MAPPING CLASS GROUPS AND CATEGORIFICATION FINAL REPORT

JASON BEHRSTOCK, ANTHONY LICATA AND ROBERT LIPSHITZ

Contents

1. Conference focus and goals

The study of mapping class groups of surfaces began in the 1920's, with the work of Dehn and Nielsen and has continued unabated since then. One theme in recent decades has been understanding to what extent mapping class groups behave like lattices. After Harvey proposed that mapping class groups might even be arithmetic, it was shown relatively quickly (by Ivanov) that they are not. Nonetheless, mapping class groups share many properties with lattices, and explaining this fact remains, three decades later, an important area of research. Some of these similarities would be explained if mapping class groups were linear groups, but linearity of mapping class groups remains open.

The field of quantum algebra originated from attempts in the 1980's to understand Witten's physical viewpoint on the Jones polynomial. The field underwent a rebirth roughly a decade ago, when Khovanov showed that the Jones polynomial

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could be "categorified," giving a bi-graded homology theory. At roughly the same time, Ozsvath-Szabo and Rasmussen gave a categorification of the Alexander polynomial. (The techniques involved in the two categorifications are completely different: Khovanov's were entirely algebraic, while Ozsvath-Szabo's and Rasmussen's involved hard analysis.) Attempts to explain these categorified knot invariants have led to various notions of categorification of quantum groups; in the last few years, this has become an extremely active field of research.

There are a number of known relations between braid groups and categorification; in particular, categorification leads to natural actions of braid groups, many of them faithful. (These actions arise from different directions, including diagramatic categorification; Fukaya categories; categories of sheaves; matrix factorizations; and gauge theory and Heegaard Floer theory.) For mapping class groups of surfaces of higher genus, fewer relations are known, though there is recent progress and much interest in the categorification community. In particular, there are faithful actions of mapping class groups of surfaces, arising from Fukaya categories and from Heegaard Floer theory. (Both can be described in elementary, combinatorial terms.)

The faithful categorified actions mentioned above do not descend to faithful linear actions; their relationship with linear actions is like the relationship between Khovanov homology and the Jones polynomial. The analogy can be pursued somewhat farther. For instance, it is conjectured that the Jones polynomial detects the unknot, but recently proved that Khovanov homology does; it is conjectured that mapping class groups are linear, but known that they are categorified-linear. It is not yet known whether group-theoretic consequences can be drawn from the fact that mapping class groups are categorified-linear.

The goal of this workshop was to bring together researchers in two related fields—geometric group theory and categorification—to improve each groups understanding of the tools and techniques of the other. Both fields are rapidly developing, and the relations between them are just starting to become apparent. The hope is that communication between the two fields will lead to new results and questions in each.

2. Program

Monday **April 8, 2013**

9:00–10:00 Christopher Leinenger-Daniel Margalit, *Mapping Class Groups and Surface Bundles, I.**
10:30–11:30 Aaron Lauda, *Getting knot invariants from representation theory via Howe duality.**
15:00–16:00 Michael Freedman, *Obstructions to embedding 3-manifolds in* $\mathbb{R}^4$.
16:30–17:30 Dylan Thurston, *Detecting rational maps.*

Tuesday **April 9, 2013**

9:30–10:30 Christopher Leininger-Daniel Margalit, *Mapping Class Groups and Surface Bundles, II.**
11:00–12:00 Sabin Cautis, *A construction of braid group actions.**
13:30–14:30 Gordana Matic, *Introduction to contact topology.**
15:00–16:00 Andrew Cotton-Clay, *Invariants from holomorphic curves in mapping tori*
16:30–17:30 J. Elisenda Grigsby, *Sutured Khovanov homology and the word problem in the braid group.*

