

Integrable Systems and Moduli Spaces

August 25-30, 2013

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday

*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday

*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday

Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)

***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in the TransCanada Pipelines Pavilion (TCPL). LCD projector and blackboards are available for presentations. Ceiling-mounted video cameras are installed in the main lecture room of 201, TCPL. To enable both half workshops to use the recording facilities during the week, one group will be assigned to Room 201 during the mornings and to Room 202 in the afternoons. The Station Manager will confirm these details with you prior to your workshop.

SCHEDULE

You are welcome to schedule lectures as you see fit, as long as you adhere to the meal times (noted above), coffee break start and end times (noted below) and take into account the welcome on Monday morning and the Banff Centre tour at 1:00 pm. We would also like to take a group photograph during your stay and recommend either immediately after the last talk of Monday or Tuesday morning, or immediately after lunch. Please meet in the foyer of TCPL (photograph will be taken outdoors so a jacket might be required).

Please email your finalized schedule and abstracts to BIRS Station Manager birmsmgr@birs.ca by Thursday morning before your arrival (at the latest) in order to allow for printing and posting to the website.

You are also encouraged to e-mail the schedule to your participants. BIRS provides the option of an electronic mail list in order to facilitate communications with your participants. When you login to the Organizer Interface at <https://www.birs.ca/orgs>, you will be prompted to create an electronic mail list for your workshop. Click "Yes" to create one and receive instructions, or "No" to decline. If you would like more information about our electronic mail lists, please e-mail help@birs.ca.

Sunday

16:00 Check-in begins (Front Desk - Professional Development Centre - open 24 hours)

17:30–19:30 Buffet Dinner, Sally Borden Building

20:00 Informal gathering in 2nd floor lounge, Corbett Hall (if desired)

Beverages and a small assortment of snacks are available on a cash honor system.

Monday

- 7:00–8:45 Breakfast
8:45–9:00 Introduction and Welcome by BIRS Station Manager, TCPL
9:00–9:40 J.Hurtubise, “*Real bundles on real varieties*”
9:55–10:35 S.Grushevsky, “*Real-normalized differentials and the geometry of the moduli space of curves*”
Coffee Break
10:50–11:30 E.Previato, “*Projective curve parameters and abelian integrals*”
11:30–13:00 Lunch
13:00–14:00 Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:00 Group Photo; meet on the front steps of Corbett Hall
14:10–14:50 F.Soloviev, “*Geometry of dented pentagram maps*”
Coffee Break
15:10–15:50 A.McIntyre, “*Chern-Simons invariants, determinant of Laplacian, and tau functions*”
16:00–16:40 D.Zakharov, “*Tautological relations via the Abel–Jacobi map*”
17:30–19:30 Dinner

Tuesday

- 7:00–9:00 Breakfast
9:00–9:40 M.Mulase, “*Quantum curves and topological recursion*”
9:55–10:35 P.Norbury, “*A quantum curve for Gromov-Witten invariants of \mathbb{P}^1* ”
Coffee Break
10:50–11:30 M.Möller, “*Commensurability of non-arithmetic lattices in $PU(1,n)$* ”
11:30–13:30 Lunch
13:30–14:20 T.Bridgeland, “*Stability conditions and quadratic differentials*”
Coffee Break
14:50–15:30 D.Chen, “*Exceptional strata of quadratic differentials in low genus*”
15:50–16:30 V.Bouchard, “*Mirror symmetry for orbifold Hurwitz numbers*”
17:00–17:15 Th. Bothner, “*Transition asymptotics for the sine-kernel determinant*”
17:15–17:30 S. Chowdhury, “*An alternative proof of Goldman’s theorem using formulas from Soliton theory*”
17:30–19:30 Dinner

Wednesday

- 7:00–9:00 Breakfast
9:00–9:40 S.Lando, “*On computation of universal polynomials for characteristic classes of singularities*”
9:55–10:35 M.Kazarian, “*Symplectic Geometry of Topological Recursion*”
Coffee Break
10:50–11:30 S.Shadrin, “*Local topological recursion and Givental theory*”
11:30–13:30 Lunch
Free Afternoon
17:30–19:30 Dinner

