Banff International Research Station

for Mathematical Innovation and Discovery

Workshop 12w5100: Recent trends in Geometric and Nonlinear Analysis
August 5th-10th, 2012

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in the new lecture theater in the TransCanada Pipelines Pavilion (TCPL). LCD projector and blackboards are available for presentations.

SCHEDULE

Sunday

16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
17:30–19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall (if desired) Beverages and a small assortment of snacks are available on a cash honor system.

Monday

7:00–8:45	Breakfast
8:45–9:00	Introduction and Welcome by BIRS Station Manager, TCPL
9:00–9:45	W.Meeks
9:45–10:30	M.Musso
10:30–11:00	Coffee Break, TCPL
11:00-11:45	Y.-H.Kim
11:45–13:30	Lunch
13:30–14:15	G.Veronelli
14:15–15:00	A.Malchiodi
15:00–15:30	Coffee Break, TCPL
15:30–16:15	P.Yang
17:30–19:30	Dinner

Tuesday

7:00–9:00	Breakfast
9:00–9:45	M.Del Pino
9:45–10:30	A.Fraser
10:30–11:00	Coffee Break, TCPL
11:00-11:45	A.Chau
11:45–13:00	Lunch
13:00–14:00	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:00	Group Photo; meet in foyer of TCPL.
14:00–14:45	L.Bandara
14:45–15:15	Coffee Break, TCPL
15:15–16:00	P.Guan
16:00–16:45	J.Vétois
17:30–19:30	Dinner

Wednesday

7:00–9:00	Breakfast
9:00–9:45	N.Ghoussoub
9:45–10:30	O.Druet
10:30–11:00	Coffee Break, TCPL
11:00-11:45	C.Gui
11:45–12:30	X.Zhang
12:30–13:30	Lunch
	Free afternoon
17:30–19:30	Dinner

Thursday

7:00–9:00	Breakfast
9:00–9:45	R.McCann
9:45–10:30	B.Daniel
10:30–11:00	Coffee Break, TCPL
11:00-11:45	C.Cowan
11:45–13:30	Lunch
13:30–14:15	P.Laurain
14:15–15:00	J.Wei
15:00–15:30	Coffee Break, TCPL
15:30–16:15	O.Hijazi
17:30–19:30	Dinner

Friday

7:00–9:00	Breakfast
9:00–9:45	R.Moser
9:45–10:30	T.Weth
10:30–11:00	Coffee Break, TCPL
11:00-11:45	F.Cirstea
11:45–13:30	Lunch

Checkout by noon

** 5-day workshop participants are welcome to use BIRS facilities (BIRS Coffee Lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

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ABSTRACTS

Lashi Bandara (ANU Canberra): *The Kato square root problem on vector bundles with generalised bounded geometry*

Abstract: We consider smooth, complete Riemannian manifolds which are exponentially locally doubling. Under a uniform Ricci curvature bound and a uniform lower bound on injectivity radius, we prove a Kato square root estimate for certain coercive operators over the bundle of finite rank tensors. These results are obtained as a special case of similar estimates on smooth vector bundles satisfying a criterion which we call generalised bounded geometry. We prove this by establishing quadratic estimates for perturbations of Dirac type operators on such bundles under an appropriate set of assumptions. This is joint work with Alan McIntosh. Arxiv link: http://arxiv.org/abs/1203.0373

Albert Chau (University of British Columbia, Vancouver): *Compact manifolds with nonnegative quadratic orthogonal bisectional curvature*

Abstract: In this talk I will discuss nonnegatively curved compact Kahler manifolds and their classification. An overview of past results will be given in the cases of bisectional and orthogonal bisectional curvature. The more recent case of quadratic orthogonal bisectional curvature will then be discussed including recent splitting theorems and the curvature of Kahler C-spaces. The talk is based on joint work with L.F. Tam.

Florica Cirstea (University of Sydney): *On complete classification of the isolated singularities for nonlinear elliptic equations*

Abstract: We consider a broad class of nonlinear elliptic equations in a punctured domain, which allow for inverse square potentials and weighted nonlinearities. We discuss new and sharp techniques which enabled us to completely classify the behaviour near the isolated singularity for all positive solutions in the context of regular variation theory. We then briefly mention how these techniques can be generalized to deal with other more general divergence form equations.

Craig Cowan (Stanford University): *Second order scalar formulation for stable solutions of fourth order problems and elliptic systems*

Abstract: We examine fourth order elliptic problems of the form $\Delta^2 u = f(u)$ and elliptic systems $-\Delta u = f(u,v)$ $-\Delta v = g(u,v)$. We show that the stability of their solutions (i.e. those whose first eigenvalue of their linearized problem is nonnegative) can be surprisingly captured by an inequality reminiscent of stability for a second order scalar equation. This inequality is then used to prove various new results regarding the regularity of the related extremal solutions on bounded domains and also the corresponding Liouville theorems on $\mathbb{R}^N$.
Some of this work is joint with Nassif Ghoussoub. This new approach was motivated by joint work with Mostafa Fazly.

