Banff International Research Station

for Mathematical Innovation and Discovery

Graph Algebras: Bridges between graph C^*-algebras and Leavitt path algebras

April 21–April 26, 2013

MEALS

Breakfast (Buffet): 7:00AM–9:30AM, Sally Borden Building, Monday–Friday
Lunch (Buffet): 11:30AM–1:30PM, Sally Borden Building, Monday–Friday
Dinner (Buffet): 5:30PM–7:30PM, Sally Borden Building, Sunday–Thursday
Note: Please remember to scan your meal card at the host/hostess station in the dining room for each meal.

MEETING ROOMS

The workshop and talks will be held in the TransCanada Pipeline Pavilion (TCPL).

- Daily Coffee Breaks for all (including observers) will be held in the foyer of the TCPL.
- Morning talks will be in Room 201 TCPL. Afternoon talks will be in Room 202 TCPL.
- Room 201 TCPL has an automated recording system, so morning talks may be recorded by BIRS.
- An LCD projector, a laptop, a document camera, and blackboards are available for all presentations.

Sunday

4:00PM	Check-in begins (Front Desk — Professional Development Centre — open 24 hours)
5:30–7:30PM	Buffet Dinner, Sally Borden Building
8:00PM	Informal gathering in 2nd floor lounge, Corbett Hall (if desired) Beverages and a small assortment of snacks are available on a cash honor system.

Monday

7:00–8:45	Breakfast
8:45–9:00	Introduction and Welcome by BIRS Station Manager, TCPL
9:00–10:00	**TALK 1: Mark Tomforde**
10:00–10:30	Coffee Break, TCPL
10:30–11:30	**TALK 2: Gene Abrams**
11:30–1:00	Lunch
1:00–2:00	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
2:00–2:10	Group Photo; meet in foyer of TCPL (photograph will be taken outdoors so a jacket might be required).
2:10–3:10	**TALK 3: Adam Sørensen**
3:10–3:30	Coffee Break, TCPL
3:30–4:30	**TALK 4: Enrique Pardo**
4:30–5:30	**TALK 5: George Elliott**
5:30–7:30	Dinner

Tuesday

7:00–9:00	Breakfast
9:00–10:00	**TALK 6: Pere Ara**
10:00–10:30	Coffee Break, TCPL
10:30–11:30	**TALK 7: Mike Boyle**
11:30–1:00	Lunch
1:00–2:00	**TALK 8: Toke Carlsen**
2:00–3:00	**TALK 9: Rasmus Bentmann**
3:00–3:30	Coffee Break, TCPL
3:30–4:30	**TALK 10: Francesc Perera**
5:30–7:30	Dinner

Wednesday

7:00–9:00	Breakfast
9:00–10:00	**TALK 11: Chris Phillips**
10:00–10:30	Coffee Break, TCPL
10:30–11:30	**TALK 12: Efren Ruiz**
11:30–12:30	**TALK 13: Gunnar Restorff**
12:30–1:30	Lunch
1:30–5:30	**Free Afternoon!**
5:30–7:30	Dinner

Thursday

7:00–9:00	Breakfast
9:00–10:00	**TALK 14: Roozbeh Hazrat**
10:00–10:30	Coffee Break, TCPL
10:30–11:30	**TALK 15: Eduard Ortega**
11:30–1:00	Lunch
1:00–2:00	**TALK 16: Sam Webster**
2:00–3:00	**TALK 17: Alex Kumjian**
3:00–3:30	Coffee Break, TCPL
3:30–4:30	**TALK 18: Aidan Sims**
5:30–7:30	Dinner

Friday

7:00–9:00	Breakfast
9:00–11:30	No Lectures. Informal meetings are encouraged. Most participants will be flying home this day.
10:00–10:30	Coffee Break, TCPL
11:30–1:30	Lunch

Check Out of Room by 12:00 Noon.

*** 5-day workshop participants are welcome to use BIRS facilities (Coffee Lounge, TCPL, and Reading Room) until 3 pm on Friday, but participants must still check out of the guest rooms by 12 noon. ***

5-DAY WORKSHOP

Graph Algebras: Bridges between graph C^*-algebras and Leavitt path algebras

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ABSTRACTS
(in alphabetical order by speaker surname)

