Outstanding Challenges in Combinatorics on Words
Feb. 19 – 24, 2012

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in the new lecture theater in the TransCanada Pipelines Pavilion (TCPL). LCD projector and blackboards are available for presentations.

SCHEDULE

Sunday

16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
17:30–19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall
	Beverages and a small assortment of snacks are available on a cash honor system.

Monday

7:00–8:45	Breakfast
8:45–9:00	Introduction and Welcome by BIRS Station Manager, TCPL
9:00–9:50	Lecture: Jason Bell, *On Automatic Sequences*
10:00–10:30	Coffee Break, TCPL
10:30–11:20	Lecture: Boris Adamczewski, *Combinatorics on words and Diophantine approximation*
11:30–13:00	Lunch
13:00–13:30	Lecture: Yuri Matiyasevich, *Arithmetization of words and related problems*
13:35–14:25	Lecture: Jamie Simpson, *Matching fractions*
14:30–15:20	Lecture: Stepan Holub, *Length types of word equations*
15:20–15:40	Coffee Break, TCPL
15:45–16:35	Lecture: Thomas Stoll, *Thue-Morse at polynomial subsequences*
16:40–17:30	Lecture: Eric Rowland, *Counting equivalence classes of words in* F_2
17:30–19:30	Dinner
20:00–20:50	Lecture: Jeffrey Shallit, *On* k*-automatic sets of rational numbers*

Tuesday

7:00–9:00	Breakfast
9:00–9:50	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
9:50	Group Photo; meet in the foyer of the TCPL
10:20–10:40	Coffee Break, TCPL
10:40–11:30	Lecture: Robert Mercas, *On Pseudo-repetitions in words*
11:30–13:30	Lunch
13:30–14:20	Lecture: Maxime Crochemore, *Maximal Exponent Repeats*
14:25–14:45	Coffee Break, TCPL
14:50–15:40	Lecture: Arturo Carpi, *Unrepetitive walks in digraphs (and the repetition threshold)*
15:45–16:35	Lecture: Golnaz Badkobeh, *Fewest repetitions vs maximal-exponent powers in infinite binary words*
16:40–17:30	Lecture: Volker Diekert, *Local Divisors*
17:30–19:30	Dinner
20:00–20:50	Lecture: Dirk Nowotka, *Avoidability under Permutations*

Wednesday

7:00–9:00	Breakfast
9:00–9:25	Lecture: Anna Frid, *Morphisms on Permutations*
9:30–10:20	Lecture: Srecko Brlek, *The last talk on the Kolakoski sequence*
10:20–10:40	Coffee Break, TCPL
10:40–11:30	Lecture: Nicolas Bédaride, *Piecewise isometries and words*
11:30–13:30	Lunch
13:30–17:30	Curling Excursion (Charter bus at precisely 13:45)
17:30–19:30	Dinner
20:00–20:50	Lecture: Juhani Karhumäki, *Combinatorics on words and k-abelian equivalence*

Thursday

7:00–9:00	Breakfast
9:00–9:50	Lecture: Luca Zamboni, *Partition regularity and words*
10:00–10:30	Coffee Break, TCPL
10:30–11:20	Lecture: Svetlana Puzynina, *Locally catenative sequences and Turtle graphics*
11:30–13:30	Lunch
13:30–14:20	Lecture: Amy Glen, *On a generalisation of trapezoidal words*
14:25–14:45	Coffee Break, TCPL
14:50–15:40	Lecture: Julien Leroy, *The S-adic conjecture)*
15:45–16:35	Lecture: Antonio Restivo, *Sturmian Words and Critical Factorization Theorem*
16:40–17:30	Lecture: Sébastien Ferenczi, *Word combinatorics of interval-exchange transformations for every permutation*
17:30–19:30	Dinner

Friday

7:00–9:00	Breakfast
9:00-11:30	Informal Discussions
10:15-10:45	Coffee Break, TCPL
11:30–13:30	Lunch
Checkout by 12 noon.	

** 5-day workshop participants are welcome to use BIRS facilities (BIRS Coffee Lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

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ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **Boris Adamczewski** (Institu Camille Jordan & CNRS)
Title: *Combinatorics on words and Diophantine approximation*
Abstract: A very fruitful interplay between combinatorics on words and Diophantine approximation comes up with the use of numeration systems.

Finite and infinite words occur naturally in Number Theory when one considers the expansion of a real number in an integer base or its continued fraction expansion. Conversely, with an infinite word w on the finite alphabet $\{0, 1, \ldots, b-1\}$ one can associate the real number ξ_w whose base-b expansion is given by w. Many problems are then concerned with the following question: how the combinatorial properties of the word w and the Diophantine properties of the number ξ_w may be related?

