Banff Conference on Waves in Fluids

'Waves in Fluids IV'[1]

Book of Abstracts

Monday

Mark Ablowitz, Colorado

Water waves: analysis and ocean observations

Historically, localized solitary waves were observed almost 180 years ago; subsequently they related to shallow, or long, water waves associated with the Korteweg-deVries (KdV) equation. Due to their special behavior KdV solitary waves were termed solitons. Interestingly, two dimensional interacting soliton waves with X,Y and ones with more complex web structure can frequently be seen in shallow water on flat beaches. These waves can be related to solutions of the unidirectional Kadomtsev-Petviashvili (KP) equations. Such two dimensional solitons are found to persist in the two-directional Benney-Luke equation which contains the KP equation as a limiting case.

David Lannes, ENS, Paris

Vorticity effects in shallow water waves

We will derive in this talk a new formulation for the water waves with vorticity, prove its well-posedness and study the behavior of the solution in the shallow water regime. In particular, we will rigorously derive asymtotic models from this new formulation. This is a joint work with Angel Castro.

J. Thomas Beale, Duke University

Computing Singular and Nearly Singular Integrals

Computations based on boundary integral formulations are often used for electromagnetics or Stokes flow as well as for water waves. We will describe a relatively simple, direct approach to computing a singular or nearly singular integral, such as a harmonic function given by a single or double layer potential

[1] I: São Paulo (2006); II: Parati (2008); III: Rio de Janeiro (2011)

on a smooth, closed curve in 2D or a surface in 3D. The present approach is suited for general use with moving interfaces since the representation of the interface is simple and the value of the integral, or related quantities, can be found at nearby points as well as on the surface. Briefly, the value of the integral is found by a standard quadrature, with the singularity replaced by a regularized version. Correction terms are then added for the errors due to regularization and discretization. These corrections are found by local analysis near the singularity. They are needed at points near the surface, since the integrals are nearly singular. For general surfaces, integrals can be computed in overlapping coordinate grids on the surface. A quadrature technique of J. Wilson allows this to be done without explicit knowledge of the coordinate charts, thus making the approach more practical. Recent work with W. Ying in 2D shows that the method can accurately handle boundaries which are close to each other, such as two drops merging, and work in 3D is in progress. Collaborators include M.-C. Lai, A. Layton, S. Tlupova, J. Wilson, and W. Ying.

Michael Siegel, NJIT

An efficient boundary integral method for 3D free-surface flow with surface tension

A nonstiff boundary integral method for 3D free-surface flow with surface tension is presented, with applications to porous media flow, water waves, and (time permitting) hydroelastic waves. The velocity of the interface is given in terms of the Birkhoff-Rott integral, and we present a new method to compute this efficiently in doubly-periodic problems by Ewald summation. The stiffness is removed by developing a small-scale decomposition, in the spirit of prior work for 2D flow by Hou, Lowengrub and Shelley. In order to develop this small scale decomposition, we formulate this problem using a generalized isothermal parameterization of the free surface. This is joint work with David Ambrose and Svetlana Tlupova.

David P. Nicholls, University of Illinois at Chicago

A Boundary Perturbation Method for Reconstruction of Layered Media via Constrained Quadratic Optimization

The scattering of linear waves by layered media constitute a fundamental model in oceanography, acoustics, electromagnetics, and the geosciences. In this talk we examine the problem of detecting the geometric properties of a two-dimensional acoustic medium where the fields are governed by the Helmholtz equation.

Building upon the success of our previous Boundary Perturbation approach (implemented with the Operator Expansions formalism) we derive a new approach which augments this with a new "smoothing" mechanism. With numerous numerical experiments we demonstrate the enhanced stability and accuracy of our new approach which further suggests not only a rigorous proof of convergence, but also a path to generalizing the algorithm to multiple layers, three dimensions, and the full equations of linear elasticity and Maxwell's equations. Joint work with Alison Malcolm (EAPS, MIT)

Jun Zhang, NYU

Geophysical Flows: from Fluid Erosion to Continental Dynamics

Geophysical fluid dynamic has been one of the most active areas in fluid research and in geological dynamics. The relevant problems often span many spatial scales and as well as many temporal scales, and often involve complex physical processes. In this talk I will discuss how an erodible structure is shaped by an open flow, and how a self-similar state emerges as the structure shrinks due to erosion. This dynamical process leads to a unique shape that is characterized by nearly uniform shear stress but with pointing front. In another experiment, we use an experimental system to model the closely coupled interactions between a floating thermal blanket and a fluid undergoing thermal convection. The latter is ubiquitous in nature, and has been one of the most studied dynamical systems in the past 30 years. We will discuss the possible connection between this coupled system and the geophysical process of continental drift.

