

# Disordered Quantum Many-Body Systems

## October 27 to November 1, 2013

### MEALS

\*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday

\*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday

\*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday

Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)

### SCHEDULE

#### Sunday

**16:00** Check-in begins (Front Desk - Professional Development Centre - open 24 hours)

**17:30–19:30** Buffet Dinner, Sally Borden Building

**20:00** Informal gathering in 2nd floor lounge, Corbett Hall (if desired)

Beverages and a small assortment of snacks are available on a cash honor system.

All lectures will be held in the lecture theater in the TransCanada Pipelines Pavilion (TCPL).

TCPL also has several breakout rooms for informal talks or group discussions.

Speakers should leave *10 minutes for questions at the end of longer talks* and *5 minutes at the end of short talks*.

#### Monday

**7:00–8:45** Breakfast

**8:45–9:00** Introduction and Welcome by BIRS Station Manager, TCPL

**9:00–10:00** Jakob Yngvason, *BEC, interactions and disorder*

**10:00–10:30** Coffee Break

**10:30–11:30** Francois Huveneers, *Asymptotic localization of energy from quenched and thermal disorder*

**11:30–13:00** Lunch

**13:00–14:00** Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall

**14:00** Group Photo; meet in foyer of TCPL (photograph will be taken outdoors so a jacket might be required).

**14:30–15:00** Coffee Break

**15:00–16:00** David Huse, *Review of many-body localization*

**16:00–16:10** Break

**16:10–16:40** Arijeet Pal, *Many-body localization phase transition: Breakdown of thermalization in highly excited states*

**16:40–17:00** Break

**17:00–18:00** John Imbrie, *A Multi-scale Jacobi Method for Anderson Localization*

**18:00–20:00** Reception sponsored by Journal of Mathematical Physics (American Institute of Physics)

## Tuesday

<b>7:00–9:00</b>	Breakfast
<b>9:00–10:00</b>	Dimitry Abanin, <i>Many-body localization: conservation laws, structure of eigenstates, and entanglement growth</i>
<b>10:00–10:30</b>	Coffee Break
<b>10:30–11:00</b>	Peter Müller, <i>Anderson's orthogonality catastrophe</i>
<b>11:00–11:10</b>	Break
<b>11:10–12:10</b>	Simone Warzel, <i>Resonant delocalization in Random Schrödinger operators</i>
<b>12:10–13:30</b>	Lunch
<b>14:30–15:00</b>	Coffee Break
<b>15:00–16:00</b>	Mathieu Lewin, <i>The mean-field approximation of stochastic crystals</i>
<b>16:00–16:15</b>	Break
<b>16:15–17:00</b>	Valentin Zagrebnov, <i>Condensation in a Disordered Bose-Hubbard Model</i>
<b>17:00–17:15</b>	Break
<b>17:15–18:00</b>	Frédéric Klopp, <i>The thermodynamic limit for one-dimensional interacting quantum fermions in a random environment</i>
<b>18:00–19:30</b>	Dinner

## Wednesday

<b>7:00–9:00</b>	Breakfast
<b>9:00–10:00</b>	Antti Knowles, <i>Random band matrices as a model of quantum disorder</i>
<b>10:00–10:30</b>	Coffee Break
<b>10:30–11:15</b>	Alain Joye, <i>Spectral Transition for Random Quantum Walks on Trees</i>
<b>11:15–11:25</b>	Break
<b>11:25–12:10</b>	Balint Virag, <i>Two results about random spectra</i>
<b>12:10–13:30</b>	Lunch
	Free Afternoon
<b>17:30–19:30</b>	Dinner