In this talk, I will survey some of the recent advances on this topic. I will also try to point out new challenges.

Speaker: **Golnaz Badkobeh** (King's College London)
Title: *Fewest repetitions vs maximal-exponent powers in infinite binary words*
Abstract: The exponent of a word is the ratio of its length over its smallest period. The repetitive threshold $r(a)$ of an a-letter alphabet is the smallest rational number for which there exists an infinite word whose finite factors have exponent at most $r(a)$. This notion was introduced in 1972 by Dejean who gave the exact values of $r(a)$ for every alphabet size a as it has been eventually proved in 2009. The finite-repetition threshold for an a-letter alphabet refines the above notion. It is the smallest rational number $\mathrm{FR}t(a)$ for which there exists an infinite word whose finite factors have exponent at most $\mathrm{FR}t(a)$ and that contain a finite number of factors with exponent $r(a)$.

It is known from Shallit (2008) that $\mathrm{FR}t(2) = 7/3$. With each finite-repetition threshold is associated the smallest number of $r(a)$-exponent factors that can be found in the corresponding infinite word.

It has been proved by Badkobeh and Crochemore (2010) that this number is 12 for infinite binary words whose maximal exponent is 7/3.

In this article we give some new results on the trade-off between the number of squares and the number of maximal-exponent powers in infinite binary words, in the three cases where the maximal exponent is 7/3, 5/2, and 3. These are the only threshold values related to the question.

Speaker: **Nicolas Bédaride** (Aix-Marseille University)
Title: *Piecewise isometries and words*
Abstract: A piecewise isometrie of the plane is a bijective map defined on the complement of several lines such that the restriction to a connected set is an isometry of the plane. In this talk, I will describe some classical piecewise isometries of the plane, and the associated language obtained by a coding.

Speaker: **Jason Bell** (Simon Fraser University)
Title: *On Automatic Sequences*

Speaker: **Srecko Brlek** (UQAM - Laboratoire de Combinatoire et d'informatique Mathématique)
Title: *The last talk on the Kolakoski sequence*

Speaker: **Arturo Carpi** (Universita Degli Studi Di Perugia)
Title: *Unrepetitive walks in digraphs (and the repetition threshold)*

Speaker: **Maxime Crochemore** (King's College London)
Title: *Maximal Exponent Repeats*

Speaker: **Volker Diekert** (University Of Stuttgart - Institut Für Formale Methoden Der Informatik)
Title: *Local Divisors*
Abstract: In my talk I will speak about several new results and simpler proofs of known results in formal language theory using the concept of a local divisor. The results encompass recent joint work with Manfred Kufleitner and Benjamin Steinberg on Krohn-Rhodes Theorem.

Speaker: **Sébastien Ferenczi** (Institut de Mathématiques de Luminy)
Title: *Word combinatorics of interval-exchange transformations for every permutation*

Speaker: **Anna Frid** (Sobolev Institute of Mathematics SB RAS)
Title: *Morphisms on Permutations*

Speaker: **Amy Glen** (Murdoch University)
Title: *On a generalisation of trapezoidal words*
Abstract: The *factor complexity function* $C_w(n)$ of a finite or infinite word w counts the number of distinct *factors* of w of length n for each $n \geq 0$. A finite word w of length $|w|$ is said to be *trapezoidal* if the graph of its factor complexity $C_w(n)$ as a function of n (for $0 \leq n \leq |w|$) is that of a regular trapezoid (or possibly an isosceles triangle); that is, $C_w(n)$ increases by 1 with each n on some interval of length r, then $C_w(n)$ is constant on some interval of length s, and finally $C_w(n)$ decreases by 1 with each n on an interval of the same length r. Necessarily $C_w(1) = 2$ (since there is one factor of length 0, namely the *empty word*), so any trapezoidal word is on a binary alphabet. Trapezoidal words were first introduced by A. de Luca (1999) when studying the behaviour of the factor complexity of *finite Sturmian words*, i.e., factors of infinite "cutting sequences", obtained by coding the sequence of cuts in an integer lattice over the positive quadrant of $\mathbf{R}^2$ made by a line of irrational slope. Every finite Sturmian word is trapezoidal, but not conversely. However, both families of words (trapezoidal and Sturmian) are special classes of so-called *rich words* – a wider class of finite and infinite words characterised by containing the maximal number of palindromes – recently introduced by the speaker, together with J. Justin, S. Widmer, and L.Q. Zamboni (2009).

