Geometry and Inverse Problems
Arriving Sunday, September 15 and departing Friday September 20, 2013

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in the lecture theater in the TransCanada Pipelines Pavilion (TCPL). An LCD projector, a laptop, a document camera, and blackboards are available for presentations.

SCHEDULE

You are welcome to schedule lectures as you see fit, as long as you adhere to the meal times (noted above), coffee break start and end times (noted below) and take into account the "welcome" on Monday morning, the Banff Centre tour at 1:00 pm, and the group photo at 2:00 pm every Monday afternoon.

Please email your finalized schedule and abstracts to BIRS Station Manager birsmgr@birs.ca by Thursday morning before your arrival (at the latest) in order to allow for printing and posting to the website.

You are also encouraged to e-mail the schedule to your participants. BIRS provides the option of an electronic mail list in order to facilitate communications with your participants. When you login to the Organizer Interface at https://www.birs.ca/orgs, you will be prompted to create an electronic mail list for your workshop. Click "Yes" to create one and receive instructions, or "No" to decline. If you would like more information about our electronic mail lists, please e-mail help@birs.ca.

Sunday

16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
17:30–19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall (if desired) Beverages and a small assortment of snacks are available on a cash honor system.

Monday

7:00–8:45	Breakfast
8:45–9:00	Introduction and Welcome by BIRS Station Manager, TCPL
9:00–9:40	M. Salo
9:45–10:25	C. Croke
	Coffee Break, TCPL - 10:30 am
10:50–11:30	A. Vasy
11:30–13:00	Lunch
13:00–14:00	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:00	Group Photo; meet in foyer of TCPL (photograph will be taken outdoors so a jacket might be required).
14:20–15:00	Y. Yang
	Coffee Break, TCPL - 15:00
15:20–16:00	A. Tamasan
16:10–17:00	H. Zhou
17:30–19:30	Dinner

Tuesday

7:00–9:00	Breakfast
9:00–9:40	M. Lassas
9:45–10:25	S. Alexakis
	Coffee Break, TCPL - 10:30
10:50–11:30	K. Krupchyk
11:30–13:30	Lunch
14:00–14:40	R. Felea
	Coffee Break, TCPL - 14:45.
15:00–15:40	F. Monard
15:45–16:25	K. Datchev
17:30–19:30	Dinner

Wednesday

7:00–9:00	Breakfast
9:00-9:40	M. Eastwood
9:45–10:25	G. Besson
	Coffee Break, TCPL - 10:30
10:50–11:30	M. Dunajski
11:30–13:30	Lunch
	Free Afternoon
17:30–19:30	Dinner

Thursday

7:00–9:00	Breakfast
9:00–9:40	A. Nachman
9:45–10:25	L. Mason
	Coffee Break, TCPL - 10:30
10:50–11:30	M. de Hoop
11:30–13:30	Lunch
14:00–14:40	A. Strohmaier
	Coffee Break, TCPL - 14:45
15:00–15:40	L. Oksanen
15:45–16:25	S. Zelditch
17:30–19:30	Dinner

Friday	
7:00–9:00	Breakfast
9:00–	Free morning for discussions Coffee Break, TCPL - 10:00
11:30–13:30	Lunch
Checkout by 12 noon.	

** 5-day workshop participants are welcome to use BIRS facilities (BIRS Coffee Lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

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ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **Spyros Alexakis** (University of Toronto)
Title: *Unique continuation from infinity for linear waves*
Abstract: We consider the problem of unique continuation from infinity for linear waves on a curved background. We derive new Carleman estimates from infinity for wave operators on Minkowski, Scharzschild and Kerr space-times and certain perturbations thereof, which allow us to conclude that solutions of a wave equation vanishing to infinite order on suitable portions of future and past null infinity imply that the solution itself vanishes in an open region of the spacetime. Surprisingly, the result in Schwarzschild (and Kerr) is stronger than the one in Minkowski. We show that our results are sharp (in particular infinite order vanishing is necessary). These results are motivated by questions in General Relativity.

Speaker: **Gérard Besson** (Université Grenoble, CNRS)
Title: *On open 3 manifolds*
Abstract: I will present some open questions on the geometry of open 3-manifolds together with results some of which were obtained using the non compact version of Perelman's Ricci flow with surgery.

