BIRS 13w5037: Interactions of gauge theory with contact and symplectic topology in dimensions 3 and 4

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1 Introduction

This workshop focussed on interactions between contact and symplectic geometry, gauge theory, and low-dimensional topology. Each of these subjects is a very active area of current research, and the interactions between them have led to breakthroughs on long standing problems. Our workshop was a follow-up to the BIRS events *Interactions of geometry and topology in low dimensions* (March 2007), *Interactions of Geometry and Topology in dimensions 3 and 4* (March 2009), and *Interactions between contact and symplectic topology and gauge theory in dimensions 3 and 4* (March 2011). Because the fields are extremely active and progressing at a rapid pace, there were many new and interesting results presented at the workshop; a number of new projects were initiated at the workshop as well.

Participants were selected from among the world experts in these areas. The organizers made an effort to balance interest between the different research areas and to ensure that the most important current trends were well represented. There was a good mixture of well-established researchers (e.g. Fintushel, Honda, Hutchings, Lisca, Matic, Stipsicz) and younger talented mathematicians (e.g. Bloom, Kutluhan, Petkova, Zarev). This stimulated many lively discussions and enabled a rich exchange of ideas in all directions.

2 Overview of the Field

Over the last several decades it has become clear that the topology of manifolds in dimensions 3 and 4 is subtly and beautifully intertwined with various flavors of geometry (hyperbolic, symplectic and contact), as well as with ideas from physics, such as gauge theories and mirror symmetry. Collaborations among people working in these diverse areas have exploded over the last few years, resulting in the solutions to venerable conjectures in topology as well as the birth of entire new sub-fields and perspectives. While much-deserved spotlight has been cast lately on the solution to many of Thurston's conjectures about 3-manifolds (using a very different set of techniques), the topological results recently obtained using inputs from gauge theory and contact and symplectic topology are no less remarkable: the characterization of fibered knots by Heegaard-Floer homology, the proof of Property P and other characterizations of the unknot, the solution to the Weinstein conjecture (and generalizations of it), and a vastly improved understanding of exotic smooth and symplectic structures on 4-manifolds. Critical to these developments has been the information provided by a vast array of new invariants whose definitions were motivated by gauge theory and topological quantum field theory. These invariants – Donaldson-Floer, Seiberg-Witten, Ozsváth-Szabó, Khovanov homology or Embedded Contact Homology to name a few – have intriguing relations among them, and a better understanding of these will lead to significant progress not only in topology but also in contact and symplectic geometry and physics. An even more promising direction is the interplay between these invariants and more constructive approaches to low-dimensional manifolds – open book decompositions of contact 3-manifolds, symplectic fillings, Lefschetz fibrations, surgery constructions among many others. This interaction between powerful invariants and constructive methods is now more than ever a major driving force in the subject. We briefly survey some of the most active directions in the field.

Unification of invariants: There has been much recent progress in showing that various invariants defined in starkly different ways actually compute the same thing. This has allowed for many striking results. For example, Taubes and Hutchings' program for identifying Seiberg-Witten Floer theory with Embedded Contact Homology has led Taubes to a proof of the 3-dimensional Weinstein Conjecture: for any compact oriented 3-manifold M and α a contact 1-form on M, the vector field that generates the kernel of the 2-form $d\alpha$ has at least one closed integral curve. Further developments in identifying the two theories have allowed for extensions and refinements of the Weinstein conjecture. Another exciting and spectacular recent instance of unification of invariants is the work of Kutluhan, Lee and Taubes relating Seiberg-Witten Floer homology to Heegaard-Floer homology, and that of Colin, Ghiggini and Honda relating Embedded Contact homology to Heegaard-Floer homology. The ramifications of such a convergence of theories are as yet unknown, but expectations are very high, considering the spectacular results that arose out of Hutchings and Taubes' program, or the manner in which progress on understanding the relation between instanton Floer homology and other Floer-type invariants allowed Kronheimer and Mrowka to solve the famous Property P conjecture: surgery on a non-trivial knot in S^3 always yields a manifold with non-trivial fundamental group.