In this talk, I will introduce a natural generalisation of trapezoidal words over an arbitrary finite alphabet A consisting of at least two distinct letters, called *generalised trapezoidal words* (or *GT-words* for short). In particular, I will discuss some combinatorial properties of this new class of words when $|A| \geq 3$ and I will show that, unlike in the binary case ($|A| = 2$), not all GT-words are rich in palindromes, but we do have a neat characterisation of those that are.

This is joint work with Florence Levé (Université de Picardie – Jules Verne, France).

Speaker: **Stepan Holub** (Charles University)
Title: *Length types of word equations*
Abstract: One of the 'rules of thumb' for solving a word equation is to guess the length vector of the solution and then construct the corresponding equivalence on the positions. The talk will comment on this method and indicate some less trivial implications.

Speaker: **Juhani Karhumäki** (University of Turku)
Title: *Combinatorics on words and k-abelian equivalence*
Abstract: We define a new equivalence relation on words which is properly in between the equality and commutative (abelian) equality. We call it k-abelian equality. We say that two words are k-abelian equal if they contain each factor of length k equally many times, and in addition start with a common prefix of length k-1. This allows to have better and better approximations of problems based on equality of words. We report basic properties of these equivalence relations, and analyze k-abelian variants of some well known problems on words. In particular we consider problems dealing local and global regularities in infinite words, as well as the existence of certain type of repetition-free words. It turns out that there exist a lot of challenging open problems in this area.

Speaker: **Julien Leroy** (Université de Liège)
Title: *The S-adic conjecture)*
Abstract: An infinite word is S-adic if it can be obtained by successive iterations of morphisms belonging to the set S. Sturmian words are well-known examples of S-adic words with $\mathrm{card}(S) = 4$. The S-adic conjecture tries to determine the link that should exist between S-adicity and sub-linear factor complexity. More precisely, it says that there is a stronger notion of S-adicity which is equivalent to sub-linear factor complexity, i.e., an infinite word would have a sub-linear complexity if and only if it is "strongly S-adic".

In this talk, I will present some recent results about that conjecture. First I will present some examples that allow to reject some natural ideas that one could have. Then I will briefly explain a general method to compute an S-adic expansion of any uniformly recurrent infinite word with sub-linear complexity. That method allows to solve the conjecture in the particular case of uniformly recurrent infinite words with first difference of complexity bounded by 2.

Speaker: **Yuri Matiyasevich** (Steklov Institute of Mathematics)
Title: *Arithmetization of words and related problems*

Speaker: **Robert Mercas** (Informatik at Otto-von-Guericke Universität at Magdeburg)
Title: *On Pseudo-repetitions in words*
Abstract: The notion of repetition of factors in words was studied already from the beginnings of the combinatorics on words area. One of the recent generalizations regarding this concept was introduced by L. Kari et al., and considers a word to be an f-repetition if it is the iterated concatenation of one of its prefixes and the image of this prefix through an anti-/morphic involution f. In this paper, we extend the notion of f-repetitions to arbitrary anti-/morphisms, and investigate a series of algorithmic problems arising in this context. Further, we present a series of results in the fashion of the Fine and Wilf theorem for f-repetitions, when f is an iso(anti)morphism.

Speaker: **Dirk Nowotka** (Kiel University)
Title: *Avoidability under Permutations*
Abstract: We present first considerations and results on avoidability where morphic and antimorphic permutations are part of the pattern.

Speaker: **Svetlana Puzynina** (University of Turku)
Title: *Locally catenative sequences and Turtle graphics*
Abstract: Motivated by striking properties of the well known Fibonacci word we consider pictures which are defined by this word and its variants via so-called turtle graphics. Such a picture can be bounded or unbounded. We characterize when the picture defined by not only the Fibonacci recurrence, but also by a general recurrence formula, is bounded, the characterization being computable.

Speaker: **Antonio Restivo** (University of Palermo)
Title: *Sturmian Words and Critical Factorization Theorem*

Abstract: We prove that characteristic Sturmian words are extremal for the Critical Factorization Theorem (CFT) in the following sense. If $px(n)$ denotes the local period of an infinite word x at point n, we prove that x is a characteristic Sturmian word if and only if $px(n)$ is smaller than or equal to $n+1$ for all $n \geq 1$ and it is equal to $n+1$ for infinitely many integers n.

