

# Mapping Class Groups and Categorification

## April 7–12, 2013

### MEALS

\*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday

\*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday

\*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday

Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)

**\*Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

### MEETING ROOMS

All lectures will be held in the lecture theater in the TransCanada Pipelines Pavilion (TCPL). An LCD projector, a laptop, a document camera, and blackboards are available for presentations.

### SCHEDULE

#### Sunday

**16:00** Check-in begins (Front Desk - Professional Development Centre - open 24 hours)

**17:30–19:30** Buffet Dinner, Sally Borden Building

**20:00** Informal gathering in 2nd floor lounge, Corbett Hall (if desired)

Beverages and a small assortment of snacks are available on a cash honor system.

#### Monday

**7:00–8:45** Breakfast

**8:45–9:00** Introduction and Welcome by BIRS Station Manager, TCPL

**9:00–10:00** Leinenger-Margalit 1

**10:00–10:30** Coffee Break, TCPL

**10:30–11:30** Lauda

**11:30–13:00** Lunch

**13:00–14:00** Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall

**14:00** Group Photo; meet in foyer of TCPL (photograph will be taken outdoors so a jacket might be required).

**14:30–15:00** Coffee Break, TCPL

**15:00–16:00** Freedman

**16:30–17:30** Thurston

**17:30–19:30** Dinner

#### Tuesday

**7:00–9:00** Breakfast

**9:30–10:30** Leinenger-Margalit 2

**10:30–11:00** Coffee Break, TCPL

**11:00–12:00** Cautis

**12:00–13:30** Lunch

**13:30–14:30** Honda-Matic 2

**14:30–15:00** Coffee Break, TCPL

**15:00–16:00** Cotton-Clay

**16:30–17:30** Grigsby

**17:30–19:30** Dinner

### **Wednesday**

**7:00–9:00** Breakfast  
**9:00–10:00** Bigelow  
**10:00–10:30** Coffee Break, TCPL  
**10:30–11:30** Korkmaz  
**11:30–13:30** Lunch  
Free Afternoon  
**17:30–19:30** Dinner

### **Thursday**

**7:00–9:00** Breakfast  
**9:00–10:00** Tian  
**10:00–10:30** Coffee Break, TCPL  
**10:30–11:30** Elias  
**11:30–13:30** Lunch  
**13:30–14:30** Brandenbursky  
**14:30–15:00** Coffee Break, TCPL  
**15:00–16:00** Dunfield  
**17:30–19:30** Dinner

### **Friday**

**7:00–9:00** Breakfast  
**9:00–10:30** Informal Discussions  
**10:30–11:00** Coffee Break, TCPL  
**11:00–11:30** Informal Discussions  
**11:30–13:30** Lunch

**Checkout by  
12 noon.**

\*\* 5-day workshop participants are welcome to use BIRS facilities (BIRS Coffee Lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. \*\*

# Mapping Class Groups and Categorification

## April 7–12, 2013

### ABSTRACTS

Speaker: **Stephen Bigelow** (University of California at Santa Barbara)

Title: *Diagrammatic knot invariants that ought to be categorified*

Abstract: To compute the Kauffman bracket of a knot diagram, the first step is to replace each crossing by a linear combination of its two smoothings. I will describe some variations of this approach, where edges may have dead ends, orientations, or colors. These give elementary definitions of Alexander and Jones polynomials that are just begging to be categorified.

Speaker: **Michael Brandenbursky** (Vanderbilt University)

Title: *Bi-invariant metrics on diffeomorphism groups*

Abstract: In this talk I will discuss various metrics on groups of diffeomorphisms of smooth manifolds, which do or do not preserve some additional structure (usually volume or symplectic form). Then I will restrict my talk to the case of the group  $G$  of area-preserving diffeomorphisms of the 2-disc. This group admits a natural bi-invariant Autonomous metric. I will show that any finitely generated free abelian group embeds bi-Lipschitz into the group  $G$ . If time permits I will discuss some results concerning groups of Hamiltonian diffeomorphisms of surfaces. The proof uses some ideas from Mapping Class Groups and Knot theory.

Speaker: **Sabin Cautis** (University of Southern California)

Title: *A construction of braid group actions*

Abstract: A categorical  $\mathfrak{sl}(n)$  action generalizes the concept of an  $\mathfrak{sl}(n)$  representation from vector spaces to categories. There are several slightly different definitions of such actions but for us any of them will suffice. After reviewing a simplified definition of a categorical  $\mathfrak{sl}(n)$  action we explain how it can be used to construct an action of the braid group on the weight space categories. This is joint work with Joel Kamnitzer.

Speaker: **Nathan Dunfield** (University of Illinois at Urbana-Champaign)

Title: *L-spaces and left-orderability: an experimental survey*

Abstract: I will discuss the results of some computer experiments on small-volume hyperbolic 3-manifolds. Specifically, for the 11,031 such manifolds in the Hodgson-Weeks census, at least 27% are L-spaces and at least 2% have left-orderable fundamental groups. So far, these two subsets are disjoint, consistent with the conjecture of Boyer-Gordon-Watson that an irreducible rational homology 3-sphere is an L-space if and only if its fundamental group is not left-orderable.

Speaker: **Ben Elias** (Massachusetts Institute of Technology)

Title: *An introduction to Soergel bimodules and Rouquier complexes*

Abstract: Soergel bimodules are a combinatorial, algebraic categorification of the Hecke algebra. Rouquier complexes are complexes of Soergel bimodules, which (conjecturally) give a categorification of the braid group. Khovanov has used Rouquier complexes to construct a triply-graded knot homology theory. We will give a gentle introduction to these topics.