Speaker: **Gene Abrams** (University of Colorado at Colorado Springs)
Title: *Primitive graph algebras*
Abstract: Let $E = (E^0, E^1, s, r)$ be an arbitrary directed graph (i.e., no restriction is placed on the cardinality of E^0, or of E^1, or of $s^{-1}(v)$ for $v \in E^0$). Let $L_K(E)$ denote the Leavitt path algebra of E with coefficients in a field K, and let $C^*(E)$ denote the graph C*-algebra of E. (Note: here $C^*(E)$ need not be separable.) We give necessary and sufficient conditions on E so that $L_K(E)$ is primitive (joint work with Jason Bell and K.M. Rangaswamy). We then show that these same conditions are precisely the necessary and sufficient conditions on E so that $C^*(E)$ is primitive (joint work with Mark Tomforde). This gives yet another example in a long and growing list of algebraic/analytic properties of the graph algebras $L_K(E)$ and $C^*(E)$ for which the graph conditions equivalent to said property are identical, but for which the proof/techniques used are significantly different. In the Leavitt path algebra setting, we show how this result allows for the easy construction of a large collection of prime, non-primitive von Neumann regular algebras (thereby giving a systematic answer to a decades-old question of Kaplansky). In the graph C*-algebra setting, we show how this result allows for the easy construction of a large collection of prime, non-primitive C*-algebras (thereby giving a systematic answer to a decades-old question of Dixmier).

Speaker: **Pere Ara** (Universitat Autonoma de Barcelona)
Title: *Leavitt path algebras of separated graphs and paradoxical decompositions*
Abstract: A separated graph is a pair (E, C), where E is a directed graph, $C = \bigsqcup_{v \in E^0} C_v$, and C_v is a partition of $r^{-1}(v)$ (into pairwise disjoint nonempty subsets) for every vertex v. (In case v is a source, we take C_v to be the empty family of subsets of $r^{-1}(v)$.) Leavitt path algebras $L_K(E, C)$ of separated graphs have been recently defined by Goodearl and the presenter [1]. They allow to incorporate the Leavitt algebras of any type (m, n) into the theory of graph algebras. Another method to obtain the Leavitt algebras of type (m, n) has been developed by Hazrat in [4]. Thanks to seminal work of George Bergman, it is possible to explicitly compute the monoids $\mathcal{V}(L_K(E, C))$ of finitely generated projective modules over these algebras.

In [3], we attach to a finite bipartite separated graph (E, C) a partial dynamical system $(\Omega(E, C), \mathbb{F}, \alpha)$ possessing a certain universal property. Here $\Omega(E, C)$ is a 0-dimensional metrizable compact space, $\mathbb{F}$ is a finitely generated free group, and α is a partial action of $\mathbb{F}$ on $\Omega(E, C)$. The corresponding crossed product algebra $C_K(\Omega(E, C)) \rtimes_{\alpha^*} \mathbb{F}$ is a certain *quotient* of $L_K(E, C)$, and we are able to compute its $\mathcal{V}$-monoid by utilizing a suitable representation as a direct limit of Leavitt path algebras of separated graphs (and the result mentioned above).

We will give an application of the theory above to a problem on paradoxical decompositions. Let G be a group acting on a set X. Then a subset E of X is said to be *G-paradoxical* if E contains two disjoint subsets E_1 and E_2 having the same G-equidecomposability type as E. This can be expressed by the relation $2[E] \leq [E]$ in the *type semigroup* $S(G, X)$. Tarski's Theorem establishes that a subset E of X is not G-paradoxical if and only if there is an additive function $\mu\colon S(G,X) \to [0,\infty]$ such that $\mu([E]) = 1$. If $\mathbb{E}$ is a G-invariant subalgebra of subsets of X, then a corresponding type semigroup $S(G, X, \mathbb{E})$ is defined, where all the sets involved are assumed to belong to the subalgebra $\mathbb{E}$. In particular, if X is a 0-dimensional compact space, one may take as $\mathbb{E}$ the family of all clopen subsets of X. Answering a question raised in two recent papers, by Rørdam and Sierakowski, and by Kerr and Novak, we show in [3] that the analogue of Tarski's Theorem fails in general for the type semigroup $S(G, X, \mathbb{E})$, where G is a countable group acting on a 0-dimensional metrizable compact space and $\mathbb{E}$ is the subalgebra of subsets of X consisting of all the clopen subsets of X.

References

[1] P. Ara, K.R. Goodearl, Leavitt path algebras of separated graphs, *Journal für die reine und angewandte Mathematik*, **669** (2012), 165–224.

[2] P. Ara, R. Exel, T. Katsura, Dynamical systems of type (m, n) and their associated C^*-algebras, *Ergodic Theory and Dyn. Systems.*

[3] P. Ara, R. Exel, Dynamical systems associated to separated graphs, graph algebras, and paradoxical decompositions, arXiv:1210.6931 [math.OA].

