Banff International Research Station
for Mathematical Innovation and Discovery

13w5055: New Perspectives on the N-body Problem
January 14-18 2013

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in the new lecture theater in the TransCanada Pipelines Pavilion (TCPL). LCD projector and blackboards are available for presentations.

Checkout by 12 noon.

** 5-day workshop participants are welcome to use BIRS facilities (BIRS Coffee Lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

SCHEDULE

Sunday

16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
17:30–19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall (if desired) Beverages and a small assortment of snacks are available on a cash honor system.

Monday

7:00–8:45	Breakfast
8:45–9:00	Introduction and Welcome by BIRS Station Manager, TCPL
9:00-9:45	**Tere M. Seara**
9:50-10:35	**Marcel Guardia**
10:40-11:00	Coffee Break, TCPL
11:00-11:45	**Gabriella Pinzari**
11:55–13:15	Lunch
13:30–14:30	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:45	Group Photo; meet in foyer of TCPL (photograph will be taken outdoors so a jacket might be required).
15:00	Coffee Break, TCPL
15:30–16:15	**Pablo Roldan**
16:20–17:25	**Jesus Palacian**
17:30–19:30	Dinner

Tuesday

7:00–8:30	Breakfast
8:30-9:15	**Giovanni Gronchi**
9:20-10:05	**Nicola Soave**
10:05-10:35	Coffee Break, TCPL
10:35-11:20	**Jinxin Xue**
11:30–13:30	Lunch
13:30	Free Discussion
15:00	Coffee Break, TCPL
	Free Discussion
17:30–19:00	Dinner
19:05–19:50	**Andreas Knauf**
19:55–20:40	**Richard Montgomery**

Wednesday

7:00–8:30	Breakfast
8:30-9:15	**Alfonso Sorrentino**
9:20-10:05	**Ezequiel Maderna**
10:05-10:35	Coffee Break, TCPL
10:35-11:20	**Andrea Venturelli**
11:30–13:30	Lunch
	Free Afternoon
17:30–19:00	Dinner
19:05–19:50	**Jacques Féjoz**
19:55–20:40	**Konstantin Khanin**

Thursday

7:00–8:30	Breakfast
8:30-9:15	**Kuo-Chang Chen**
9:20-10:05	**Vivina Barutello**
10:05-10:35	Coffee Break, TCPL
10:35-11:20	**Davide L. Ferrario**
11:30–13:30	Lunch
14:15–15:00	**Xiaoxiao Zhao**
15:00	Coffee Break
15:30–16:15	**Dan Offin**
16:20–17:25	**Zhihong Jeff Xia**
18:15–19:30	Dinner

Friday

7:00–8:30	Breakfast
8:30-9:15	**Rafael Ortega**
9:20-10:05	**Cristina Stoica**
10:05-10:35	Coffee Break, TCPL
10:35-11:20	**Mitsuru Shibayama**
11:25:12:10	**Yiming Long**
12:20–13:30	Lunch

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ABSTRACTS

Vivina Barutello (Università di Torino, vivina.barutello@unito.it)
Parabolic trajectories as minimal phase transitions
Abstract: We consider the conservative dynamical system

$$\ddot{x}(t) = \nabla V(x(t)), \qquad x \in \mathbb{R}^d \setminus \mathcal{X}, \tag{1}$$

where $d \geq 2$, the potential V is smooth outside –and goes to infinity near– the collision set $\mathcal{X}$. A (global) *parabolic trajectory* for (1) is a collisionless solution which has null energy. In the Kepler problem ($V(x) = 1/|x|$) all global zero-energy trajectories are indeed parabola; for the class of anisotropic Kepler problems in $\mathbb{R}^d \setminus \{0\}$ with homogeneous potentials, we seek parabolic trajectories having prescribed asymptotic directions at infinity and which, in addition, are Morse minimizing geodesics for the Jacobi metric. Such trajectories correspond to saddle heteroclinics on the collision manifold, are structurally unstable and appear only for a codimension-one submanifold of such potentials. We give them a variational characterization in terms of the behavior of the parameter-free minimizers of an associated obstacle problem. We then give a full characterization of such a codimension-one manifold of potentials and we show how to parameterize it with respect to the degree of homogeneity.