Benoît Daniel (Université de Lorraine, Nancy): *Constant mean curvature surfaces in homogeneous manifolds*

Abstract: Constant mean curvature (CMC) surfaces are surfaces that locally minimize area under a certain volume constraint. They are closely related to the isoperimetric problem. A celebrated theorem of H. Hopf states that the only CMC topological spheres in Euclidean 3-space are round spheres. We will present some generalisations of this result in some other ambient homogeneous 3-manifolds.

Manuel del Pino (Universidad de Chile): *Minimal surfaces and the Allen-Cahn equation*

Abstract: We review some results on construction of solutions to the Allen Cahn equation with its nodal set close to a given, largely dilated minimal surface. In particular we present a counterexample to De Giorgi's conjecture in dimensions 9 or higher.

Olivier Druet (Ecole Normale Supérieure de Lyon): *Conformal eigenvalues of the spheres*

Abstract: We shall study the minimisation problem of various eigenvalues of the Laplacian in the conformal class of a given manifold. We will focus mainly on the case of the conformal class of the standard sphere.

Ailana Fraser (University of British Columbia, Vancouver): *An extremal eigenvalue problem for surfaces with boundary*

Abstract: Beginning with the work of J. Hersch for the two sphere and that of P. Li and S. T. Yau for more general surfaces, the question of determining surfaces of fixed area which maximize the first eigenvalue has been actively studied. In this talk we will describe recent work with R. Schoen concerning extremal eigenvalue questions for surfaces with boundary. In both cases the eigenvalue problems are related to minimal surface questions. For closed surfaces these are minimal surfaces in spheres while for surfaces with boundary they are related to minimal surfaces in the ball satisfying a natural boundary condition. We will describe the extremal surfaces in the genus zero case.

Nassif Ghoussoub (University of British Columbia, Vancouver): *De Giorgi type results for elliptic systems*

Abstract: In low dimensions, and under various conditions on the nonlinearity, the solution of an elliptic system is necessarily one-dimensional whenever each one of its components is monotone in one direction. An extension of a geometric Poincaré inequality to systems is used to establish De Giorgi type results for stable solutions as well as additional rigidity properties stating that the gradients of the various components of the solutions must be parallel. The concept of "an orientable system", which seems to be key for dealing with systems of three or more equations, is introduced. For such systems, the notion of a stable solution in a variational sense coincide with the pointwise (or spectral) concept of stability. This is joint work with M. Fazly.

Changfeng Gui (University of Connecticut): *Traveling wave solutions to reaction diffusion equations with fractional Laplacians*

Abstract: In this talk, I will discuss the existence and asymptotic behavior of traveling wave solutions to Allen-Cahn equation with fractional Laplacians where the double well potenotial has unequal depths. A key ingredient is the estimate of the speed of the traveling wave in terms of the potential, which seems new even for the classical Allen-Cahn equation. I will also discuss nonexistence of traveling wave solutions to a

nonlocal combustion model. The talk is based on recent results obtained jointly with Tingting Huan and with Mingfeng Zhao respectively.

Pengfei Guan (McGill University, Montréal): *A new C^2 estimate for curvature equations of hypersurfaces*

Abstract: We consider the global curvature estimates for curvature equation in general form $\sigma_k(\kappa) = f(X, \nu)$, where ν, κ are the outer-normal and principal curvatures of hypersurface M respectively. This type of equation arsing from various geometric problems, like the Christoffel-Minkowski problem, prescribing curvature measure problem, etc. A longstanding problem is the C^2 estimates for the equation when f is depending on ν, for $1 < k < n$. We establish such estimate for admissible solutions when $k = 2$, and for convex solutions of equation when $2 < k < n$.

Oussama Hijazi (Université de Lorraine, Nancy): *A Positive Mass type theorem on Spin Manifolds*

Abstract: This is a report on some recent work with Sebastian Montiel in which we get a Shi-Tam type inequality, implying the Positive Mass Theorem. We then establish a Holographic Principle for the existence of parallel spinors.