## Thursday

<b>7:00–9:00</b>	Breakfast
<b>9:00–10:00</b>	Yoshiko Ogata, <i>A classification of finitely correlated state</i>
<b>10:00–10:30</b>	Coffee Break
<b>10:30–11:15</b>	Benjamin Schlein, <i>Hartree-Fock dynamics for weakly interacting fermions</i>
<b>11:15–11:25</b>	Break
<b>11:25–12:10</b>	Abel Klein <i>Localization for multi-particle Anderson Hamiltonians &amp; Unique continuation principle for spectral projections</i>
<b>12:10–13:30</b>	Lunch
<b>14:30–15:00</b>	Coffee Break
<b>15:00–15:30</b>	Peter Hislop, <i>Applications of the unique continuation principle for spectral projections</i>
<b>15:30–15:35</b>	Break
<b>15:35–15:55</b>	Son Nguyen, <i>Localization for multi-particle Anderson Hamiltonians</i>
<b>15:55–16:05</b>	Break
<b>16:05–16:35</b>	Stephen Dias Barreto, <i>Disordered Graded Quantum Lattice Systems</i>
<b>16:35–16:40</b>	Break
<b>16:40–17:10</b>	Mira Shamis, <i>Resonant delocalization on the Bethe strip</i>
<b>17:10–17:20</b>	Break
<b>17:20–17:40</b>	Daniel Schmidt, <i>Eigenvalue statistics for two-particle models</i>
<b>17:40–17:45</b>	Break
<b>17:45–18:05</b>	Christian Sadel, <i>SDE limits for transfer matrices with hyperbolic channels and limiting eigenvalue processes</i>
<b>18:05–19:30</b>	Dinner

## Friday

<b>7:00–9:00</b>	Breakfast
<b>9:00–</b>	Informal Discussions
<b>11:30–13:30</b>	Lunch
<b>Checkout by</b>	
<b>12 noon.</b>	

\*\* 5-day workshop participants are welcome to use BIRS facilities (BIRS Coffee Lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout by 12 noon. \*\*

# Disordered quantum many-body systems

## October 27 to November 1, 2013

### ABSTRACTS

(in alphabetic order by speaker surname)

Speaker: **Dimitry Abanin** (Perimeter Institute for Theoretical Physics)

Title: *Many-body localization: conservation laws, structure of eigenstates, and entanglement growth*

Abstract: We construct a complete set of local integrals of motion that characterize the many body localized phase. Our approach relies on the assumption that local perturbations only act locally on the eigenstates in the interacting localized phase, which we support with numerical simulations of a random field XXZ spin chain. Our study provides a description of the structure of the many body localized states. We discuss the implications of the local conservation laws for quantum dynamics in the interacting localized phase, showing that they underlie an unusual, logarithmic in time, growth of entanglement entropy. We also argue that the many-body localization can be used to protect coherence in the system by preventing relaxation between eigenstates with different local integrals of motion.

Speaker: **Stephen Dias Barreto** (Padre Conceicao College of Engineering)

Title: *Disordered Graded Quantum Lattice Systems* (Joint work with Francesco Fidaleo, University of Roma, Tor Vergata)

Abstract: We study Fermion systems on a lattice with random interactions through their dynamics and the associated KMS states. These systems require a more complex approach compared with the standard spin systems on a lattice, on account of the difference in commutation rules between these two systems for the local algebras corresponding to disjoint regions. It is for this reason that some of the known formulations and proofs in the case of the spin lattice systems with random interactions do not automatically go over to the case of disordered Fermion lattice systems. We extend to the disordered CAR algebra some standard results concerning the spectral properties exhibited by temperature states of disordered quantum spin systems. We investigate the Arveson spectrum, known to physicists as the set of Bohr frequencies. Further we also establish its connection with the Connes and Borchers spectra, and with the associated invariants for such  $W_*$ -dynamical systems which determine the type of von Neumann algebras generated by a temperature state. We prove that all such spectra are independent of the disorder. As a natural application, we show that a temperature state can generate only a type III von Neumann algebra (with the type III<sub>0</sub> component excluded). In the case of the pure thermodynamic phase, the associated von Neumann algebra is of type III<sub>λ</sub> for some  $\lambda \in (0, 1]$ , independent of the disorder.

All such results are in accordance with the principle of self-averaging which affirms that the physically relevant quantities do not depend on the disorder.

Speaker: **Peter Hislop** (University of Kentucky)

Title: *Applications of the unique continuation principle for spectral projections*

Abstract: I will begin by reviewing A. Klein's application of the unique continuation principle for spectral projections to one-body random Schrödinger operators. I then will present joint work with F. Klopp, and extensions by A. Klein and S. Nguyen, on applications to one-region and two-region Wegner estimates for interacting  $N$  body random Schrödinger operators, applications to Delone-Anderson models, and to the integrated density of states.