We then focus on the planar case: we deepen and complete the analysis for homogeneous singular potentials characterizing all parabolic orbits connecting two minimal central configurations as free-time Morse minimizers (in a given homotopy class of paths). These may occur for at most one value of the homogeneity exponent. In addition, we link this threshold of existence of parabolic trajectories with the absence of collisions for all the minimizers of fixed-ends problems. Also the existence of action minimizing periodic trajectories with nontrivial homotopy type can be related with the same threshold.

Kuo-Chang Chen (National Tsing Hua University, kchen@math.nthu.edu.tw)
Keplerian action functional, convex optimization, and n-body problems with only boundary and topological constraints
Abstract: Variational methods have been applied to construct various types of solutions for the n-body problem, under various types of symmetry constraints. However, there were not much success with similar approaches for n-body problems without symmetry and equal-mass constraints, especially when $n > 3$. In this talk we will introduce an apparatus to construct periodic solutions for the n-body problem with only boundary and topological constraints. This approach is a combination of variational arguments in our works on retrograde orbits for the three-body problem and certain constraint convex optimization problems. Our method has no restriction on equal masses. We will illustrate this approach by constructing relative periodic solutions for the planar four-body problems within several topological classes.

Jacques Féjoz (Université Paris-Dauphine, fejoz@ceremade.dauphine.fr)
Punctured invariant tori in the spatial three-body problem (after ZHAO Lei)

Abstract: We will describe the proof of existence of solutions of the Lunar spatial three-body problem along which the two inner bodies undergo infinitely many almost collisions, and which, after regularization, densely fill invariant tori where collisions form a set of codimension 2. These motions are bounded in positions and oscillating in velocities. This work is part of the PhD thesis of Zhao Lei.

Davide L. Ferrario (Università di Milano Bicocca, davide.ferrario@unimib.it)
Dynamics of some symmetric n-body problems
Abstract: We will study n-body problems which are symmetric with respect to the action of suitable extensions of finite rotation groups. The space of symmetric configurations is the complement of an arrangement of linear subspaces in a Euclidean space, and blow-up, McGehee coordinates and variational methods can be in some cases used to understand local dynamics (around the space of collisions) and some properties of periodic orbits.

Giovanni Gronchi (Università di Pisa, gronchi@dm.unipi.it)
The evolution of the orbit distance in the double averaged restricted 3-body problem
Abstract: We study the long term evolution of the distance between two Keplerian confocal trajectories in the framework of the averaged restricted 3-body problem. The bodies may represent the Sun, a solar system planet and an asteroid. The secular evolution of the orbital elements of the asteroid is computed by averaging the equations of motion over the mean anomalies of the asteroid and the planet. When an orbit crossing with the planet occurs the averaged equations become singular. However, it is possible to define piecewise differentiable solutions by extending the averaged vector field beyond the singularity from both sides of the orbit crossing set and to compute an explicit formula for the difference of the extended vector fields at planet crossings. We generalize the previous results by improving the singularity extraction technique and show that the extended vector fields are Lipschitz-continuous. Moreover, we consider the distance between the Keplerian trajectories of the small body and the planet. Apart from exceptional cases, we can select a sign for this distance so that it becomes an analytic map of the orbital elements near to crossing configurations. We prove that the evolution of the 'signed' distance along the averaged vector field is more regular than that of the elements in a neighborhood of the crossing times. A comparison between averaged and non-averaged evolutions and applications of these results are shown using orbits of near-Earth asteroids.