Speaker: **Chris Croke** (University of Pennsylvania)
Title: *Scattering and Lens rigidity*
Abstract: We will consider compact Riemannian manifolds M with boundary N. We let IN be the unit vectors to M whose base point is on N and point inwards towards M. Similarly we define OUT. The scattering data (loosely speaking) of a Riemannian manifold with boundary is map from IN to OUT which assigns to each unit vector V of IN a the unit vector W in OUT. W will be the tangent vector to the geodesic determined by V when that geodesic first hits the boundary N again. This may not be defined for all V since the geodesic might be trapped (i.e. never hits the boundary again). A manifold is said to be scattering rigid if for any Riemannian manifold Q with boundary isometric to N and with the same scattering data the boundary isometry extends to an isometry of the manifolds. The lens data includes not only the scattering data but also the lengths of the geodesics. In this talk we will discuss some recent results on the scattering (and lens) rigidity problems. One thing we will discuss is recent work of my graduate student Haomin Weh on the relation between scattering data and lens data for surfaces.

There are a number of manifolds that are known to be lens rigid and there are examples that are not scattering or lens rigid. All of the known examples of non-rigidity have trapped geodesics in them. In this talk we will see that the flat solid torus is scattering rigid. This is the first global scattering (or lens) rigidity result for a manifold that has a trapped geodesic. The main issue is to show that the unit vectors tangent to trapped geodesics in any such Q have measure 0 in the unit tangent bundle of Q. We will also consider scattering rigidity of a number of two dimensional manifolds (joint work with Pilar Herreros) which have trapped geodesics.

Speaker: **Kiril Datchev** (MIT)
Title: *Quantitative limiting absorption principle in the semiclassical limit*
Abstract: We give an elementary proof of Burq's resolvent bounds for long range semiclassical Schrodinger operators. Globally, the resolvent norm grows exponentially in the inverse semiclassical parameter, and near infinity it grows linearly. We also weaken the regularity assumptions on the potential.

Speaker: **Maarten de Hoop** (Purdue University)
Title: *Recovery of the metric of a Riemannian manifold from local boundary diffraction travel times*
Abstract: We consider a region M in $\mathbb{R}^n$ with boundary ∂M and a Riemannian metric g. We analyze the inverse problem, originally formulated by Dix, of reconstructing g from boundary measurements associated with the single scattering of seismic waves in this region. In our formulation, the measurements determine the shape operator of wavefronts outside of M originating at diffraction points within M. Even without diffractors, these measurements can, under appropriate conditions, be obtained from the Neumann-to-Dirichlet map using techniques from Boundary Control theory. In the presence of conormal singularities and single scattering, we discuss a relationship with what seismologists refer to as 'time migration' and a technique to generate local boundary diffraction travel times, in the absence of conjugate points.

We develop an explicit reconstruction procedure which consists of two steps. In the first step we reconstruct the directional curvatures and the metric in what are essentially Riemmanian normal coordinates; in the second step, where we assume that the metric is conformal to the Euclidean metric, we develop a conversion to Cartesian coordinates. We admit the presence of conjugate points. In dimension $n \geq 3$ both steps involve the solution of a system of ordinary differential equations. In dimension $n = 2$ the same is true for the first step, but the second step requires the solution of a Cauchy problem for an elliptic operator which is unstable in general.

Joint research with S.F. Holman, E. Iversen, M. Lassas and B. Ursin.

Speaker: **Maciej Dunajski** (University of Cambridge)
Title: *A problem of Roger Liouville*
Abstract: Cover an open set of a plane with curves, one curve through each point in each direction. How can you tell whether these curves are the unparametrised geodesics of some metric? This inverse problem gives rise to a certain closed system of partial differential equations and hence to obstructions to finding such a metric. I shall present a twistor inspired solution obtained jointly with Robert Bryant and Mike Eastwood. I will also review a more recent work aiming to solve this problem in dimension three.

Speaker: **Michael Eastwood** (Australian National University)
Title: *The projective rigidity of projective space*
Abstract: In 2002, LeBrun and Mason showed that the only projective structure on real projective two-space with all geodesics being simple closed curves is the standard one. I shall show that the corresponding infinitesimal statement holds in all dimensions. This boils down to identifying the kernel of a suitable X-ray transform. I'll explain how this goes, starting from scratch. I'll also explain how one might approach similar questions on complex projective space. This is joint work with Laurent Stolovitch.

Speaker: **Raluca Felea** (Rochester Institute of Technology)
Title: *Microlocal analysis of SAR with moving objects*
Abstract: We consider four particular cases of Synthetic Aperture Radar imaging with moving objects. In each case we analyze the forward operator F which maps the image to the data and the normal operator F^*F which is used to recover the image. In general, by applying the backprojection operator F^* to the data, artifacts appear in the reconstructed image. We describe these artifacts and show how to microlocally reduce their strength to obtain a better image.