This result is extremal with respect to the CFT since a consequence of the CFT is that, for any infinite recurrent word x, either the function $px(n)$ is bounded, and in such a case x is periodic, or $px(n) \geq n+1$ for infinitely many integers n. As a byproduct of the techniques used in the paper we extend a result of of Harju and Nowotka stating that any finite Fibonacci word f_n for $n \geq 5$ has only one critical point. Indeed we determine the exact number of critical points in any finite standard Sturmian word.

Speaker: **Eric Rowland** (UQAM)
Title: *Counting equivalence classes of words in* F_2
Abstract: In the last decade several papers have appeared concerning the size of an equivalence class of words in a free group under its automorphism group. A central theme of the area is that information about the equivalence class of a word can be obtained from statistics of its (contiguous) subwords. Here we are interested in the free group on 2 generators. We give a new characterization of words of minimal length, and we introduce a natural operation that "grows" words from smaller words. The growth operation gives rise to a notion of maximally minimal words (in a way that can be made precise), which we call root words. Equivalence classes containing root words have special structure, and the hope is that understanding this structure will lead to an exact enumeration of equivalence classes in F_2 containing a minimal word of length n.

Speaker: **Jeffrey Shallit** (University of Waterloo)
Title: *On k-automatic sets of rational numbers*
Abstract: In this talk I will describe how automata can accept sets of rational numbers. Applications include deciding various questions about automatic sequences and the enumeration of various properties. There are many open questions.

Speaker: **Jamie Simpson** (Curtin University)
Title: *Matching fractions*
Abstract: If x is an infinite word and k is a non-negative integer then the kth matching fraction of x, written $\mu(k)$, is the following limit, if it exists:

$$\mu(k) = lim_{n\to\infty} \frac{|\{i \in [1,n] : x[i] = x[i+k]\}|}{n}. \tag{1}$$

The idea of a matching fraction was suggested by Peter Pleasants and has been investigated by Keith Tognetti and colleagues. In particular they showed that if x is a Sturmian word on alphabet $\{a, b\}$ whose density of as is α then

$$\mu(k) = \max(|1 - 2\{\alpha\}|, |1 - 2\{k\alpha\}|). \tag{2}$$

Here $\{\alpha\}$ is the fractional part of α.

I will review what's known about matching fractions and suggest some areas for investigation.

Speaker: **Thomas Stoll** (Université d'Aix-Marseille)
Title: *Thue-Morse at polynomial subsequences*
Abstract: Consider the Thue-Morse sequence (t_n) on the symbols $\{0, 1\}$. A celebrated result of Gelfond (1967/68) gives an asymptotic formula for the number of 0's and 1's in any fixed linear subsequence of TM, i.e., (t_{an+b}). In quite recent work, Mauduit and Rivat (2009) found precise formulas for quadratic polynomials. The cases of cubes and higher-degree polynomials remain elusive so far. From work of Dartyge and Tenenbaum (2006) it follows that there are $\gg N^{2/h!}$ symbols "0" (or "1") in any subsequence indexed

by a polynomial of degree h. The aim of the present talk is to give an overview about the various results known in this area, pose some (old and new) conjectures, and – to improve by elementary/combinatorial means the general lower bound to $\gg N^{4/(3h+1)}$.

Speaker: **Luca Zamboni** (Université Lyon 1 & University of Turku)
Title: *Partition regularity and words*
Abstract: Van der Waerden's theorem states that given a finite partition of the natural numbers N, at least one element of the partition contains arbitrarily long arithmetic progressions (i.e., is a.p.-rich). In fact, in the hypothesis N can be replaced by any subset X of N which is itself a.p.-rich. In other words, the property of being a.p.-rich is partition regular, i.e., cannot be destroyed under finite partition. However given a partition regularity property P (e.g., being a.p.-rich) and a subset X of N (e.g., X =set of all prime numbers), the question of determining whether X has property P is often quite difficult (even if either X or its complement must have property P). In the context of combinatorics on words it is natural to consider X to be the set of all occurrences of a given factor in some nice infinite word (i.e., the set of all occurrences of 0 in the Thue-Morse word, or the set of all occurrences of 0100 in the Fibonacci word). In this talk we will consider different partition regularity properties in the context of words. It turns out that many well known words (including Sturmian words and words generated by substitution rules) provide a rich setting for studying different partition regularity properties. And conversely, some deep dynamical and or arithmetical questions concerning these words may be reformulated in terms of partition regularity. One such example is the strong coincidence condition (SCC) conjectured for irreducible primitive Pisot substitutions. We will describe an equivalent reformulation of SCC whose difficulty lies in the arithmetic properties of the associated Thomas-Dumont numeration systems. This is based on joint work with M. Bucci and S. Puzynina.