Wooyoung Choi, NJIT/ KAIST

Unsteady Stokes expansion and its generalization

Since the pioneering work of Stokes (1847, Trans. Camb. Phil. Soc., p. 441) for nonlinear periodic gravity waves, the method of asymptotic expansions has served as a powerful tool to describe nonlinear water waves. In recent years, under the small steepness assumption, a formulation based on the Dirichlet-to-Neumann operator has been used frequently to study the time evolution of nonlinear water waves, in particular, with wide-band spectrum. In this talk, this formulation for time-dependent problems will be discussed in connection with the original Stokes expansion and its generalization to solve a wide range of water wave problems will be presented.

Bernard Deconinck, U. Washington

The inverse water wave problem

The inverse water wave problem is the problem of bathymetry reconstruction, given properties of the water wave surface. I will discuss what a minimal set of surface data consists of so that bathymetry reconstruction is possible. Our approach uses ideas from the recent reformulation of the water wave problem, due to Ablowitz, Fokas, and Musslimani. A set of algebraic equations is derived, the numerical solution of which results in the knowledge of the bathymetry. Numerous examples will be shown. This is joint work with Vishal Vasan.

Zhan Wang, UCL

Focusing phenomenon in three dimensional capillary-gravity waves

The capillary-gravity wave problem exhibits a large variety of phenomena, one of which is the combination of geometric and nonlinear self-intersection focusing. The underlying mathematical language is the focusing 2+1 nonlinear Schrodinger equation (NLS) in the small amplitude limit. In this talk, we perform simulation of the water wave equations, using the cubic truncation of the Dirichlet to Neumann operator. Three examples are shown: transverse instability of plane solitary waves which finally evolve into fully localized structures or an intermediate state between plane solitary waves and lumps, super Gaussian initial data and high-energy solitary waves, both of which break up into a complex set of localized structures.

J. H. Duncan, J. D. Diorio and X. Liu, Department of Mechanical Engineering, University of Maryland, USA
A. Dimas, Department of Civil Engineering, University of Patras, Greece

<u>The Cross-Stream Structure of the Crests of Breaking Waves</u>

The development of cross-stream structure on the crests of spilling breaking water waves is studied through experiments and linear stability analysis. In the experiments, spilling breakers with wavelengths 80-120 cm are generated in a tank that is 12.8 m long and 1.2 m wide with water depth about equal to the nominal length of the breaking wave. A programmable wave maker is used to generate Froude-scaled wave packets (central frequencies 1.15 - 1.40 Hz and various wave maker amplitudes) that create breakers via dispersive focusing. A cinematic 2D LIF technique is used to measure the crest profile histories both in stream-wise and cross-stream planes. It is found that the cross-stream averaged amplitude undergoes periodic oscillations due to the passage of large streamwise (oriented parallel to the wave crest) ripples. Cross-stream ripples, while initially small, grow rapidly as breaking develops. These cross-stream ripples are in the range of 1-4 cm in wavelength and can have amplitudes comparable in size to the streamwise ripples. The amplitude of the cross-stream ripples grows with the gravity wavelength to the third power.
In order to explore the physics of these cross-stream ripples, the three-dimensional structure of the linear instability of wake-type shear flows is studied based on the numerical solution of Rayleighs equation subject to the appropriate free surface boundary conditions. For a given mean velocity profile and Froude number of the shear flow, and complex wavenumber of the instability, the finite-difference discretization of Rayleighs equation results in a generalized eigenvalue problem for the complex frequency of the instability, which is solved using a standard Q-Z algorithm. The instability exhibits two frequency ranges of unstable waves for each Froude number: one at low frequencies, called Branch I, and one at higher ones, called Branch II. The wavenumbers of the most-unstable waves are compared to the wavenumbers of the ripples found in the experiments.
Duncan, Diorio and Liu gratefully acknowledge the support of the National Science Foundation, Division of Ocean Sciences under grant OCE751853. Dimas gratefully acknowledges support under project ARISTEIA I - 1718 (programme Education and Lifelong Learining) cofinanced by the European Union (European Social Fund) and Hellenic Republic funds.

