Workshop 13w5060
Interplay of convex geometry and Banach space theory
March 10 - March 15, 2013

Sunday

16:00 Check-in begins
(Front Desk Professional Development Centre - open 24 hours)

17:30 - 19:30 Buffet Dinner

20:00 Informal gathering in 2nd floor lounge, Corbett Hall. Beverages and small assortment of snacks are available on a cash honor system.

Monday

7:00 - 8:45 Breakfast

8:45 - 9:00 Introduction and Welcome by BIRS Station Manager, TCPL

9:00 - 9:30 Monika Ludwig: "Anisotropic Fractional Perimeters"

9:40 - 10:10 Andreas Bernig: "Minimality of k-planes in normed spaces"

10:10 - 10:45 Coffee Break, TCPL

10:45 - 11:15 Elizabeth Meckes: "The spectra of powers of random unitary matrices"

11:30 - 13:30 Lunch

13:00 - 14:00 Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall

14:00 Group Photo; meet in foyer of TCPL (photograph will be taken outdoors so a jacket might be required)

14:15 - 14:45 Vitali Milman: "Extension of Minkowski's polarization result to quasi-concave functions and related geometric inequalities"

14:45 - 15:15 Coffee Break, TCPL

15:15 - 15:45 Liran Rotem: "Geometric constructions and inequalities for α-concave functions"

15:55 - 16:25 Susanna Spektor: " Khinchine inequality for slightly dependent random variables"

16:35 - 17:05 Dima Ryabogin: "On the continual Rubik's cube"

17:30 - 19:30 Dinner

Tuesday
7:00 - 9:00 Breakfast

9:00 - 9:30 Imre Barany: "Jarnik's convex lattice n-gon for non-symmetric norms"

9:40 - 10:10 Rolf Schneider "Lipschitz continuous diametric completions"

10:10 - 10:45 Coffee Break, TCPL

10:45 - 11:15 Alexander Koldobsky: "Complex intersection bodies"

11:30 - 13:30 Lunch

13:30 - 14:00 Andrea Colesanti: "Functional notions of quermassintegrals"

14:10 - 14:40 Alina Stancu: "From convexity to centro-affine curvature flows and back"

14:40 - 15:15 Coffee Break, TCPL

15:15 - 15:45 Manuel Weberndorfer: "Shadow systems of asymmetric L_p zonotopes"

15:55 - 16:25 Ohad Giladi: "Bourgain's discretization Theorem"

16:35 - 17:05 Joscha Prochno: "Combinatorial Inequalities and Subspaces of L_1"

17:30 - 19:30 Dinner

20:00 Artem Zvavitch: "Stability of the reverse Blaschke-Santalo inequality for unconditional convex bodies"

Wednesday

7:00 - 9:00 Breakfast

9:00 - 9:30 Karoly Bezdek: "On the affine plank problem via successive inradii"

9:40 - 10:10 Mathieu Meyer: "Affine Invariant points"

10:10 - 10:45 Coffee Break, TCPL

10:45 - 11:15 Mark Meckes: "The magnitude of metric spaces"

11:30 - 13:30 Lunch

Free Afternoon

17:30 - 19:30 Dinner

Thursday

7:00 - 9:00 Breakfast

9:00 - 9:30 Gideon Schechtman: "A quantitative version of the commutator theorem for zero trace matrices II"

9:40 - 10:10 Alexander Litvak: "Vertex index of convex bodies"

10:10 - 10:45 Coffee Break, TCPL

10:45 - 11:15 Shiri Artstein: "Weighted covering numbers"

11:30 - 13:30 Lunch

13:30 - 14:00 Peter Pivoravov: "A central limit theorem for projections of the cube"

14:10 - 14:40 Gabriele Bianchi: "An update on the covariogram problem"

14:40 - 15:15 Coffee Break, TCPL

15:15 - 15:45 Jiazu Zhou: "Bonnesen-style mixed isohomothetic inequalities"

