

**Report on the Program:
Operator algebras and dynamical systems from number theory
Nov 24 - Nov 29, 2013**

FORMAT

The format adopted was chosen to increase the chances of interaction and collaboration. There were fewer formal lectures than normally encountered in such workshops however the time created by the more relaxed program was filled with small group discussions and active collaboration on current research topics. In the opinion of the organisers it was very successful and the feedback from the participants supported our view.

The organisers also deliberately included a number of research students in the group, both advanced and beginning. There was a clear major benefit to these students from the small group interactions in which they were able to participate as well as the advanced lectures from the speakers.

SCIENTIFIC OVERVIEW

The workshop was about current work on analyzing symmetries and equilibrium states of operator theoretic dynamical systems, on their interaction with number theory and on number theoretic questions to which the operator algebra/dynamical system approach is relevant. Several constructions of noncommutative spaces and of dynamical systems from number theory are now available. Their study reveals more and more of the deep connection between the operator algebraic approach from mathematical physics and the abstract systems arising from number theoretic data. The hope is of course that the use of these new techniques in number theory will open new directions and may yield answers to some long-standing problems in number theory. Conversely we expect number theory to continue to provide interesting structures and questions about these new dynamical systems and others, informing the developments of operator algebraic techniques.

Results announced at the workshop provided evidence that this hope is being realised. Illustrative examples were the announcements of Cuntz, Li, the discussion of recent work of Cornelissen and Marcolli and the recent use of nonselfadjoint techniques for number theoretic systems by Katsoulis and others. We will discuss this in more detail below.

On the operator algebra side recent constructions of new algebras such as that of Carey-Rennie-Phillips-Putnam were understood in a deeper fashion using the results of Xin Li exposed in the workshop. The study of the equilibrium states and symmetries of the Toeplitz extensions and their realization in terms of product systems of Hilbert bimodules was discussed extensively. New results of Laca-Neshveyev-Trifkovic on induction of Bost-Connes systems, with applications to the connection with Hecke algebras and to functoriality were also discussed. We expand on some of these matters in the remainder of the report.

MAJOR RESEARCH THEMES AND ANNOUNCEMENTS

(i) Joachim Cuntz announced his latest results (joint with Christopher Deninger) which give a new natural construction of the ring of Witt vectors in characteristic p . He also described the relationship between the new approach to Witt vectors and cyclic theory. This research seems to herald a novel interaction between ideas from noncommutative geometry and algebraic geometry that may promote developments in both fields. In a separate short presentation, Cuntz also gave a quick review of the traditional approach to Witt vectors and discussed the significance and efficiency of the alternative approach in contrast to the old one.

(ii) A new development that was not apparent at the time of the organisation of the workshop but emerged in time for inclusion was the theme of nonselfadjoint operator algebra techniques in the area of noncommutative dynamical systems from number theory. The organisers were able to include a representative talk on this theme by Katsoulis. A number of very clear connections with the work of Xin Li and of Cornelissen and Marcolli were identified. We expect to see concrete results directly traceable to the workshop interactions along these lines.

(iii) New approaches to operator algebras arising in the dynamical systems approach to number theory were announced. This theme included Xin Li's talk in which he discussed the realization of the Q_λ algebras of Carey-Phillips-Putnam-Rennie as commutator ideals in semigroup algebras. This resolved a five year old

question about the existence of a ‘Toeplitz’ approach to the Q_λ ’s. These new methods also give an alternative description of their K-theory. Li also announced a complete classification of the boundary quotient C*-algebras introduced by Crisp and Laca a decade ago (significantly, this includes the nonsimple case), and he also discussed various other results on semigroup algebras that tie in to Brownlowe’s talk and indicates a fertile area for further investigations.

(iv) Many talks developed the ongoing cross-fertilisation of techniques for computing KMS states from systems arising in number theory to systems based on graph algebras especially through the focus on Toeplitz extensions.

(v) Recent work on induction motivated by the study of functoriality of Bost-Connes systems with respect to algebraic number fields was presented by Trifkovic. As a byproduct, this approach also yields a clear path between the two approaches to phase transitions from number theory, namely Bost-Connes systems and systems based on Hecke algebras.