Thursday

- 7:00–9:00** Breakfast
- 9:00–9:40** A.Voronov, “*The BV Formalism and the Cohomology of Moduli Spaces*”
- 9:55–10:35** D.Zvonkine, “*Cohomological relations on $\overline{\mathcal{M}}_{g,n}$ via 3-spin structures*”
Coffee Break
- 10:50–11:30** G.Borot, “*Why \hbar^{2g-2+n} and (p, q) minimal models*”
- 11:30–13:30** Lunch
- 13:30–14:20** G.van der Geer, “*Hurwitz spaces and divisors on moduli spaces of curves*”
Coffee Break
- 14:50–15:40** G.Farkas, “*The universal theta divisor over the moduli space of curves*”
- 16:00–16:40** R. de Jong, “*Torus bundles and 2-forms on the universal family of Riemann surfaces*”
- 17:00–17:15** M.Basok, “*Tau function and moduli of spin curves*”
- 17:15–17:30** Ch.Norton, TBA
- 17:30–19:30** Dinner

Friday

- 7:00–9:00** Breakfast
- 9:00–9:40** A.Schmitt, “*Motives of moduli spaces of Higgs bundles and holomorphic chains*”
- 9:55–10:35** J. Harnad, “*Tau functions as generators of integrable systems, random processes and moduli space invariants*”
Coffee Break
- 11:30–13:30** Lunch
- Checkout by
12 noon.**

** 5-day workshop participants are welcome to use BIRS facilities (BIRS 2nd floor lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

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ABSTRACTS

(in alphabetic order by speaker surname)

Speaker: **Gaetan Borot** (MPI Bonn)

Title: *Why \hbar^{2g-2+n} and (p, q) minimal models*

Abstract: To any solution of a linear differential system with rational coefficients depending on a dispersion parameter \hbar , we associate a kernel, correlation function obtained by determinantal formulas from the kernel, and shown to satisfy loop equations. A non-trivial question is then to know when the correlators do have an expansion of topological type in the small dispersion limit, i.e. the n -point correlators is of order $O(\hbar^{n-2})$, and has an expansion in powers of \hbar with parity $(-1)^n$. In this case, the coefficients of expansion are given by the topological recursion developed by Eynard and Orantin for the semi-classical spectral curve. For formal solutions of the (p, q) minimal model, we show by introducing an appropriate "insertion operator" that the expansion is indeed of topological nature. The 1-matrix model in the one-cut regime provides another example where the expansion has to be of topological nature. Incidentally (or not ?), both examples are actually embedded in a family of compatible differential systems, i.e. are integrable systems.

This is joint work with B. Eynard and M.C. Bergere.

Speaker: **Vincent Bouchard** (U of Alberta)

Title: *Mirror symmetry for orbifold Hurwitz numbers*

Abstract: In recent years, it has been found that many enumerative geometric problems have a common feature: they have a mirror symmetric counterpart which is governed by a universal integral recursion formula due to Eynard and Orantin. The key ingredient to the mirror theory is the existence of a spectral curve (also known as "mirror curve" in this context). Once the spectral curve mirror to a given counting problem is determined, the integral recursion uniquely calculates all generating functions of the corresponding enumerative invariants. In this talk I will show that the mirror counterparts to orbifold Hurwitz numbers satisfy the integral recursion, with spectral curve given by the "r-Lambert curve". I will also argue that orbifold Hurwitz numbers can be obtained in the "infinite framing limit" of orbifold Gromov-Witten theory of $[C^3/(\mathbb{Z}/r\mathbb{Z})]$, thus shedding some light on the appearance of the recursion for orbifold Hurwitz numbers and its relation with the so-called remodeling conjecture for Gromov-Witten theory on toric orbifolds.

This is joint work with D. Hernandez Serrano, X. Liu and M. Mulase.

Speaker: **Tom A Bridgeland** (University of Sheffield)

Title: *Stability conditions and quadratic differentials*

Abstract: Associated to any triangulated category D is a complex manifold $\text{Stab}(D)$ called the space of stability conditions. In examples these spaces $\text{Stab}(D)$ often turn out to have natural Frobenius structures, although there is no general understanding of why this should be the case. I will give a basic overview of stability conditions, focusing on a recent class of examples (joint with Ivan Smith) in which moduli spaces of meromorphic quadratic differentials show up.

Speaker: **Dawei Chen** (Boston College)

Title: *Exceptional strata of quadratic differentials in low genus*

Abstract: Connected components of strata of abelian and quadratic differentials are classified by Kontsevich-Zorich and Lanneau, respectively. In general, parities to distinguish these components arise from hyperelliptic and spin structures. However, there exist several exceptional strata of quadratic differentials in low

genus, for which a parity that geometrically distinguishes their connected components was not known. In this talk, I will discuss how to construct such parities using tools in algebraic geometry. As a by-product, we verify a numerically invariant property for Teichmueller curves in these strata, which was first observed and conjectured by Kontsevich-Zorich. This is part of a joint work with Martin Möller.