Mark J. Cooker (University of East Anglia, England)

Violent Wave Motion on a Short Time Scale.

This talk embraces a few of the workshop's declared themes: three-dimensional flow, breaking waves, and waves excited by forcing. During the overturning and breaking of a water wave the fluid particles often undergo large accelerations in short periods of time. The material derivative of the velocity vector may have a magnitude many times greater than gravitational acceleration g = 9.8 m/s^2. The acceleration is particularly important to consider for the free-surface fluid particles. In the lunging motion under the overturning arch of a plunging breaker, it is not unusual for a surface particle to accelerate at 4g. Moreover, at the surface of a wave breaking against a seawall an acceleration of THOUSANDS of g can briefly occur. Such violent flows make tough demands on computational schemes that claim to track the trajectory of surface particles. E.g. it may require careful treatment of the higher derivatives in a Taylor-series approximation (with respect to time step) of both the velocity potential and the particle position coordinates in a Lagrangian description of the free-surface motion. For Euler's equations of motion to be balanced, a large fluid acceleration must coincide with a large gradient in pressure (especially near the free surface). Such considerations lead to predictions of (impulsive) pressure fields which contain transient spatial maxima very near the free surface. The order of magnitude of the maximum (in time) of the pressure in an incompressible fluid during impact, can be written as: $p_m = O(\rho UL/T)$, where rho is the fluid density, L is the length-scale of the flow (depth or significant wavelength), U is the speed of collision (e.g. $[gL]^{[1/2]}$), and T is a time-scale much briefer than $(L/g)^{(1/2)}$. A linear mixed boundary-value problem can be solved to approximate the distribution of maximum pressure p_m in the fluid domain at the time of impact. The calculation also indicates the sudden change in velocity field that the impulsive pressure-gradient brings about (see [1]). The velocity and acceleration evaluated at the free surface indicate the presence of conditions leading to a high-acceleration jet, such as might occur in wave overturning, or the ascending splash made after wave collision. An example, to be presented, illustrates a simply forced initial-value problem, with a given initial free surface position. The example demonstrates the strong influence of the free surface SHAPE on the presence, absence, position and violence of any subsequent jet.

[1] M.J. Cooker (2002) Unsteady pressure fields which precede the launch of free-surface liquid jets. Proc. R. Soc. Lond. A 458, pp473–488.

Lyubov Chumakova (Dept. Mathematics, MIT)
Rodolfo R. Rosales (Dept. Mathematics, MIT)
Esteban G. Tabak (Courant Institute of Mathematical Sciences, NYU).

Leaky rigid lid – the simplest case.

Much of our understanding of the tropical tropospheric dynamics is based on the concept of discrete internal modes. However, discrete modes are the signature of systems of finite extent, while the atmosphere should be modeled as infinite and "is characterized by a single isolated eigenmode and a continuous spectrum" [1]. Is it then unphysical to use the discrete internal rigid lid modes to model atmospheric phenomena? We propose the following approach to resolve this debate. First we obtain a radiation condition at the tropopause, using the incompressible and in hydrostatic balance approximation. This radiation condition can be formulated as a boundary condition at the tropopause, which involves a pseudo-differential operator in the horizontal direction (a Hilbert transform). Then we use this boundary condition to compute a new set of vertical modes: the leaky rigid lid modes. These modes decay in time. For realistic values of the stratification, the decay time-scale for the first few modes ranges from an hour to a week. This suggests that the rate of energy loss through upwards propagating waves may be an important factor in setting the time scale for some atmospheric phenomena. Finally, the leaky rigid lid modes are not orthogonal, but they are complete, and there is a simple way to project initial conditions onto them.
[1] Lindzen, R., The interaction of waves and convection in the tropics. J. Atmos. Sci., 60, 3009-3020, 2003.
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Lyubov Chumakova (Dept. Mathematics, MIT)
Rodolfo R. Rosales (Dept. Mathematics, MIT)
Esteban G. Tabak (Courant Institute of Mathematical Sciences, NYU).