In conjunction with Geordie Williamson, the author has recently proven a number of facts about Soergel bimodules and Rouquier complexes en route to proving the Soergel conjecture. Several of these facts (the Hodge-Riemann bilinear relations and the "diagonal miracle") have yet to be exploited in connection to knot theory. We discuss the diagonal miracle and some related open questions.

Speaker: **J. Elisenda Grigsby** (Boston College)

Title: *Sutured Khovanov homology and the word problem in the braid group*

Abstract: Khovanov homology is an invariant of links in the three-sphere. Sutured (annular) Khovanov homology is a closely-related invariant of links in the solid torus originally defined by Asaeda-Przytycki-Sikora and later related to Heegaard Floer homology by Lawrence Roberts. I will present a combinatorial proof that sutured annular Khovanov homology detects the trivial braid conjugacy class, hence provides (yet) another solution to the word problem in the braid group. The proof involves an explicit relationship between Plamenevskaya's invariant of transverse braids and Dehornoy's left-invariant order on the braid group. This is joint work with John Baldwin.

Speaker: **Ko Honda** (University of Southern California) and **Gordana Matic** (University of Georgia)

Title: *Introduction to contact topology I and II*

Abstract: In the sequence of two talks, we will give a brief introduction to contact topology, highlighting the aspects that are most relevant to mapping class groups: convex surface theory, tight vs overtwisted dichotomy, open book decompositions, and the contact category.

Speaker: **Mustafa Korkmaz** (Middle East Technical University)

Title: *Low dimensional linear representations of mapping class groups*

Abstract: The action of the mapping class group  $\text{Mod}_g$  of an orientable surface of genus  $g$  on the first homology of the surface gives the  $2g$ -dimensional classical symplectic representation. I will show that there is no nontrivial representation of  $\text{Mod}_g$  into  $GL(n, C)$  if  $n < 2g$ . The second result is that any nontrivial representation into  $GL(2g, C)$  is conjugate to the symplectic representation. I will also show that there is no faithful linear representation of  $\text{Mod}_g$  in dimensions  $\leq 3g - 3$ . Finally, I will give a few corollaries of these results.

Speaker: **Aaron Lauda** (University of Southern California)

Title: *Getting knot invariants from representation theory via Howe duality*

Abstract: It is a well understood story that one can extract link invariants associated to simple Lie algebras. These invariants are called Reshetikhin-Turaev invariants and the famous Jones polynomial is the simplest example. Kauffman showed that the Jones polynomial could be described very simply by replacing crossings in a knot diagram by various smoothings. In this talk we will explain Cautis-Kamnitzer-Licata's simple new approach to understanding these invariants using basic representation theory and the quantum Weyl group action. Their approach is based on a version of Howe duality for exterior algebras called skew-Howe duality. Even the graphical (or skein theory) description of these invariants can be recovered in an elementary way from this data. The advantage of this approach is that it suggests a 'categorification' where knot homology theories arise in an elementary way from higher representation theory and the structure of categorified quantum groups.

Speaker: **Christopher Leininger** (University of Illinois at Urbana-Champaign) and **Daniel Margalit** (Georgia Institute of Technology)

Title: *Mapping Class Groups and Surface Bundles I and II*

Abstract: A surface bundle is completely determined by the associated monodromy from the fundamental group of the base to the mapping class group of the fiber. Therefore, we stand to gain much information about surface bundles by studying the algebraic and geometric properties of the mapping class group of a surface.

In the first talk of this two-part series, we will discuss the cohomology of the mapping class group. Each such cohomology class can be thought of as a characteristic class for surface bundles. We will start by describing some classical results on the low-dimensional cohomology of the mapping class group. Then we will discuss some of the recent dramatic progress, most notably the resolution of the Mumford Conjecture by Madsen and Weiss, which completely determines the stable cohomology of the mapping class group. We will also indicate many of the remaining open problems and mysteries.

In the second talk, we turn our attention to geometric aspects of the mapping class group. We will describe a fascinating connection between the coarse geometry of a surface bundle and the geometry of actions of the mapping class group. This will begin with some preliminary discussion of some of the canonical spaces on which the mapping class group acts, after which we will explain the connection between coarse hyperbolicity of surface bundles and the notion of convex cocompactness for subgroups of the mapping class group as defined by Farb and Mosher. We end with a discussion of some of the open questions and partial results.

Speaker: **Yin Tian** (University of Southern California)

Title: *A categorification of  $U_q\mathfrak{sl}(1|1)$  as an algebra*

Abstract: Representation theory of quantum groups has profound applications to low-dimensional topology in the framework of Reshetikhin-Turaev invariants. What is more interesting is the categorical picture. On one hand, the categorification of  $U_q\mathfrak{sl}_2$  is closely related to Khovanov homology of knots which categorifies the Jones polynomial. On the other hand, the Alexander polynomial of knots can be recovered from the representation theory of super quantum group  $U_q\mathfrak{sl}(1|1)$ ; moreover, the knot Floer homology gives rise to a categorification of the Alexander polynomial. In this talk, we will construct a triangulated category motivated from the contact topology to give a categorification of an integral version of  $U_q\mathfrak{sl}(1|1)$ .