Marcel Guardia (University of Maryland, marcel.guardia@upc.edu)
Oscillatory motions for the restricted planar circular three body problem
Abstract: In 1980 J. Llibre and C. Simó proved the existence of oscillatory motions for the restricted planar circular three body problem, that is, of orbits which leave every bounded region but which return infinitely often to some fixed bounded region. To prove their existence they had to assume that the ratio between the masses of the two primaries was exponentially small with respect to the Jacobi constant. In the present work, we generalize their work proving the existence of oscillatory motions for any value of the mass ratio.

We show that, for any mass ratio and large enough Jacobi constant, there exist transversal intersections between the stable and unstable manifolds of infinity which guarantee the existence of a symbolic dynamics that creates the so called oscillatory orbits. The main achievement is to rigorously prove the transversality of the invariant manifolds without assuming the mass ratio small, since then this transversality can not be checked by using classical perturbation theory respect to the mass ratio. Finally, we show that in a curve in the two dimensional parameter space formed by the mass ratio and the Jacobi constant, the invariant manifolds of infinity undergo a cubic tangency. This is a joint work with P. Martin and T. M. Seara.

Konstantin Khanin (University of Toronto, khanin@math.toronto.edu)
TBA
Abstract:

Andreas Knauf (Erlangen-Nüremberg University, knauf@mi.uni-erlangen.de)
Motion in Random Potentials
Abstract: We consider the motion of a particle under the influence of a random potential on Euclidean space, in particular the distribution of asymptotic velocities and the question of ergodicity of time evolution. Whereas, e.g., for smooth Poisson potentials the Hamiltonian flow is a.s. not ergodic, it is topologically transitive for a class of random potentials with coulombic singularities and for large energies. Then the compactified flow is ergodic. (with Ch. Schumacher; to appear in 'Ergodic Theory and Dynamical Systems')

Yiming Long (Nankai University, longym@nankai.edu.cn)
Stability of elliptic Lagrangian solutions of the classical three body problem via index theory
Abstract: Lagrange found his famous elliptic equilateral triangle solutions of the classical planar three body problem in 1772 which depend on the mass parameter and eccentricity of the ellipse. Linear stability of such solutions has been investigated by perturbation methods or numerical methods. But we are not aware of any rigorous analytical method which relate this stability to the parameters in their full range. In this lecture, I shall give a brief introduction on the new rigorous analytical method and recent results jointly obtained by Xijun Hu, Shanzhong Sun and myself on this linear stability problem for the full range of the masses and eccentricity via index theories for symplectic matrix paths. We proved the existence of three curves in the parameter rectangle, which separate the full parameter domain precisely according to the linear stability of elliptic Lagrangian solutions.

Ezequiel Maderna (Universidad de la República, emaderna@cmat.edu.uy)
Abundance of complete parabolic motions via weak KAM theory
Abstract: In this talk I will consider the general N-body problem. I will show a Hölder estimate for the minimal action which enable us to prove the existence of weak solutions for the Hamilton-Jacobi equation. The calibrating curves of a given such solution produces a lamination of the configuration space composed by rays of free time minimizers which, as we will see, they are complete parabolic motions.

Richard Montgomery (University of California at Santa Cruz, rmont@ucsc.edu)
Paradoxes and Opportunities from Global Regularization
Abstract: Levi-Civita's regularization, when applied systematically and democratically to each binary collision of the planar N-body problem, yields a system on a modified phase space free of binary collision singularities. Rick Moeckel and I applied symplectic reduction to this regularized system to obtain a regularized reduced system which we call the 'global regularization'. (This global regularization has singularities along triple and higher collisions. When $N = 3$, and perhaps 4 the higher collision singularities can be McGehee-blown-up to yield a complete flow.) Apparent paradoxes and opportunities now appear in the light of global regularization. One paradox concerns the Martinez-Simo simultaneous binary collision singularity result for $N = 4$. A variational opportunity arises because global regularization creates new discrete symmetries. Combined with the Jacobi-Maupertuis metric formulation of dynamics, these new symmetries could possibly yield new 'designer orbits' of brake type. The new symmetries arise directly out of the 2:1 branched cover nature of the original Levi-Civita transformation. The symmetry groups are products of a large number $d(N)$ copies of Z_2's.