Speaker: **David Huse** (Princeton University)

Title: *Review of many-body localization*

Abstract: I will review various cases of systems exhibiting localization, focusing on the case that is now known as "many-body localization", namely localization in a many-body quantum system at an energy

density that corresponds to nonzero temperature. I will review the present "conjectured" understanding of many-body localization, including some discussion of aspects where there is little or no such conjectured understanding.

Speaker: **Francois Huveneers** (Université de Paris Dauphine)

Title: *Asymptotic localization of energy from quenched and thermal disorder*

Abstract: I will discuss a theoretical approach to energy localization in both quantum and classical chains of oscillators. Quenched disorder is known to reduce, or sometimes completely suppress, energy transport in systems like spin chains. At high temperature, randomness coming from the Gibbs state potentially plays the same role when dealing with strongly anharmonic oscillators. However, randomness evolves then with time, allowing resonant spots to travel into the system. Still, I will present some results on asymptotic localization of energy, valid for both quenched and thermal disorder. (joint work with Wojciech De Roeck)

Speaker: **John Imbrie** (University of Virginia)

Title: *A Multi-scale Jacobi Method for Anderson Localization*

Abstract: I will present a new method for proving exponential localization. An explicit construction of eigenvalues and eigenfunctions is given, using convergent expansions that exhibit local dependence on the random potential. It is based on the idea of sequential diagonalization, a la Jacobi, with a mixture of perturbative and non-perturbative estimates. I will discuss prospects for the method as a tool for studying many-body localization in spin chains. (Joint work with Tom Spencer)

Speaker: **Alain Joye** (Institut Fourier, Université Grenoble 1)

Title: *Spectral Transition for Random Quantum Walks on Trees*

Abstract: We will discuss random quantum walks on a homogeneous tree of degree 3 describing the discrete time evolution of a quantum particle with internal degree of freedom in  $\mathbb{C}^3$  hopping on the neighboring sites of the tree in presence of static disorder. The one time step random unitary evolution operator of the particle depends on a unitary matrix  $C \in U(3)$  which monitors the strength of the disorder. We show the existence of open sets of matrices in  $U(3)$  for which the random evolution has either pure point spectrum almost surely or purely absolutely continuous spectrum. We also establish properties of the spectral diagram which provide a description of the spectral transition driven by  $C \in U(3)$ . (Joint work with Eman Hamza)

Speaker: **Abel Klein** (University of California, Irvine)

Title: *Localization for multi-particle Anderson Hamiltonians & Unique continuation principle for spectral projections*

Abstract: We extend the bootstrap multiscale analysis to multi-particle continuous Anderson Hamiltonians, obtaining Anderson localization with finite multiplicity of eigenvalues, a strong form of dynamical localization, and decay of eigenfunction correlations. (Joint work with Son Nguyen.) In this talk I will review our localization results for multi-particle continuous Anderson Hamiltonians. The bootstrap multiscale analysis will be the subject of Son Nguyen's talk.

The bootstrap multiscale analysis requires Wegner estimates. These estimates use a unique continuation principle for spectral projections of Schrödinger operators. Given a Schrödinger operator  $H = -\Delta + V$  on  $L^2(\mathbb{R}^d)$ , let  $H_\Lambda$  denote its restriction to a finite rectangle  $\Lambda$  with either Dirichlet or periodic boundary condition. We will discuss unique continuation estimates of the type

$$\chi_I(H_\Lambda)W\chi_I(H_\Lambda) \geq \kappa \chi_I(H_\Lambda) \quad \text{with} \quad \kappa > 0$$

for appropriate potentials  $W \geq 0$  and intervals  $I$ . These estimates play an important role in the proofs of Wegner estimates for one-particle and multi-particle continuous Anderson Hamiltonians, the subject of Peter Hislop's talk.

Speaker: **Frédéric Klopp** (Université Pierre et Marie Curie, Paris)

Title: *The thermodynamic limit for one-dimensional interacting quantum fermions in a random environment*

Abstract: In this talk, we present a simple model of one dimensional interacting electrons in a disordered environment and describe its thermodynamic limit. We shall describe both the ground state energy per particle and the ground state itself. Our main parameter to control the system are the density of particles (or equivalently the density of random scatterers): we assume the density of particles to be small (or equivalently the density of random scatterers to be large). The electrons interact through a repulsive potential. We will in particular be interested in the effect of the interaction on the localization properties of the ground state.

The results were obtained in collaboration with N. Veniaminov.