Speaker: **Gavril Farkas** (HU Berlin)

Title: *The universal theta divisor over the moduli space of curves*

Abstract: I will discuss a complete birational classification of the universal theta divisor over the moduli space of curves of genus g , in particular explaining how the nature of the theta divisor changes at genus 12.

Speaker: **Gerard van der Geer** (University of Amsterdam)

Title: *Hurwitz spaces and divisors on moduli spaces of curves*

Abstract: Hurwitz spaces give rise to maps and correspondences between moduli spaces of curves. In this talk we give examples of this and study the induced maps on divisor classes on moduli spaces of curves. We calculate some divisor classes and also discuss the Hodge bundle of Hurwitz spaces. This is based on joint work with Alexis Kouvidakis.

Speaker: **Samuel Grushevsky** (Stony Brook)

Title: *Real-normalized differentials and the geometry of the moduli space of curves*

Abstract: We will survey some of the ideas, constructions, and results on the geometry of the moduli spaces of curves obtained in joint work with Igor Krichever, using meromorphic differentials with real periods, and motivated by perturbation theory of integrable systems

Speaker: **John Harnad** (Concordia)

Title: *Tau functions as generators of integrable systems, random processes and moduli space invariants*

Abstract: The first part consists of a brief introduction to the Sato-Segal-Wilson theory of Tau functions, viewed as abelian group representations on sections of dual determinantal line bundles over infinite Grassmannians and flag manifolds or, equivalently, as fermionic vacuum state matrix elements. The second part consists of applications, including: 1) Random matrix integrals, subjected to linear exponential deformations in the measure. This include both the standard self-coupled matrix models and externally coupled ones, like the Brezin-Hikami models and the Kontsevich integral, which is a generating function for Gromov-Witten invariants. The two are related by a “convolution symmetry” on the Grassmannian. 2) Donaldson-Thomas generating functions for Calabi-Yau 3-folds, which may be realized as special values of Tau functions of the type occurring in Schur processes, and counting of Young diagrams. 3) Deformations of the latter, which determine probability measures on the path space of random partitions. These define determinantal random point processes of random partitions, which are special cases of Schur processes, that may also be viewed as convolution symmetry flows. These have been used (by Okounkov, Reshetikhin, Takasaki and others) as models for crystal growth/melting.

Speaker: **Jacques Hurtubise** (McGill)

Title: *Real bundles on real varieties*

Abstract: We give a survey of recent results on real and pseudo-real bundles on algebraic curves, as well as on higher dimensional real varieties. Particular emphasis is on the topology of the moduli. (Joint work with Indranil Biswas, Johannes Huisman, Oscar Garcia-Prada and Florent Schaffhauser.)

Speaker: **Robin de Jong** (Leiden University)

Title: *Torus bundles and 2-forms on the universal family of Riemann surfaces*

Abstract: We revisit two results due to S. Morita expressing certain natural integral cohomology classes on the universal family of Riemann surfaces \mathcal{C}_g , coming from the canonical flat symplectic form on the universal jacobian, in terms of the tautological classes e and e_1 . Our discussion will be on the level of

the natural 2-forms representing the relevant cohomology classes, and involves a comparison with other natural 2-forms representing e , e_1 coming from the Arakelov metric on the relative tangent bundle of \mathcal{C}_g over \mathcal{M}_g . A secondary object called a_g occurs, which was discovered and studied by N. Kawazumi around 2008. We discuss some properties of this conformal invariant, including its asymptotic behavior towards the boundary in the Deligne-Mumford compactification of \mathcal{M}_g , and its relation with the Faltings delta- and the Hain-Reed beta-invariant.

Speaker: **Maxim Kazarian** (Independent University, Moscow)

Title: *Symplectic Geometry of Topological Recursion*

Abstract: The topological recursion of Chekhov-Eynard-Orantin is a procedure allowing one to reconstruct partition functions in numerous problems of mathematical physics. The power of this method is justified by a huge number of manifesting examples, however, in most cases its validity is usually considered as a miraculous coincidence without a proper explanation why it should work.

In the talk, we describe a geometric background to the topological recursion providing a better understanding of its nature. We show that the possible partition functions obtained by this recursion are in one-to-one correspondence with the Lagrangian subspaces in the infinite dimensional symplectic vector space. The symplectic group acts on the space of partition functions by mean of ‘quantization of quadratic Hamiltonians’. As an immediate corollary we obtain that the abstract Gromov-Witten potentials introduced by Givental are particular examples of general partition functions, therefore, they automatically always satisfy relations of topological recursion.