RESEARCH ACTIVITIES ARISING FROM THE WORKSHOP

A major feature of the workshop was the formation of collaborative teams. These teams took advantage of the discussion sessions to discuss incomplete research and new ideas that emerged from the talks presented. These included the following groups:

(i) Brownlowe-Larsen-Stammeier.

They worked on two projects concerning operator algebras arising from semigroup crossed product constructions. They expect to produce two papers in the new year.

(ii) an Huef, Laca, Raeburn, Sims.

This team is spread over 3 countries and the workshop has been one of the few times where they have managed to be in the same place together. They were able to bring two joint papers near to completion, both are likely to be submitted in the subsequent months.

(iii) Bruce, Laca, Sims

A project on KMS states of graph algebras that emerged from the connection between two of the talks was identified and discussed by this group as an excellent opportunity to introduce Bruce (a new student at UVic) to research in the field. Joint supervision on this project will take place in the summer.

(iv) Cornelissen, Li

Discussions about the proof of the Cornelissen-Marcolli theorem. Li has a new, shorter version, that uses monoids and there was some useful progress.

(v) Cornelissen, Katsoulis,

Cornelissen continued to work on his project involving replacing non-involutive algebras by C*-algebras with distinguished automorphisms.

(vi) Carey, Rennie, Phillips

Spectral flow is used as an invariant for the graph algebras that arise from Mumford curves in the work of Carey-Marcolli-Rennie. These algebras are nonunital and there is no general theory of this invariant in this setting. The problem is avoided in the Mumford curve case by invoking local units: the spectral flow invariants in question can be calculated in a unital subalgebra of the nonunital graph algebra. This special situation cannot be expected to persist in all cases.

At this workshop progress was made in finalising a long standing project aimed at addressing this question. The idea is to use Kasparov theory to define spectral flow and to use the local index formula and the Carey-Phillips nonunital spectral flow formula to define and calculate spectral flow in the general nonunital setting. A draft was prepared that is close to completion.

Carey and Rennie also observed that Xin Li's work was relevant to the C^* dynamical systems that may be associated to the iterated function systems currently being studied by Michael Barnsley and his team.

(vii) Li, Katsoulis

One late addition to the themes of the workshop was to investigate the relevance of non-selfadjoint algebras to those dynamical systems that appear in number theoretic applications. The work of Xin Li announced at the workshop led to interesting discussions on the comparison between C^* -algebras and non-self adjoint operator algebras, as well as the reconstruction of number fields and function fields from quantum statistical mechanical systems.

FEEDBACK FOR BIRS

Participant comments

There was a general agreement amongst the participants that the facilities at the Banff Centre were excellent. The lecture room and the meeting rooms were excellent and the accommodation of high quality. Feedback to the organisers on the schedule of talks and their intention to promote interaction were very positive. The connections with other research areas were discussed much more than would normally take place at a workshop. Mathematically there were comments about the new results presented including those of Cuntz and Li, the comparison between C^* -algebras and non-self adjoint operator algebras, as well as the reconstruction of number fields and function fields from quantum statistical mechanical systems.

Organisational issues

There were some problems with the change of participants due to the long lead time between the submission of the application and actual workshop. A number of initially proposed participants were unable to come to the detriment of some of the connections with number theory. Partly the dates affected us negatively with the US-based researchers Marcolli and Consani due to the clash with the US Thanksgiving week. On the other hand, this also gave us the opportunity to adjust to changes in focus and interest, thus including a nonselfadjoint expert and several students, newcomers to the field. We were able to run the workshop at near full capacity.