Young-Heon Kim (University of British Columbia, Vancouver): *Hölder continuity of optimal transport maps*

Abstract: Optimal transport theory considers phenomena when mass distributions are matched in a most efficient way, where the efficiency is measured against a transportation cost. In a family of interesting cases, such matching is reduced to a mapping between two domains, called an optimal map. A fundamental issue is regularity of optimal maps, and it is reduced to studying a fully nonlinear second order elliptic PDE of Monge-Ampere type. We present a Hoelder continuity result for the optimal map when the mass distributions are rough, i.e. assumed only bounded above and below, under a sharp condition, called Ma-Trudinger-Wang condition. This is joint work with Alessio Figalli and Robert McCann.

Paul Laurain (Université Paris 7): *Quantization phenomena for solutions of conformally invariant problems*

Abstract: After introducing conservation laws discovered by T. Rivière for the solutions of elliptic systems with antisymmetric potential. We will show how, with the help of generalized Wente inequalities, these conservation laws allow us to highlight a quantization phenomena for the energy for sequences of solutions of a large class of elliptic problems in dimension 2. In particular, this includes critical points of conformally invariant functional and allows us to give new proofs of results such as the compactness of J-holomorphic curves in manifold of low regularity.

Robert McCann (University of Toronto): *Higher-order time asymptotics of fast diffusion in Euclidean space (via dynamical systems methods)*

Abstract: With Jochen Denzler (UT Knoxville) and Herbert Koch (Bonn), we quantify the speed of convergence and higher asymptotics of fast diffusion dynamics on Euclidean space to the Barenblatt (self similar) profile. The degeneracy in the parabolicity of the equation is cured by re-expressing the dynamics on a manifold with a cylindrical end, called the cigar. The nonlinear evolution semigroup becomes differentiable with respect to Hoelder initial data on the cigar. The linearization of the dynamics is given by Laplace-Beltrami operator plus a drift term (which can be suppressed by the introduction of appropriate

weights into the function space norm), plus a finite-depth potential well with a universal profile. In the limiting case of the (linear) heat equation, the depth diverges, the number of eigenstates increases without bound, and the continuous spectrum recedes to infinity. We provide a detailed study of the linear and nonlinear problems in Hoelder spaces on the cigar, including a sharp boundedness estimate for the semigroup, and use this as a tool to obtain sharp convergence results toward the Barenblatt solution. In finer convergence results (after modding out symmetries of the problem), a subtle interplay between convergence rates and tail behavior is revealed. The difficulties involved in choosing the right functional analytic spaces in which to carry out the analysis can be interpreted as genuine features of the equation rather than mere annoying technicalities.

http://www.math.toronto.edu/mccann/papers/DKM.pdf

Andrea Malchiodi (SISSA, Trieste): *Variational analysis for singular Liouville equations*

Abstract: We consider singular Liouville equations on compact surfaces, motivated by the prescription of Gaussian curvatures for metrics with conical singularities, and from the study of abelian Chern-Simons models. We tackle the problem of existence variationally, though some new-scaling invariant-improved Moser-Trudinger inequalities, jointly with topological methods.

William Meeks (University of Massachussett): *H-surfaces in homogeneous 3-manifolds*

Abstract: Recently Meeks, Mira, Perez and Ros have classified the moduli space of constant mean curvature spheres in a homogeneous 3-manifolds X. When the manifold X is simply connected and not diffeomorphic to $\mathbb{R}^3$, they prove that there is a unique such sphere for every value of the mean curvature. When X is diffeomorphic to $\mathbb{R}^3$, then they prove that there exists a unique constant mean curvature H sphere for every value H greater than $Ch(X)/2$, where $Ch(X)$ is the Cheeger constant of X (and no such $H-$sphere for other values H of mean curvature).

Roger Moser (University of Bath): *Singular perturbation problems involving curvature*

Abstract: For a surface $M \subset \mathbb{R}^3$ with second fundamental form A and normal vector ν, and for $\epsilon > 0$, consider the functional

$$E_\epsilon(M) = \int_M \left(|A|^2 + \frac{1}{\epsilon}\Phi(\nu) \right) d\sigma,$$

where $\Phi : \mathbb{S}^2 \to [0, +\infty)$ is a given function. This is a geometric version of functionals from the Modica-Mortola theory of phase transitions or the theory of Ginzburg- Landau vortices (depending on the structure of Φ), and it arises in models for faceting crystal surfaces. How do the functionals and their minimisers, critical points, gradient ows, etc., behave when we let $\epsilon \to 0$? The question is largely open, but I will describe some preliminary results for special cases.

Monica Musso (Pontificia Universidad Católica de Chile, Santiago): *Critical equations in* $\mathbb{R}^2$

Abstract: We review some new results on construction of single and multiple bubbling solutions for variational problems leading to semlinear equations with nonlinearities of Trudinger Moser type. This is joint work with M. del Pino, S. Deng, I. Guerra and B. Ruf.

