

New Trends in Noncommutative Algebra and Algebraic Geometry

October 28, 2012–November 2, 2012

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday

*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday

*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday

Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)

***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in the new lecture theater in the TransCanada Pipelines Pavilion (TCPL). LCD projector and blackboards are available for presentations.

SCHEDULE

Sunday

16:00 Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
17:30–19:30 Buffet Dinner, Sally Borden Building
20:00 Informal gathering in 2nd floor lounge, Corbett Hall
Beverages and a small assortment of snacks are available on a cash honour system.

Monday

7:00–8:45 Breakfast
8:45–9:00 Introduction and Welcome by BIRS Station Manager, TCPL
9:00–10:00 Dan Rogalski
10:00–10:30 Coffee Break, TCPL
10:30–11:30 Kenneth Chan
11:30–13:00 Lunch
13:00–14:00 Milen Yakimov
14:00 Group Photo; meet in foyer of TCPL (photograph will be taken outdoors so a jacket might be required).
15:00–15:30 Coffee Break, TCPL
15:30–16:30 David Saltman
16:30–17:30 Chelsea Walton
17:30–19:30 Dinner

Tuesday

7:00–9:00 Breakfast
9:00–10:00 Susan Sierra
10:00–10:30 Coffee Break, TCPL
10:30–11:30 Louis Rowen
11:30–13:30 Lunch
13:00–14:00 Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:00–15:00 Toby Stafford
15:00–15:30 Coffee Break, TCPL
15:30–16:30 Ken Brown
16:30–17:30 Raf Bocklandt
17:30–19:30 Dinner

Wednesday

7:00–9:00 Breakfast
9:00–10:00 Stéphane Launois
10:00–10:30 Coffee Break, TCPL
10:30–11:30 Ellen Kirkman
11:30–13:30 Lunch
Free Afternoon
17:30–19:30 Dinner

Thursday

7:00–9:00 Breakfast
9:00–10:00 Quanshui Wu
10:00–10:30 Coffee Break, TCPL
10:30–11:30 Birge Huisgen-Zimmermann
11:30–13:30 Lunch
14:00–15:00 Daniel Chan
15:00–15:30 Coffee Break, TCPL
15:30–16:30 Ken Goodearl
16:30–17:30 Rajesh Kulkarni
17:30–19:30 Dinner

Friday

7:00–9:00 Breakfast
9:00–10:00 Informal Discussions
10:00–10:30 Coffee Break, TCPL
11:30–13:30 Lunch

**Checkout by
12 noon.**

** 5-day workshop participants are welcome to use BIRS facilities (BIRS Coffee Lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

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ABSTRACTS (in alphabetic order by speaker surname)

Speaker: **Raf Bocklandt** (Newcastle University)

Title: *Noncommutative projective geometry through the looking glass*

Abstract: We will discuss how certain features of noncommutative projective geometry and Artin-Schelter regular algebras can be interpreted using recent developments in mirror symmetry for Riemann surfaces.

Speaker: **Ken Brown** (University of Glasgow)

Title: *Noncommutative unipotent groups in characteristic 0*

Abstract: Let k be algebraically closed of char 0 and let n be a positive integer. Which Hopf k -algebras are deformations of the commutative polynomial algebra in n variables over k ? I'll outline recent results around this topic, of myself, my student Steven O'Hagan and Guangbin Zhuang (student of James Zhang), and discuss some open questions.

Speaker: **Daniel Chan** (University of New South Wales)

Title: *Hopf algebra actions on projective curves*

Abstract: There has been considerable interest in studying actions of Hopf algebras on graded algebras, but it seems little has been studied concerning analogous actions on projective varieties. In this talk we reflect on some thoughts and observations regarding Hopf algebra actions on projective curves and their twisted homogeneous co-ordinate rings. This is a report on some preliminary investigations carried out with Kenneth Chan.

