Algebraic Geometry and Geometric Modeling
27 January — 1 February 2013

Monday

7:00–8:45	Breakfast
8:45–9:00	Introduction and Welcome by BIRS Station Manager, TCPL
9:00–9:40	Chandrajit Bajaj, Geometric Modeling Tales Born from Two Sciences: Algebra & Geometry
9:50–10:30	Jorg Peters, Refinability of splines derived from regular tessellations
10:30–11:00	Coffee Break, TCPL
11:00–11:40	Wenping Wang, All-hex Meshing Using Singularity-restricted Field
11:40–13:00	Lunch
13:00–14:00	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:00	Group Photo; meet in foyer of TCPL (photograph will be taken outdoors so a jacket might be required).
14:10–14:50	Ming C. Lin, Roles of Algebraic Geometry in Physics-based Simulation
14:50–15:20	Coffee Break, TCPL
15:20–16:00	Minho Kim, GPU Isosurface Raycasting of Volume Datasets Based On Box-Splines
16:10–16:50	Gershon Elber, Multivariate (Geometric) Constraints Solving using Subdivision based Solvers
17:00–17:40	Ileana Streinu, Algebraic equations with real roots arising in Origami design
17:40–19:30	Dinner
19:40–20:20	Tom Sederberg, Lessons Learned while Commercializing a CAD Technology

Tuesday

7:00–9:00	Breakfast
9:00–9:40	Helmut Pottmann, Structures from Circular Arcs
9:50–10:30	Gabriel Taubin, Non-Convex Hull Surfaces
10:30–11:00	Coffee Break, TCPL
11:00–11:40	R. Krasauskas and S. Zube, Clifford-Bezier curves and surfaces
11:30–13:30	Lunch
14:00–14:40	Bernard Mourrain, Spline spaces on planar and volume subdivisions
14:40–15:10	Coffee Break, TCPL
15:10–15:50	Falai Chen, Modified T-Splines
16:00–16:40	Jiansong Deng, Spline Spaces over T-meshes
16:50–17:30	Tor Dokken, Locally Refined B-splines and Linear Independence
17:30–19:30	Dinner
19:40–20:20	Chandrajit Bajaj, Rememberances of Prof. S. Abhyankar

Wednesday

7:00–9:00	Breakfast
9:00–9:40	Ragni Piene, Higher order self-dual toric varieties
9:50–10:30	Luis Garcia, The control polyhedron of a rational Bezier surface
10:30–11:00	Coffee Break, TCPL
11:00–11:40	Stephen Mann, Error in Multivariate Polynomial Interpolation
11:40–13:30	Lunch
	Free Afternoon
17:30–19:30	Dinner

Thursday	
7:00–9:00	Breakfast
9:00–9:40	Carlos D'Andrea, Rational Plane curves with $\mu = 2$
9:50–10:30	Xuhui Wang, μ-Bases for Complex Rational Curves
10:30–11:00	Coffee Break, TCPL
11:00–11:40	Laurent Busé, Matrix representations of parameterized curves and surfaces
11:40–13:30	Lunch
14:00–14:40	Bert Juttler, Derivatives of isogeometric test functions
14:40–15:10	Coffee Break, TCPL
15:10–15:50	Jianmin Zheng, Foldover-free surface reparameterization with hard constraints
16:00–16:40	Scott Schaefer, Generalized Barycentric Coordinates
16:50–17:30	Hal Schenck, Geometry of Wachpress surfaces
17:30–19:30	Dinner
Friday	
7:00–9:00	Breakfast
9:00–9:40	Ciprian Borcea, Volume frameworks and deformation varieties
9:50–10:30	Gregory G. Smith, Nonnegative sections and sums of squares
10:30–11:00	Coffee Break, TCPL
11:00–11:40	
11:40–13:30	Lunch
Checkout by 12 noon.	

** 5-day workshop participants are welcome to use BIRS facilities (BIRS Coffee Lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

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ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **Chandrajit Bajaj** (University of Texas)
Title: *Geometric Modeling Tales Born from Two Sciences: Algebra & Geometry*
Abstract: I shall describe the solution to two problems where conversant knowledge of algebra & geometry paves the way for computational efficient solutions in geometric modeling.