Leaky rigid lid – the rotating case.

This work continues the study of modes arising from radiation conditions at the tropopause. The question is now: How is the effect of wave energy loss from the troposphere to the stratosphere modified in the presence of planetary rotation? Do the leaky rigid lid modes survive, possibly with some modifications? A formal calculation shows that the modes persist, albeit with a modified time dependence: they both move, and decay, slower as the Coriolis parameter increases. However, non-trivial mathematical difficulties arise. The effective boundary condition now requires information that is not "stored" at the tropopause, namely data related to how much energy has escaped. This leads to non-local operators in time (i.e. involving the past), which is problematic for solving initial value problems. However, we can recover this information from the variable values away from the interface, via an appropriately defined pseudo-differential operator in space. This removes history dependent terms from the effective boundary condition. The question of how to project the initial conditions (or forcing terms) into the new modes is still under investigation.

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Triantaphyllos Akylas, MIT

Oblique nonlinear interactions of internal wave beams and associated resonances

Quadratic nonlinear interactions between two colliding internal gravity wave beams in a uniformly stratified fluid, and the resulting secondary beams with frequencies equal to the sum and difference of those of the primary beams, are studied theoretically. The analysis centers on oblique interactions, involving beams that propagate in different vertical planes. It is pointed out that, for certain oblique collision configurations, radiated secondary beams with frequency equal to the difference of the primary frequencies have unbounded steady-state amplitude. This resonance is interpreted physically and its potential consequences are discussed. (Joint work with my PhD student Hussain Karimi.)

Rich McLaughlin, UNC

Falling bodies and fluids in sharp stratification

The motion of bodies and fluids moving through a stratified background fluid arises naturally in the context of carbon (marine snow) settling in the ocean, as well as less naturally in the context of the recent gulf oil spill. The details of the settling rates may play a role in assessing the role of the ocean in the earth's carbon cycle. We discuss two examples: First, at very low Reynolds number, a first principled theory is possible for solid bodies, and extended to porous bodies. Second, at higher Reynolds numbers, full DNS are presented showing the behavior of dense, miscible vortex rings penetrating strong stratification. Both cases are compared to detailed experimental measurements.

Paul A. Milewski, **Esteban G. Tabak, NYU**

Conservation Law Modeling of Mixing in Layered Hydrostatic Flows

This talk explores a framework to quantify the large-scale effects of fluid mixing without resolving the associated small scale motion. The equations of motion for hydrostatic flows adopt the form of a hyperbolic system of nonlinear equations, which typically yield breaking waves. In order to model the shock waves that ensue, one needs to involve integral conserved quantities, such as mass and momentum. Yet in a system composed of layers that may mix, first physical principles do not provide a set of conserved quantities large enough to completely determine the flow. Our proposal is to replace the conventional conservations laws of each layer's mass and momentum, invalid after shocks form, by others, such as energy, in a way that provides a natural description of the mixing process.

Roberto Camassa, UNC

Some fundamental issues in internal wave dynamics

One of the simplest physical setups supporting internal wave motion is that of a stratified incompressible Euler fluid in a channel. This talk will discuss asymptotic models capable of describing large amplitude wave propagation in this environment, and in particular of predicting the occurrence of self-induced shear instability in the waves' dynamics for continuously stratified fluids. Some curious properties of the Euler setup revealed by the models will be presented.

