WORKSHOP "METRIC GEOMETRY, GEOMETRIC TOPOLOGY AND GROUPS"
Final Report

Thirty five participants of the workshop represented different areas of metric geometry, geometric topology and geometric group theory. There were many participants whose work helped to define the modern face of these areas. In particular, eight of the participants were previously chosen as invited speakers at the International Congresses of Mathematicians; some others were recipients of various prestigious awards and are editors of leading mathematical journals. On the other hand many of the participants were young mathematicians. In particular, six of the participants were either Ph.D. students or post-docs; five of them gave (very successful) talks about their work which undoubtedly will improve their chances of finding post-doctoral and tenure-track positions at leading universities.

The general level of the talks was very high. In particular, many of the talks presented solutions of known open problems; the quality of most presentations was superb. For example, Mohammad Ghomi (Georgia Tech) presented a solution of a problem posed by Croft, Falconer and Guy which is also closely related to the famous Dürer unfoldability problem. Among other things his excellent and highly entertaining talk touched on some not widely known facts from the history of the medieval art explaining the origins of the problem. Dürer's problem asks whether it is always possible to cut a convex polyhedral surface along a spanning tree of its edges so that it can then be isometrically embedded in the Euclidean 3-space; Mohammad proved that it is always possible after an affine transformation of the surface.

Igor Mineyev (University of Illinois at Urbana-Champaigne) spoke about his solution of the celebrated Hanna Neumann problem in group theory. Robert Young (University of Toronto) announced his affirmative solution of an old problem posed by one of the founders of geometric measure theory Laurence Chisholm Young. The subject of this deceptively simply looking problem is the comparison between the volumes of optimal fillings of a cycle in an Euclidean space and the double of this cycle. Yevgeny Liokumovich (University of Toronto) explained a solution of a fifteen years old problem posed by Frankel and M. Katz which is a modified version of an earlier problem by M. Gromov. His theorem proven jointly with A. Nabutovsky and R. Rotman asserts that a boundary of a Riemannian 2-disc can be contracted to a point through closed curves with length bounded above by an explicit function of the length of the boundary of the disc, its diameter and its area. This theorem is an example of a result in *quantitative topology* pioneered by Gromov. The goal of quantitative topology is a study of geometric properties of maps (in particular, homotopies) which are known to exist by topological reasons.

Herman Gluck (University of Pennsylvania), Fedor Manin (University of Chicago) and Greg Chambers (University of Toronto) also gave talks pertaining to quantitative topology. Herman presented a beautiful proof of a recent theorem by H. Wen asserting that the group multiplication on the 3-sphere has the minimal Lipschitz constant in its homotopy class. Fedor discussed his interesting recent results on optimal Lipschitz constants of integer multiples of a fixed element of a homotopy group. Greg explored some simply looking but difficult questions in quantitative topology of surfaces. In particular, he presented his joint result with Y. Liokumovich that informally means that contracting the double of a loop on a Riemannian 2-sphere cannot be more difficult than the contracting the loop itself. He also presented a joint result with Regina Rotman with a sharp bound on the comparative difficulty of contracting a closed curve on a surface as a free loop and as a loop based at one of its points.

One of the goals of the workshop was to further foster interactions between interrelated areas of metric geometry, geometric group theory and geometric topology such as systolic geometry, quantitative topology, rigidity, embedding of Cayley graphs into Banach spaces, manifolds of negative curvature, manifolds of positive scalar curvature, and cohomological properties of groups. This last area was represented, in particular, by an interesting talk by Piotr Nowak (University of Warsaw).

All these areas are united not only by direct applications to each other but also by a set of common ideas and methods such that (non)amenability, rigidity, hyperbolicity, randomness, expanders. A talk of John Roe (Penn State) was devoted to amenability. He proved that Property A of bounded geometry metric space is equivalent to non-existence of certain linear operators ("non-compact ghosts"). (Property A is an amenability property that plays an important role in geometric topology and geometric group theory that has been recently extensively studied.)