Wednesday **April 10, 2013**

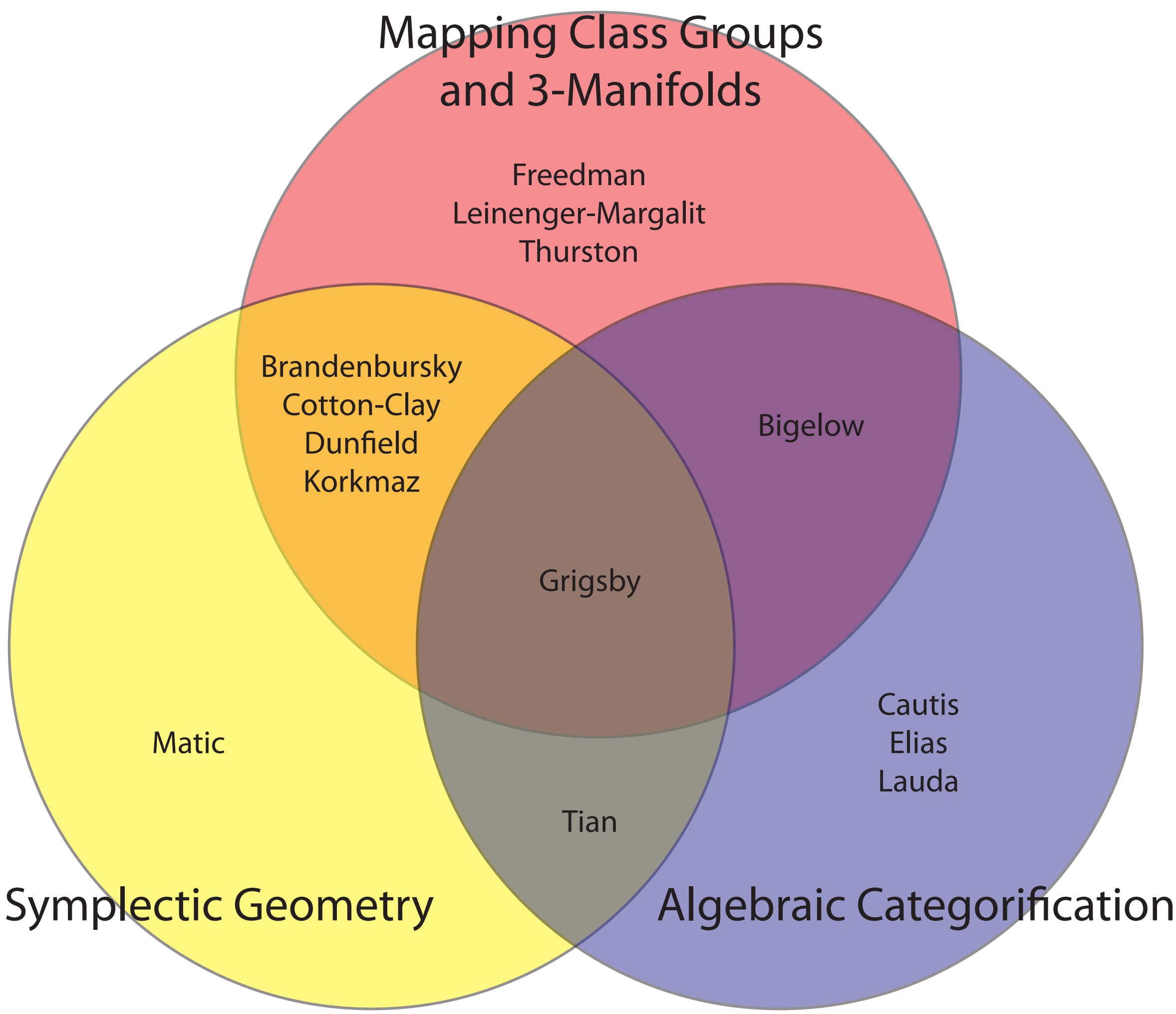
9:00–10:00 Stephen Bigelow, *Diagrammatic knot invariants that ought to be categorified.*
10:30–11:30 Mustafa Korkmaz, *Low dimensional linear representations of mapping class groups.*

Thursday **April 11, 2013**

9:30–10:30 Yin Tian, *A categorification of* $U_q(\mathfrak{gl}(1|1))$ *as an algebra.*
11:00–12:00 Benjamin Elias, *An introduction to Soergel bimodules and Rouquier complexes.*
2:00–3:00 Michael Brandenbursky, *Bi-invariant metrics on groups of Hamiltonian diffeomorphisms.*
3:30–4:30 Nathan Dunfield, *L-spaces and left-orderability: an experimental survey.*

Five of the talks, marked with asterisks above, were primarily expository, aimed at introducing researchers in one field to key questions and techniques in another. The remaining talks focused on new developments, with the aim of explaining their significance to non-experts.