Another important goal is to understand the relationship between the various Floer-type invariants for knots and 3-manifolds and Khovanov homology. Khovanov homology was constructed as a categorification of the Jones polynomial of knots and its nature is very algebraic (rather than geometric). Ozsváth and Szabó showed that Khovanov's homology of a link is related to the Heegaard Floer homology of its double branched cover by a spectral sequence. The progress accomplished on combinatorial Heegaard-Floer homology has already enabled Manolescu and Ozsváth to explore further the relationship between the two theories, through the notion of homological thinness. This should remain an active area of research for the coming years, as it also relates to the link invariants constructed by Seidel and Smith using the symplectic geometry of nilpotent slices, and recent progress in understanding the structure of Fukaya categories suggests new lines of approach. In another direction, Kronheimer and Mrowka have established an intriguing relationship between Khovanov homology and knot instanton Floer homology, again via a spectral sequence, and their new work builds on their foundational results on singular instantons to affirmatively answer the question of whether Khovanov homology detects the unknot. (The same question for the Jones polynomial remains open.) Finally, recent developments arising from theoretical physics, such as Witten's work recasting Khovanov homology in terms of 4-dimensional gauge theory, and Aganagic and Vafa's work relating deformations of the A-polynomial to mirror symmetry, while not yet well understood by mathematicians, most likely will serve as a catalyst for exciting new developments.

TQFTs and the algebraic structure of invariants: It appears that many invariants of 3-manifolds or links can be extended to invariants of 3-manifolds with boundary or tangles, forming "extended topological field theories". While this was a built-in feature of Khovanov homology, the discovery of similar structures in Floer-type invariants is more recent. While Juhász's "sutured" Heegaard-Floer homology already led to exciting applications (such as detecting fiberedness), the introduction by Lipshitz, Ozsváth and Thurston of "bordered Floer homology" has led to a much richer algebraic picture of Heegaard-Floer theory. This not only opens new perspectives for computations (see below), but also provides insight into the structure of Heegaard-Floer theory and its relation to Khovanov homology. Recent work of Douglas and Manolescu suggests that the story extends even further, to surfaces with boundary and 3-manifolds with corners. Bordered and sutured versions of other Floer-type invariants have not yet been as thoroughly developed, but appear full of promise and should be a hotbed of future activity; for instance, sutured instanton Floer homology plays a key role in Kronheimer and Mrowka's work mentioned above. In a different direction, recent progress on algebraic structures in Fukaya categories and Legendrian contact homology (by Bourgeois, Ekholm, Eliashberg; Abouzaid, Seidel; Ganatra) is likely to have significant applications in low-dimensional topology.

Developing computational techniques: Most of the invariants arising from gauge theory and contact / symplectic topology involve spaces of solutions to geometric PDEs, which makes explicit computations particularly difficult. In the past few years there has been dramatic progress in several directions. The problem of combinatorially constructing Heegaard-Floer groups without counting pseudo-holomorphic curves has taken a very promising turn as knot Floer homology was given a purely combinatorial interpretation by Manolescu, Ozsváth and Sarkar. This has already led to progress in the classification of transverse knots in contact manifolds as well as work by Ng on bounds for the Thurston-Bennequin invariant of Legendrian knots. While combinatorial Heegaard-Floer homology continues to develop at a spectacular pace, Lipshitz, Ozsváth and Thurston's bordered Floer homology offers a different approach to computing Heegaard-Floer homology by decomposing a 3-manifold into a sequence of elementary cobordisms between oriented surfaces. In a different direction, a result of Bourgeois, Ekholm and Eliashberg gives a way to compute the contact homology of a contact manifold obtained from another one by Legendrian surgery. This construction is particularly simple in dimension 3 where there is essentially an algorithm for writing down the contact homology of a contact 3-manifold in terms of its Legendrian surgery description. With recent progress on the classification of Legendrian knots this could yield a flood of information about contact 3-manifolds.