Hyperbolicity played a prominent role in several talks. For example, Stephane Sabourau (University of Paris Est-Creteil) compared the exponential growth rate of a group G with exponential growth rates of its quotients by infinite normal subgroups in the case when G acts properly and co-compactly on a proper geodesic δ-hyperbolic metric space. As the result, he was able to solve a problem posed by A. Sambusetti asking to compare the volume growth rate of a normal cover of a Riemannian manifold M with the volume growth of the universal cover of M in the case when M admits a Riemannian metric with negative curvature. Negatively curved and more general non-positiovely curved manifolds were the subject of talks by Igor Belegradek (Georgia Tech), Ursula Hämenstaedt (University of Bonn) and Grigory Avramidi (University of Chicago). Igor gave an exposition of many importnt recent advances in geometry of complete non-positively curved manifolds. A talk by Grigori presented some new results on geometry at infinity of complete non-compact negatively curved manifolds of dimension four (joint with Tam Nguyen Phan and Yunhui Wu). It is interesting that their methods work only in dimension four. Finally, Ursula Hamestädt announced a proof of a theorem asserting that closed hyperbolic manifolds of dimension four cannot be surface bundles. Her proof of this long-sought threorem uses diverse results of many mathematicians.

Rigidity was a central subject of a talk by Tullia Dymarz (University of Wisconsin) who explained her surprising result that all quasi-symmetric maps of products of Euclidean spaces with the p-adics are Bi-Lipschitz.

A survey talk by Matthew Kahle (Ohio State University) was devoted to fundamental groups of random two-dimensional simplicial complexes. A foundational talk by Dmitry Jakobson (McGill University) presented an answer found by the speaker and his collaborators to the following fundamental question: What is a "good" defintion of a random Riemannian metric on a compact smooth manifold of a high dimension?

Finally, note that expanders play a truly central role in many of these areas ias they appear in constructions of important (counter)examples in systolic geometry, geometry of manifolds of negative curvature, geometric group theory and geometric topology. They also have important applications in computer science. On the other hand their study involves probabilistic methods as well as ideas coming from the theory of arithmetic groups. An interesting generalization of the definition of expanders was discussed in a talk by Assaf Naor (Courant Institute) entitled "Superexpanders". Jointly with M. Mendel Assaf proved that superexpanders exist. In particular, there exists a sequence of larger and larger 3-regular graphs that does not admit a coarse embedding into any uniformly convex Banach space.

Note that over the past several decades, a powerful thrust in geometry has been the synthetic, as opposed to the analytic, and, as a result, many of the themes of Riemannian geometry, such as geodesic geometry, harmonic maps, and negative (or nonpositive or nonnegative) curvature have been liberated from the hypothesis of being generated by an appropriate 2-tensor: indeed subriemannian geometry and geometric group theory are vibrant areas that both have analogies to and contribute to classical geometry. An interesting talk of Enrico Le Donne (University of Jyväskylä) was devoted to recent connections between subriemannian (and subfinsler) geometry and geometric group theory, and presented the joint results of the speaker with Breuillard, Leonardi, Monti and Vittone. Moreover, these ideas have subtle interactions with geometric topology because of the introduction of controlled methods, and more inevitably, via the peculiar role of the fundamental group made clear by the h-cobordism theorem and surgery theories. The fundamental group also plays an unexpectedly prominent role in metric geometry of Riemannian manifolds and, in particular, in study of manifolds of positive scalar curvature, where it role is enhanced through appearance of methods and ideas from geometric topology and even geometric functional analysis including those related to the Novikov conjecture (a prominent rigidity conjecture in geometric topology). These themes played a prominent role in the talk of Alexander Dranishnikov (University of Florida) who recently achieved an important breakthrough towards a proof of an important conjecture by Gromov describing geometry of universal coverings of manifolds of positive scalar curvature.

One of the recurring themes of many of these areas of geometry is that a study of discrete objects can be sometimes reduced to a study of geometry of continuous objects after passing to an appropriate limit. On the other hand problems about geometry of smooth objects (e.g. Riemannian manifolds) can be sometimes reduced to a study of singular objects. Typically this happens after passing to a Gromov-Hausdorff limit as in the talk by Burkhard Wilking (University of Muenster) who managed to replace the required upper bound for the sectional curvature in the celebrated Gromov theorem on almost flat manifolds by a (much weaker) upper bound for the L^1-norm of the Ricci curvature scaled by an appropriate power of the volume. (This impressive result was obtained jointly with E. Cabezas-Rivas.) More unepectedly this theme appeared in the talk by Anton Petrunin (Penn State). Among other results Anton explained how a well-known theorem by C. Taubes asserting that all finitely presented groups appear as fundamental groups of compact complex 3-manifolds can be proved through studying fundamental groups of hyperbolic 4-orbifolds. (This is his joint result with D. Panov.)