As it is shown in the recent joint paper with Dunin-Barkowski, Orantin, Shadrin, and Spitz, the above correspondence applied to the simplest non-trivial case of Hurvitz potential provides an independent proof of the Bouchard-Marino conjecture and the ELSV formula.

Speaker: **Sergey Lando** (High school of economy, Moscow)

Title: *On computation of universal polynomials for characteristic classes of singularities*

Abstract: Any space of meromorphic functions is stratified according to the number of critical values of these functions. The cohomology classes Poincare dual to the strata of such a stratification can be described in terms of certain universal classes. The standard tool for the description are Thom polynomials. Their computation is a complicated problem, which usually is done step-by-step and requires the knowledge of a complete classification of singularities.

However, in some cases computation of characteristic classes for spaces of meromorphic functions on algebraic curves can be done explicitly for large series of classes. Certain corresponding generating functions are solutions to integrable hierarchies. Further computations pose interesting problems of both geometric and combinatorial nature.

The talk is based on a joint work with Maxim Kazarian and, partly, Dimitri Zvonkine.

Speaker: **Andrew McIntyre** (Bennington College)

Title: *Chern-Simons invariants, determinant of Laplacian, and tau functions*

Abstract: Suppose X is a compact 2-manifold, of fixed genus 2 or more, with hyperbolic metric. It is known (Belavin-Knizhnik, Bost, Takhtajan-Zograf) that the determinant of the Laplacian on X is the modulus squared of a holomorphic function F on the Teichmuller space of such X , times a “conformal anomaly”. It has been gradually understood (Polyakov, Krasnov, Takhtajan-Teo, Schlenker) that the conformal anomaly is the exponential of a regularized volume of a certain infinite-volume hyperbolic 3-manifold M whose conformal boundary is X . It is a result of Zograf that the function F may be written as a Selberg zeta-like product for the 3-manifold M . (These results are a baby case of physicists’ conjectured “holography”.) This raises the question of the meaning of the phase of F . Park realized that the phase of F may be interpreted in terms of a regularized Atiyah-Patodi-Singer eta invariant of M .

In this joint work with Jinsung Park, KIAS, we generalize constructions of Meyerhoff and Yoshida to define a regularized Chern-Simons invariant for M , which forms a natural complexification of the regularized volume. We relate it to the regularized eta invariant. The definition involves a framing that is singular

along curves in M , reminiscent of Witten's work on the Jones polynomial. The Bergman tau function, introduced and studied by Kokotov-Korotkin, makes a surprise appearance.

Speaker: **Gabriele Mondello** (University of Roma 1)

Title: *Abelian differentials on curves of genus 3*

Abstract: The moduli space of curves endowed with a nonzero Abelian differential is naturally stratified according to the configuration of zeroes of the differential. In general, such strata are topologically and geometrically interesting though mysterious. For instance, Kontsevich and Zorich raised the problem of whether such strata are $K(G,1)$. We will provide a description of all strata in genus 3, which allows to partially answer such conjecture. Time permitting we will also sketch an approach for the smallest strata in higher genus. This is joint work with Eduard Looijenga.

Speaker: **Martin Möller** (Uni Frankfurt)

Title: *Commensurability of non-arithmetic lattices in $PU(1,n)$*

Abstract: All presently known non-arithmetic ball quotients arise up to commensurability via the covering construction of Deligne-Mostow. We give a complete classification of commensurability classes by determining the Lyapunov spectrum of the geodesic flow.

Speaker: **Motohico Mulase** (UC Davis)

Title: *Quantum curves and topological recursion*

Abstract: Several mathematical examples of quantum curve equations and topological recursion, based on algebraic geometry problems, will be presented in this introductory talk.

Speaker: **Paul Norbury** (U of Melbourne)

Title: *A quantum curve for Gromov-Witten invariants of \mathbb{P}^1 .*

Abstract: I will describe a wave function given by a specialisation of the free energy for stationary Gromov-Witten invariants of \mathbb{P}^1 which satisfies a linear difference equation closely related to the Toda equation. The wave function has an asymptotic expansion with coefficients given by analytic functions on the Eynard-Orantin spectral curve thus giving another example of a construction of the quantum curve described by Gukov-Sulkowski. This is joint work with Mulase.

Speaker: **Emma Previato** (Boston University)

Title: *Projective curve parameters and abelian integrals*

Abstract: We investigate integral-inversion formulas, especially in terms of "Kleinian sigma functions", in order to connect moduli spaces and integrable equations. We present limiting formulas over Wirtinger-type varieties and an algebraic solution of the dispersionless KP hierarchy. The direction of intersection theory in moduli space is indicated by equations of Gauss-Manin type and monodromy.