Speaker: **Antti Knowles** (New York University)

Title: *Random band matrices as a model of quantum disorder*

Abstract: I introduce random band matrices, which constitute an alternative to random Schrodinger operators for describing disordered quantum systems. Then I describe (a) the key conjectures with supporting evidence; (b) what is currently known rigorously, along with the main ingredients of the proofs; and (c) some open problems. I focus on the large-time asymptotics of the dynamics, eigenvectors localization/delocalization, microscopic eigenvalue statistics, and mesoscopic eigenvalue statistics.

Speaker: **Mathieu Lewin** (CNRS / University of Cergy-Pontoise)

Title: *The mean-field approximation of stochastic crystals*

Abstract: We introduce and study a Hartree-Fock-type model for a crystal in which the nuclei are thrown in space in a random fashion around a reference lattice, and the interacting electrons are purely quantum. After having settled the short range case, we particularly discuss open problems associated with the Coulomb interaction. Joint work with Eric Cancès (Paris) and Salma Lahbabi (Cergy).

Speaker: **Peter Müller** (University of Munich)

Title: *Anderson's orthogonality catastrophe*

Abstract: We give an upper bound on the modulus of the ground-state overlap of two non-interacting fermionic quantum systems with  $N$  particles in a large but finite volume  $L^d$  of  $d$ -dimensional Euclidean space. The underlying one-particle Hamiltonians of the two systems are standard Schrödinger operators that differ by a non-negative compactly supported scalar potential. In the thermodynamic limit, the bound exhibits an asymptotic power-law decay in the system size  $L$ , showing that the ground-state overlap vanishes for macroscopic systems. The decay exponent can be interpreted in terms of the total scattering cross section averaged over all incident directions. The result confirms and generalises P. W. Anderson's informal computation [Phys. Rev. Lett. **18**, 1049–1051 (1967)]. This is joint work with Martin Gebert and Heinrich Küttler.

Speaker: **Son Nguyen** (University of Missouri)

Title: *Localization for multi-particle Anderson Hamiltonians*

Abstract: We extend the bootstrap multiscale analysis to multi-particle continuous Anderson Hamiltonians, obtaining Anderson localization with finite multiplicity of eigenvalues, a strong form of dynamical localization, and decay of eigenfunction correlations. (Joint work with Abel Klein)

Speaker: **Yoshiko Ogata** (Tokyo University)

Title: *A classification of finitely correlated state*

Abstract: I would like to talk about classification of finitely correlated state. This is a joint work with Sven Bachmann. Recently, Bachmann and Nachtergaele wrote an interesting paper, classifying a class of finitely correlated states called product vacua. Our work is a part of project to generalize it. We consider pure finitely correlated states given by auxiliary systems with some fixed dimension, with minimal representation.

Under some conditions, we can construct some Hamiltonian which has the finitely correlated state as a unique grand state. We showed that any Hamiltonian given by this can be connected by a path of smooth local Hamiltonians keeping uniform gap.

Speaker: **Arijeet Pal** (Harvard University)

Title: *Many-body localization phase transition: Breakdown of thermalization in highly excited states*

Abstract: We explore the dynamical properties of an isolated, interacting quantum spin system in the presence of disorder. We find a dynamical quantum phase transition at high energy densities between the thermal phase which thermally equilibrates and the localized phase which fails to equilibrate or to serve as its own heat bath. In this talk I will motivate and define the different measures which we use to distinguish the 2 phases and the transition between them. Based on our numerical study I will discuss the evidence which suggests that the critical point may have infinite randomness.

Speaker: **Christian Sadel** (University of British Columbia)

Title: *SDE limits for transfer matrices with hyperbolic channels and limiting eigenvalue processes* (joint work with B. Virag)

Abstract: We consider the transfer matrices of a random Schroedinger operator  $H_\lambda$  on a strip with disorder  $\lambda$ . In the critical scaling  $n \sim \lambda^{-2}$  we obtain a limiting SDE for  $n \rightarrow \infty$  for a certain Schur complement of the process of transfer matrices. As it turns out, the Schur complement eliminates the hyperbolic channels in the correct way. This limit SDE also leads to limiting processes for the eigenvalue process of the restriction of  $H_\lambda$  to finite pieces in the critical scaling. This generalizes a former result by Valko and Virag that could only deal with elliptic channels.