Dan Offin (Queen's University, offind@mast.queensu.ca)
Dynamics, symmetry and hyperbolicity.
Abstract: We begin by discussing the connection between absolute minimization of the action and hyperbolic structures on an invariant set for a convex Lagrangian system on a compact complete manifold. Limiting cases of absolutely minimizing periodic orbits and the resulting hyperbolic structure are considered. Next we consider conditionally minimizing periodic orbits in the N-body problem with symmetry constraints. Although this context does not the earlier case of absolutely minimizing trajectories on compact manifolds, we show how the argument may be modifed to apply in certain cases of the N-body problem.

We give a sufficient condition using the spatio-temporal structure of symmetric minimizing curves for the N-body problem to guarantee hyperbolicity of the orbits.
This is a joint work with Gonzalo Contreras.

Rafael Ortega (Universidad de Granada, rortega@ugr.es)
Stable periodic solutions in the forced pendulum equation
Abstract: Consider the equation

$$x'' + \beta \sin x = f(t)$$

where the forcing satisfies

$$f(t + 2\pi) = f(t), \quad \int_0^{2\pi} f(t)dt = 0.$$

I present the following result: assuming

$$0 < \beta \leq \frac{1}{4},$$

for almost every forcing (in the sense of prevalence) there exists a stable 2π-periodic solution.

- The sentence "for almost every forcing" is needed, the result is false for some forcings.
- The number $\frac{1}{4}$ is sharp.

Jesus Palacian (Universidad Publica de Navarra, palacian@unavarra.es)
Singular Reduction in the Three- and in the N-Body Problem: Invariant Tori Reconstructed from the Relative Equilibria
Abstract: We start with the spatial three-body problem, applying successive reductions to it in order to get the simplest reduced Hamiltonian, i.e. the one where all the continuous symmetries have been reduced out. We use Deprits variables of the n body problem, normalising over the mean anomalies of the inner and outer ellipses in a region outside the resonance regime.

After truncating higher-order terms the resulting system is expressed in the corresponding invariants that define the reduced space, which is a regular manifold of dimension eight. Then we reduce with respect to the symmetries of the modulus of the total angular momentum and its projection onto the vertical axis of the inertial frame, obtaining the invariants and expressing the Hamiltonian in terms of them. The reduced space has dimension six and is singular for some combinations of the parameters. Next, as the normalised Hamiltonian is independent of one of the pericentres, we reduce the system with respect to the symmetry related with the modulus of the angular momentum of the outer ellipse. The reduced space is two-dimensional and may have up to three singular points. In this space we study the corresponding reduced system analysing the relative equilibria, stabilities and bifurcations.

We reconstruct the relative equilibria of the successive reduced spaces that are of elliptic type, establishing the existence of KAM tori for the spatial three-body problem. Because of the degeneracy of the system we have used a theorem by Han, Li and Yi to deal with it. We get several types of invariant tori from the different reduced spaces using adequate combinations of Deprit's variables.

Finally we discuss the implications of our treatment to the spatial n-body problem, pointing out how to construct some of the reduced spaces and their invariants and how to get some of the KAM tori of the n-body problem, relating our work to the one of Chierchia and Pinzari.

This is a joint work with Flora Sayas and Patricia Yanguas.

Gabriella Pinzari (Università Roma Tre, pinzari@mat.uniroma3.it)
On certain instabilities occurring in the planetary three–body problem, with closely spaced planets
Abstract: As it is known to all, the stability of semi–major axes up to exponentially long times in the planetary N–body problem has been proved by [N. N. Nekhorossev, 1977]. The stability of the whole system, comprehending also eccentricities and mutual inclinations is a more subtle question, deeply related

to mean–motion resonances. In a region of phase space where these are absent, stability con be proven at least for polynomially long times, as shown in a previous work with L. Chierchia.