Gerassimos Athanassoulis, NTU Athens

A new, easily implemented representation of the Dirichlet-to-Neumann operator for the nonlinear water wave problem over arbitrary topographies

Since the recognition by Zakharov that the free surface elevation and the trace of the velocity potential form a set of canonical variables for the nonlinear water wave problem, a lot of work has been performed in order to exploit the Hamiltonian structure for the theoretical and numerical study of the problem. The Dirichlet-to-Neumann (DtN) operator, used to implement the internal kinematics as an essential condition, in accordance with the premises of Hamiltons principle. The main difficulty in this approach is the treatment of DtN operator. In the case of constant depth, the functional (Volterra-Taylor) series expansion of DtN, implemented in the Fourier space has been proved an efficient tool for the numerical simulations as well as the asymptotic study of the problem. However, in the case of a general bathymetry, this approach becomes complicated, and has been implemented up to now only for a periodically varying bottom and for long waves over random topography. Luke introduced another variational principle for the nonlinear water-wave problem, free of the essential constraint of the internal kinematics. As a first application, we present a unified numerical investigation of nonlinear steady travelling waves, corresponding to a wide range of wave amplitudes and water depths. *NOTE: Due to time constraints, this abstract was abridged by the organiser.*

Benjamin Akers, Air Force Institute of Technology

Overturned Interfacial Traveling Waves

A traveling wave ansatz is developed for computing overturning traveling waves on interfaces parameterized by arclength. Periodic traveling waves between two constant density fluids are computed, including waves which are multivalued functions of the horizontal coordinate. A host of branches of traveling waves are computed, whose limits include Stokes' waves, Crapper's waves, and Wilton's ripples . The largest wave's profile and amplitude are studied a function of Bond and Atwood number.

B. Akers, D.M Ambrose, and J.D. Wright. Traveling waves from the arc length parameterization : Vortex sheets with surface tension. *preprint.*
B. Akers and W. Gao. Wilton ripples in weakly nonlinear model equations. *Comm. Math. Sci.*, 10:1015–1024, 2012.

Wednesday

Jean-Marc Vanden-Broeck, UCL

Periodic and non periodic pure gravity waves in water of infinite depth.

Two-dimensional water waves travelling at a constant velocity at the surface of a fluid of infinite depth are considered. Gravity is taken into account but surface tension is neglected. The problem is solved numerically by a mixture of boundary integral equation methods and series truncation methods. Numerical evidence for the existence of new solitary waves (i.e. non periodic waves) is presented. In addition new families of periodic waves are presented and discussed.

Sunao Murashige, Future University Hakodate

Conformal mapping for two-dimensional steady motion of solitary waves

This work proposes a conformal mapping which is suitable for analysis of two-dimensional steady motion of solitary waves. This mapping takes the decay rate at the outskirts into account. In this mapped plane, we can obtain approximate analytic solutions with the exact decay rate, and develop a method of fully nonlinear computation. Numerical examples show effectivity of this mapping for solitary waves in the one- and two-layer fluid systems.

Jon Wilkening, UC Berkeley

Traveling-Standing Water Waves and Microseisms

We study a two-parameter family of solutions of the surface Euler equations in which solutions return to a spatial translation of their initial condition at a later time. Pure standing waves and pure traveling waves emerge as special cases at fixed values of one of the parameters. Resonant effects cause many disconnections in the bifurcation curves at large amplitude. Some of these resonant disconnections persist all the way to zero-amplitude. We find many examples of wave crests that nearly sharpen to a corner, with corner angles close to 120 degrees near the traveling wave of greatest height, and close to 90 degrees for large-amplitude pure standing waves. However, aside from the traveling case, we do not believe any of these solutions approach a limiting extreme wave that forms a perfect corner.
We also compute nonlinear wave packets, or breathers, which can take the form of NLS-type solitary waves or counterpropagating wave trains of nearly equal

wavelength. The main challenge in the former case is resolving algebraically decaying tails. In the latter case, an interesting phenomenon occurs in which the pressure develops a large DC component that varies in time but not space, or at least varies slowly in space compared to the wavelength of the surface waves. These large-scale pressure zones can move very rapidly since they travel at the envelope speed, and may be partially responsible for microseisms, the background noise observed in earthquake seismographs. This is joint work with Paul Milewski and Frederic Dias.

Philippe Guyenne, University of Delaware

Surface signature of internal waves

Based on a Hamiltonian formulation of a two-layer ocean, we consider the situation in which the internal waves are treated in the long-wave regime while the surface waves are described in the modulational regime. We derive an asymptotic model for surface-internal wave interactions, in which the nonlinear internal waves evolve according to a KdV equation while the smaller-amplitude surface waves propagate at a resonant group velocity and their envelope is described by a linear Schrodinger equation. In the case of an internal soliton of depression for small depth and density ratios of the two layers, the Schrodinger equation is shown to be in the semi-classical regime and thus admits localized bound states. This leads to the phenomenon of trapped surface modes which propagate as the signature of the internal wave, and thus it is proposed as a possible explanation for bands of surface roughness above internal waves in the ocean. Numerical simulations taking oceanic parameters into account are also performed to illustrate this phenomenon. This is joint work with Walter Craig and Catherine Sulem.

