Motivation

Stochastic processing networks: Design simple & efficient scheduling policies

Consider simplest set-up: Jackson-like queueing network

Back-pressure is maximally stable

- Delay can be really bad
- Fixes within frame-work exist

Can we approach design differently?

Details

- Discrete-time queueing network
- Set of N queues
- F is the set of flows: source $s(f)$ & destination $d(f)$ for flow $f \in F$
- $Q_n^f(t)$ be the queue-length of flow f at node n (at time t)
- Rate matrix R: number of (whole) packets that can be received in each time-unit
- Assume no interference constraints
- Max amount served from n of flow f at t to m: $\min(Q_n^f(t), R_{nm})$
- Stochastic arrivals

Back-pressure

- At every node n define for flow f and node m

$$W^f_{n,m}(t) = R_{nm}\left(Q^f_t(n) - Q^f_t(m)\right).$$

- Use this to define a per-flow metric

$$F^f_n(t) = \max_{m \in N} W^f_{n,m}(t)$$

  with the maximiser given by $m^*_{f,n}(t)$.

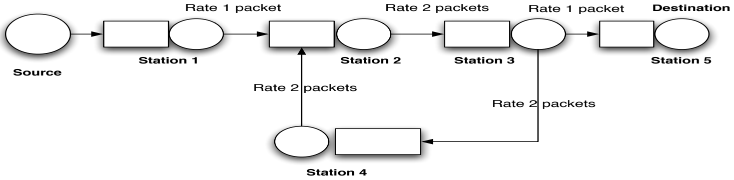
- Now define a node metric

$$G_n(t) = \max_{f \in F} F^f_n(t)$$

  with the maximiser given by $f^*_n(t)$.

- **Scheduling algorithm**: At node n
  - If $G_n(t) > 0$, serve flow $f^*_n(t)$ and route $\min\left(Q^{f^*_n(t)}_n(t), R_{nm^*_{f^*_n(t),n}}\right)$ packets to $m^*_{f^*_n(t),n}(t)$.
  - Else serve no flow.

Issues



- Will transmit packets to nodes not connected to destination node
- Cannot recognize loops so delay performance can be bad

Alternate Policy[1]

At node n for flow f define metric $V_t^f(n)$ as follows:

- If $n = d(f)$, then $V_t^f(d(f)) = 0$.
- If n communicates with $d(f)$, then $V_t^f(n) = \min_{m \in N} \frac{Q_n^f(t)}{R_{nm}} + V_t^f(m)$.
- Else $V_t^f(n) = +\infty$.

Comments:

- Vs surrogate to draining time
- Vs measure of net downstream load
- Uses shortest-path ideas

[1] S. and Leith, Draining-time based scheduling algorithm, CDC'07

New Algorithm

Similar to Back-Pressure perform the following computations

- At every node n define for flow f and node m

$$W^f_{n,m}(t) = \begin{cases} R_{nm}\left(V^f_t(n) - V^f_t(m)\right) & V^f_t(n), V^f_t(m) < +\infty,\ R_{nm} > 0 \\ -\infty & \text{else} \end{cases}$$

- Use this to define a flow metric $F^f_n(t) = \max_{m \in N} W^f_{n,m}(t)$
  Maximiser given by $m^*_{f,n}(t)$.
- Now define a node metric

$$G_n(t) = \max_{f \in F} F^f_n(t)$$

  with the maximiser given by $f^*_n(t)$.
- **Scheduling algorithm**:
  - If $G_n(t) > 0$, serve flow $f^*_n(t)$ & route $\min\left(Q^{f^*_n(t)}_n(t), R_{nm^*_{f^*_n(t),n}}\right)$ packets to $m^*_{f^*_n(t),n}(t)$.
  - Else serve no flow at node n

Alternate View

Alternate View:

- If n communicates with $d(f)$, then $F_n^f(t) = \max_{m \in N} W_{n,m}^f(t) = Q_n^f(t)$, and $G_n(t) = \max_{f \in F} Q_n^f(t)$
- Scheduling Policy - serve **longest queue** at every node.

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- Scheduling Policy - serve **longest queue** at every node.
- For every node n we have $m_{f,n}^*(t) = \arg\min_{m \in M} \frac{Q_n^f(t)}{R_{nm}} + V_t^f(m)$
- Routing Policy - route along **dynamic shortest paths** to destination with link metric $\frac{Q_n^f(t)}{R_{nm}}$
- *This is work-conserving*

Open problem: Is this maximally stable?

Comments

S. & Leith, CDC'07: simple cases, quadratic Lyapunov fn exists

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Quadratic Lyapunov functions will not work in general

Tandem queues: 1 flow, work-conserving FCFS

- Maximally stable
- Quadratic Lyapunov fns will not show this
  KumarMeyn, Trans. AC, 1995
- Piece-wise linear Lyapunov fns work
  DownMeyn, QueueingSystems, 1997

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$R \equiv$ constant, piece-wise quadratic Lyapunov works
NaghshvarZhuangJavidi, Trans. IT, 2012
DiekerShin'12 (arxiv) could be related

Open problem: If maximally stable, Lyapunov fn?