ABSTRACTS OF TALKS
WHITNEY WORKSHOP, BANFF 2013, AS OF APRIL 24.

Charles Fefferman. Finiteness Principle I, II, III.

Abstract: These expository talks motivate the proof of the Brudnyi-Shvartsman finiteness principle for $C^m(\mathbf{R}^n)$ starting with the simplest nontrivial case, $C^2(\mathbf{R}^2)$, and then passing to the general case.

Charles Fefferman. Whitney problems for Sobolev functions.

Abstract: Extension of functions from finite subsets of $\mathbf{R}^n$ to functions in a Sobolev space. Theorems (proven and checked), algorithms (being written up, probably correct), open problems.

Vladimir Goldshtein. Capacities in Sobolev spaces I, II. CANCELLED!

Abstract: There are two main notions of capacities on Sobolev spaces: Sobolev capacity of compact sets and variational capacity of condensors. Sobolev capacity has found a number of applications in the theory of partial differential equations via the theory of Sobolev spaces. Variational capacity is a generalization of the conformal capacity and has found applications in the geometric function theory (in the theory of quasiconformal mappings and its generalizations).

We discuss the following subjects:

Polar sets. A set is said to be polar if it has locally zero variational capacity. Polar sets are negligible sets for Sobolev spaces.

Quasi-continuity (continuity outside of a set of arbitrary small capacity). A Lusin type theorem for the capacity says that every Sobolev function has a quasi-continuous representative. The property of quasicontinuity permits to redefine Sobolev spaces with the help of quasi-continuous representatives.

Capacity as an outer measure. Choquet properties for the variational capacity.

Existence of extremal functions and metric estimates for capacities.

Capacitary properties of composition and extension operators.

Vladimir Goldshtein. Quasiconformal Whitney Partition. CANCELLED!

Abstract: A classical Whitney partition is a partition of a bounded Euclidean domain Ω into dyadic cubes with disjoint interiors, and with edges comparable to the distance to $\partial\Omega$. Its modern generalization that is called a Whitney partition is a partition into convex polyhedra with uniformly bounded ratios of their exterior to interior radii, and with diameters comparable to the distance to $\partial\Omega$. We propose a quasiconformal generalization of a Whitney partition where convex polyhedra are changed to quasiconformal images of balls under quasiconformal homeomorphisms of $\mathbf{R}^n$ onto itself, with uniformly bounded capacitory dilatations. We prove that the quasiconformal image of a quasiconformal Whitney family is a quasiconformal Whitney family. Capacitory estimates are the main tools in this study.

Ritva Hurri-Syrjanen. On fractional Poincare and Hardy inequalities.

Abstract: Fractional Poincare inequalities and fractional Hardy inequalities come from the classical Poincare inequality and the classical Hardy inequality respectively, where the pth power of the absolute value of the function's gradient has been replaced by a fractional integral. In this talk we will review some known results on these inequalities. We will also discuss some recent developments of fractional inequalities in bounded irregular domains. Joint work with Antti V. Vahakangas.

Arie Israel. Computational Aspects of the Sobolev Extension Problem: Part 2.

Abstract: In this talk we describe some algorithms for manipulating dyadic decompositions of Euclidean space. These procedures are important components in a current work on near-linear time algorithms for computing Sobolev extensions. This is joint work with Charles Fefferman, Boaz Klartag, and Garving Luli.

Erwan Le Gruyer. Some results on minimal Lipschitz extensions for $m-$jets from $\mathbb{R}^D$ to $\mathbb{R}^n$ under restrictive assumptions.

Abstract: The purpose of the talk is to present some results involving the Lipschitz constant for $m-$jets defined on a non-empty subset of $\mathbb{R}^D$ with values in $\mathbb{R}^n$ under restrictive assumptions. In particular we restrict ourselves to functions $F \in C^{m,1}(\mathbb{R}^D, \mathbb{R}^n)$ of the form

$$F = \sum_{k=1}^{D} F_k(x_k), F_k : \mathbb{R} \longmapsto \mathbb{R}^n, \ \forall k.$$

We produce minimal extensions and associated formulas which generalize Glaesers formula when ($D = 1$, $n = 1$) and also generalize a recently derived formula when ($D = 1$, n is arbitrary) (with Matthew Hirn).

Kevin Luli. Computational Aspects of the Sobolev Extension Problem: Part 1.

Abstract: Given a real-valued function f defined on a finite subset E of a Euclidean space $\mathbf{R}^n$, how can one find a near-optimal extension of f belonging to the Sobolev space $L^{m,p}(\mathbf{R}^n)$? How small can one take the Sobolev norm of an extension?

These questions are an instance of the Whitney extension problem. In previous work of Fefferman-Israel-Luli it was shown that there exists a bounded linear extension operator, and a formula was given for the near-optimal norm. The bounded linear extension operator and the approximate formula for the optimal norm were interesting theoretically, but the proof provided no means to construct these objects.

In this talk, we will review the recent results in the Fefferman-Israel-Luli paper. We will describe in broad strokes some of the key ingredients in the proofs and explain how the procedure can be modified to yield an efficient algorithm.