The presence of resonances in general changes drastically the aspect of the effective Hamiltonian and instability may occur: [J. Féjoz, M. Guàrdia, V. Kaloshin and P. Roldán, preprint 2011] prove a displacement of the eccentricity in the planar, restricted, elliptic three–body problem in correspondence of the resonance $1:7$.

I shall talk about a case of instability occurring in the planetary three–body problem, where the planets are very close one to the other and revolve in opposite verses. This corresponds to the very special resonance $1:1$, with the mean motions $n_1 \sim n_2$. Along this resonance the equilibrium point of the effective perturbing function bifurcates from elliptic to partially hyperbolic and hence calls for lower dimensional hyperbolic KAM tori. Such tori should have co–dimension 1 in the planar problem, 2 in the spatial problem, with hyperbolic directions corresponding to one of the eccentricities and the mutual inclination. Remarkably, in a similar situation, some diffusion for semi–axes seems to be numerically detected [A. Quillen, 2011 and cited works].

I shall discuss a Graff–type ([S. Graff, 1974]) normal form for this system. The major difficulty is to avoid collision singularity.

Pablo Roldan (Universitat Politècnica de Catalunya, pablo.roldan@upc.edu)
Numerical study of a normally hyperbolic cylinder in the RTBP
Abstract: We consider the circular planar Restricted Three-Body Problem modeling a Sun-Jupiter-Asteroid system, and we show the existence of a normally hyperbolic cylinder composed of resonant periodic orbits using numerical methods. Then we study the splitting of the associated invariant manifolds.

This is a key step in our proof of diffusion along mean-motion resonance in the elliptic planar RTBP.

Tere M. Seara (Universitat Politècnica de Catalunya, tere.m-seara@upc.edu)
Orbits with increasing angular momentum in the elliptic restricted three body problem: combining two scattering maps
Abstract: The goal of the talk is to show the existence of global instability in the elliptic restricted three body problem. In this model, we find orbits whose angular momentum changes between two a priori stablished values. The main tool is to combine two different scattering maps associated to the normally parabolic manifold of infinity to build trajectories whose angular momentum increases. This is a joint work with A. Delshams, V. Kaloshin and A. de la Rosa.

Nicola Soave (Università di Milano Bicocca)
Symbolic dynamics: from the N-centre to the $(N+1)$-body problem
Abstract: In a previous paper with S. Terracini, we proved the existence of periodic solutions with negative energies for the planar N-centre problem. That result is based upon a broken geodesics method, which, roughly speaking, consists in gluing together different arcs of solutions of some fixed ends problems. In this talk we discuss the generalization of the method on an ideal (i.e. different from the physical one) restricted $(N+1)$-body problem, with $N \geq 3$. We prove the existence of infinitely many collision-free periodic solutions with negative and small *Jacobi constant* and small values of the angular velocity, for any initial configuration of the centres. We will introduce a Maupertuis' type variational principle; major difficulties arise from the fact that, contrary to the classical Jacobi length, the related functional does not come from a Riemaniann structure but from a Finslerian one. Our existence result allows us to characterize the associated dynamical system with a symbolic dynamics, where the symbols are given partitions of the centres in two non-empty sets. Further developments towards an application to the real restricted $(N+1)$-body problem are discussed.

Alfonso Sorrentino (Università di Roma Tre, Sorrentino@mat.uniroma3.it)
Symplectic and variational methods for the study of invariant Lagrangian graphs.

Abstract: In this talk I would like to describe some variational and symplectic approaches to the study of the existence of invariant Lagrangian graphs for Tonelli Hamiltonian systems.

Cristina Stoica (Wilfrid Laurier University, c.w.stoica@gmail.com)
Poincaré-Birkhoff normal forms near relative equilibria in N-body problems
Abstract: The Poincaré-Birkhoff normal forms near equilibria are a well-established tool in studying the dynamics of Hamiltonian systems. For systems with symmetry, normal forms near relative equilibria are still under development.