Thursday

John Bush, MIT

Pilot-wave hydrodynamics

Yves Couder and coworkers have recently discovered that droplets walking on a vibrating fluid bath exhibit several features previously thought to be peculiar to the microscopic, quantum realm. These walking droplets propel themselves by way of a resonant interaction with their own wave field, and represent a macroscopic realization of the pilot-wave dynamics envisaged by Louis de Broglie, which was superseded by the Copenhagen Interpretation as the standard view of quantum mechanics. New theoretical developments provide rationale for the complex behavior of the bouncing droplets, and yield a trajectory equation for the walking droplets. Experimental and theoretical results in turn reveal and rationalize the emergence of quantization and wave-like statistics from pilot-wave dynamics in a number of settings. The relation between this fluid system and de Broglies pilot-wave theory is discussed.

Paul Milewski, University of Bath

Wave modelling of pilot wave-droplet coupling in a Faraday Problem

Recent experiments by two groups, Yves Couder (Paris) and John Bush (MIT) have shown experimentally that droplets will bounce on the surface of a vertically vibrated bath (instead of coalescing with it), generating a Faraday-type wavefield at every bounce. From this state, a pitchfork symmetry breaking bifurcation leads to a "walking" state whereby the bouncing droplet is "guided" by the self-generated wavefield - the droplet's pilot wave. Once this state is achieved a large array of interesting dynamics ensues with surprising analogies to quantum mechanical behaviour. We will present a coupled particle-fluid model that can can be used simulate the dynamics of this problem. This is joint work with John Bush, Andre Nachbin (IMPA) and Carlos Galeano Rios(IMPA).

Diane Henderson, Penn State

Surface waves and dissipation

Surface waves at an air-water interface are usually modeled using inviscid dynamics. However, water is a viscous fluid, and resulting dissipative effects, though small, can play an important role in the wave dynamics when the waves propagate over long distances. Previous experiments have shown that the dissipation rate of waves is strongly affected by conditions at the free surface. So to derive a model that predicts dissipation rate, one usually allows for weak viscosity and chooses among various possible boundary conditions at the surface. The surface is typically considered to be an interface between water and a vacuum. In that case possible boundary conditions are that (i) the surface is shear-free between water and a vacuum, also referred to as "clean", (ii) the surface admits no tangential velocities, also referred to as "inextensible", or (iii) the surface has a visco-elastic film with its own constitutive law, such that stresses in the water match the stresses in the film. Another approach is to consider the surface to be an interface between water and (dynamic) air across which the stresses are continuous. Here we generalize that model to allow for a visco-elastic film at the air-water interface. We discuss experiments within the context of these models in an effort to better understand the boundary condition at the air-water interface and the ranges of applicability of these models.

Harvey Segur, University of Colorado

The nonlinear Schrdinger equation, dissipation and ocean swell

The focus of this talk is less about how to solve a particular mathematical model, and more about how to find the right model of a physical problem. The nonlinear Schrdinger (NLS) equation was discovered as an approximate model of wave propagation in several branches of physics in the 1960s. It has become one of the most studied models in mathematical physics, because of its interesting mathematical structure and because of its wide applicability it arises naturally as an approximate model of surface water waves, nonlinear optics, Bose-Einstein condensates and plasma physics. In every physical application, the derivation of NLS requires that one neglect the (small) dissipation that exists in the physical problem. But our studies of water waves (including freely propagating ocean waves, called swell) have shown that even though dissipation is small, neglecting it can give qualitatively incorrect results. This talk describes an ongoing quest to find an appropriate generalization of NLS that correctly predicts experimental data for ocean swell. As will be shown, adding a dissipative term to the usual NLS model gives correct predictions in some situations. In other situations, both NLS and dissipative NLS give incorrect predictions, and the right model is still to be found.