Speaker: **Kenneth Chan** (University of Washington)

Title: *Hopf algebra actions on filtered algebras*

Abstract: The Hopf algebra actions on a filtered algebra is closely related to the Hopf algebra actions on its associated graded algebra. We use results in the graded case, as discussed in Chelsea Walton's talk, to obtain some results in the ungraded case. In particular, we study the Hopf algebra actions on the Weyl algebra. This is joint work with Chelsea Walton, Yan Hua Wang and James Zhang

Speaker: **Birge Huisgen-Zimmerman** (University of California, Santa Barbara)

Title: *Representation-tame algebras need not be homologically tame*

Abstract: I will start with a mini-survey of pre-2000 results on the Finitistic Dimension Conjectures for finite dimensional algebras (which date back to 1960), and then sketch some of the progress made post-2000. In particular, I will discuss the representation theory and homology of biserial algebras, a class of algebras first studied in depth by Gelfand and Ponomarev in light of the fact that they encode the representation theory of the Lorentz group (they were later seen to also surface in other parts of mathematical nature). Their finite dimensional representations are completely understood, but their homology still holds surprises. I will conclude by addressing "generic finitistic dimensions", which are obtained by focusing on the projective dimensions attained on irreducible components of module varieties.

Speaker: **Ken Goodearl** (University of California, Santa Barbara)

Title: *Unique Factorization and Quantum Clusters in Quantum Algebras*

Abstract: The topics of this talk are unique factorization domains and cluster structures in the context of quantum algebras. Classical results in algebraic geometry show that many rings of functions on affine varieties are UFDs. Moreover, in the development of cluster algebras, it has been shown that many classical coordinate rings are cluster algebras, and that cluster algebras in turn are often (but not always) UFDs. On the quantum side, it has been established that many quantized coordinate rings are noncommutative UFDs, but relatively few of them have been identified as quantum cluster algebras so far. We will discuss recent progress in this connection obtained in joint work with Milen Yakimov: For a large class of quantum algebras, our results secure the existence of initial quantum clusters, made up of homogeneous irreducible elements.

Speaker: **Ellen Kirkman** (Wake Forest University)

Title: *Noncommutative Complete Intersections (joint with J. Kuzmanovich, J. Zhang)*

Abstract: Let A be a connected graded finitely generated k -algebra. Gulliksen proved that when A is commutative the following conditions are equivalent:

1. A is a complete intersection.
2. The Ext-algebra $E(A) := \bigoplus_{n=0}^{\infty} \text{Ext}_A^n(k, k)$ of A has finite Gelfand-Kirillov dimension.
3. The Ext-algebra $E(A)$ is noetherian.

We consider the relationship between these conditions when A is noncommutative, particularly for rings $A = R^G$ of invariants under the action of a finite group G of graded automorphisms of an AS-regular graded algebra R . When R is commutative, Kac-Watanabe and Gordeev proved that a necessary condition for R^G to be a complete intersection is that G is generated by bireflections; we present some examples suggesting that there is an analogous result for noncommutative AS-regular algebras.

Speaker: **Rajesh Kulkarni** (Michigan State University)

Title: *Ulrich bundles on projective varieties and representations of Clifford algebras*

Abstract: The representations of Clifford algebras of homogeneous forms are in bijection with certain vector bundles on associated hypersurfaces called the Ulrich bundles. This connection has been exploited recently in our work in studying vector bundles on cubic and quartic surfaces. One consequence is that any smooth quartic surface is a linear Pfaffian. We will also discuss current work which allows construction of low-rank stable Ulrich bundles on certain 3-folds using twisted tensor product construction in Clifford algebras.

Speaker: **Stéphane Launois** (University of Kent)

Title: *Efficient recognition of tnn cells*

Abstract: In this talk, I will explain how one can use tools developed to study the prime spectrum of quantum matrices in order to study totally nonnegative matrices.

Speaker: **Daniel Rogalski** (University of California, San Diego)

Title: *Skew Calabi-Yau algebras and Nakayama automorphisms*

Abstract: Skew (or twisted) Calabi-Yau algebras are algebras with some especially nice homological properties. In the connected graded setting, they are essentially just the Artin-Schelter regular algebras, which have a dualizing complex equal to a complex shift of the algebra, possibly twisted on one side by an automorphism (the Nakayama automorphism). We discuss some nontrivial formulas for what happens to the Nakayama automorphism when one takes a smash product with a Hopf algebra action or does a graded twist. Our methods rely on graded local cohomology and the formulas involve the notion of the homological determinant of an automorphism. We also note that in wide generality, the homological determinant of the Nakayama automorphism is equal to 1.