[4] R. Hazrat, The graded structure of Leavitt path algebras, *Israel J. Math.*

Speaker: **Rasmuss Bentmann** (University of Copenhagen)
Title: *Circle actions on C^*-algebras up to KK-equivalence*

Abstract: We classify circle actions on C^*-algebras and C^*-algebras over unique path spaces up to equivariant KK-equivalence under suitable bootstrap assumptions. For instance, gauge actions on Cuntz–Krieger algebras are KK-equivalent if and only if the corresponding matrices are shift equivalent over the integers. Both results follow from a general investigation of the relevant spectral sequences based on ideas of Bousfield. This is joint work with Ralf Meyer.

Speaker: **Mike Boyle** (University of Maryland)
Title: *Strong shift equivalence of matrices over a ring*

Abstract: This is joint work with Scott Schmieding.

Let R be a ring. Two square matrices A,B are elementary strong shift equivalent (ESSE-R) over R if there are matrices U,V over R such that $A = RS$ and $B = SR$. Strong shift equivalence over (SSE-R) is the equivalence relation generated by ESSE-R. Shift equivalence over R (SE-R) is a tractable equivalence relation which is refined by SSE-R. The refinement is trivial if $R = \mathbb{Z}$ (Williams), a principal ideal domain (Effros 1981) or a Dedekind domain (Boyle-Handelman 1993). No results have appeared since 1993.

It turns out that this refinement is captured precisely by a certain quotient group of the group $NK_1(R)$ of algebraic K-theory. It follows that for very many (not all) rings R, the relations SE-R and SSE-R are the same. For the class of nilpotent matrices over R (nilpotent matrices are those shift equivalent to [0]), this quotient group is $NK_1(R)$ itself. When $NK_1(R)$ is not trivial, it is not finally generated (Farrell, 1977).

I will try to explain background and motivation for SSE in terms of classification of symbolic dynamical systems and inverse problems for nonnegative matrices.

Speaker: **Toke Carlsen** (Norwegian University of Science and Technology)
Title: *Orbit equivalence and graph C^*-algebras*
Abstract: I will in this talk report on some work in progress with Nathan Brownlowe and Michael Whittaker from the University of Wollongong in which we study the relationship between when there is an isomorphism between two graph C^*-algebras that maps the diagonal subalgebra of one graph C^*-algebra onto the diagonal subalgebra of the other graph C^*-algebra, and a certain kind of orbit equivalence of the infinite path spaces of the two graphs.

Speaker: **George Elliott** (University of Toronto)
Title: *Classification—a paradigm for the twenty-first century?*
Abstract: The classification of AF C*-algebras and, subsequently, AF von Neumann algebras, both close to forty years ago, opened the season on set-theoretically difficult to classify objects. (No one says that finite groups are easy to classify, but this combinatorial question pales in comparison with even just the case of locally finite groups, not to mention arbitrary countable groups, or, for that matter, arbitrary separable C*-algebras—all of which can as it happens be classified by a reasonably gratifying classification functor—abstract to begin with—but, in the last analysis, every functor is concrete!) (I should perhaps say that I am talking about what is sometimes referred to as the Elliott intertwining argument—and the maps in the abstract classifying category are the approximately inner equivalence classes of maps in the given category. Unfortunately, even for the category of finite simple groups—as opposed, say, to the category of finite-dimensional C*-algebras—, this classifying category is not well understood.)

Speaker: **Roozbeh Hazrat** (University of Western Sydney)
Title: *Using the grading of Leavitt path algebras*
Abstract: Leavitt path algebras have a natural $\mathbb{Z}$-graded structure. We study the graded structure, characterising acyclic and comets graphs completely. Then we study the graded Grothendieck group as a capable invariant for the classification of these algebras.

Speaker: **Alex Kumjian** (University of Nevada, Reno)
Title: *Twisted Higher Rank Graph C^*-algebras*
Abstract: We define the C^*-algebra of a higher rank graph twisted by a two-cocycle which takes values in the circle and derive some basic properties. Any noncommutative torus and the crossed product of $\mathcal{O}_n$ by a quasifree automorphism are examples of C^*-algebras that can arise from this construction. We also discuss the cohomology theory that gives rise to the twisting cocycle. This is joint work with David Pask and Aidan Sims.