Andre Nachbin, IMPA

Solitary waves in branching channels

The dynamics of solitary waves is studied in two-dimensional open chan- nels with branching points. We rationalize the wave characteristics at branching points by using the Jacobian J of a Schwarz-Christoffel transformation. It is observed that J acts in a similar fashion as a topography, as in other wave models previously studied. Computational results illustrate the two-dimensional wave dynamics at a channel's branching point, which are then compared to a reduced one-dimensional model describing solitary waves on a graph. An approximate compatibility condition is used at the node of the 1D graph. Numerical experiments show that the one-dimensional graph-like model captures well the effective reflection and transmission properties of the solitary wave, as it crosses the branching point, where different angles and different channel widths have been considered.

Aslan Kasimov, KAUST

The dynamics of circular and polygonal hydraulic jumps

The talk describes the circular hydraulic jump that occurs when a vertical jet of fluid strikes a horizontal plate. In sufficiently viscous fluids, the circular symmetry of the jump can break to give rise to stationary polygonal patterns. We have recently discovered that these polygons can in fact rotate, resulting in striking spiral waves in the flow downstream. In addition to these observations, a theory for the stationary circular jump and for its instability based on Whitham's shock dynamics will also be presented.

Ricardo Barros, IMPA

Strongly nonlinear internal waves in two-layer flows with a top free surface.

We will focus in this talk on the traveling-wave solutions for the strongly nonlinear model for two-layer flows, with a top free surface, derived by Choi & Camassa (1996) and Barros et al. (2007). The model generalizes the Green-Naghdi equations for a single layer and has a Hamiltonian structure. Such structure is naturally preserved by its traveling-wave solutions, being these described by a Hamiltonian system with two degrees of freedom. The dynamics, in the baroclinic regime, takes place around a saddle-center equilibrium and generalized solitary waves are usually expected. These are characterized by ripples in the far field in addition to the solitary pulse. Here, we are particularly interested in the case when all ripples vanish. True solitary waves for this system will be investigated numerically, and a whole variety of solutions will be presented.

David Ambrose, Drexel

Ill-Posedness Issues for Truncated Series Models of Water Waves

Some numerical methods for water waves, such as the Craig-Sulem method, involve expanding terms in the water wave evolution equations as series, truncating those series, and then simulating the resulting equations. For one such scheme, we present analytical evidence that the truncated system is in fact ill-posed; this involves further reducing the evolution equations to a model for which we can prove ill-posedness. We then present numerical evidence that the full truncated system is ill-posed, showing that arbitrarily small data can lead to arbitrarily fast blowup. We present this numerical evidence for multiple levels of truncation. We are able to prove that by adding a viscosity to the system, we instead arrive at a well-posed initial value problem. This is joint work with Jerry Bona and David Nicholls.

Olga Trichtchenko and Bernard Deconinck, University of Washington

Stability of water waves in the presence of surface tension

The aim of the project is to examine the perturbative effect of surface tension on the slow-growing oscillatory instabilities that are present with emphasis of stability in finite depth and possible change in stability behaviour in shallow water, specifically on the bubble instabilities found by Deconinck and Oliveras [5]. Recently, Ablowitz, Fokas and Musslimani presented a reformulation of the surface water wave problem [1]. This formulation uses only surface variables, allowing for greatly improved computational efficiency. Building on previous work [2,4], we use this formulation to compute stationary periodic solutions of the one-dimensional problem, in the presence of small surface tension. The spectral stability of these solutions is examined, using Hills method [3,4].
References:
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2. B. Akers and D. P. Nicholls, Traveling waves in deep water with gravity and surface tension. SIAM J. Appl. Math., Vol. 70 (2010), pp. 2373-2389.
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4. B. Deconinck and J. N. Kutz. Computing spectra of linear operators using Hills method. Journal of Computational Physics, Vol. 219 (2006), pp. 296321.
5. B. Deconinck and K. Oliveras, The instability of periodic surface gravity waves. J. Fluid Mech., Vol 675 (2011), pp.141-167.