Exploiting interactions between constructions and invariants: The emergence of invariants of embeddings from contact homology is another very promising avenue of research. Given a manifold embedded in Euclidean space, one can look at its unit conormal bundle in the unit cotangent bundle of Euclidean space to get a Legendrian submanifold. The contact homology of this Legendrian gives an invariant of the original embedding. Ekholm, Etnyre, Ng and Sullivan have recently given a rigorous computation of this invariant for knots in 3-space and shown it is equal to a very powerful combinatorial invariant defined by Ng. This new invariant has surprising connections with many classical knot invariants and seems quite strong; it also appears to give new information about transverse knots in contact manifolds. Furthermore, knot contact homology appears to be related to deformations of the A-polynomial via a very recent physical construction of Aganagic and Vafa using mirror symmetry and gauge-string dualities. Exploring the properties of knot contact homology and extending it to other situations should be a fruitful line of research for years to come. Moreover, contact homology is only the tip of the iceberg of Symplectic Field Theory (SFT). This theory, introduced by Eliashberg, Givental and Hofer, has been an inspirational and driving force in symplectic geometry for over a decade now, and recent advances in its rigorous definition suggest that a precise formulation of the relative version will emerge in the coming years. In spite of recent progress by Bourgeois, Ekholm and Eliashberg, there is still much work to do to extract computable information that can be used in applications. In the end though, it is expected that the theory will be invaluable and provide more invariants, not only for Legendrian knots in contact manifolds and Lagrangian cobordisms between them, but also for topological knots by considering the conormal construction mentioned above. Evidence for this comes from Abouzaid's recent demonstration that the symplectic geometry of cotangent bundles can be used to distinguish exotic smooth structures on spheres of high dimension, and further results of his about exact Lagrangians in cotangent bundles. Can such ideas be exploited in dimension 4 to attack the smooth Poincaré conjecture?

Indeed, one of the driving open problems in 4-dimensional topology is the smooth Poincaré conjecture. Recent work of Freedman, Gompf, Morrison and Walker reveals that Khovanov homology can be used to give an obstruction to specific handle decompositions of homotopy 4-spheres being the actual 4-sphere. This development sparked considerable interest as a method for identifying counterexamples to the smooth 4-d Poincaré conjecture, assuming they exist! Akbulut and Gompf subsequently proved that many potential counterexamples to the Poincaré conjecture are actually the standard sphere. Another approach to such problems is to try to build exotic smooth structures on 4-manifolds that are as "small" as possible. After Freedman and Donaldson's work in the early 1980's gave the first examples of exotic smooth structures, and Kotschick's result for $\mathbb{CP}^2\#8\overline{\mathbb{CP}}^2$, there was little progress until J. Park's breakthrough in 2003. There has since been a flurry of activity on existence of exotic smooth structures on small symplectic 4-manifolds by different teams of researchers (Akhmedov-Park, Baldridge-Kirk, and Fintushel-Stern-Park). The advances are made by exploiting a certain tension between constructions and invariants. Using clever new cut-and-paste constructions such as knot or rim surgery or Luttinger surgery along particularly well-chosen embedded surfaces, together with an intimate understanding of their effect on Seiberg-Witten invariants, one can often deduce the presence of several (generally infinitely many) exotic smooth structures (only some of which carry symplectic forms). It is reasonable to expect further progress on this important problem for other small symplectic 4-manifolds (e.g. $\mathbb{CP}^2\#\overline{\mathbb{CP}}^2$ or $S^2 \times S^2$) via this approach.

Contact structures on 3-manifolds and Heegaard-Floer theory: The existence of tight contact structures on 3-manifolds has been an important subject of investigation for a long time, and one on which significant progress has been made in the last decade. This fundamental question has potential applications not only to contact geometry but also to low-dimensional topology and dynamics. It also illustrates very well the natural interactions between the invariants described above and constructive methods. After incremental steps by a number of mathematicians, Lisca and Stipsicz have completely classified which Seifert fibered 3-manifolds admit a tight contact structure. Their approach relies heavily on Heegaard-Floer homology through a non-vanishing criterion for the contact invariant of Ozsváth and Szabó. On the other hand, geometric methods reminiscent of the theory of normal surfaces of Haken and Kneser have enabled Colin, Giroux and Honda to establish general results such as: (1) Every 3-manifold has only finitely many homotopy classes of 2-plane fields which carry tight contact structures. (2) Every closed atoroidal 3-manifold carries finitely many isotopy classes of tight contact structures. One of the outstanding and fundamental questions here is the understanding of tight contact structures on hyperbolic 3-manifolds. Work of Kazez, Honda and Matić has led to a characterization of tight 3-manifolds in terms of right-veering diffeomorphisms, a step which should make calculations in contact homology and Heegaard Floer homology manageable, but thus far the condition of a manifold being hyperbolic has not been properly understood in this context. It is hoped that the current wide-ranging technology will help elucidate the problem of tight structures on 3-manifolds.