Roughly, these talks can be divided into talks on mapping class group theory and thee-manifold topology; symplectic geometry and symplectic approaches to categorification; and algebraic approaches to categorification. But several of the talks took place at the interface between two or more of these fields:



3. Presentations

3.1. **Stephen Bigelow.** Spoke on the subject of "Diagrammatic knot invariants that ought to be categorified."

This talk is about $U_q(\mathfrak{sl}_2)$. Probably most of the audience is already familiar with this algebra. I hope the definition I will describe gives you a useful new way to think about it.

The *classical* Temperley-Lieb category gives a diagrammatic way to represent morphisms between certain representations of $SL(2, \mathbf{C})$. Let V be $\mathbf{C}^2$ with the obvious action of $SL(2, \mathbf{C})$. A "cap" represents the determinant map $V \otimes V \to \mathbf{C}$. A "cup" is a dual to this. An "X" represents the map $u \otimes v \mapsto v \otimes u$. Stack diagrams vertically or horizontally to represent compositions or tensor products.

Next, we introduce *orientations*. An oriented vertical strand represents a projection of V onto an axis. A diagram in which all strands are oriented then represents a scalar times an elementary linear map between tensor powers of V. This scalar turns out to depend only on the turning number of the diagram.

To define the *quantum* version of this, let an oriented closed loop evaluate to $q^{\pm 1}$, depending on the orientation, as opposed to -1 in the classical case. Also, specify the over and under stands for each crossing, and resolve them using a Kauffman skein relation.

It will be useful to allow endpoints on the *sides* of the diagram. An oriented strand with both endpoints on the sides can be deleted, up to a scalar to account for its turning number.

Finally, we introduce a *pole*. This is a special strand that is unoriented, straight, and vertical. We require all other endpoints of strands to be on the sides of the diagram. Strands are allowed to cross over or under the pole. We obtain an associative algebra, with multiplication given by vertical stacking.

The main result of this talk is that the above diagrammatic algebra, modulo a few additional relations, is $U_q(\mathfrak{sl}_2)$. I will list and motivate the relations, explain the Hopf algebra structure in terms of diagrams, and show some of the necessary computations.

The action of $U_q(\mathfrak{sl}_2)$ on $V^{\otimes n}$ is given by threading n parallel strands in place of the pole. It is immediately "visually" obvious that this action commutes with the action of the Temperley-Lieb category.

3.2. **Michael Brandenbursky.** Spoke on "Bi-invariant metrics on groups of Hamiltonian diffeomorphisms."

It is well known that every compactly-supported Hamiltonian diffeomorphism of a symplectic manifold (M, ω) is a composition of finitely many autonomous diffeomorphisms. How many? In [1, 2], the speaker (and his coauthor) studied the geometry of this question. They defined the *autonomous norm* on the group $\mathrm{Ham}(M)$ of Hamiltonian diffeomorphisms of M

$$\|f\|_{\mathrm{Aut}} := \min \{ m \in \mathbb{N} \mid f = h_1 \cdots h_m \text{ where each } h_i \text{ is autonomous} \}.$$

The associated bi-invariant metric, defined by $\mathbf{d}_{\mathrm{Aut}}(f, g) := \|fg^{-1}\|_{\mathrm{Aut}}$, is the most natural word metric on $\mathrm{Ham}(M)$. It is particulary interesting in the two-dimensional case due to the following observation.

