



(b) Agent paths.

Figure 10: Simulation for a multimodal Gaussian profile.

Summary and Remarks

- ✔ Swarms composed of agents with double integrator dynamics
- ✔ Potential functions based approach
- ✔ Earlier results for swarms with single integrator dynamics recovered
- ✔ Aggregation
  - ➔ Stability (cohesiveness) is achieved.
  - ➔ Explicit bounds on the swarm size obtained.
- ✔ Formation control
  - ➔ Desired geometric shape is achieved locally.
- ✔ Social foraging
  - ➔ Plane, quadratic, Gaussian resource profiles.
  - ➔ Convergence to more favorable regions shown.
  - ➔ Cohesiveness preserved.