Andreea Nicoara. A Nullstellensatz for Lojasiewicz Ideals

Abstract: I will talk about recent progress on the Bochnak Nullstellensatz Conjecture, namely that a finitely generated ideal of smooth functions on an n-dimensional manifold equals the ideal of functions vanishing on its zero set iff the ideal is closed in the Whitney topology and real. Time permitting, I will indicate a possible way of completely settling this conjecture. (Joint work with Francesca Acquistapace and Fabrizio Broglia.)

Pavel Shvartsman. Extensions of BMO-functions and fixed points of contractive mappings in L_2 I, II.

Abstract: Let E be a closed subset of $\mathbf{R}^n$ of positive Lebesgue measure. We discuss a constructive algorithm which to every function f defined on E assigns its almost optimal extension to a function $F(f) \in BMO(\mathbf{R}^n)$. We obtain the extension $F(f)$ as a fixed point of a certain contractive mapping $T_f : L_2(\mathbf{R}^n) \to L_2(\mathbf{R}^n)$.

The extension operator $f \to F(f)$ is non-linear, and in general it is not known whether there exists a continuous linear extension operator

$$BMO(\mathbf{R}^n)|_E \to BMO(\mathbf{R}^n)$$

for an arbitrary set E.

In these talk we present a rather wide family of sets for which such extension operators exist. In particular, this family contains closures of domains with arbitrary internal and external cusps. The proof of this result is based on a solution to a similar problem for spaces of Lipschitz functions defined on subsets of a hyperbolic space.

Ignacio Uriarte-tuero. Two conjectures of Astala on distortion of sets under quasiconformal maps and related removability problems. NEW TALK!

Abstract: Quasiconformal maps are a certain generalization of analytic maps that have nice distortion properties. They appear in elasticity, inverse problems, geometry (e.g. Mostow's rigidity theorem)... among other places.

In his celebrated paper on area distortion under planar quasiconformal mappings (Acta 1994), Astala proved that if E is a compact set of Hausdorff dimension d and f is K-quasiconformal, then fE has Hausdorff dimension at most $d' = \frac{2Kd}{2+(K-1)d}$, and that this result is sharp. He conjectured (Question 4.4) that if the Hausdorff measure $\mathcal{H}^d(E) = 0$, then $\mathcal{H}^{d'}(fE) = 0$.

First it was shown that Astala's conjecture is sharp in the class of all Hausdorff gauge functions (UT, IMRN, 2008).

Lacey, Sawyer and UT jointly proved completely Astala's conjecture in all dimensions (Acta, 2010). The proof uses Astala's 1994 approach, geometric measure theory, and new weighted norm inequalities for Calderón-Zygmund singular integral operators which cannot be deduced from the classical Muckenhoupt A_p theory.

These results are related to removability problems for various classes of quasiregular maps. I will mention sharp removability results for bounded K-quasiregular maps (i.e. the quasiconformal analogue of the classical Painleve problem) recently obtained jointly by Tolsa and UT.

I will further mention recent results related to another conjecture of Astala on Hausdorff dimension of quasicircles obtained jointly by Prause, Tolsa and UT.

The talk will be self-contained.

Yosef Yomdin. Fourier Sampling of Piecewise-Smooth Functions, Johnson– Lindenstrauss lemma, and Turan-Nazarov Inequality (with D. Batenkov).

Abstract: A periodic C^d smooth function f can be reconstructed from its first N Fourier coefficients with an error of order $1/N^d$. However, for f only piecewise C^d smooth the classical Fourier approximation has an error of order $1/N$, no matter how large d is. There is a long-standing open problem (Eckhoff Conjecture) concerning a possibility to gain the smooth accuracy rate $1/N^d$ via a non-linear manipulations with the first N Fourier coefficients of any piecewise C^d smooth function

f. This problem was recently solved by D. Batenkov, via Algebraic Sampling approach. The key point was a proper choice of the samples among the first N Fourier coefficients of the function f. On the other hand, the problem of estimating robustness of the Fourier sampling on a given sampling set S is addressed by a discrete version of the well-known Turan-Nazarov inequality for exponential polynomials. It turns out to give rather accurate estimates and challenging predictions. There is another interesting connection between the Eckhoff problem and its versions with the general bounds on sampling accuracy of functions in given functional classes. It turns out that such bounds can be provided by a combination of Kolmogorovs ϵ-entropy and Johnson-Lindenstrauss dimensionality reduction. These bounds are accurate enough to imply a non-effective solution of the Eckhoff Conjecture, as well as many similar results for piecewise-analytic and other natural classes of non-regular functions. On the other hand, this approach raises basic problems related to the role of randomness in the dimensionality reduction algorithms, as well as the required accuracy of the measurements.

We plan to discuss these and some other results and related open questions.

Nahum Zobin. Sobolev extension domains, I, II. NEW TALK!

Abstract: We announce a solution of an old problem of description of planar finitely connected bounded Sobolev extension domains for $p > 2$, and for any smoothness.We present several new tools which allow to prove that the natural condition on subhyperbolic metric, which has been proven by Pavel Shvartsman to be sufficient for such a domain to be a Sobolev extension domain, is actually also necessary. The heart of the proof is an explicit construction of almost fats growing functions in such a domain (with Pavel Shvartsman).