3 Highlights from the Workshop

A variety of geometric approaches to low-dimensional topology were represented, and several high-profile recent results in the field were featured prominently in the workshop. Some key very recent developments that were presented at the workshop include: the construction of new TQFTs (topological quantum field theories) providing invariants of 3- and 4-manifolds such as bordered monopole Floer homology, the embedded contact homology TQFT, or the 2+1+1-dimensional TQFTs built from quilted Floer theory; the emergence of new knot invariants such as quantum-deformed A-polynomials and knot contact homology and the conjectured relations among them; and new perspectives on contact topology coming from generalizations of Heegaard-Floer homology and the flexibility properties of "loose" Legendrian knots.

The TQFT-style properties of invariants of 3- and 4-manifolds (i.e., the ability to associate invariants to manifolds with boundary or cobordisms, and to recover invariants of closed manifolds from those of simpler pieces via gluing theorems) have been a major source of progress in low-dimensional topology. Three remarkable new instances of such TQFT-type constructions were presented at the workshop:

- Jonathan Bloom and John Baldwin presented their work in progress constructing "bordered monopole Floer homology", an extension of monopole Floer homology to Riemann surfaces and 3-manifolds with boundary that ultimately should enrich Seiberg-Witten theory into a 2+1+1-dimensional TQFT.

- Michael Hutchings presented recent results about the TQFT properties of embedded contact homology, an advance which furthers our understanding of the relation between Seiberg-Witten theory and symplectic cobordisms between contact 3-manifolds.

- Katrin Wehrheim presented a framework for the construction of 2+1+1-dimensional TQFTs using Gay and Kirby's Morse 2-functions [GK] and the functoriality properties of quilted Floer theory (the so-called "symplectic category"); this framework, which expands on her earlier work with Woodward [WW], is expected to ultimately provide symplectic interpretations of Donaldson and Seiberg-Witten invariants.

Tobias Ekholm and Lenny Ng gave two talks presenting the latest results of their very recent collaboration with physicists Mina Aganagic and Cumrun Vafa [AENV], aiming to provide a mathematical understanding of recent physical conjectures made by the latter [AV]. These conjectures relate knot theory, gauge theory, and string theory, by constructing a new quantum deformation of the A-polynomial for knots and links which can be understood in terms of open string theory and mirror symmetry. Aganagic and Vafa's quantum-deformed A-polynomial appears to coincide with another knot invariant recently defined by Ekholm and Ng, the so-called "augmentation polynomial" for knot contact homology, which arises out of symplectic field theory for the conormal bundle of the knot. This remarkable and unexpected development at the interface of knot theory, symplectic geometry, and mathematical physics will certainly lead to new insights in all three subjects.

Vincent Colin presented new work in progress with Ko Honda (who was also at the workshop) that extends Heegaard-Floer homology from 3-manifolds to higher dimensional contact manifolds. This is a remarkable development, confirming that the deep connections between Heegaard-Floer theory and contact topology in dimension 3 are by no means accidental and suggesting that the recent advances made in dimension 3 can perhaps be leveraged in higher dimensions. On the other hand, Olga Plamenevskaya reported on recent joint work with Emmy Murphy, Klaus Niederkruger, and Andras Stipsicz (who was also present), showing that higher-dimensional contact manifolds which contain a certain geometric structure (the "plastikstufe") exhibit flexibility properties that largely reduce their contact topology to classical homotopy theory [MNPS].

There were a number of other remarkable talks on topics ranging from the applications of Heegaard-Floer theory to the study of 3-manifolds and knots in them to gauge-theoretic invariants of smooth 4-manifolds and much more.

4 Featured Talks

What follows is a list of the 22 talks featured at the workshop. The central themes were (some talks fit into more than one theme):

- **Heegaard-Floer and monopole Floer homologies and applications.** *(Talks 1, 2, 8, 10, 11, 13, 16, 18, 21)*
- **Knots and invariants.** *(Talks 8, 9, 11, 13, 14, 15, 21)*
- **4-manifolds and their invariants.** *(Talks 3, 5, 6, 10, 18, 19, 20, 22)*
- **Contact manifolds and Legendrian submanifolds.** *(Talks 4, 12, 14, 15, 17)*
- **Lagrangian Floer theory.** *(Talks 7, 22)*

Below is a detailed list of speakers, titles, and brief descriptions of their talks.