LIST OF PARTICIPANTS

an Huef, Astrid; University of Otago, New Zealand
 Brenken, Berndt; University of Calgary
 Brownlowe, Nathan; University of Wollongong, Australia
 Bruce, Chris; University of Victoria
 Carey, Alan; Australian National University
 Cornelissen, Gunther; Utrecht University, Netherlands
 Cuntz, Joachim; University of Münster, Germany
 Duwenig, Anna; University of Victoria
 Echterhoff, Siegfried; Mathematisches Institut, Universität Münster, Germany
 Karemaker, Valentijn; Utrecht University, Netherlands
 Katsoulis, Elias; East Carolina University, U.S.
 Khalkhali, Masoud; University of Western Ontario
 Laca, Marcelo; University of Victoria
 Larsen, Nadia; University of Oslo, Norway
 Li, Xin; Queen Mary University, London, U.K.
 Phillips, John; University of Victoria
 Raeburn, Iain; University of Otago, New Zealand

Rennie, Adam; University of Wollongong, Australia
 Sims, Aidan; University of Wollongong, Australia
 Stammeier, Nicolai; University of Muenster, Germany
 Trifkovic, Mak; University of Victoria

SCHEDULE

Sunday

20:00 Informal gathering in 2nd floor lounge, Corbett Hall
 Discussions on the program of lectures.

Monday

8:45–9:00 Introduction and Welcome by BIRS Station Manager, TCPL 201
 9:00–10:00 Joachim Cuntz: *Periodic cyclic homology in positive characteristic / An alternative to Witt vectors*
 10:00–10:30 Coffee Break, TCPL
 10:30–11:30 Siegfried Echterhoff: *K-theory of crossed products by group actions on totally disconnected spaces and semi-group algebras*
 11:30–13:00 Lunch
 14:00–15:00 Astrid an Huef: *KMS states on the C*-algebras associated to finite graphs*
 15:00–15:30 Coffee Break, TCPL
 15:30–17:30 Informal discussion, TCPL
 17:30–19:30 Dinner
 19:30– Informal discussion, TCPL / Corbett Hall lounge

Tuesday

9:00–10:00 Gunther Cornelissen: *Noncommutative dynamical systems and point counting*
 10:00–10:30 Coffee Break, TCPL
 10:30–11:30 Elias Katsoulis: *Piecewise conjugacy as an isomorphism invariant for operator algebras*
 11:30 Group Photo, on the steps outside TCPL (meet in the foyer).
 11:40–13:30 Lunch
 14:00–15:00 Aidan Sims: *KMS states on C*-algebras associated to k-graphs*
 15:00–15:30 Coffee Break, TCPL
 15:30–16:00 Joachim Cuntz: *An alternative to Witt vectors*
 16:00–17:30 Informal discussion
 17:30–19:30 Dinner
 19:30– Informal discussion, TCPL / Corbett Hall lounge

Wednesday

9:00–10:00 Iain Raeburn: *Operator-algebraic dynamical systems associated to self-similar groups.*
 10:00–10:30 Coffee Break, TCPL
 10:30–11:30 Xin Li: *C*-algebras, monoids, and dynamical systems*
 11:30–13:30 Lunch
 Excursion
 17:30–19:30 Dinner
 19:30– Informal discussion, TCPL / Corbett Hall lounge

Thursday

- 9:00–10:00** Nadia Larsen: *C*-algebras associated to graphs, path spaces and equilibrium states*
- 10:00–10:30** Coffee Break, TCPL
- 10:30–11:30** Mak Trifkovic: *Bost-Connes systems and induction*
- 11:30–13:30** Lunch
- 14:00–15:00** Masoud Khalkhali: *A noncommutative view of zeta regularized determinants and analytic torsion*
- 15:00–15:30** Coffee Break, TCPL
- 15:30–16:30** Nicolai Stammeier: *C*-algebras associated to irreversible algebraic dynamics*
- 17:30–19:30** Dinner
- 19:30–** Informal discussion, TCPL / Corbett Hall lounge

Friday

- 7:00–9:00** Breakfast
- 9:00–10:00** Nathan Brownlowe: *Zappa-Szep products of semigroups and their C*-algebras*
- 10:00–10:30** Coffee Break, TCPL
- 11:30–13:30** Lunch

ABSTRACTS

(in alphabetic order by speaker surname)

Speaker: **Astrid an Huef** (University of Otago)Title: *KMS states on the C*-algebras associated to finite graphs.*

Abstract: In 1984, Enomoto, Fujii and Watatani proved that if E is a strongly connected finite directed graph which is not a simple cycle, then the associated graph C^* -algebra admits a unique KMS state for the gauge action. I will give an introduction to KMS states on the C^* -algebras associated to finite graphs by proving the Enomoto-Fujii-Watatani Theorem using elementary methods. The main idea is the philosophy of Exel-Laca and Laca-Neshveyev that the Toeplitz algebra of the graph has a much richer supply of KMS states. This is joint work with Marcelo Laca, Iain Raeburn and Aidan Sims.