15:55 - 16:25 Susanna Dann: "The lower dimensional Busemann Petty problem in the complex hyperbolic space"

16:35 - 17:05 Xiao, Jie: "Capacity, surface area, and graphic ADM mass"

17:30 - 19:30 Dinner

Friday

7:00 - 9:00 Breakfast

9:00 - 9:30 Paul F.X. Müller: "Davis and Garsia Inequalities for Hardy Martingales and dyadic Perturbations"

9:40 - 10:10 Vlad Yaskin: "Counterexamples to convexity of k-intersection bodies"

10:10 - 10:45 Coffee Break, TCPL

11:30 - 13:30 Lunch

ABSTRACTS

Shiri Artstein: Weighted covering numbers

Abstract: We will discuss the notion of weighted covering/separation numbers and some inequalities comparing classical covering numbers with weighted ones. We show an application where we give a new proof for Rogers' bound for the covering number of a convex body by slightly smaller copies of itself.

Based on joint work with Orit Raz and on work in progress with Boaz Slomka.

Imre Baranyi: Jarnik's convex lattice n-gon for non-symmetric norms

Abstract: What is the minimal perimeter L_n that a convex lattice polygon with n vertices can have? Jarnik proved in 1926 that $L_n = \frac{\sqrt{6\pi}}{9} n^{3/2} + O(n^{3/4})$. The aim of this paper is to extend this result to all, not necessarily symmetric, norms in the plane. We also determine the limiting shape of the convex lattice n-gon minimizing the perimeter. This is joint work with N. Enriquez from Paris.

Andreas Bernig: Minimality of k-planes in normed spaces

We introduce a new volume definition on normed vector spaces. We show that the induced k-area functionals are convex for all k. In the particular case $k = 2$, our theorem implies that Busemann's 2-density is convex, which was recently shown by Burago-Ivanov. We also show how the new volume definition is related to the centroid body.

Gabriele Bianchi: An update on the covariogram problem

Abstract: The covariogram g_K of a set K in R^d is the function that to each x in R^d associates the volume of the intersection of K and K+x.

When the dimension d is larger than 2 very little is known regarding whether g_K determines K or not. We present a positive result for very smooth convex bodies in any dimension d. The proof relies on some results by T. Kobayashi on the Fourier transform of the characteristic function of K, seen as a function of several complex variables.

The same problem, whether g_K determines K or not, is considered also

when in the definition of g_K the volume is substituted by the width. In a joint paper with Gennadiy Averkov, we prove that the width-covariogram determines any convex polygon (as it is in the case of the ordinary covariogram) but, (contrary to the ordinary covariogram) it does not determine convex polytopes already in dimension 3.

Andrea Colesanti: Functional notions of quermassintegrals

Abstract: This talk is about a recent work in collaboration with S. Bobkov and I. Fragala', in which we introduced a notion of quermassintegral for the class of quasi-concave functions. In the talk I will present the construction of functional quermassintegrals, trying to motivate it with analogies with properties of classical quermassintegrals such as Steiner type formulas, inequalities of Brunn-Minkowski type and integral geometric fomulas.

Susanna Dann: The lower dimensional Busemann-Petty problem in the complex hyperbolic space

Abstract. The lower dimensional Busemann-Petty problem asks whether origin-symmetric convex bodies in $\mathbb{R}^n$ with smaller volume of all k-dimensional sections necessarily have smaller volume. The answer is negative for $k > 3$. The problem is still open for k = 2; 3. We study this problem in the complex hyperbolic n-space $\mathbb{H}^n_{\mathbb{C}}$ and prove that the answer is affirmative only for sections of complex dimension one and negative for sections of higher dimensions.

Ohad Giladi: Bourgain's discretization Theorem

Abstract: A theorem by Ribe states that every two uniformly homeomorphic Banach spaces have the same local structure. In 1987, Bourgain proved a quantitative version of this theorem, and recently there has been some progress in this direction.