Let Σ_g be a closed surface, $H \colon \Sigma_g \to \mathbb{R}$ a Morse function and h a Hamiltonian diffeomorphism generated by H. After cutting the surface Σ_g along critical level sets one is left with a finite number of regions, so that each region is diffeomorphic to the annulus. By the Arnol'd-Liouville theorem, there exist angle-action symplectic coordinates on each of these annuli, so that h rotates each point on a regular level curve with the same speed, i.e. the speed depends only on the level curve. It follows that a generic Hamiltonian diffeomorphism of Σ_g may be written as a finite composition of autonomous diffeomorphisms, such that each of these diffeomorphisms is "almost everywhere rotation" in the right coordinates, and hence relatively simple. (Of course this decomposition is neither unique nor canonical, but it is plausible that it might be useful in dynamical systems.)

Let $\mathbb{D}^2$ be an open unit disc in the plane. This talk described joint work of the speaker and Jarek Kedra [1], in which they proved:

Theorem 1 ([1]). *For every natural number $k \in \mathbb{N}$ there exists an injective homomorphism $\mathbb{Z}^k \to \mathrm{Ham}(\mathbb{D}^2)$ which is bi-Lipschitz with respect to the word metric on $\mathbb{Z}^k$ and the autonomous metric on $\mathrm{Ham}(\mathbb{D}^2)$.*

In particular, the result implies that the autonomous metric is unbounded on $\mathrm{Ham}(\mathbb{D}^2)$. Brandenbusrky also touched on more recent work [2] proving

Theorem 2 ([2]). *Let $g > 1$. Then the metric group $(\mathrm{Ham}(\not\prec_g), {}_{\mathrm{Aut}})$ is unbounded.*

The main tool in the proofs of both results is a construction of certain quasi-morphisms on mapping class groups of punctured surfaces.

3.3. **Sabin Cautis.** Described "A construction of braid group actions." A categorical $\mathfrak{sl}_n$ action generalizes the concept of an $\mathfrak{sl}_n$ representation from vector spaces to categories. The first example of such an action was described in 2004 by Chuang and Rouquier in the case when $n = 2$. They used this action to prove a version of Broué's abelian defect group conjecture for the symmetric group.

More precisely, given a representation V of $\mathfrak{sl}_2$ the weight spaces $V(n)$ and $V(-n)$ are isomorphic via an explicit isomorphism constructed using the $\mathfrak{sl}_2$ action. Likewise, given an action of $\mathfrak{sl}_2$ on categories, one can construct an equivalence of categories $C(n) \xrightarrow{\sim} C(-n)$. Chuang and Rouquier described an action of $\mathfrak{sl}_2$ between the blocks of the symmetric group in positive characteristic and used this resulting isomorphism to show that certain blocks are equivalent (thus proving Broué's conjecture).

More recently, in joint work with Joel Kamnitzer we show that a categorical $\mathfrak{sl}_n$ action leads to a series of equivalences which satisfy the braid relations (of the braid group on n strands). There are several slightly different definitions of what it means to have such a categorical $\mathfrak{sl}_n$ action but for the purposes of constructing braid group actions any of them suffice.

Braid group actions resulting from such categorical $\mathfrak{sl}_n$ actions can be used to construct various homological knot invariants. In particular, one can categorify all the Reshetikhin-Turaev knot invariants of type A this way.

In his talk, Cautis concentrated on explaining what it means to have a categorical $\mathfrak{sl}_n$ action and how one can use this to define the action of the braid group. This was illustrated with an ongoing example when $n = 3$ and where the categories involved are modules over polynomial algebras (this example categorifies the representation $\mathrm{Sym}^3 W$ where W is the standard representation of $\mathfrak{sl}_3$).

Although there was not enough time to discuss them, there are many examples of such $\mathfrak{sl}_n$ actions, arising from ranging from algebra and representation theory (actions on modules over certain algebras), modular representation theory (such as the example of Chuang and Rouquier), algebraic geometry (actions on categories of coherent sheaves or D-modules on certain varieties) and, somewhat conjecturally, symplectic geometry (actions on Fukaya categories).