Speaker: **Nathan Brownlowe** (University of Wollongong)Title: *Zappa-Szep products of semigroups and their C*-algebras.*

Abstract: We examine a class of Zappa-Szep products of semigroups which generalise both the self-similar actions of Nekrashevych and the quasi-lattice ordered groups of Nica. We consider the C^* -algebras of these products in the sense of Li, and we give an alternative presentation involving isometric representations of the semigroups. We discuss examples including self-similar actions of groups and semigroups, the semigroups $\mathbb{N} \rtimes \mathbb{N}^\times$ and $\mathbb{Z} \rtimes \mathbb{Z}^\times$, the Baumslag-Solitar groups, and products of self-similar groups. We also define a quotient C^* -algebra we call the boundary quotient. This is joint work with Jacqui Ramagge, David Robertson and Michael Whittaker.

Speaker: **Gunther Cornelissen** (Utrecht University)Title: *Noncommutative dynamical systems and point counting*

Abstract: Counting the points of a curve over finite fields doesn't determine the curve up to isomorphism; this is analogous to the famous fact that you cannot "hear the shape of a drum". I will show how to remedy this by counting points in a weighted way (namely, using characters). This weighted point counting arises naturally from considering the KMS-states of a noncommutative dynamical system associated to the curve.

Speaker: **Joachim Cuntz** (University of Münster)Title: *Periodic cyclic homology in positive characteristic / An alternative to Witt vectors*

Abstract: The ring of Witt vectors associated to a ring R is a classical tool in algebra. We give a construction which is much simpler than the usual one. This is also of interest in view of a possible version of periodic cyclic homology for algebras over finite fields. This is joint work with Christopher Deninger.

Speaker: **Siegfried Echterhoff** (University of Münster)

Title: *K-theory of crossed products by group actions on totally disconnected spaces and semi-group algebras*

Abstract: In this lecture we give a report on joint work with Joachim Cuntz and Xin Li on the computation of the K-theory for crossed products by certain actions of groups on totally disconnected spaces. We apply the results to the computation of the K-theory for certain semi-group C*-algebras. In particular, we obtain explicit computations for the $ax + b$ -semigroups $R \rtimes R^*$, where R is the ring of integers in a number field.

Speaker: **Elias Katsoulis** (East Carolina University)

Title: *Piecewise conjugacy as an isomorphism invariant for operator algebras*

Abstract: The concept of piecewise conjugacy for classical multivariable dynamical systems arose from the work of Davidson and Katsoulis on non-selfadjoint operator algebras and was used recently by Cornelissen and Marcolli. In this talk we will review several results from the original work of Davidson and Katsoulis on piecewise conjugacy for classical dynamical systems. We will also present a recent generalization of piecewise conjugacy for multivariable C*-dynamical systems, due to Kakariadis and Katsoulis. We will show that this "generalized" piecewise conjugacy continues to be an isomorphism invariant for certain operator algebras associated with such systems. We will also discuss the completeness of piecewise conjugacy as an isomorphism invariant.

Speaker: **Masoud Khalkhali** (University of Western Ontario)

Title: *A noncommutative view of zeta regularized determinants and analytic torsion*

Abstract: I shall first recall the classical theory of Ray-Singer analytic torsion, and conformal anomaly, for families of elliptic operators. I will mostly focus on families of elliptic operators on Riemann surfaces. The methods used here are based on ideas of spectral geometry and hence stand a chance of extension to a noncommutative setting. The extensions, when possible, are however quite nontrivial and involve many new elements and difficult computations. I shall then look at some known examples of noncommutative Riemann surfaces, the noncommutative elliptic curves equipped with curved metrics, and sketch the progress made in the last few years in understanding their conformal and spectral geometry. Scalar curvature can be defined by study of special values of spectral zeta functions. In particular I shall explain a formula for scalar curvature obtained in my joint work with F. Fathizadeh (and independently by Connes and Moscovici). This formula plays an important role for further study of noncommutative spectral geometry of noncommutative tori.