Broadly speaking, systolic geometry is a study of geometry of length and volume functionals. Sometimes it leads to beautiful inequalities such as the celebrated Gromov inequality providing an upper bound for the length of a shortest non-contractible closed curve on an essential Riemannian manifold in terms of only its volume and dimension. (Essentiality is a topological property of nonsimply-connected manifolds. Essential manifolds include projective spaces, lens spaces and all compact nonsimply-connected surfaces.) Systolic geometry draws its methods, in particular, from quantitative topology and is very closely related with the geometry of infinite-dimensional spaces naturally associated with Riemannian manifolds such as spaces of free and based loops and spaces of cycles. These themes were present in the talk of Regina Rotman (University of Toronto). In particular, Regina established a quantitative version of a well-known theorem proven by J.P. Serre that asserts that every pair of points on a closed Riemannian manifold can be connected by an infinite set of distinct geodesics. Regina found explicit upper bounds on the length of the mth of these geodesics that depend only on m, the diameter and the dimension of the ambient manifold (joint work with A. Nabutovsky). Another talk pertaining to systolic geometry was given by Florent Balacheff (University of Lille I). In his recent joint work with J.C. Alvarez Paiva Florent discovered amazing applications of contact geometry to the classification of local minima of volume on the space of smooth Finsler manifolds for which the length of a shortest periodic geodesic is equal to some prescribed constant. In his talk at the workshop Florent also presented further unexpected interplay between systolic geometry, convex geometry and geometry of numbers that were discovered in his joint work with J.C. Alvarez Paiva and K. Tsanev. The talk of Egor Shelukhin (University of Montreal) was devoted to geometry of the space of volume preserving diffeomorphisms of closed surfaces. The main result of Egor's talk asserts that for each $p \geq 1$ the L^p-metric of the group of volume-preserving diffeomorphisms of the 2-sphere is unbounded. This result completes the investigation of the diameter of groups of volume-preserving diffeomorphisms that was started almost thirty years ago by A. Shnirelman and continued by Y. Eliashberg and T. Ratiu.

Finally, we would like to mention an interesting talk by Michael Yampolsky (University of Toronto). This talk explored relationships between algirithmic solvability of Thurston equivalence in dynamics and geometry of branched covers, and presented joint results of the speaker with M. Braverman, S. Bonnot and N. Selinger.

We conclude with the list of the titles and the abstracts of all talks given during the workshop.

Grigori Avramidi (University of Chicago)
Ends of negatively curved manifolds
Abstract:

A theorem of Gromov says that a complete finite volume Riemannian manifold of bounded negative curvature is the interior of a compact manifold with boundary. If, in addition, the curvature is bounded away from zero then the boundary is an aspherical manifold with virtually nilpotent fundamental group. I will discuss examples showing this is no longer true if the curvature is allowed to approach zero. Then, I will explain a recent result showing that in dimension four the boundary is aspherical. This is joint work with Yunhui Wu and Tam Nguyen Phan.

Florent Balacheff (University of Lille I)
Isosystolic inequalities for optical hypersurfaces

Abstract:

In this talk, we will explain how the study of systolic geometry for non-reversible Finsler metrics allows us to find surprising and nice relations between metric geometry, convex geometry and the geometry of the numbers. This is joint work with Juan-Carlos Alvarez Paiva and Kroum Tzanev.

Igor Belegradek (Georgia Institute of Technology)
Topology of open nonpositively curved manifolds
Abstract:

I will survey recent results on rank one open nonpositively curved manifolds focusing on the topology of ends and restrictions on the fundamental group. The talk is based on http://arxiv.org/abs/1306.1256.