First, through the use of the theory of polyhedral symmetric groups, root systems, and their extensions, one is able to generate all regular and semi-regular symmetric tilings (tesselations) of a sphere. This characterization enables a 6D parameterization of the search space via symmetric decorations of periodic and aperiodic planar tilings, and thereby the first polynomial time solution to automated prediction and design of 3D shell assemblies of varying sizes. Moreover, this theory provide a generalized subdivision procedure for spherical polyhedra, wherein increased facet complexity preserves local symmetries.

In a second brief tale, I'll show how desingularization theory of algebraic curves, and especially the constructive application of monoidal transformations to blowup a singularity, allows for a general procedure to achieve robust numerical quadrature/cubature of singular/hyper singular integrands (kernels). I'll show how effective use of this provides for a stable solution of the boundary derivative solutions of the Poisson-Boltzmann equation using algebraic splines in an isogeometric sense.

Speaker: **Ciprian Borcea** (Rider University,)
Title: *Volume frameworks and deformation varieties*
Abstract: Like their kindred bar-and-joint frameworks, volume frameworks may lead to interesting deformation spaces. We explore singularities and deformation varieties for cyclic volume frameworks associated to polygons, with particular regard to heptagons and K3 surfaces. This is joint work with Ileana Streinu.

Speaker: **Laurent Busé** (INRIA Sophia Antipolis)
Title: *Matrix representations of parameterized curves and surfaces*
Abstract: In geometric modeling, parameterized curves and surfaces are used intensively. To manipulate them, it is useful to have an implicit representation, in addition to their given parametric representation. Indeed, a parametric representation is for instance well adapted for visualization purposes whereas an implicit representation allows significant improvements in the computation of intersections. Nevertheless, implicit representations are known to be very hard to compute. To overcome this difficulty, general matrix-based implicit representations of parameterized curves and surfaces (hereafter called *matrix representations*) will be discussed in this talk, as well as their application to intersection problems in geometric modeling.

Roughly speaking, a matrix representation consists in a matrix depending on the implicit variables and whose rank drops exactly on the associated parameterized object. We will first describe a very simple method to compute these matrix representations. They can be seen as an extension of the method of moving lines and moving surfaces that has been initiated by Sederberg and Chen in the case of plane curves. Indeed, matrix representations are in general *singular matrices* whereas the method of moving lines and surfaces was developed with the constraint of building a non-singular matrix. The gain of this extension is that matrix representations are valid for a dramatically larger class of parameterized curves and surfaces, actually any parameterized curves (including space curves) and almost all parameterized surfaces. Moreover, these matrix representations allow to treat intersection problems with some classical and well established tools of numerical linear algebra (such as the singular value decomposition and generalized eigenvalues computations), opening in this way the door to a more stable and robust numerical treatment of intersection problems.

In the second part of the talk we will show that matrix representations also contain a lot of geometric properties of their associated parameterization. Indeed, we will show that the inversion problem can be solved very simply with a matrix representation. More generally, we will show that many properties of the singularities of a parameterization can be described from its matrix representations. This can be seen as an extension of the notion of *singular factors* introduced by Chen, Goldman and others for plane curves to the case of space curves and surfaces (this is a joint work with Nicolas Botbol, University of Buenos Aires and Marc Chardin, University of Paris VI).

Speaker: **Falai Chen** (University of Science and Technology of China)
Title: *Modified T-Splines*
Abstract: T-splines are a generalization of NURBS surfaces, the control meshes of which allow T-junctions. T-splines can significantly reduce the number of superfluous control points in NURBS surfaces, and provide valuable operations such as local refinement and merging of several B-splines surfaces in a consistent framework. In this talk, we propose a variant of T-splines called Modified T-splines. The basic idea is to construct a set of basis functions for a given T-mesh that have the following nice properties: non-negativity, linear independency, partition of unity and compact support. The basis functions are constructed as linear combinations of the B-spline basis functions over the extended tensor product mesh of the given T-mesh. Due to the good properties of the basis functions, the Modified T-splines are favorable both in adaptive geometric modeling and iso-geometric analysis.

Speaker: **Carlos D'Andrea** (University of Barcelona)
Title: *Rational Plane curves with* $\mu = 2$
Abstract: We describe sets of minimal generators of the defining ideal of the Rees Algebra associated to the ideal of three bivariate homogeneous polynomials parametrizing a proper rational curve in projective plane, having a minimal syzygy of degree 2. Joint work with Teresa Cortadellas.