Speaker: **Louis Rowen** (Bar Ilan University)

Title: *Identities of algebras with involution*

Abstract: We survey the recent theory of polynomial identities of algebras with involution, focusing on Specht's problem and minimal identities for matrix algebras with involution.

Speaker: **David Saltman** (CCR, Princeton)

Title: *Tensor Products of Division Algebras and Fields*

Abstract: This is joint work with Louis Rowen. It began as an investigation of the question of whether $D_1 \otimes_F D_2$ is a domain where the D_i are division algebras and F is an algebraically closed field contained in their centers. We present an example where the answer is "no", and also study the Picard group and Brauer group properties of $F_1 \otimes_F F_2$ where the F_i are fields. Finally, as part of our example, we have results about division algebras and Brauer groups over curves. Specifically, we give a splitting criterion for certain Brauer group elements on the product of two curves over F .

Speaker: **Susan Sierra** (University of Edinburgh)

Title: *Maximal orders in the Sklyanin algebra.*

Abstract: We study maximal orders contained in the 3-Veronese of a generic Sklyanin algebra S . We show that if R is such an algebra, then R may be thought of as the blow-up of S at a (possibly non-effective) divisor on the elliptic curve E contained in $\text{Qgr } S$. We classify all such R .

As a consequence, we show that any order in the localized Sklyanin algebra has finitely many ideals.

This is joint work with Dan Rogalski and Toby Stafford.

Speaker: **Toby Stafford** (University of Manchester)

Title: *Equidimensionality of characteristic varieties over Cherednik algebras and Z -algebras.*

Abstract: A technique from noncommutative projective geometry that has proved very useful for Cherednik and related algebras is the notion of Z -algebras; these are certain rings of infinite, lower triangular matrices. The reason is that, with a careful choice of the Z -algebra B , one can obtain a well-behaved (and, in particular, smooth) commutative associated graded ring both for B and hence for the Cherednik algebra H .

In this lecture we will show how to develop a homological machinery for Z -algebras. As an application we will prove that the associated varieties of H -modules are equidimensional. This generalises Gabber's result for unitary algebras and resolves a conjecture in our earlier work.

This is all joint work with Iain Gordon.

Speaker: **Chelsea Walton** (University of Washington)

Title: *Hopf actions and Nakayama automorphisms*

Abstract: Let H be a Hopf algebra with antipode S , and let A be an N -Koszul Artin-Schelter regular algebra. We study connections between the Nakayama automorphism of A and S^2 of H when H coacts on A inner faithfully. Several applications pertaining to Hopf actions on Artin-Schelter regular algebras are given.

Speaker: **Quanshui Wu** (Fudan University)

Title: *Three dimensional Calabi-Yau algebras and their homologies*

Abstract: In this talk, I will first remind the audience of some definitions such as Hochschild (co)homology, cyclic (co)homology and Poisson (co)homology, then I will sketch a way following literature to calculate the Hochschild (co)homology and cyclic homology for 3-dimensional graded Calabi-Yau algebras by using the Poisson (co)homology of Poisson algebras.

Speaker: **Milen Yakimov** (Louisiana State University)

Title: *Rigidity of quantum tori and description of automorphism groups*

Abstract: The automorphisms of noncommutative associate algebras are often difficult to classify. Andruskiewitsch-Dumas and Launois-Lenagan made two conjectures on the description of the automorphism groups of two large families of quantum algebras.

In this talk we will prove a general rigidity theorem for quantum tori. It leads to a scheme that can be used to classify the automorphism groups of algebras that admit just one quantum cluster (i.e. can be squeezed by a quantum affine space algebra and the corresponding quantum torus). The technique has a broad range of applications and in particular settles the above mentioned conjectures.