Greg Chambers (University of Toronto)
Optimal homotopies of curves on surfaces.
Abstract:
In this talk, we will prove the following theorems. For any $\epsilon > 0$, we have that:
(1) If two simple closed curves on a 2-dimensional Riemannian manifold are homotopic through loops of length at most L, then they are also homotopic through simple closed curves of length at most $L + \epsilon$ (joint work with Y. Liokumovich).
(2) If the boundary of a Riemannian 2-disc can be contracted through closed curves of length at most L, then it can be contracted through based loops of length at most $L+2D+\epsilon$, where D is the diameter of the 2-disc (joint work with R. Rotman). This result can be generalized for simple closed curves on Riemannian 2-manifolds.
(3) A closed curve on an orientable Riemannian 2-manifold can be contracted through loops of length at most $L + \epsilon$ if the curve formed by traversing twice can be contracted through loops of length at most L (joint work with Y. Liokumovich). This can be seen as a quantitative version of the fact that the fundamental group of an orientable surface contains no elements of order 2.

Alexander Dranishnikov (University of Florida)
Scalar curvature, macroscopic dimension and inessential manifolds.
Abstract:

In his book dedicated to Gelfand's 80th anniversary [G] Gromov introduced the notion of macroscopic dimension and proposed a conjecture: The macroscopic dimension $dim_{mc}\tilde{M}$ of the universal cover $\tilde{M}$ of a closed n-manifold with positive scalar curvature is at most $n-2$. We proved this conjecture in [BD] for spin manifolds whose fundamental group satisfies the Analytic Novikov Conjecture and the following K-theoretic condition (the Rosenberg-Stolz condition [RS]): $ko_*(\pi) \longrightarrow KO_*(\pi)$ is a monomorphism. In this presentation we will discuss the inequality $dim_{mc}\tilde{M} < n$ for for closed positive scalar curvature n-manifolds M. In particular, we prove it for manifolds whose fundamental group satisfies the Analytic Novikov Conjecture and the weaker K-theoretic condition: $ko^{lf}(E\pi) \longrightarrow KO^{lf}(E\pi)$ is a monomorphism. This allows to to prove the Gromov Conjecture for manifolds with the fundamental groups satisfying the Novikov conjecture which are duality groups. The inequality $dim_{mc}\tilde{M} < n$ is related to Gong-Yu's concept of a macroscopically large manifold as well as to Gromov's notion of inessential manifolds. We show that this inequality means exactly that $\tilde{M}$ is macroscopically large integrally. The large obstacle on the way to the Gromov conjecture is the difference between rational and integral versions of these concepts. The rational inessentiality means that $f_*([M]) = 0$ in $H_n(B\pi; Q)$. In the case of a spin manifold with positive scalar curvature the rational inessentiality follows from Rosenberg's theorem [R] and the K-homology Chern character. In [G] Gromov conjectured that the condition $dim_{mc}\tilde{M} < n$ implies the rational inessentiality. It turns out that this his conjecture is closely related to the question of amenability of the fundamental group [Dr2] and generally has a counterexample [Dr3].

References:
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Tullia Dymarz (University of Wisconsin)
Quasisymmetric vs Bi-Lipschitz maps
Abstract:

On a metric space, there are various classes of functions which respect aspects of the metric space structure. One of the most basic classes is the bi-Lipschitz maps Another possibly much larger class consists of the so-called quasisymmetric maps. On both Euclidean space and the p-adics, there are many quasisymmetric maps which are not bi-Lipschitz. However, on the product of Euclidean space with the p-adics, we show that all quasisymmetric maps are bi-Lipschitz. Furthermore, our proof does not use any direct analysis but instead uses results on quasi-isometries and spaces of negative curvature.

Mohammad Ghomi (Georgia Institute of Technology)
Affine unfoldings of convex polyhedra
Abstract:

A well-known problem in geometry, which may be traced back to the Renaissance artist Albrecht Durer, is concerned with cutting a convex polyhedral surface along some spanning tree of its edges so that it may be isometrically embedded into the plane. We show that this is always possible after an affine transformation of the surface. In particular, unfoldability of a convex polyhedron does not depend on its combinatorial structure, which

settles a problem of Croft, Falconer, and Guy. Among other techniques, the proof employs a topological characterization for embeddings among immersed planar disks.