Giona Veronelli (Université de Cergy-Pontoise): *On the homotopy Dirichlet problem for p-harmonic maps*

Abstract: We will deal with the relative homotopy Dirichlet problem for p-harmonic maps from compact manifolds with boundary to manifolds of non-positive sectional curvature. The problem is well-understood in the harmonic case thanks to works by Hamilton, Schoen-Yau and Hildebrandt-Kaul-Widman, but generalizations of their approaches to the p-harmonic realm are not available so far. We present a complete solution to the problem in case the target manifold is either compact, rotationally symmetric or two dimensional and simply connected. The proof of the compact case uses some ideas of White to define the relative d-homotopy type of Sobolev maps, and the regularity theory by Hardt and Lin. To deal with non-compact targets, we introduce a periodization procedure which permits to compactify the codomain and reduce the problem to the previous one. This is a joint work with Stefano Pigola.

Jérôme Vétois (Université de Nice-Sophia Antipolis): *Instability of the Yamabe Equation*

Abstract: On a compact Riemannian manifold, we discuss the question of stability (or instability) of solutions to equations with critical Sobolev growth. We prove, in a joint work with Angela Pistoia and Pierpaolo Esposito, a sharp result of instability for the Yamabe equation.

Juncheng Wei (Chinese University of Hong-Kong): *Classification and nondegeneracy of Toda system*

Abstract: We consider the following Toda system

$$\Delta u_i + \sum_{j=1}^{n} a_{ij} e^{u_j} = 4\pi\gamma_i \delta_0 \ \text{ in } \mathbb{R}^2, \quad \int_{\mathbb{R}^2} e^{u_i} dx < \infty, \ \ \forall\, 1 \le i \le n,$$

where $\gamma_i > -1$, δ_0 is Dirac measure at 0,and the coefficients a_{ij} form the standard tri-diagonal Cartan matrix. (i) We completely classify the solutions and obtain the quantization result:

$$\sum_{j=1}^{n} a_{ij} \int_{\mathbb{R}^2} e^{u_j} dx = 4\pi(2 + \gamma_i + \gamma_{n+1-i}), \ \ \forall\, 1 \le i \le n.$$

This generalizes the classification result by Jost and Wang for $\gamma_i = 0$, $\forall\ 1 \le i \le n$.(ii) We prove that if $\gamma_i + \gamma_{i+1} + \ldots + \gamma_j \notin \mathbb{Z}$ for all $1 \le i \le j \le n$, then any solution u_i is *radially symmetric* w.r.t. 0.(iii) We prove that the linearized equation at any solution is *non-degenerate*. Then we apply these results to give a complete classification of Toda system of rank 2 (A_2, B_2, G_2), and construct nontopological solutions for A_2 and B_2 Chern-Simons system. (Joint work with CSLin-Ye, Ao-Lin).

Tobias Weth (Frankfurt Universität): *On the Szegö-Weinberger profile in Riemannian manifolds*

Abstract: We consider the geometric variational problem of analyzing the local Szegö-Weinberger isoperimetric profile in a Riemannian manifold. This profile is obtained by maximizing the first nontrivial Neumann eigenvalue of the Laplace-Beltrami Operator among subdomains with fixed volume. As a corollary of our analysis, we deduce an isoperimetric comparison principle relative to the profile in the corresponding space form of constant sectional curvature. A major difficulty in this problem is the effect of (possible) degeneracy of the first nontrivial Neumann eigenvalue on extremal domains. This is joint work with Moustapha Fall.

Paul Yang (Princeton): *A conformally invariant operator on the pluriharmonics*

Abstract: In CR geometry, the presence of a large space of pluriharmonic functions is a difficult topic to study. I report on a joint project with Jeffrey Case to extend the operator of Branson-Fontana-Morpurgo to

general CR strucures in 3-D. It is closely related to the 4-th order operator studied by Hirachi. A number of interesting questions arise that has no analogue in conformal geometry. I will give a short survey of the background material and then about the possible applications of this operator to several questions on CR geometry in 3-D.

Xiangwen Zhang (McGill University, Montréal): *Some estimates for complex Monge-Ampère equation on Hermitian manifolds*

Abstract: In the talk, some regularity estimates for the complex Monge-Ampère equation will be presented. We will prove that any $C^{1,1}$ plurisubharmonic solution u to the complex Monge-Ampère equation $\det(u_{i\bar{j}}) = f$ on $\mathbb{C}^n$ with f strictly positive and Hölder continuous has in fact Hölder continuous second derivatives. Moreover, to show this regularity result holds on more general complex manifolds, we will establish the Bedford-Taylor's interior C^2 and a local Calabi's C^3 estimates in Hermitian setting.