Speaker: **Katya Krupchyk** (University of Helsinki)
Title: *L^p resolvent estimates for elliptic operators on compact manifolds.*
Abstract: We establish uniform L^p estimates for resolvents of elliptic self-adjoint differential operators on compact manifolds without boundary. We also show that the spectral regions in our resolvent estimates are optimal in general. This is joint work with Gunther Uhlmann.

Speaker: **Matti Lassas** (University of Helsinki)
Title: *Inverse problems for non-linear wave equations and the Einstein equation*

Abstract: We consider inverse problem for a non-linear wave equation with a time-depending metric tensor on manifolds. In addition, we study the question, do the observation of the solutions of coupled Einstein equations and matter field equations in an open subset U of the space-time M corresponding to sources supported in U determine the properties of the metric in a larger domain $W \subset M$ containing U. To study these problems we define the concept of light observation sets and show that these sets determine the conformal class of the metric. The results have been done in collaboration with Yaroslav Kurylev and Gunther Uhlmann.

Speaker: **Lionel Mason** (Oxford University)
Title: *Scattering of null geodesics in asymptotically de Sitter 2+1 Einstein-Weyl spaces and holomorphic discs (joint with Claude LeBrun)*
Abstract: I start by discussing integral formulae for the 2+1 wave equation. The scattering of null geodesics through the space-time is encoded in a diffeomporphism of the sphere to itself modulo mobius transformations. We show that given such a diffeomorphism, the Einstein-Weyl space can be reconstructed as the space of homolomorphic discs with boundary on a particular subset of the 2-quadric $\mathbb{C}P^1 \times \mathbb{C}P^1$. This construction also leads to a large data approach to the problem of constructing Zoll metrics on the sphere.

Speaker: **Francois Monard** (University of Washington)
Title: *Recent progress on the explicit inversion of geodesic X-ray transforms*
Abstract: We review recent progress made by the author on some inverse problems involving geodesic X-ray transforms on Riemannian surfaces with boundary. We are concerned with the reconstruction of functions, or more generally, of symmetric solenoidal tensor fields from knowledge of their X-Ray transform. Recalling some results known in the simple case (Fredholm equations for functions and solenoidal vector fields, s-injectivity of the ray transform for tensors of any order), we then explain how to reconstruct other sections of certain bundles (k-differentials for k an integer), which in some cases coincide with solenoidal tensor fields, from knowledge of their ray transform. Such reconstruction formulas take the form of Fredholm equations when the metric is simple. Furthermore, the error is proved to be a contraction when the gaussian curvature is small in C^1 norm, in which case the unknowns can be reconstructed via Neumann series. Second, we present numerical implementation of these formulas, one-shot in the constant curvature case, iterative in the close-to-constant case. We observe that, while the borderline cases where the error operators cease to be contractions are not well known quantitatively, numerics indicate that, on the examples treated, the Neumann series converges for a family of metrics that is arbitrarily close to non-simple.

Speaker: **Adrian Nachman** (University of Toronto)
Title: *Imaging an Anisotropic Conductivity in a Known Conformal Class and Minimal Surfaces*
Abstract: We present recent results on determining an anisotropic conductivity in a known conformal class from knowledge of one current. We'll briefly explain how the data needed can be obtained using Magnetic Resonance Imagers. Utilizing geometric measure theory methods, we show that the corresponding electric potential is the unique solution of a constrained minimization problem with respect to a weighted total variation functional defined entirely in terms of the physical data. Further, we show that the associated equipotential surfaces are area minimizing with respect to a Riemannian metric obtained from the data.

This is joint work with Nicholas Hoell, Robert Jerrard and Amir Moradifam. The experimental results are joint work with Weijing Ma, Nahla Elsaid, Michael Joy and Tim DeMonte.

Speaker: **Lauri Oksanen** (University of Helsinki)
Title: *Inverse problem for the wave equation with a white noise source*
Abstract: We consider the wave equation for a Laplace-Beltrami operator with a stochastic source that has the white noise distribution supported on the boundary of a smooth compact domain M. We prove that the boundary trace of a solution corresponding to a single known realization of the white noise determines the scattering relation of M almost surely under a non-trapping assumption. The talk is based on joint work with Tapio Helin and Matti Lassas, University of Helsinki.

Speaker: **Mikko Salo** (University of Jyväskylä)
Title: *Geometric inverse problems*
Abstract: We discuss a selection of inverse problems related to geometry that are of current interest, pointing out certain connections and open questions.