Herman Gluck (University of Pennsylvania)
Lipschitz minimality of group multiplication on the three-sphere.
Abstract:

I will discuss the new result of Haomin Wen, a graduate student at Penn, that group multiplication on the three-sphere is a Lipschitz constant minimizer in its homotopy class, unique up to composition with isometries of domain and range. The proof is based on some insightful new geometric inequalities.

In the talk I will provide the background for Haomin's result, and will end with a list of problems that point the way to future work.

Ursula Hamenstädt (University of Bonn)
Surface bundles over surfaces do not admit real hyperbolic metrics
Abstract:

We show that the signature of a surface bundle over a surface with word hyperbolic fundamental group does not vanish. As a consequence, a closed real hyperbolic 4-manifold does not admit the structure of a surface bundle.

Dmitry Jakobson (McGill University)
Gaussian measures no spaces of metrics.
Abstract:

The first part of the talk is joint work in progress with B. Clarke, N. Kamran, L. Silberman and J. Taylor. We define Gaussian measures on manifolds of metrics with the fixed volume form. We next compute the moment generating function for the L^2 (Ebin) distance to the reference metric.

The second part of the talk is joint work with Linan Chen. We use the two-dimensional approach of Duplantier-Sheffield to define a 4-dimensional analogue of Gaussian Free Field, and use it to construct a canonical measure on a conformal class of metrics in R^4. We derive a KPZ type relation for those measures.

Matthew Kahle (Ohio State University)
Recent progress on random groups
Abstract:

In this talk I will discuss recent work by Hoffman, Paquette, and myself, where we find a sharp threshold for the fundamental group of random 2-complexes to have Kazhdan's property (T). Our result depends on a spectral gap criterion due to Zuk, but we also require new concentration results for spectral gaps of Erdos-Renyi random graphs. I will also put this work in the context of other work on random fundamental groups (hyperbolicity, cohomological dimension, etc.), and compare what is know for this model with what is known for Gromov's density model and Zuk's triangular model.

Enrico Le Donne (ETH, Zurich)
Growth rate of nilpotent groups and subFinsler geometry.

Abstract:

This talk is introductory. We explain the role of Carnot groups and their subFinsler geometry in Geometric Group Theory, recalling whole of the terminology. We discuss a link between fine volume estimates, rate of convergence to asymptotic cones, and presence of abnormal geodesics. The results on rate of convergence for nilpotent groups are in collaboration with Breuillard (Paris). The results on abnormal curves in Carnot groups are in collaboration with Leonardi (Modena), Monti (Padova), and Vittone (Padova).

Yevgeny Liokumovich (University of Toronto)
Slicing of Riemannian 2-surfaces by short curves
Abstract:

Consider Riemannian 2-sphere M of area A and diameter d. We prove that there exists a slicing of M by loops of length $\leq 200d \max\{1, \log(A/d^2)\}$. We construct examples showing that this bound is optimal up to a constant factor. This is a joint work with A. Nabutovsky and R. Rotman. Related questions about sweep-outs and slicings of surfaces will also be discussed.

Fedor Manin (University of Chicago)
Algebraic versus geometric complexity in homotopy groups
Abstract:

Given a metric space X and an element $\alpha \in \pi_n(X)$, how does the minimal geometric complexity of a representative of $k\alpha$ grow as a function of k? If we can find a representative simpler than the "obvious" one, as quantified by measures such as Lipschitz constant or volume, we say, by analogy to the π_1 case, that α is distorted. Asymptotically, distortion functions are topological invariants of nice compact spaces. For such spaces, we discuss various ways in which distortion can arise and establish conditions on X under which homotopy elements are or are not distorted. Methods include rational homotopy theory, filling functions, and L^∞ cohomology.

Igor Mineyev (University of Illinois at Urbana-Champaign)
Generalizing the Hanna Neumann conjecture

Assaf Naor (Courant Institute, NYU)
Superexpanders

Piotr Nowak (University of Warsaw)
Cohomology of deformations
Abstract:

The theme of this talk will be the question of how the cohomology of a fixed group with coefficients in a representation on a Hilbert space behaves under deformations of the representation. We will show that under certain conditions the vanishing is preserved by epsilon-deformations on the generators. The techniques we use allow to give an estimate on this epsilon in terms of n-dimensional Kazhdan-type constants which we introduce and which are natural generalizations of the classical Kazhdan constants. We will also discuss some examples and applications. This is based on joint work with Uri Bader.