Speaker: **Jiansong Deng** (University of Science and Technology of China)
Title: *Spline Spaces over T-meshes*
Abstract: A T-mesh is a rectangular grid that allows T-junctions. Some types of spline spaces over T-meshes have been considered in the literature, including hierarchical B-splines, T-splines and LR splines. In the talk, I will explain why we need T-meshes in geometric modeling and how to define spline spaces over T-meshes. In the talk, I will introduce a new type of spline spaces over T-meshes and given their dimension formula and basis function construction. The applications in computer graphics, image processing and isogeometric analysis are reviewed as well.

Speaker: **Tor Dokken** (SINTEF, Oslo)
Title: *Locally Refined B-splines and Linear Independence*
Abstract: We will address local refinement of a tensor product grid specified as a sequence of inserted line segments parallel to the knot lines. The line segments are assigned multiplicities to model the continuity across each line segments individually. We obtain a quadrilateral grid with T-junctions in the parameter domain, and a collection of tensor product B-splines on this mesh here named an LR-mesh. The approach applies equally well in dimensions higher than two. By refining according to a hand-in-hand principle between the dimensions of the spline space over the LR-mesh, the spline space spanned by the Locally Refined B-splines and the number of locally refined B-splines the LR B-splines are linear independent and form a basis. Alternatively linear independence is not check during refinement, but the "pealing algorithm" is used to check if the resulting collection of LR B-splines is linear independent.

This is joint work with Tom Lyche and Kjell Fredrik Pettersen.

Speaker: **Gershon Elber** (Technion)
Title: *Multivariate (Geometric) Constraints Solving using Subdivision based Solvers*
Abstract: We explore a subdivision based paradigm to solve a set of (piecewise) rational constraints represented by (piecewise) rational multivariate spline functions. While the basic approach is robust, it can also be slow. In this talk, we will examine and survey several recently presented schemes to alleviate these computational costs. The addition of inequality filters, single solution isolation techniques, orthogonalization and domain reduction, and the minimization of the memory explosion that results from the exponential dependency on the number of variables will be discussed.

With this machinery, we will demonstrate that this type of solvers can be successfully used to solve a large variety of (geometric) problems above the (piecewise) rationals domain. This set of problems includes point-curve and curve-curve bi-tangents, convex hulls and kernels of planar curves and 3-space surfaces, ray-traps (bouncing billiard balls) between planar curves, the 10th Apollonius problem (a circle tangent to given three circles) and its generalization, bounding circles and spheres, bisectors and Voronoi regions, mold design, visibility and accessibility, sweeps and envelopes, and self-intersection computation and trimming in offset approximations of curves and surfaces.

Speaker: **Luis Garcia** (Sam Houston State University)
Title: *The control polyhedron of a rational Bezier surface*
Abstract: Algebraic geometry investigates the algebraic and geometric properties of polynomials. Geometric modeling uses polynomials to build computer models for industrial design and manufacture from basic units, called patches, such as, Bezier curves and surfaces. Bezier curves are governed by their control points. The polygon formed by connecting the control points with line segments is called the control polygon. This polygon is unique and determines many important features of the curve, thus validating its name.

A Bezier surface is also intuitively governed by control points; in particular, the surface lies within the convex hull o its control points. This convex hull is often indicated by drawing some edges between the control points, the resulting structure is called a "control mesh". Unlike curves, there is no unique choice of control mesh for a surface. So it is not clear in which way these meshes "control" the Bezier surface. In this talk, we will present one possible answer to this uestion. Our results rely upon the geometry of toric varieties.

This is joint work with Frank Sottile and Chungang Zhu.

Speaker: **Bert Jüttler** (JKU Linz)
Title: *Derivatives of isogeometric test functions*
Abstract: This talk is motivated by the representation of the test functions in Isogeometric Analysis (IgA). IgA is a numerical method that uses the NURBS-based representation of a CAD model to generate the finite-dimensional space of test functions which is used for the simulation. More precisely, the test functions are obtained by composing the inverse of the domain parametrization (also called the geometry mapping) with the NURBS (rational B-spline) basis functions. We derive a representation of the derivatives of these test functions in NURBS form. More precisely, given a (possibly piecewise) rational geometry mapping and a rational test functions defined on it, we present a method to compute the derivatives of the test functions with respect to the global coordinate system. The derivatives are again given as a rational function defined on the rational geometry mapping. All computations can be described compactly using homogeneous coordinates. We then use these results to derive conditions on the isogeometric test functions which guarantee C_k smoothness, in particular for the interesting case of singularly parametrized domains. The conditions depend heavily on the given geometry mapping. We present C_0, C_1 and C_2 smoothness results for a special class of singularly parametrized domains and compare them with existing H_1 and H_2 regularity results. The framework can be applied to all types of singularities and to derivatives of higher order.