1. **Jonathan Bloom** (MIT) *A bordered monopole Floer theory I*
2. **John Baldwin** (Boston College) *A bordered monopole Floer theory II*
   This series of two talks discusses work in progress toward constructing a gauge theoretic analogue of the Fukaya category and monopole Floer theoretic invariants of bordered 3-manifolds. Our construction associates an A-infinity category to a surface, an A-infinity functor to a bordered 3-manifold, and an A-infinity natural transformation to a 4-dimensional cobordism of bordered 3-manifolds. The first talk describes the basic ideas that go into constructing these categories, functors, and natural transformations. The second talk focuses more on the topological aspects of our construction. In particular, we describe a finite set of bordered handlebodies which "generate" our category. The morphism spaces for this generating set define a finitely generated A-infinity algebra. A bordered 3-manifold then gives rise to a module over this algebra, and a cobordism of bordered 3-manifolds defines a map of modules. We describe how one proves a pairing theorem and how the homology of our algebra is related to Khovanov's H^n algebra.

3. **R. Inanc Baykur** (MPIM Bonn) *Topological complexity of symplectic 4-manifolds and Stein fillings*
   Following the ground-breaking works of Donaldson and Giroux, Lefschetz pencils and open books have become central tools in the study of symplectic 4-manifolds and contact 3-manifolds. An open question at the heart of this relationship is whether or not there exists an a priori bound on the topological complexity of a symplectic 4-manifold, coming from the genus of a compatible Lefschetz pencil on it, and a similar question inquires if there is such a bound on any Stein filling of a fixed contact 3-manifold, coming from the genus of a compatible open book. We will present our solutions to both questions, making heroic use of positive factorizations in surface mapping class groups of various flavors. This is joint work with J. Van Horn-Morris.

4. **Vincent Colin** (Univ. Nantes) *An extension of Heegaard Floer homology to higher dimensions*
   In dimension three, Heegaard Floer homology can be computed from the page and the monodromy of an open book decomposition supporting a contact structure. In a joint work in progress with Yasha Eliashberg and Ko Honda, we explain how to extend the definition of $\widehat{\mathrm{HF}}$ to contact manifolds of arbitrary odd dimension.

5. **David Gay** (U. Georgia) *Morse 2-functions on and trisections of 4-manifolds*
   I'll outline how to use Morse 2-functions (stable maps to dimension 2) to get a perfect analogue in dimension 4 of the existence and uniqueness of Heegaard splittings in dimension 3. The analogue of a Heegaard splitting of a 3-manifold is a trisection of a 4-manifold, a splitting into three boundary connect sums of $S^1 \times B^3$'s suitably glued together. The parallels with the Heegaard theory is striking enough, but there is also a strong connection with open book decompositions of 3-manifolds. In fact, extending the definition of a trisection to dimension 3 in the obvious way, a trisection of a 3-manifold is precisely an open book decomposition. There is also an obvious definition of a k-section of an n-manifold, with the most interesting cases being $k = n - 1$ and $k = n$. Sadly, we have no higher dimensional existence or uniqueness results, but the idea is still intriguing. This is joint work with Rob Kirby.

6. **Eric Harper** (McMaster) *Instanton homology of corks W_n*
   The corks W_n recently constructed by Akbulut and Yasui give rise to exotic structures on smooth 4-manifolds via an involution of their boundary homology 3-sphere. The cork W_1, also known as Akbulut's cork, gave rise to the first example of a diffeomorphism acting non-trivially on the instanton homology of an irreducible homology 3-sphere. In this talk, we will show that each cork W_n has a diffeomorphism on the boundary that acts non-trivially on the instanton homology and we will compute the instanton homology groups in the cases $n = 2, 3$.

7. **Andriy Haydys** (U. Bielefeld) *Fukaya-Seidel category and gauge theory*
   In this talk I will outline a new construction of the Fukaya-Seidel category, which is associated to a symplectic manifold equipped with a compatible almost complex structure J and a J-holomorphic Morse function. For the infinite dimensional case of the holomorphic Chern-Simons functional this construction conjecturally associates a Fukaya-Seidel-type category to a smooth three-manifold.