More generally, it would be interesting to understand what sort of categorical actions can be used to define actions of more general mapping class groups (the braid group being the MCG associated to a punctured disk). Such actions would hopefully allow one to generalize the Reshetikhin-Turaev homological invariants from knots in S^3 to knots in arbitrary 3-manifolds. This should also shed light on the Heegaard-Floer homology of 3-manifolds as this can be thought of as the Reshetikhin-Turaev 3-manifold invariant associated with the super Lie algebra $\mathfrak{gl}(1|1)$.

3.4. **Andrew Cotton-Clay.** Gave a talk on "Invariants from holomorphic curves in mapping tori." He started by describing relationships between several low-dimensional Floer theories: Heegaard Floer homology (Ozsváth-Szabó), monopole Floer homology (Kronheimer-Mrowka and others), embedded contact homology (Hutchings) and periodic Floer homology (Hutchings-Thaddeus). In particular, it is now known (due to work of Taubes, Kutluhan-Lee-Taubes and Colin-Ghiggini-Honda) that corresponding variants of these theories are all isomorphic (when defined). Cotton-Clay then focused on periodic Floer homology, which is defined from a surface diffeomorphism ϕ. He described a computation of the periodic Floer homology in the second-to-lowest spin^c-structure in terms of the geometry of the mapping class of ϕ. He then concluded with further results on the moduli spaces of holomorphic curves involved in the definition of periodic Floer homology, and a computation of the periodic Floer homology in other spin^c structures in certain examples.

3.5. **Nathan Dunfield.** Spoke on "L-spaces and left-orderability: an experimental survey." Dunfield discussed the results of some computer experiments on small-volume hyperbolic 3-manifolds. Specifically, for the 11,031 such manifolds in the Hodgson-Weeks census, at least 27% are L-spaces and at least 2% have left-orderable fundamental groups. So far, these two subsets are disjoint, consistent with the conjecture that an irreducible rational homology 3-sphere is an L-space if and only if its fundamental group is not left-orderable.

3.6. **Ben Elias.** Provided "An introduction to Soergel bimodules and Rouquier complexes." Soergel bimodules are a combinatorial, algebraic categorification of the Hecke algebra. Rouquier complexes are complexes of Soergel bimodules, which (conjecturally) give a categorification of the braid group. Khovanov has used Rouquier complexes to construct a triply-graded knot homology theory.

In conjunction with Geordie Williamson, Elias has recently proved a number of facts about Soergel bimodules and Rouquier complexes en route to proving the Soergel conjecture. Several of these facts (the Hodge-Riemann bilinear relations and the "diagonal miracle") have yet to be exploited in connection to knot theory. This talk discussed the diagonal miracle and some related open questions.

3.7. **Michael H. Freedman.** Most of this talk described joint work with Ian Agol and Nathan Dunfield. The speaker and his coauthors found obstructions to embedding certain three-manifolds into $\mathbb{R}^4$. These obstructions can be detected via a Heegaard splitting, but are not invariant under stabilization; Freedman raised the question of whether their construction can be modified to survive stabilization.

Freedman also introduced a space which he called the *conflict hypergraph*, gave an example of three vertices which were in conflict even though pairwise they are not in conflict, and raised questions about the basic properties of this hypergraph.

3.8. **J. Elisenda Grigsby.** Spoke on "Sutured Khovanov homology and the word problem in the braid group." Khovanov homology is an invariant of links in the three-sphere that was recently shown by Kronheimer-Mrowka to detect the unknot. Further work of Hedden-Ni and Batson-Seed implies that Khovanov homology detects the unlink (in contrast to its graded Euler characteristic, which is known not to detect the unlink when the number of components is at least 2).