Anoton Petrunin (Penn State University)
Telescopic actions (joint work with D. Panov).
Abstract:

A group action H on X is called "telescopic" if for any finitely presented group G, there exists a subgroup H' in H such that G is isomorphic to the fundamental group of X/H'.

We construct examples of telescopic actions on some CAT[-1] spaces, in particular on 3 and 4-dimensional hyperbolic spaces. As applications we give new proofs of the following statements:

(1) Aitchison's theorem: Every finitely presented group G can appear as the fundamental group of M/J, where M is a compact 3-manifold and J is an involution which has only isolated fixed points;

(2) Taubes' theorem: Every finitely presented group G can appear as the fundamental group of a compact complex 3-manifold.

John Roe (Penn State University)
Ghostbusting and property A.
Abstract:

Guoliang Yu's property A for metric spaces is a weak version of amenability that has found applications in geometry, topology and analysis. Recently, it has become clear that property A is in fact equivalent to several other natural regularity conditions. In this talk I'll report on these developments, with particular reference to the existence or not of "ghost operators" ... operators in the Roe algebra whose behavior at infinity is trivial, but which are not compact. This is joint work with Rufus Willett (Hawai'i).

Regina Rotman (University of Toronto)
Short geodesic segments on closed Riemannian manifolds.
Abstract:

A well-known result of J. P. Serre states that for an arbitrary pair of points on a closed Riemannian manifold there exist infinitely many geodesics connecting these points. A natural question is whether one can estimate the length of the "k-th" geodesic in terms of the diameter of a manifold. We will demonstrate that given any pair of points on a closed Riemannian manifold M of dimension n and diameter d, there always exist at least k geodesics of length at most $4nk^2d$ connecting them. We will also demonstrate that for any two points of a manifold that is diffeomorphic to the 2-sphere, there always exist at least k geodesics between them of length at most $22kd$. (Joint with A. Nabutovsky).

Stephane Sabourau (University Paris-Est Creteille)
Growth of quotients of isometry groups.
Abstract:

We will present some gap properties regarding the exponential growth rate of quotient of isometry groups in geometry group theory and Riemannian geometry.

Egor Shelukhin (University of Montreal)
Braids and L^p-norms of area-preserving diffeomorphisms

Abstract:

We consider large-scale metric properties of groups area-preserving diffeomorphisms of surfaces endowed with the hydrodynamic L2-metric (or more generally the L^p-metric). The first such property is the unboundedness of the metric, which we establish for the last open case among compact surfaces, the two-sphere. Our methods involve quasimorphisms on braid groups and differential forms on configuration spaces. This talk is based on a joint work with Michael Brandenbursky.

Burkhard Wilking (University of Münster)
A generalization of Gromov almost flat manifold theorem
Abstract:

By Gromov a manifold with diameter 1 and sectional curvature sufficiently small in absolute value is finitely covered by a nilmanifold. We generalize the theorem by relaxing the curvature condition in the theorem.

Michael Yampolsky (University of Toronto)
Geometrization of branched coverings of the sphere and decidability of Thurston equivalence
Abstract:

I will discuss a recent result obtained jointly with M. Braverman and S. Bonnot on the algorithmic decidability of the question whether a postcritically finite branched covering map of the 2-sphere is Thurston equivalent to a rational map. I will further describe the connection of the general question of algorithmic decidability of Thurston equivalence with geometrization of branched coverings, and will present a new theorem joint with N. Selinger on an algorithmic construction of a canonical geometrization of a branched covering.

Robert Young (University of Toronto)
Filling multiples of embedded curves.
Abstract:

Filling a curve with an oriented surface can sometimes be "cheaper by the dozen". For example, L. C. Young constructed a smooth curve drawn on a Klein bottle in R^n which is only about 1.3 times as hard to fill twice as it is to fill once and asked whether this ratio can be bounded below. We will use methods from geometric measure theory to answer this question and pose some open questions about systolic inequalities for surfaces embedded in R^n.