Speaker: **Alexander Schmitt** (FU Berlin)

Title: *Motives of moduli spaces of Higgs bundles and holomorphic chains*

Abstract: Let X be a compact Riemann surface and Γ the canonical central extension of its fundamental group. For a given positive integer $r > 0$, the equivalence classes of representations of Γ in $GL_n(\mathbb{C})$ mapping the central element to $\exp(2\pi i/n)$ are parameterized by a smooth affine algebraic variety, the character variety. Hausel and Rodriguez-Villegas counted points of the character variety over finite fields and derived an important conjecture on its Betti numbers. By work of Hitchin, Simpson and many others, the character variety is homeomorphic to the moduli space of Higgs bundles of rank r and degree 1 on X . The latter space carries a much richer geometric structure. For example, it comes with a \mathbb{C}^* -action. The computation of the Betti numbers of the moduli space of Higgs bundles is reduced to the computation of Betti numbers of the fixed point locus. This consists of moduli spaces of holomorphic chains. Using this strategy, Hitchin computed the Betti numbers in the case $r = 2$ and Gothen for $r = 3$. In joint work with Garcia-Prada and Heinloth, we combined Hitchin's strategy with the approach of Harder and Narasimhan

to determine the Betti numbers of the moduli space of vector bundles in order to find the Betti numbers in the case $r = 4$. We expressed the result in terms of motives in the Grothendieck ring of varieties. In the talk, I will report on this work and briefly digress on some fundamental results on stability of holomorphic chains.

Speaker: **Sergey Shadrin** (U of Amsterdam)

Title: *Local topological recursion and Givental theory*

Abstract: I'll explain a precise correspondence between two different theories: the Chekhov-Eynard-Orantin topological recursion, coming from the change of variables in a particular type of matrix models, and the Givental formula for the so-called ancestor potential that is a key ingredient in the theory of genus expansions of Frobenius manifolds.

There are many applications of this correspondence, and, if time permits, I'll try to explain how one can obtain some formulas for the usual Hurwitz numbers and Hurwitz numbers with completed cycles in terms of the intersection theory of the moduli space of curves (the ELSV formula, and Zvonkine's conjectural r-ELSV formula).

The talk will be based on joint works with P. Dunin-Barkowski, M. Kazarian, N. Orantin, L. Spitz, and D. Zvonkine.

Speaker: **Fedor Soloviev** (Toronto)

Title: *Geometry of dented pentagram maps*

Abstract: We propose a new family of natural generalizations of the pentagram map from 2D to higher dimensions and prove their integrability on generic twisted and closed polygons. In dimension d there are $d-1$ such generalizations called dented pentagram maps, and we describe their geometry, continuous limit, and Lax representations with a spectral parameter. We prove algebraic-geometric integrability of dented maps in the 3D case and show that the dimension of the invariant tori in the dented case coincides with that for the higher pentagram map constructed with help of short diagonal hyperplanes. When restricted to corrugated polygons, the dented pentagram maps coincide between themselves and with the corresponding corrugated pentagram map. We also prove integrability for a larger variety of dented-type pentagram maps in higher dimensions.

This is a joint work with Boris Khesin.

Speaker: **Alexander A. Voronov** (University of Minnesota)

Title: *The BV Formalism and the Cohomology of Moduli Spaces*

Abstract: This is a report on a work of Domenico D'Alessandro and myself, in which we use the Batalin-Vilkovisky formalism to give a new proof of Costello's theorem on the existence and uniqueness of a solution to the Quantum Master Equation. We also make a physically motivated conjecture on the cohomology of moduli spaces.

Speaker: **Dmitry Zakharov** (Stony Brook)

Title: *Tautological relations via the Abel–Jacobi map*

Abstract: Given a curve with marked points and integer weights on the markings, we consider the corresponding divisor. This defines an Abel–Jacobi map from the moduli space of marked curves to the universal Jacobian variety. This map extends to the space of curves of compact type. Pulling back relations from the universal Jacobian for various values of the weights gives a plethora of tautological relations on the space of curves of compact type. These relations include those discovered by Faber, Getzler, Belorousski and Pandharipande, and Tavakol.

Speaker: **Dimitri Zvonkine** (CNRS)

Title: *Cohomological relations on $\overline{\mathcal{M}}_{g,n}$ via 3-spin structures*

Abstract: We construct a family of relations between tautological cohomology classes on the moduli space $\overline{\mathcal{M}}_{g,n}$. This family contains all relations known to this day and is expected to be complete. The construction uses the Frobenius manifold of the A2 singularity and the 3-spin Witten class.