This is joint work with Thomas Takacs.

Speaker: **Minho Kim** (University of Seoul)
Title: *GPU Isosurface Raycasting of Volume Datasets Based On Box-Splines*
Abstract: In this talk, I will discuss the techniques for GPU isosurface volume raycasting on the BCC and FCC datasets reconstructed by box-splines. The six-direction cubic box-spline and the seven-direction quartic box-spline are used for reconstruction of FCC and BCC datasets, respectively. For real-time isosurface raycasting, the evaluation procedures should be optimized for graphics hardware. Specifically, conditional branches and lookup tables, which are natural choices for implementation, need to be avoided. Other techniques for performance improvements and fast and accurate normal computation are also discussed. The results show both performance and quality improvements compared to previous compatible methods.

Speaker: **R. Krasauskas and S. Zube** (Vilnius University)
Title: *Clifford-Bezier curves and surfaces*
Abstract: We survey several cases of quaternion and geometric (Clifford) algebra based generalizations of Bezier curves and surfaces which were recently introduced into geometric modeling:
- complex curves on the plane,
- quaternion representation of principal Dupin cyclide patches with Willmore energy formulas,
- general bilinear quaternionic patches of Darboux cyclides (with up to 6 famillies of circles),
- a parallel theory of isotropic cyclides based on geometric algebra over the isotropic space $\mathbb{R}^3$ with signature ++0.

All these constructions are invariant with respect to certain groups of classical geometric transformations, e.g. Moebius or Laguerre transformations, so they are naturally related to special kinds of rational surfaces that are important to geometric modeling, e.g. Pythagorean-normal surfaces. We introduce the concept of Clifford-Bezier curves and surfaces, with the aim to unify various before mentioned constructions. Finally we will discuss existing and potential applications, and natural limits of this approach.

Speaker: **Ming C. Lin** (UNC)
Title: *Roles of Algebraic Geometry in Physics-based Simula1tion*
Abstract: From turbulent fluids to granular flows, many phenomena observed in nature and in society show complex emergent behavior on different scales. The modeling and simulation of such phenomena continues to intrigue scientists and researchers across different fields. Understanding and reproducing the visual appearance and dynamic behavior of such complex phenomena through simulation is valuable for enhancing the realism of virtual scenes and for improving the efficiency of design evaluation. This is especially important for applications, where it is impossible to manually animate all the possible interactions and responses beforehand. In this talk, we discuss the roles and applications of algebraic geometry used in geometric modeling of complex surfaces to solving inequality arising from various constraints in simulating such phenomena. Some of the example dynamical systems that I will describe include turbulent fluids, deformable tissues, granular flows, and crowd simulation. I conclude by discussing some research challenges in algebraic geometry and geometric modeling for physics-based simulation.

Speaker: **Stephen Mann** (University of Waterloo)
Title: *Error in Multivariate Polynomial Interpolation*
Abstract: One approach to multivariate data interpolation is to select a polynomial basis, construct a Vandermonde matrix for the basis and data points, where the inverse of this matrix gives the polynomial coefficients of the interpolant. In this talk, we show that any fixed choice of basis has corresponding sets of data that result in poor approximations in the interpolant. In particular, we point out that approximation power is lost when data points a near common zero's of this fixed basis, and we explain why schemes such as the Least usually produce good interpolants.

Speaker: **Bernard Mourrain** (INRIA Sophia-Antipolis)
Title: *Spline spaces on planar and volume subdivisions*
Abstract: We will consider spline functions over a partition of a domain, and describe algebraic techniques to analyse the dimension of these spaces and basis. Different examples for planar meshes including T-mesh and triangular meshes will be discussed. Extension to 3D will discussed too, in relation with some conjectures in algebraic geometry.

Speaker: **Jorg Peters** (University of Florida)
Title: *Refinability of splines derived from regular tessellations*
Abstract: Splines can be constructed by convolving the indicator functions of a cell whose shifts tessellate $\mathbb{R}^d$. This paper presents simple, non-algebraic criteria that imply that, for regular shift-invariant spaces, only a small subset of such spline families yield nested spaces: primarily the well-known tensor-product and box splines. Among the many non-refinable constructions are hex-splines and their generalization to the Voronoi cells of non-Cartesian root lattices.