Legena Henry, University of the West Indies

An analytical approach to Statistical Moments associated with Large Ocean Waves

Non-Gaussian characteristics in ocean surface elevation have been associated with physical phenomena such as instability in wind waves, non-linear energy transfer between waves, and diffractive focusing in long-crested wave fields. These give rise to large waves which are destructive to freight vessels, offshore oil platforms, offshore wind farms and other ocean-based structures. Still, the scientific community has not completely analytically captured the exact mechanisms behind freak wave occurrence. One way of quantifying the prevalence of freak waves and other non-Gaussianity in a wave field is through changes in statistical moments of ocean surface elevation, such as kurtosis. This talk is therefore concerned with quantifying the statistical moments of ocean surface elevation, towards understanding the dynamics of rare events such as freak waves. It is widely accepted that elevation statistics deviate from Gaussianity, but there is little consensus on how precisely they do so, and what physical mechanisms mostly explain this. The proposed work will investigate the surface elevation moment equations (with and without the effect of directionality), and test whether focusing or blow-up in the surface elevation moment equations is consistent with high kurtosis in 2D and 3D ocean surface wave fields' surface elevation. The main semi-analytical tool to be implemented for arriving at such a conclusion is Wiener Chaos Expansions, a system of moment equations used for capturing the time-dependent random effect of wind forcing on surface elevation. The proposed study will be of benefit to applications where linear models are currently used for surface wave statistics; e.g. in MIKE, WAMIT, SWAN, and other programs used for ocean wave simulations, naval engineering, sea-faring and offshore design.

Friday

Nathan Kutz, University of Washnigton

Characterizing flow around a cylinder with compressive sensing and machine learning

Compressive sensing (CS) is used to determine the flow characteristics around a cylinder (Reynolds number and pressure/flow field) from a sparse number of pressure measurements on the cylinder. Using a supervised machine learning (ML) strategy, library elements encoding the dimensionally reduced dynamics are computed for various Reynolds numbers. The use of convex L^1 optimization is then used with a limited number of pressure measurements on the cylinder to reconstruct the full pressure field and the resulting flow field around the cylinder. Aside from the highly turbulent regime (large Reynolds number) where only the Reynolds number can be identified, accurate reconstruction of the pressure field, flow field and Reynolds number are achieved.

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Finite-depth effects on solitary waves in a floating ice sheet

The two-dimensional problem of nonlinear waves travelling at the interface between a thin ice sheet and an ideal fluid of finite depth is considered. The ice-sheet model used here is based on the special Cosserat theory of hyperelastic shells satisfying Kirchhoffs hypothesis, which yields a conservative and nonlinear expression for the bending force. A Hamiltonian formulation for this hydroelastic problem is proposed in terms of quantities evaluated at the fluidice interface. A nonlinear Schrodinger equation and a 5th order KdV equation are derived at some critical speeds. The numerical scheme for computing steady waves for the fully-nonlinear problem is based on a boundary-integral method. Solitary waves of elevation and depression with decaying oscillations are computed when the phase speed of the waves is lower than the minimum of the dispersion relation. Overhanging waves with bubble-shaped profiles are found when the speed is low. A high-order spectral method is used to calculate the time-evolution of these waves. Some results are also presented when the fluid beneath the ice sheet is not uniform, but consists of two superposed layer of fluids of different densities.

Mark Blyth, University of East Anglia

Hydroelastic waves on fluid sheets and on a closed elastic cell

In the first part of the talk we discuss periodic travelling waves on a sheet of water sandwiched between two thin elastic plates. The plates are capable of supporting an in-plane elastic tension and a normal shear component and bending moment. This is an extension of the problem studied by Kinnersley (1976) for water waves on a fluid sheet. Here the constant surface tension capillary condition applied by Kinnersley at the upper and lower boundaries of the sheet is replaced with a balance of elastic stresses. We identify both symmetric and antisymmetric travelling-wave solution branches, as well as a bifurcation off the symmetric branch to asymmetric wave solutions. In the limit as the elastic modulus becomes large, the solutions approach limiting configurations in which the boundary waves correspond to static solutions valid in the absence of fluid forcing. In the second part of the talk we discuss periodic waves on the boundary of a thin-walled elastic cell placed into a steady ambient flow. We uncover a richly populated solution space with multiple solution branches describing steady wave modes of different wavelengths.