Alexander Koldobsky: Complex intersection bodies

Abstract: Complex intersection bodies in $\mathbb{C}^n$ can be interpreted as 2-intersection bodies in $\mathbb{R}^{2n}$ with complex structure, i.e. invariant with respect to rotations of pairs of coordinates by the same angle. We show how adding complex structure affects 2-intersection bodies and makes the properties of complex intersection bodies similar to those of their real counterparts. This is joint work with G.Paouris and M.Zymonopoulou.

Alexander Litvak: Vertex index of convex bodies

Abstract: We show several results on the vertex index of a given d-dimensional centrally symmetric convex body, which, in a sense, measures how well the body can be inscribed into a convex polytope with small number of vertices. This index is closely connected to the illumination parameter of a body, introduced earlier by Karoly Bezdek, and, thus, related to the famous conjecture in Convex Geometry about covering of a d-dimensional body by 2^d smaller positively homothetic copies. We provide estimates of this index and relate the lower bound with the outer volume ratio. We also discuss the sharpness of the bounds, providing examples. Finally, we discuss the non-symmetric case. The talk is based on joint works with K.Bezdek and E.D.Gluskin.

Monika Ludwig: Anisotropic fractional perimeters

Abstract: For a Borel set $E \subset \mathbb{R}^n$ and $0 < s < 1$, the fractional s-perimeter of E is given by

$$P_s(E) = \int_E \int_{E^c} \frac{1}{|x-y|^{n+s}}\, dx\, dy, \tag{1}$$

where E^c denotes the complement of E in $\mathbb{R}^n$ and $|\cdot|$ the Euclidean norm on $\mathbb{R}^n$. The functional P_s is an $(n-s)$-dimensional perimeter on Borel sets on $\mathbb{R}^n$, as $P_s(\lambda E) = \lambda^{n-s} P_s(E)$ for $\lambda > 0$. It is non-local in the sense that it is not determined by the behavior of E in a neighborhood of ∂E.

The limiting behavior of fractional s-perimeters as $s \to 1^-$ and $s \to 0^+$ turns out to be very interesting. Dávila, extending results by Bourgain, Brezis & Mironescu from 2002, shows that for a bounded Borel set $E \subset \mathbb{R}^n$ of finite perimeter,

$$\lim_{s \to 1^-} (1-s) P_s(E) = \alpha_n P(E),$$

where $P(E)$ is the perimeter of E and α_n is a constant depending on n.

Anisotropic fractional s-perimeter is a natural generalization of the Euclidean notion of fractional perimeter obtained by replacing the Euclidean norm $|\cdot|$ in (1) by an arbitrary norm $\|\cdot\|_L$ with unit ball L. In the talk

the limiting behavior of anisotropic s-perimeters as $s \to 1^-$ and $s \to 0^+$ is discussed. Applications to anisotropic fractional isoperimetric inequalities and anisotropic fractional Sobolev norms are also presented.

Elizabeth Meckes: The spectra of powers of random unitary matrices

Abstract: It has long been recognized that the eigenvalues of random unitary matrices look very different from i.i.d. points on the circle: they are very evenly spaced. As you start raising a random unitary matrix to powers, the eigenvalues start to clump together, and by the time you raise a random matrix in U(N) to the Nth power, the eigenvalues look exactly like i.i.d. points on the circle. There is a good reason for this: Rains showed that the distributions are exactly the same, and that there is a kind of smooth interpolation between the two extremes. In joint work with M. Meckes, we quantify this phenomenon by proving sharp non-asymptotic estimates on the means and tails of the L_p Wasserstein distances between spectral measures of powers of random unitary matrices and the uniform measure on the circle. The bounds also allow us to obtain sharp almost-sure convergence rates as the size of the matrix tends to infinity. Along the way, we needed the long-suspected fact that the full unitary group satisfies the same type of log-Sobolev inequality as the special unitary group; I will sketch the proof of this result, which should be of independent interest.