Grigsby spoke about joint work with John Baldwin involving sutured (annular) Khovanov homology, a closely-related invariant of links in the solid torus originally defined by Asaeda-Przytycki-Sikora and later related to Heegaard Floer homology by Lawrence Roberts. She presented a combinatorial proof (requiring no gauge theory or holomorphic curves) that sutured annular Khovanov homology detects the trivial braid conjugacy class, and hence provides (yet) another solution to the word problem in the braid group. The proof relies on an explicit relationship between Plamenevskaya's invariant of transverse braids, braid dynamics, and Dehornoy's left-invariant order on the braid group.

3.9. **Mustafa Korkmaz.** Spoke on "Low dimensional linear representations of mapping class groups."

Let S denote a compact connected orientable surface of genus g and let $\mathrm{Mod}(S)$ denote the mapping class group of it, the group of isotopy classes of diffeomorphisms $S \to S$ which are identity on the boundary of S. If $\bar{S}$ is the closed surface obtained by gluing a disk along each boundary component, after fixing a symplectic basis of $H_1(\bar{S};\mathbb{Z})$, the action of $\mathrm{Mod}(\bar{S})$ on the first homology group $H_1(\bar{S};\mathbb{Z})$ gives rise to a homomorphism $\mathrm{Mod}(\bar{S}) \to \mathrm{Sp}(2g,\mathbb{Z})$ onto the symplectic group. Thus, one gets a homomorphism $P : \mathrm{Mod}(S) \to \mathrm{GL}(n,\mathbb{C})$ which is the composition of the following homomorphism

$$\mathrm{Mod}(S) \to \mathrm{Mod}(\bar{S}) \to \mathrm{Sp}(2g,\mathbb{Z}) \hookrightarrow \mathrm{GL}(2g,\mathbb{C}).$$

In his lecture, Korkmaz outlined proofs of the following three theorems:

Theorem 3. *(Franks-Handel, Korkmaz) Let $g \geq 1$ and let $n \leq 2g-1$. Let ϕ : $\mathrm{Mod}(S) \to \mathrm{GL}(n,\mathbb{C})$ be a homomorphism. Then ϕ factors through $\mathrm{Mod}(S) \to H_1(\mathrm{Mod}(S);\mathbb{Z})$. In particular, the image of ϕ is*

(i) *trivial if $g \geq 3$, and*
(ii) *a quotient of the cyclic group $\mathbb{Z}_{10}$ of order 10 if $g = 2$.*

Theorem 4. *(Korkmaz) Let $g \geq 3$ and let $\phi : \mathrm{Mod}(S) \to \mathrm{GL}(2g,\mathbb{C})$ be a group homomorphism. Then ϕ is either trivial or conjugate to the homomorphism P : $\mathrm{Mod}(S) \to \mathrm{GL}(2g,\mathbb{C})$.*

Theorem 5. *(Korkmaz) Let $g \geq 3$ and let $n \leq 3g-3$. Then there is no injective homomorphism* $\mathrm{Mod}(S) \to \mathrm{GL}(n, \mathbb{C})$*.*

One of the outstanding unsolved problems in the theory of mapping class groups is the existence of a faithful linear representation $\mathrm{Mod}(S) \to \mathrm{GL}(n, \mathbb{C})$ for some n. It is known that the braid groups, the mapping class group of the sphere with marked points and the hyperelliptic mapping class groups are linear. The third theorem shows that in dimensions $n \leq 3g-3$, there is no faithful linear representation of the mapping class group. Previously, this was known for $n \leq \sqrt{g+1}$.

He also discussed a few applications of these theorems, including some algebraic consequences.

3.10. **Aaron Lauda.** Described techniques for "Getting knot invariants from representation theory via Howe duality." It is a well-understood story that one can extract link invariants associated to simple Lie algebras. These invariants are called Reshetikhin-Turaev invariants; the Jones polynomial is the simplest example. Kauffman showed that the Jones polynomial admits a simple description in terms of smoothings of a knot diagram. In this talk, Lauda explained Cautis-Kamnitzer-Licata's simple new approach to understanding Reshetikhin-Turaev invariants using basic representation theory and the quantum Weyl group action. Their approach is based on a version of Howe duality for exterior algebras called skew-Howe duality. Even the graphical (or skein theory) description of these invariants can be recovered in an elementary way from this data. The advantage of this approach is that it suggests a 'categorification' where knot homology theories arise in an elementary way from higher representation theory and the structure of categorified quantum groups.