Speaker: **Ragni Piene** (University of Oslo)
Title: *Higher order self-dual toric varieties*
Abstract: A lattice point configuration $\mathcal{A} \subset \mathbb{Z}^n$ defines a (real or complex) toric embedding in $\mathbb{P}^N$, where $N = \#\mathcal{A} - 1$. We want to characterize those varieties that are isomorphic to one of its higher order dual varieties, in particular find conditions on the configuration $\mathcal{A}$ or its convex lattice polytope $P = \mathrm{Conv}(\mathcal{A})$ for this to happen. This is joint work with Alicia Dickenstein, and generalizes previous work by Bourel, Dickenstein, Rittatore in the case of ordinary self dual toric varieties.

Speaker: **Helmut Pottmann** (King Abdullah University of Science and Technology)
Title: *Structures from Circular Arcs*
Abstract: We present novel structures which are composed of circular arcs and are motivated by potential applications in architecture: Looking for remarkable spatial designs from circles, we investigate the families of circles on Darboux cyclides and show how they can be arranged in form of hexagonal webs. Moreover, we discuss meshes whose edges are circular arcs and all whose nodes are congruent. Finally, we briefly address the kinematics of arc splines and illustrate how it can be used for surface design, surface approximation and non-static architecture.

Speaker: **Tom Sederberg** (Bringham Young)
Title: *Lessons Learned while Commercializing a CAD Technology*
Abstract: The press release for this workshop states, “These applications of geometric modeling to computer-aided geometric design and computer graphics are profoundly important to the world economy.” My understanding of what that means has become more clear because of the experience I have had over the past nine years of working with some former students to commercialize a new CAD technology called T-splines. The talk will discuss the challenges of such an endeavor and express some thoughts of the role that the underlying math plays in the overall commercialization effort. Two problems related to algebraic geometry that have arisen from the T-Splines research will be presented.

Speaker: **Scott Schaefer** (Texas A&M University)
Title: *Generalized Barycentric Coordinates*
Abstract: This talk explores the motivation and development of generalized barycentric coordinates. Starting from Wachspress’s 2D construction, we show generalizations to arbitrary dimension, smooth shapes, and newer generalizations of barycentric coordinates. The talk ends with a new construction for a family of barycentric coordinates for arbitrary 2D shapes and hints at the possibility of a closed-form construction for positive coordinates for arbitrary shapes.

Speaker: **Hal Schenck** (University of Illinois, Urbana-Champaign)
Title: *Geometry of Wachpress surfaces*
Abstract: Wachpress barycentric coordinates are a generalization of the usual barycentric coordinates for a simplex to a nonsimplicial polytope, and were introduced by Wachpress some 30 years ago in his work on finite elements. In general they are not unique, but Warren showed that they are unique if we require minimal degree coordinates. We study the Wachpress coordinates for a convex polygon with d vertices, and interpret them as a rational map from the projective plane to projective $d-1$ space. We prove the image is a smooth surface, and obtain an explicit description of the equations defining the surface (they are quadrics and cubics). We also determine a number of interesting algebraic invariants (Grobner basis, Castelnuovo-Mumford regularity, graded betti numbers) associated to the surface. This involves a connection to combinatorics via a Stanley-Reisner ideal.

This is joint work with Corey Irving, Santa Clara University.

Speaker: **Gregory G. Smith** (Queen's University)
Title: *Nonnegative sections and sums of squares*
Abstract: A polynomial with real coefficients is nonnegative if it takes on only nonnegative values. For example, any sum of squares is obviously nonnegative. For a homogeneous polynomial with respect to the standard grading, Hilbert famously characterized when the converse statement hold, i.e. when every nonnegative homogeneous polynomial is a sum of squares. In this talk, we will examine this converse for homogenous polynomials with respect to a positive multigrading. In particular, we will provide many new examples in which every nonnegative homogeneous polynomial is a sum of squares.