This talk presents recent results in the Poincaré-Birkhoff normal forms theory for cotangent bundle systems with lifted $SO(3)$ symmetry. We use the phase-space description given by an appropriate constructive co-tangent bundle slice theorem: for non-zero momentum, the reduced space is locally described by the product of $so(2)^*$ and a canonical cotangent bundle system. The theory is applied to N-body problems.

Mitsuru Shibayama (Osaka University, sibayama@kurims.kyoto-u.ac.jp)
Non-integrability criterion for homogeneous Hamiltonian systems via blowing-up theory
Abstract: AIt is a big problem in the Hamiltonian systems to decide whether a given Hamiltonian system is integrable. We provide a sufficient condition for the non-integrability of some Hamiltonian systems. We prove it by using the blowing-up theory which McGehee established in the collinear three-body problem. Moreover we compare our result with the differential Galois theory which is the most powerful tool for showing the non-integrability.

Andrea Venturelli (Université Avignon, andrea.venturelli@univ-avignon.fr)
A family of quasiperiodic collisions solutions in the spatial isosceles three-body problem.
Abstract: In the spatial isosceles three-body problem, by minimizing the lagrangian action on a well chosen class of paths, we find a one parameter family of quasiperiodic solutions with regularized double collisions. In fact, these soolutions are periodic on a rotating frame. The main difficulty is to show that our solutions are free of triple collisions, and that angular momentum is not zero. It is a joint work with E. Mateus and C. Vidal.

Zhihong Jeff Xia (Northwestern University, xia@math.northwestern.edu)
TBA
Abstract:

Jinxin Xue (University of Maryland, jinxinxue@gmail.com)
Noncollision singularities in a simplified four-body problem
Abstract: In this work we study a model of simplified four-body problem called planar two-center-two-body problem. In the plane, we have two fixed centers $Q_1 = (-\chi, 0), Q_2 = (0, 0)$ of masses 1, and two moving bodies Q_3 and Q_4 of masses $\mu \ll 1$. They interact via Newtonian potential. Q_3 is captured by Q_2, and Q_4 travels back and forth between two centers. Based on a model of Gerver, we prove that there is a Cantor set of initial conditions which lead to solutions of the Hamiltonian system whose velocities are accelerated to infinity within finite time avoiding all early collisions. We consider this model as a simplified model for the planar four-body problem case of the Painlevé conjecture. This is a joint work with Dmitry Dolgopyat.

Xiaoxiao Zhao (Sichuan University, zxgg2007@163.com)
Non-planar Periodic Solutions for Spatial Restricted N+1-body Problems
Abstract: We use variational minimizing methods and Jacobi's theory for local minimizers to study spatial restricted N+1-body problems with a sufficiently small mass moving on the vertical axis of the moving circular orbit plane for the first N bodies, here the vertical axis passes through the center of masses for the N primaries.

Firstly, for $2 \leq N \leq 10^{10}$, we study the spatial restricted N+1-body problems. N primary bodies with equal masses are located at the vertices of a regular polygon, we prove that the minimizer of the Lagrangian action on the anti-T/2 or odd symmetric loop space must be a non-planar periodic solution.

Secondly, we prove the existence of non-planar periodic solutions for the following spatial restricted 3-body and 4-body problems: for $N = 2$ or 3, given any positive masses $m_1, \cdots, m_N$ in a central configuration(for $N = 2$, two bodies are in a Euler configuration; for $N = 3$, three bodies are in a Lagrange configuration), the mass points of $m_1, \cdots, m_N$ move in the plane of N circular obits centered at the center of masses. Using variational minimizing methods, we establish the existence of the minimizers of the Lagrangian action on anti-T/2 or odd symmetric loop spaces, moreover, we prove these minimizers are non-planar periodic solutions by using the Jacobi's Necessary Condition for local minimizers.

Here a key point is that we need to generalize the classical Jacobi's Necessary Condition for local minimizers with the fixed end conditions to the case with periodic boundary conditions. This is a joint work with Fengying Li and Shiqing Zhang