Speaker: **Alex Strohmaier** (University of Loughborough)
Title: *Analytic properties of resolvent and S-matrix on manifolds of even rank*
Abstract: I will explain an extension of the meromorphic Fredholm theorem to an extended class of functions, namely Hahn holomorphic functions. As an application I will show that the S-matrix as well as the resolvent of the Laplace operator on compactly perturbed even dimensional $\mathbb{R}^n$ as well as on compactly perturbed symmetric spaces of even rank extend to Hahn meromorphic functions near the bottom of the essential spectrum. This, for example, gives a simple explanation of non-accumulation of resonances near zero (joint work with J. Mueller).

Speaker: **Alexandru Tamasan** (University of Central Florida)
Title: *On the Range characterization of data in X-ray tomography*
Abstract: In this work I will explicit a Hilbert transform associated with A-analytic maps and characterize the range of the attenuated X-ray transform on strictly convex planar sets. As an application, I will explain how X-ray data of an isotropic source can also be obtained from the Doppler data of a solenoidal vector field. This is work done by Kamran Sadiq in collaboration with me.

Speaker: **András Vasy** (Stanford University)
Title: *The local inverse problem for the geodesic X-ray transform and boundary rigidity in a conformal class (a.k.a. of sound speed)*
Abstract: In this talk, based on joint work with Gunther Uhlmann, I consider the geodesic X-ray transform on a Riemannian manifold with boundary. I will explain how, under a convexity assumption on the boundary, one can invert the local geodesic X-ray transform in a stable manner. Here the local transform means that one would like to recover a function in a suitable neighborhood of a point on the boundary of the manifold given its integral along geodesic segments that stay in this neighborhood (i.e. with both endpoints on the boundary of the manifold). Our method relies on the introduction of an artificial boundary at which the 'microlocal normal operator' we construct is (essentially) a scattering pseudodifferential operator in the sense of Melrose's scattering calculus. I will then also explain how, under the assumption of the existence of a strictly convex family of hypersurfaces foliating the manifold, this gives immediately the solution of the global inverse problem by a stable 'layer stripping' type construction. Finally, based on joint work with Plamen Stefanov and Gunther Uhlmann, I will explain how this approach can be used to determine a Riemannian metric in a given conformal class given the restriction of its distance function to the boundary.

Speaker: **Yang Yang** (University of Washington)
Title: *An inverse boundary value problem for the transport equation on a magnetized Riemannian manifold*
Abstract: We study the reconstruction of the attenuation and absorption coefficients in stationary linear transport equation from knowledge of albedo operator in dimension $n > 2$ on a Riemannian manifold in the presence of a magnetic field. We show the forward boundary value problem is well-posed under two types of sub-critical conditions. We obtain uniqueness and non-uniqueness results of the reconstruction under some restrictions. Finally, stability estimates are also established.

Speaker: **Steve Zelditch** (Northwestern University)
Title: *Manifolds with maximal eigenfunction growth*
Abstract: The pointwise Weyl law implies that the sup norm of an L2 normalized eigenfunction of the Laplacian of eigenvalue lambda on a compact Riemannian manifold (M, g) is bounded by $C_b \lambda^{(n-1)/4}$. On the round sphere and on any surface of revolution, the zonal eigenfunctions achieve this bound. We say that (M, g) has maximal eigenfunction growth if it has a sequence of eigenfunctions attaining the bound.

In 2002 Sogge and I proved that such a manifold must have a self-focal point, i.e a point p for which a positive measure of the geodesics starting at p loop back to p at some time (measure in the usual sense on S_p^*M). However this is not sharp since umbilic points of ellipsoids are self-focal but the eigenfunctions do not have maximal sup norms. In recent work (still being written), Sogge and I prove that if (M, g) is real analytic and has maximal eigenfunction growth, then the first return map on S_p^*M for geodesic loops must have an invariant L^2 function. This is in a certain sense a sharp result because there is a converse for quasi-modes.

Speaker: **Hanming Zhou** (University of Washington)
Title: *The boundary rigidity problem in the presence of a magnetic field and a potential*
Abstract: In this paper, we consider a compact Riemannian manifold with boundary, endowed with a magnetic potential α and a potential U. For brevity, this type of systems are called $\mathcal{MP}$-systems. On simple $\mathcal{MP}$-systems, we consider both the boundary rigidity problem and scattering rigidity problem. We show that these two problems are equivalent on simple $\mathcal{MP}$-systems. Unlike the cases of geodesic or magnetic systems, knowing boundary action functions or scattering relations for only one energy level is insufficient to uniquely determine a simple $\mathcal{MP}$-system, even under the assumption that we know the restriction of the system on the boundary ∂M, and we provide some counterexamples. These problems can only be solved up to gauge equivalence. We prove rigidity results for metrics in a given conformal class, for simple real analytic $\mathcal{MP}$-systems and for simple two-dimensional $\mathcal{MP}$-systems.