Speaker: **Ileana Streinu** (Smith College)
Title: *Algebraic equations with real roots arising in Origami design*
Abstract: In mid 1990's, Robert Lang proposed a beautiful and very original method for designing origami shapes with an underlying metric tree structure and implemented it in a freely available software called TreeMaker. Lang's method takes as input the metric tree and a polygonal region (the "piece of paper"). It starts with a non-linear optimization phase which—when successful—produces a decomposition into special convex pieces (which we call Lang polygons) of the input region. The second phase computes crease patterns for the Lang polygons. The main bottleneck in the applicability of Lang's method is the first phase, which often fails: how can that be avoided?

I will show that the optimization phase can be replaced by the construction of a very special Lang polygon directly from the input tree, and this can be achieved by setting up a system of algebraic equations that can be solved inductively. For the base case, we get a single equation whose coeffcients are (some of) the standard symmetric functions expressed in terms of the edge lengths of the tree. I will show that all the roots of this system are real, but only one—the largest, which is positive—is relevant for the origami design problem. An inductive argument then shows the existence of a real solution for the entire system. Along the way, I will relate the solutions of this algebraic system to the intrinsic curvature of a piecewise linear surface (the folded origami) and to the medial axis of the resulting convex polygon.

Speaker: **Gabriel Taubin** (Brown University)
Title: *Non-Convex Hull Surfaces*
Abstract: We present a new algorithm to reconstruct an approximating watertight surface from a finite oriented point cloud sampled from the smooth boundary surface of a solid object. The Convex Hull (CH) of an arbitrary set of points is the intersection of all the supporting linear half spaces. The CH boundary surface is piecewise linear and watertight, but since it cannot represent concavities, it is usually a poor approximation of the sampled surface. We introduce the Non-Convex Hull (NCH) of an oriented point cloud as the intersection of complementary spherical half spaces; one per point. The boundary surface of this set is a piecewise quadratic interpolating surface, which can also be described as the zero level set of the NCH Signed Distance function. Instead of developing a combinatorial algorithm to reconstruct this Non Convex Hull Surface as a union of quadratic and linear patches, we evaluate the NCH Signed Distance function

on the vertices of a volumetric mesh, regular or adaptive, and generate an approximating polygonal mesh for the NCH Surface using an isosurface algorithm. Despite its simplicity, this naïve algorithm produces high quality polygon meshes competitive with those generated by state-of-the-art algorithms. It is able to deal with moderate irregular sampling, and it is massively parallelizable. Since interpolating surfaces are not always desirable, we also propose an octree-based sampling scheme to construct a bounded-error approximating NCH Signed Distance function, which significantly speed-up the computation, but can produce meshes of the same quality.

Speaker: **Wenping Wang** (Univ. of Hong Kong)
Title: *All-hex Meshing Using Singularity-restricted Field*
Abstract: I shall present in a new method for computing an all-hex mesh of a 3D volume. Decomposing a volume into high-quality hexahedral cells is a challenging task in geometric modeling and computational geometry. Inspired by the recent use of frame fields in quad meshing and the CubeCover approach to hex meshing, we present a complete all-hex meshing framework based on the singularity-restricted field that induces a valid all-hex structure. Given a volume represented by a tetrahedral mesh and a 2D frame field on its boundary surface, we first compute a boundary-aligned 3D frame field inside it, then convert the frame field to be singularity-restricted by novel topological operations, and finally use the CubeCover method to compute the volume parametrization. Experimental results show that our algorithm is capable of generating high-quality all-hex meshes of a variety of 3D volumes robustly and efficiently.

Speaker: **Xuhui Wang** (Rice University)
Title: *μ-Bases for Complex Rational Curves*
Abstract: We present a fast algorithm for finding a μ-basis for any rational planar curve that has a complex rational parametrization. We begin by identifying two canonical syzygies that can be extracted directly from any complex rational parametrization without performing any additional calculations. For generic complex rational parametrizations, these two special syzygies form a μ-basis for the corresponding real rational curve. In those anomalous cases where these two canonical syzygies do not form a mu-basis, we show how to quickly calculate a mu-basis by performing Gaussian elimination on these two special syzygies. We also present an algorithm to determine if a real rational planar curve has a complex rational parametrization. Examples are provided to illustrate our methods.

Speaker: **Jianmin Zheng** (Nanyang Technological University)
Title: *Foldover-free surface reparameterization with hard constraints*
Abstract: This talk presents an algorithm for surface reparameterization by constructing a foldover-free 2D triangular mesh transformation subject to hard positional constraints. The method is based on iterative RBF-based warping and a subdivision scheme. It is shown to always work and have low distortion. We show one application of the algorithm in surface texture mapping.