8. **Matt Hedden** (Michigan State) *Taut foliations, left-orderability, and L-spaces*
   I will discuss joint work with Adam Levine on the Floer homology of manifolds obtained by gluing together knot complements. In the case that the knots are torus knots, we obtain certain graph manifolds for which the questions of existence of left-orderings of the fundamental group and of taut foliations can be dealt with explicitly.

9. **Chris Herald** (U. Nevada Reno) *The pillowcase and perturbations of traceless representations of knot groups*
   I'll discuss joint work with Matt Hedden and Paul Kirk which aims to explicitly understand the generating set for Kronheimer and Mrowka's singular instanton chain complex for a knot. A priori, the set of generators is not clear, since the relevant flat moduli space contains positive dimensional components. I'll show how to explicitly perturb the Chern-Simons functional, which is Bott-Morse but not Morse, to obtain a Morse function, ensuring a finite set of generators for the complex, whilst preserving the desirable feature of being able to identify generators in terms of particular representations of the (augmented) knot group. Along the way, I'll describe two different surfaces along which we can decompose the complement of a knot with the earring, as in the KM singular instanton configuration space. Each gives rise to a Lagrangian intersection picture in a pillowcase, i.e., a fiber product descriptions, for the flat moduli space of singular connections and sheds light on the chain complex generators.

10. **Michael Hutchings** (UC Berkeley) *Embedded contact homology as a (symplectic) field theory*
    We explain how ECH can be extended to a kind of TQFT which recovers the Seiberg-Witten invariants of closed symplectic four-manifolds cut along contact type hypersurfaces. While the construction uses Seiberg-Witten theory, it has applications to contact geometry. For example, one corollary is that the contact invariant (in ECH, HM, or HF) is functorial under strong symplectic cobordisms.

11. **Cagatay Kutluhan** (Harvard) *Holonomy filtration and knots*
I will describe a $\mathbb{Z} \oplus \mathbb{Z}$-filtered monopole knot homology isomorphic to Ozsvath-Szabo's knot Floer homology.

12. **Paolo Lisca** (Pisa) *Stein fillable contact 3-manifolds and positive open books of genus one*
A two-dimensional open book (S, h) determines a closed, oriented three-manifold $Y(S, h)$ and a contact structure $C(S, h)$ on $Y(S, h)$. The contact structure $C(S, h)$ is Stein fillable if h is positive, i.e. h can be written as a product of right-handed Dehn twists. Work of Wendl implies that when S has genus zero the converse statement holds, that is if $C(S, h)$ is Stein fillable then h is positive. On the other hand, Wand as well as Baker, Etnyre and Van Horn-Morris constructed counterexamples to the converse statement with S of genus two. In this talk I will present a proof of the converse statement under the assumption that S is a one-holed torus and $Y(S, h)$ is a Heegaard Floer L-space. If time permits I will describe a (still conjectural) classification up to diffeomorphisms of the Stein fillings of $(Y(S, h), C(S, h))$, where S is a one-holed torus, h is positive and $Y(S, h)$ is a Heegaard Floer L-space.

13. **Thomas Mark** (Virginia) *Floer homology and the fractional Dehn twist coefficient*
An open book decomposition of a 3-manifold Y is equivalent to a choice of fibered link embedded in Y. For a fibered knot (i.e. a one-component link), the monodromy of the fibration on the complement gives rise to a rational number called the fractional Dehn twist coefficient. This number measures the twisting of the monodromy around the boundary of the fiber surface. I will describe how the Heegaard Floer homology of a 3-manifold Y provides bounds for the fractional Dehn twist coefficient of any open book decomposition of Y with connected binding, and discuss applications to knot theory and contact topology. One consequence is the existence of an a priori upper bound on the absolute value of the twist coefficient of any fibered knot in a given 3-manifold. This is joint work with Matthew Hedden.