Mark Meckes: The magnitude of metric spaces

Magnitude is a partially defined numerical invariant of metric spaces introduced recently by Tom Leinster, motivated by considerations from category theory, which generalizes the cardinality of a finite set. I will discuss the motivation of the definition and some of what is known and not known about magnitude, highlighting connections with harmonic analysis, intrinsic volumes (in both convex and Riemannian geometry), and biodiversity. This is work of Tom Leinster, Simon Willerton, and myself.

Mathieu Meyer: Affine invariant points

Abstract: We answer in the negative a question by Grünbaum who asked if there exists a finite basis of affine invariant points. We give a positive answer to another question by Grünbaum about the “size” of the set of all affine invariant points. Related, we show that the set of all convex bodies

K, for which the set of affine invariant points is all of $\mathbb{R}^n$, is dense in the set of convex bodies. Crucial to establish these results, are new affine invariant points, not previously considered in the literature.

Paul F.X. Müller: Davis and Garsia Inequalities for Hardy Martingales and dyadic Perturbations

Abstract: For Hardy martingales we improve and extend the square function estimates and the Davis-Garsia inequalities. We use the general iteration principle introduced by Jean Bourgain in: Embedding L_1 to L_1/H_1, TAMS 278 (1983)

Peter Pivoravov: A central limit theorem for projections of the cube

Abstract: I will discuss a central limit theorem for the volume of projections of the N-cube onto a random subspace of dimension n, when n is fixed and N tends to infinity. Randomness in this case is with respect to the Haar measure on the Grassmannian manifold.

Joscha Prochno: Combinatorial Inequalities and Subspaces of L_1

Abstact:The structure and variety of subspaces of L_1 is very rich. Over the years, tremendous efforts has been put in characterizing subspaces of L_1. Although there are a number of sophisticated criteria at hand now, it might turn out to be nontrivial to decide for a specific Banach space whether it is isomorphic to a subspace of L_1. Let M_1 and M_2 be N-Functions (i.e., Orlicz functions up to a simple additional assumption). We establish some combinatorial inequalities and show that the spaces $\ell^n_{M_1}(\ell^n_{M_2})$ are uniformly isomorphic to subspaces of L_1 if M_1 and M_2 are "separated" by a function t^r, $1 < r < 2$.

Liran Rotem: Geometric constructions and inequalities for α-concave functions:

Following the work of Borell, α-concave functions are as usually treated as densities of special classes of measures. However, one may also examine these functions from a geometric point of view, as a generalization of convex bodies. In this talk, various notions about convex bodies will be extended to the functional setting, including the support function, the Minkowski addition, the mean width, and several notions of duality. Next, we will briefly discuss the special case of quasi-concave functions and a functional extension of Minkowski's theorem on mixed volumes (based on joint work with V. Milman). Finally, we will describe a new transform on α-concave functions, and present a new Blaschke-Santalo type inequality for this class.

Dima Ryabogin: On the continual Rubik's cube

Abstract: Let f and g be two continuous functions on the unit sphere S^{n-1} in $\mathbb{R}^n$, $n \geq 3$, and let their restrictions to any one-dimensional great circle E coincide after some rotation ϕ_E of this circle: $f(\phi_E(\theta)) = g(\theta)$ $\forall \theta \in E$. We prove that in this case $f(\theta) = g(\theta)$ or $f(\theta) = g(-\theta)$ for all $\theta \in S^{n-1}$.

Gideon Schechtman: A quantitative version of the commutator theorem for zero trace matrices II

Abstract: As is well known, a complex $m \times m$ matrix A is a commutator (i.e., there are matrices B and C of the same dimensions as A such that $A = [B, C] = BC - CB$) if and only if A has zero trace. If $\| \cdot \|$ is the operator norm from ℓ_2^m to itself and $| \cdot |$ any ideal norm on $m \times m$ matrices then clearly for any A, B, C as above $|A| \leq 2\|B\||C|$.
Does the converse hold? That is, if A has zero trace are there $m \times m$ matrices B and C such that $A = [B, C]$ and $\|B\||C| \leq K|A|$ for some absolute constant K? If not, what is the behavior of the best K as a function of m? A year ago I spoke at the same location on the case of $| \cdot |$ being the operator case. This time I'll speak mostly about the easier case of $| \cdot |$ = the Hilbert–Schmidt norm for which we (in this case Johnson and myself) have a basically complete answer.