Speaker: **Eduard Ortega** (University of Trondheim (NTNU))
Title: *Simple Cuntz-Pimsner rings*
Abstract: Together with Carlsen we Introduced the Cuntz-Pimsner rings, a pure algebraic analog of the Cuntz-Pimsner algebras. We showed that they generalized, for example, the crossed products and the Leavitt path algebras. In this talk we are going to give necessary and sufficient conditions for when every non-zero ideal in a relative Cuntz-Pimsner ring contains a non-zero graded ideal, when a relative Cuntz-Pimsner ring is simple, and when every ideal in a relative Cuntz-Pimsner ring is graded. A "Cuntz-Krieger uniqueness theorem" for relative Cuntz-Pimsner rings is also given and Condition (L) and Condition (K) for relative Cuntz-Pimsner rings are introduced. As applications of these results, a uniqueness result for the Toeplitz algebra of a directed graph and characterizations of when crossed products of a ring by a single automorphism and fractional skew monoid rings of a single corner isomorphism are simple, are obtained. This is a joint work with T.M.Carlsen and E.Pardo.

Speaker: **Enrique Pardo** (University of Cádiz)
Title: *A symbolic dynamics approach to Kirchberg algebras*
Abstract: In this talk, we will study a combinatorial model of Kirchberg algebras, due to Takeshi Katsura [2], from a point of view connecting combinatorics with symbolic dynamics [1]. We will show how this approach applies to characterize some properties of these algebras in a different way to that used by Katsura [3], by using dynamical properties of partial actions of inverse semigroups on topological spaces and its associated groupoids.

Also, we will try to explain how this picture could be applied in a purely algebraic context to study algebraic analogs of Katsura algebras. In particular, this should allow us to extend the problem of classification of (purely infinite simple) Leavitt path algebras to a wider class of algebras. The final goal will be the search of an algebraic analog of the Kirchberg-Phillips' Classification Theorem [4, 5].

The central part of this talk is contained in a joint work with Ruy Exel (Departamento de Matemática, Universidade Federal de Santa Catarina, Florianópolis, Brazil),

R. Exel, E. Pardo, Representing Kirchberg algebras as inverse semigroup crossed products, arXiv:1303.6268v1 (2013)

submited to *Indiana University Mathematical Journal*.

References

[1] R. Exel, Inverse semigroups and combinatorial C^*-algebras, *Bull. Braz. Math. Soc. (N.S.)* **39** (2008), 191-313.

[2] T. Katsura, A construction of actions on Kirchberg algebras which induce given actions on their K-groups, *J. reine angew. Math.* **617** (2008), 27–65.

[3] T. Katsura, A class of C^*-algebras generalizing both graph algebras and homeomorphism C^*-algebras IV, pure infiniteness, *J. Funct. Anal.* **254** (2008), 1161–1187.

[4] E. Kirchberg, The classification of purely infinite C^*-algebras using Kasparov theory, preprint.

[5] N.C. Phillips, A classification theorem for nuclear purely infinite simple C^*-algebras, *Doc. Math.* **5** (2000), 49-114.

Speaker: **Francesc Perera** (Universitat Autonoma de Barcelona)
Title: *Geometric structure of dimension functions on some $C(X)$-algebras*
Abstract: In this talk we will analyse the structure of the set of dimension functions of certain $C(X)$-algebras, via the study of decomposition properties of their Cuntz semigroup. As a consequence, the Blackadar-Handelman conjectures hold for these examples. This is joint work with Ramon Antoine and Joan Bosa (UAB).

Speaker: **N. Christopher Phillips** (University of Oregon)
Title: *Analogs of Cuntz algebras on L^p spaces*
Abstract: Graph algebras have been intensively studied in the C* and purely algebraic contexts. We propose another context: Banach algebras of operators on L^p spaces. We report on results on the analogs in this context of Cuntz algebras and UHF algebras. (The analogs of UHF algebras are needed for the proofs of some of the results on the analogs of Cuntz algebras.)

For $p \in [1, \infty)$ and $d \in \{2, 3, \ldots\}$, we identify a "good" analog $\mathcal{O}_d^p$ of the Cuntz algebra $\mathcal{O}_d$ which acts on spaces of the form $L^p(X, \mu)$. We prove uniqueness and simplicity of $\mathcal{O}_d^p$. Unlike the C* case, these two results seem to be independent, and our proofs of the two results are entirely unrelated. We further prove that $\mathcal{O}_d^p$ is purely infinite and amenable as a Banach algebra, and we compute its topological K-theory, getting the expected answer. We prove that for $d_1, d_2 \in \{2, 3, \ldots\}$ and for $p_1 \neq p_2$, there is no nonzero continuous homomorphism from $\mathcal{O}_{d_1}^{p_1}$ to $\mathcal{O}_{d_2}^{p_2}$. We leave a number of problems open. In particular, we do not know whether the L^p spatial tensor product $\mathcal{O}_2^p \otimes_p \mathcal{O}_2^p$ is isomorphic to $\mathcal{O}_2^p$ for $p \neq 2$.