14. **Lenny Ng** (Duke) *Topological strings and knot contact homology I*

15. **Tobias Ekholm** (Uppsala) *Topological strings and knot contact homology II*
Knot contact homology is an invariant of topological knots and links given by the Legendrian contact homology of the conormal lift. Recently, knot contact homology, or more specifically a subsidiary invariant called the augmentation polynomial, has been connected in surprising ways to other areas of knot theory and even string theory. I will introduce the augmentation polynomial (and its link generalization, the augmentation variety) and present some of its properties. These include an interpretation as a "stable A-polynomial" derived from certain representations of the knot group, and a conjectural interpretation as a recurrence relation for colored HOMFLY polynomials.
After a very brief description of some aspects of topological string theory and its relation to Chern-Simons theory, we discuss possible geometric explanations of the recently observed relation between knot contact homology and open topological strings. A key role is played by Lagrangian fillings of the conormal lift of a link that we will use in several ways. For example, we discuss a certain class of non-exact fillings that when equipped with a flat U(1)-connection induce augmentations and thereby parameterize branches of the augmentation variety that agree with branches of the corresponding variety defined through topological string theory.

16. **Ina Petkova** (Rice) *An absolute $\mathbb{Z}/2$ grading on bordered Floer homology*
Bordered Floer homology is a TQFT-type generalization of Heegaard Floer homology to 3-manifolds with boundary. It is related to HF homology via a gluing formula. In its original form, this formula only recovers the homological HF grading as a relative grading, and within a choice of spin^c structures for the bordered manifolds.
After a brief description of bordered Floer homology and its original grading by sets with a Heisenberg group action, I will discuss how to obtain an absolute $\mathbb{Z}/2$ grading, simultaneously for all spin^c structures, that recovers the absolute $\mathbb{Z}/2$ grading after gluing. As a motivating example, with this $\mathbb{Z}/2$ grading, bordered Floer homology categorifies the kernel of the homology map induced by the inclusion of the boundary into the 3-manifold.

17. **Olga Plamenevskaya** (Stony Brook) *Looking for flexibility in higher-dimensional contact manifolds*
By a classical result of Eliashberg, contact manifolds in dimension 3 come in two flavors: tight (rigid) and overtwisted (flexible). In higher dimensions, a class of flexible contact structures is yet to be found. However, some attempts to generalize the notion of an overtwisted disk have been made. One such object is a "plastikstufe" introduced by Niederkruger following some ideas of Gromov. We show that under certain conditions, the presence of plastikstufe leads to the following flexibility phenomena: 1) Legendrian knots become "loose", i.e. satisfy an h-principle, and 2) non-isotopic contact structures become isotopic after connect-summing with a manifold containing a plastikstufe. This is based on previous work of Murphy and Cieliebak-Eliashberg. (Joint with E. Murphy, K. Niederkruger, and A. Stipsicz.)

18. **Daniel Ruberman** (Brandeis) *Embeddings of non-orientable surfaces in* $M^3 \times I$
We use the correction terms from Heegaard Floer homology to obtain obstructions to embedding closed, non-orientable surfaces in 3- and 4-dimensional manifolds, focusing on embeddings in a lens space or the product of a lens space and an interval. For instance, we show that if the projective plane or the Klein bottle embeds nontrivially in $L(p,q) \times I$, then it must also embed in $L(p,q)$; it is reasonable to conjecture that the same is true for nonorientable surfaces of any genus. This is joint work with Adam Levine and Saso Strle.

19. **Nikolai Saveliev** (Univ. Miami) *Index theory of the de Rham complex on manifolds with periodic ends*
The analytic index of the de Rham complex on a compact orientable manifold is known to equal its Euler characteristic; the same holds for manifolds with product ends, for a properly understood L^2 index. We show that this is no longer true for more general manifolds with periodic ends, by providing an explicit formula for the difference between the L^2 index of the de Rham complex and the Euler characteristic of the manifold in terms of topology of the end. This research is a continuation of the joint project with T. Mrowka and D. Ruberman studying index theory on manifolds with periodic ends and its applications in low-dimensional topology and gauge theory.

20. **Stefano Vidussi** (UC Riverside) *On the topology of SCY 4-manifolds*
I will discuss some new (or gently used) results and conjectures on the structure of symplectic 4-manifolds with trivial canonical class, focusing on those with positive first Betti number.