Rolf Schneider: Lipschitz continuous diametric completions

Abstract: A nonempty bounded subset M of a metric space is (diametrically) complete if it cannot be enlarged without increasing its diameter. A completion of M is any complete subset of the space that contains M and has the same diameter. For example, in a Euclidean space, a completion of a set M of diameter d is any convex body of constant width d that contains M. By Zorn's lemma, any nonempty bounded subset of a metric space has a completion, in fact many, in general. The talk deals with constructive completion procedures in normed spaces, which lead to unique completions and for which continuity properties with respect to the Hausdorff metric can be shown. (Joint work with Jose Pedro Moreno)

Susanna Spektor: Khinchine inequality for slightly dependent random variables

Abstract: We prove Khinchine inequality under assumptions on the sum of Rademacher random variables and show some applications. (Joint work with A. E. Litvak and M. Rudelson).

Alina Stancu: From Convexity to Centro-affine Flows and back

I will start by showing how centro-affine curvature flows appear naturally in within the theory of convex bodies. Long term existence and uniqueness of solutions to these flows employ main features of the latter theory. To come full circle, I will present an application of the flows to convex geometry. Part of the results presented will be based on joint work with M. N. Ivaki.

Manuel Weberndorfer: Shadow systems of asymmetric L_p zonotopes

Abstract: New inequalities for the asymmetric L_p volume product and the asymmetric L_p volume ratio are presented, together with their equality conditions. These inequalities are established using properties of shadow systems and have Reisners volume product inequality for L_1 zonotopes as a special case. The symmetric version of our inequalities has been previously obtained by Campi and Gronchi. Our results also settle the question of uniqueness of the extremals in their inequalities.

Vlad Yaskin: Counterexamples to convexity of k-intersection bodies.

Abstract: It is a well-known result due to Busemann that the intersection body of an origin-symmetric convex body is also convex. Koldobsky introduced the notion of k-intersection bodies. We show that the k-intersection body of an origin-symmetric convex body is not necessarily convex if $k > 1$.

Xiao Jie: Capacity, surface area, and graphic ADM mass

Abstract: Starting with the Polya-Szego conjecture - of all convex bodies

with a given surface area, the round ball has the minimum electrostatic capacity, we will address some inequalities linking the variational capacity, the surface area and the graphic ADM mass.

Jiazu Zhou: Bonnesen-style mixed isohomothetic inequalities

Abstract: By the translative invariant containment measure in integral geometry, we derive some Bonnesen-style mixed isohomothetic inequalities that imply the known Mimkowski inequality for mixed area of two planar convex sets. The method could lead to Bonnesen-style mixed isohomothetic inequalities that generalize Alaxandrov-Fenchel inequality for mixed volumes of convex bodies in space.

Artem Zvavitch: Stability of the reverse Blaschke-Santalo inequality for unconditional convex bodies.

Abstract: Mahler's conjecture asks whether the cube is a minimizer for the volume product of a body and its polar in the class of symmetric convex bodies in $\mathbb{R}^n$. The corresponding inequality to the conjecture is sometimes called the the reverse Blaschke-Santaló inequality. The conjecture is known in $\mathbb{R}^2$ and in several special cases. In the class of unconditional convex bodies, Saint Raymond confirmed the conjecture, and Meyer and Reisner, independently, characterized the equality case. In talk we will present a joint work with Jaegil Kim on stability version of these results and also show that any symmetric convex body, which is sufficiently close to an unconditional body, satisfies the the reverse Blaschke-Santaló inequality.