We prove that the "good" L^p analogs of UHF algebras are unique in a suitable sense, simple, amenable, have a unique continuous trace, and have the expected K-theory. We show that for each p and each supernatural number, there are uncountably many mutually nonisomorphic L^p UHF algebras with the same supernatural number and having all the properties given above except uniqueness and amenability.

This talk will be primarily a survey, but will explain some of the key ideas.

Speaker: **Gunnar Restorff** (University of Faroe Islands)
Title: *Classification of Cuntz-Krieger algebras and Graph C^*-algebras*
Abstract: The purely infinite Cuntz-Krieger algebras were internally classified during the period 1994–2004. Since (internal) classification results for purely infinite Cuntz-Krieger often were followed up by (external) strong classification results for purely infinite C^*-algebras, the complete classification of purely infinite Cuntz-Krieger algebras raised a number of questions. In recent years a number of people have contributed to answering these questions (Arklint, Bentmann, Bonkat, Eilers, Katsura, Kirchberg, Khler, Meyer, Nest, R, Ruiz, Tomforde). I will try to give an overview of the current state of these questions, and provide some new results concerning the strong classification question.

Speaker: **Efren Ruiz** (University of Hawai'i at Hilo)
Title: *Permanence properties for graph algebras*
Abstract: It is well known that hereditary sub-algebras of a graph C^*-algebra are not necessarily graph C^*-algebras. In fact, there are unital hereditary sub-algebras of a graph C^*-algebra that are not graph C^*-algebras. So it is natural to ask the question "are there sub-classes of the class of graph C^*-algebras in which hereditary sub-algebras are graph C^*-algebras?". In this talk, I will discuss two classes of graph C^*-algebras (the class of Cuntz-Krieger algebras and the class of unital graph C^*-algebras). We show that these classes have nice permanence properties. We use our results to determine when the unitization of a non-unital graph C^*-algebra is a graph C^*-algebra. Many of the results translate directly to Leavitt path algebras.

Speaker: **Aidan Sims** (University of Wollongong)
Title: *K-theory of twisted k-graph algebras*
Abstract: In 1984, Elliott famously proved that the K-groups of a noncommutative torus are independent of the angles of rotation appearing in the commutation relations. The fundamental idea of Elliott's beautiful argument is that for each non-commutative torus A_z, one can construct a continuous field of C^*-algebras over $[0, 1]$ whose fibre over 0 is $C(\mathbb{T}^k)$, and whose fibre over 1 is A_z. An elegant induction using the Pimsner-Voiculescu sequence then proves the desired result. In this talk, I will discuss how Elliott's idea can be applied to twisted k-graph C^*-algebras.

Speaker: **Adam Sørensen** (University of Wollongong)
Title: *Geometric Classification of Graph Algebras*
Abstract: Given a (directed, countable) graph G we can associate to it a C^*-algebra, $C^*(G)$. Inspired by Franks' classification of irreducible shifts of finite type we will introduce a short list of moves on graphs that do not change the Morita equivalence class of the associated algebras. We will discuss situations where we can get theorems of the form $C^*(G)$ is Morita equivalent to $C^*(E)$ if and only if G can be transformed into E using our allowed moves.

Speaker: **Mark Tomforde** (University of Houston)
Title: *Classification of Leavitt path algebras: How to use tools from the classification of C^*-algebras in the Algebra setting*
Abstract: One of the main programs in the theory of C^*-algebras is to classify C^*-algebras using invariants from K-theory. In this talk I'll discuss how similar efforts have been attempted for noncommutative algebras, particularly the Leavitt path algebras of a directed graph. These classification results have connections with the symbolic dynamics of the directed graph, and I will discuss the successes, limitations, and current status of these efforts.

Speaker: **Sam Webster** (University of Wollongong)
Title: *Path spaces of a graph*
Abstract: One can view graph C^*-algebras spatially by considering the image of the natural left-regular representation on the Hilbert space associated to an invariant subspace of the path space of a graph. Using the finite-path space, one obtains the Toeplitz-Cuntz-Krieger algebra of the graph. There is also a notion of the boundary of the path space which yields the Cuntz-Krieger algebra. For locally finite graphs the boundary paths are the right-infinite paths. These path spaces may be endowed with a locally compact Hausdorff topology. When Kumjian, Pask, Raeburn and Renault first studied graph C^*-algebras they used the infinite-path space of a locally finite graph to build a locally compact groupoid, giving rise to a C^*-algebra. A similar approach may be used for higher-rank graphs. We will present different characterisations of the path spaces of arbitrary directed graphs and higher-rank graphs, and show how the topology naturally occurs. Time permitting, we will show how the boundary-path space is affected by combinatorial constructions on the graph called desingularisations.