21. **Liam Watson** (UCLA) *Bordered Heegaard Floer homology and the Alexander module*
The Alexander module is an invariant for knots arising from the module structure of the universal abelian cover of the knot exterior. This is a natural setting in which to define the Alexander polynomial, and as such it is natural to ask how the Alexander module arises in Heegaard Floer theory. Bordered Heegaard Floer homology provide the right tool to answer this question. This talk will explain how bordered invariants determine the Alexander module, categorifying the Seifert form along the way. In particular, appealing to some joint work with Matt Hedden, we'll show that there are examples of knots with identical Alexander module and identical knot Floer homology, that are distinguished by the relevant bordered invariants. This is part of a joint project with Jen Hom, Sam Lewallen and Tye Lidman.

22. **Katrin Wehrheim** (MIT) *How to construct* 2+1+1 *topological field theories via the symplectic category*
In previous work with Chris Woodward, we axiomatized and constructed 2+1 (connected) topological field theories that factor through the (monotone) symplectic category. Based on a representation of 4-cobordisms by Morse 2-functions and a reduction of the Gay-Kirby moves to "strip shrinking moves", I can now argue that the extension of such 2+1 theories to 2+1+1 dimensions hinges on a single "quilt axiom" – an identity of pseudoholomorphic quilt invariants associated to crossings in the Morse diagram. In the case of 2+1 theories arising from certain representation spaces resp. symmetric products, this yields conjectural symplectic constructions of Donaldson resp. Seiberg-Witten invariants, and in particular an approach to proving invariance of Perutz' Lagrangian matching 'invariants'.

5 Scientific Progress Made

The workshop brought together leading experts from several different areas, and this sparked much scientific interaction. Despite the many exciting talks that were presented, the schedule of the workshop was designed to allow ample time for informal scientific discussions in order to facilitate interactions between the subject areas. This was accomplished by scheduling enough break time throughout the day and some longer breaks to encourage as much informal open-ended discussions as possible. The evenings provided collaborating teams of researchers time to meet and discuss their research projects, and opportunities for discussions leading to new collaborations.

The workshop participants reported on a number of fruitful scientific interactions, progress on existing collaborations, and new collaborations that arose of the workshop:

– Tobias Ekholm and Lenny Ng made further progress on their project to understand the connections between topological string theory and knot contact homology, and in fact came close to finishing the paper [AENV] during the workshop; as mentioned above this is a high profile new development at the interface of knot theory, symplectic geometry and mathematical physics. Ekholm also reports that he had a number of discussions with Honda and Colin on their higher-dimensional analogue of Heegaard-Floer homology which should lead to progress on the foundational aspects of that theory.

– David Gay and Joan Licata began a new collaboration, which aims to find "universal" analogues of front projections for Legendrian knots in arbitrary contact 3-manifolds using open book decompositions. If successful this should be of great value in the study of these knots.

– David Gay and Katrin Wehrheim had a number of fruitful discussions on the relations between Morse 2-functions and Wehrheim's new proposal for 2+1+1-dimensional TQFTs; Wehrheim reports that these led to a simplification of the set of axioms involved in her construction.

– Daniel Ruberman and Saso Strle made progress on their joint program to understand embeddings of non-orientable surfaces into 3- and 4-manifolds, and were able to obtain a new lower bound for the genus of a topological embedding of a surface in a product manifold. Ruberman also reports that he and his long-time collaborator Nikolai Saveliev made some progress on one of the central conjectures in their long-term program connecting gauge theory to Rokhlin's invariant.

– David Duncan and Eric Harper began collaborating on a project aiming to better understand a symplectic interpretation of the Casson-Lin invariant. Duncan also became interested in the Floer-theoretic aspects of the "pillowcase" description of moduli spaces of singular connections presented by Chris Herald in his talk, and discussed them extensively with Chris Herald and Matt Hedden.

– Liam Watson made progress on understanding a class of bordered 3-manifolds that behave like Heegaard-Floer homology solid tori, and reports that conversations at the workshop allowed him to finish this project which will soon lead to a paper.

– Ina Petkova reports that discussions with Rumen Zarev allowed her to complete a project on gradings in bordered Floer homology, and discussions with Bulent Tosun will lead to a new collaboration.

– finally, Paolo Lisca reports that he began two new collaborative projects with Andras Stipsicz and Paolo Ghiggini during the workshop.

References

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