3.11. **Christopher Leininger and Daniel Margalit.** Provided a two-part series aimed at a broad audience on "Mapping Class Groups and Surface Bundles."

A surface bundle is completely determined by the associated monodromy from the fundamental group of the base to the mapping class group of the fiber. Therefore, we stand to gain much information about surface bundles by studying the algebraic and geometric properties of the mapping class group of a surface.

In the first talk of this two-part series, Margalit discussed the cohomology of the mapping class group. Each such cohomology class can be thought of as a characteristic class for surface bundles. The talk began by describing some classical results on the low-dimensional cohomology of the mapping class group. First, he showed that the first cohomology of the mapping class group is trivial in most cases, and so there turns out to be no characteristic classes for surface bundles over the circle. The story for two-dimensional classes is quite different. Margalit gave three very different descriptions of two-dimensional cohomology classes: the first Morita–Mumford–Miller class, the Weil–Petersson 2-form, and the signature. It turns out that the second cohomology of the mapping class is cyclic in most cases. Therefore, all three of these cohomology classes are multiples of each other.

The talk will also discussed some of the recent dramatic progress, most notably the resolution of the Mumford Conjecture by Madsen and Weiss, which shows that the stable cohomology of the mapping class group is generated by the Morita–Mumford–Miller classes. He also indicated many of the remaining open problems and mysteries. For instance, very little is known about the unstable cohomology classes of the mapping class group, despite a great abundance of such elements. Based on the two-dimensional story explained above, one can expect that some of these classes will be equally intriguing.

In the second talk, Leininger turned his (and our) attention to geometric aspects of the mapping class group. He described a fascinating connection between the coarse geometry of a surface bundle and the geometry of actions of the mapping class group. This began with some preliminary discussion of some of the canonical spaces on which the mapping class group acts, after which he explained the connection between coarse hyperbolicity of surface bundles and the notion of convex cocompactness for subgroups of the mapping class group as defined by Farb and Mosher. This talk also ended with a discussion of a number of open questions and partial results.

3.12. **Gordana Matic.** Gave a general audience "Introduction to contact topology." In this talk, she gave a brief introduction to contact topology, highlighting the aspects that are most relevant to mapping class groups: convex surface theory, the tight-versus-overtwisted dichotomy, open book decompositions, and the contact category.

3.13. **Dylan Thurston.** Spoke on "Detecting rational maps."

3.14. **Yin Tian.** Spoke about "A categorification of $U_q(\mathfrak{gl}(1|1))$ as an algebra."

In the framework of Reshetikhin-Turaev invariants, the super quantum group $U_q(\mathfrak{sl}(1|1))$ gives rise to the Alexander polynomial of links just as the quantum group $U_q(\mathfrak{sl}_2)$ gives rise to the Jones polynomials. In the last decade it was discovered that the connection between quantum groups and knot invariants can be refined further: it can be lifted to the categorical level. (The existence of such a lifting process, called categorification, was conjectured by Crane and Frenkel.) Two poineering examples appeared around the turn of the century: Khovanov homology, defined by Khovanov, which categorifies the Jones polynomial and knot Floer homology, defined independently by Ozsváth-Szabó and Rasmussen, which categorifies the Alexander polynomial. In this talk, Tian discussed the construction of a triangulated category motivated from 3-dimensional contact topology, that gives a categorification of $U_q(\mathfrak{sl}(1|1))$.

The motivation is a $(2+1)$-dimensional TQFT, the Honda's "contact category." More precisely, the contact category $\mathcal{C}(\Sigma)$ associated to an oriented surface Σ, studies contact structures on the 3-manifold $\Sigma \times [0,1]$ and the induced object called dividing sets on the surface Σ. The main feature of $\mathcal{C}(\Sigma)$ is the existence of distinguished triangles. Motivated from the connection between the contact category and bordered Heegaard Floer theory defined by Lipshitz-Ozsváth-Thurston, Tian defined a differential graded category to reformulate the contact category $\mathcal{C}(\Sigma)$ of an annulus Σ. In related work, he also gave a categorification of a Clifford algebra via the contact category of a disk.