Speaker: **Benjamin Schlein** (Bonn University)

Title: *Hartree-Fock dynamics for weakly interacting fermions*

Abstract: The mean field regime of fermionic systems is naturally linked to a semiclassical limit. Asymptotically, the quantum evolution converges towards the classical Vlasov dynamics. A better approximation to the solution of the many-body Schroedinger equation is given by the Hartree-Fock equation. In this talk, we will present precise bounds on the rate of the convergence of the Schroedinger evolution towards the Hartree-Fock dynamics, for initial data close to Slater determinants with the correct semiclassical structure. This is a joint work with Niels Benedikter and Marcello Porta.

Speaker: **Daniel Schmidt** (Virginia Tech)

Title: *Eigenvalue statistics for two-particle models*

Abstract: We derive an upper bound on the probability that a certain type of local Hamiltonian has at least  $n$  eigenvalues in a given energy interval (an  $n$ -level Wegner estimate). We demonstrate the usefulness of this bound by verifying the input conditions for electron-hole Hamiltonians that arise in the Bogoliubov-de Gennes theory of dirty superconductors. The same bound can be applied to the Hamiltonian for an electron-hole pair in the Wegner orbital model.

Speaker: **Mira Shamis** (Princeton University)

Title: *Resonant delocalization on the Bethe strip*

Abstract: Recently, Aizenman and Warzel discovered a mechanism for the appearance of absolutely continuous spectrum for random Schrödinger operators on the Bethe lattice through rare resonances (resonant delocalization). We extend their analysis to operators with matrix-valued random potentials drawn from ensembles such as the Gaussian Orthogonal Ensemble. These operators can be viewed as random operators on the Bethe strip, a graph (lattice) with loops.

Speaker: **Balint Virag** (University of Toronto)

Title: *Two results about random spectra*

Abstract: 1. We show that the IDS for supercritical bond percolation on  $\mathbb{Z}^2$  has a continuous part. The

same holds for percolation on regular trees, and more general Galton Watson trees, but  $\mathbb{Z}^3$  is open. Joint work with Charles Bordenave and Arnab Sen.

2. The  $l^2$  mass of eigenvectors of the 1d random Schrödinger operator takes a certain shape at low noise. This shape is universal and is built from an exponentially decaying part and a random part. Joint work with Ben Rifkind.

Speaker: **Simone Warzel** (TU Munich)

Title: *Resonant delocalization in Random Schrödinger operators*

Abstract: We shall discuss recent results on the existence of extended states in the presence of disorder, and in particular the occurrence of energy regimes in which extended states are formed from resonating local quasi-modes. Such mechanism was found to play a key role for the phase diagram of Random Schroedinger operators on tree graphs. The corresponding eigenstates are non-ergodic, in the sense that they violate a heuristic version of the equidistribution principle, yet they do not exhibit Anderson localization. Similar resonant delocalization may play a significant role in conduction within systems of many particles. This mechanism is currently being explored further within the context of graphs of exponential volume growth (joint work with Michael Aizenman and Mira Shamis).

Speaker: **Jakob Yngvason** (University of Vienna)

Title: *BEC, interactions and disorder*

Abstract: The rigorous study of the interplay between interactions and disorder in Bose Einstein Condensation has to face the fact that our knowledge about BEC in interacting systems, even without disorder, is quite limited. In the talk I shall give a survey of what I know about the subject, in particular the effects of disorder on dilute Bose gases in the Gross-Pitaevskii limit.

Speaker: **Valentin Zagrebnov** (Université d'Aix-Marseille)

Title: *Condensation in a Disordered Bose-Hubbard Model*

Abstract: We study Bose-Einstein Condensation (BEC) in the infinite-range hopping Bose- Hubbard model with repulsive on-site particle interaction and in the presence of an ergodic random one-site potential. We show that interaction produces a new phenomenon: In contrast to usual enhancement of BEC for perfect bosons, for constant on-site repulsion and certain discrete single-site random potentials there is suppression of BEC at some fractional densities. This suppression appears with increasing disorder. On the other hand, the custom BEC suppression at integer densities may disappear, if disorder increases. It is found that for a continuous distribution the BEC critical temperature decreases for small on-site repulsion while the BEC is suppressed at integer values of density for large repulsions.