4. Other outcomes

Many of the participants have commented on how focused the talks were on the conference theme, and how much effort the speakers made to communicate to researchers in other fields. (The organizers agree, and appreciate how cooperative the speakers were.) Several participants also noted specific outcomes, including new and ongoing collaborations and research projects.

At the conference, Jason Behrstock, Michael Freedman, and Saul Schleimer resolved some of the questions raised in Freedman's talk about the "conflict hypergraph." They are in the process of writing up their results.

Conversations with Nathan Dunfield allowed Corrin Clarkson, a graduate student, to significantly strengthen a theorem in a paper she is writing. She also reports helpful (and encouraging) conversations with Elisenda Grigsby, Daniel Margalit and Dylan Thurston.

Elisenda Grigsby reports speaking a great deal with Anthony Licata white at BIRS about a joint project with Stephan Wehrli aimed at proving a conjecture relating sutured annular Khovanov homology to the Hochschild homology of certain bimodules appearing in Catharina Stroppel's work on Category O. In direct response to a question from Sabin Cautis and Tony Licata about representability of alternating knots as closures of alternating braids, she also formulated a small problem to suggest to some undergraduate students working with me at Boston College this summer. She is also optimistic about Michael Freedman's talk leading to new research projects, and was pleased to meet young researchers including Yin Tian.

Slava Krushkal found it particularly productive talking with Michael Freedman: they had some new ideas on thickness (in the sense of Gromov) and distortion of complexes embedded in Euclidean spaces, exhibiting some new phenomena when the ambient dimension is twice the dimension of the complex. They plan to write a paper on "Embedding thickness below the stable range" describing these ideas.

Timothy Perutz was inspired to formulate a family of problems in higher-dimensional symplectic topology, based on ideas in mapping class group theory. The mapping class group $Mod(S, \partial S)$ of diffeomorphisms with compact support in the interior of a surface S with non-empty boundary can be understood as the symplectic mapping class group of an exact symplectic form $d\theta$, that is, the group of path-components of the group of self-diffeomorphisms ϕ with compact support in the interior of S such that $\phi^*\theta - \theta$ is an exact 1-form. This perspective highlights a generalization of mapping class groups, to mapping class groups $\pi_0\mathrm{Aut}(M, \omega)$ of higher-dimensional exact symplectic manifolds (M, ω) with contact-type boundary. There is a subgroup $\Gamma_M \subset \pi_0\mathrm{Aut}(M, \omega)$ generated by higher-dimensional Dehn twists, along Lagrangian spheres in M. Intersection numbers of Lagrangian spheres categorify to Floer cohomology groups, which are morphisms in the exact Fukaya category. Recent work of Ailsa Keating (http://arxiv.org/abs/1204.2851) shows one similarity between Γ_M and $Mod(S, \partial S)$. Perutz talked with Behrstock, Margalit, and others about identifying properties of $Mod(S, \partial S)$ which might be shared by Γ_M, and formulated several concrete conjectures which he hopes to incorporate into his research program.

Liam Watson reports that conversations at the conference motivated him to write up an application of Khovanov homology to the symmetry group of a knot (i.e., the mapping class group of a knot exterior). In particular, he can define an invariant that distinguishes strong inversions, and can be used to say things about other symmetries (including free periods and chierality).

References

[1] Brandenbursky M., Kedra J.: *On the autonomous metric on the group of area-preserving diffeomorphisms of the 2-disc*, Algebraic & Geometric Topology, **13** (2013), 795–816.

[2] Brandenbursky M.: *On the autonomous metric on groups of Hamiltonian diffeomorphisms of closed hyperbolic surfaces,* arXiv:1305.2757, 2013.