Speaker: **Nadia Larsen** (University of Oslo)

Title: *C*-algebras associated to graphs, path spaces and equilibrium states*

Abstract: Given a directed graph E , the graph algebra is the universal C*-algebra for a generating family of partial isometries and projections satisfying the Cuntz-Krieger relations. There are other C*-algebras associated to E , such as the Toeplitz graph algebra. We view graph C*-algebras as crossed products where the free group on the edge set acts by partial homeomorphisms on a space of boundary paths of the graph. For a function on the edge set that induces a time evolution on a graph C*-algebra, equilibrium (or KMS) states therefore can be characterised using a general theorem of Exel and Laca. Guided by their work on KMS states for Toeplitz-Cuntz-Krieger type algebras associated to infinite matrices, we describe in case of graph C*-algebras the convex sets of KMS states of finite type and of KMS states of infinite type whose associated measures are supported on recurrent infinite paths. This is joint work with Toke M. Carlsen (NTNU, Norway).

Speaker: **Xin Li** (Queen Mary University)

Title: *C*-algebras, monoids, and dynamical systems*

Abstract: This talk is about semigroup C*-algebras on the one hand, and the interplay between C*-algebras and dynamical systems on the other hand. I will highlight the connections between these topics and the themes of the workshop.

Speaker: **Iain Raeburn** (University of Otago)

Title: *Operator-algebraic dynamical systems associated to self-similar groups.*

Abstract: We discuss the notion of a self-similar group action, illustrating with examples from dynamics. Nekrashevych has shown that each such action has a Cuntz-Pimsner algebra, and hence it has a Toeplitz algebra too. Both algebras carry natural dynamics (that is, actions of the real line), and hence it makes sense

to study the KMS states for these dynamics. We will discuss joint work on this topic carried out with Marcelo Laca, Jacqui Ramagge and Mike Whittaker.

Speaker: **Aidan Sims** (University of Wollongong)

Title: *KMS states on C^* -algebras associated to k -graphs*

Abstract: The k -graphs introduced by Kumjian and Pask in 2000 are k -dimensional analogues of directed graphs. The C^* -algebra of a k -graph is generated by copies of the Cuntz-Krieger algebras of its coordinate graphs subject to commutation relations encoded by its path structure; the Toeplitz algebra of the k -graph is assembled from the Toeplitz algebras of the coordinate graphs in a similar fashion. I will describe the KMS states for 1-parameter subgroups of the gauge action on the Toeplitz algebra of a finite k -graph, including an analogue of Enomoto, Fujii and Watatani's theorem that the Cuntz-Krieger algebra of an irreducible directed graph admits a unique KMS state.

Speaker: **Nicolai Stammeier** (University of Münster)

Title: *C^* -algebras associated to irreversible algebraic dynamics*

Abstract: An irreversible algebraic dynamics is a triple (G, P, θ) consisting of an infinite, countable, discrete, amenable group G , a countably generated, free abelian monoid P together with an action θ of P on G by injective group endomorphisms satisfying certain regularity assumptions. This dynamical system is studied in terms of the semidirect product $G \rtimes_{\theta} P$, which semigroup turns out to satisfy the left-Ore condition. The structure of the family of constructible right ideals of $G \rtimes_{\theta} P$ will enable us to deduce results about the full semigroup C^* -algebra $C^*(G \rtimes_{\theta} P)$ and its canonical quotient $\mathcal{O}(G \rtimes_{\theta} P)$. In this talk, we will mainly focus on $\mathcal{O}(G \rtimes_{\theta} P)$. The spectrum of its diagonal will be identified as a completion of G with respect to (P, θ) . The core of $\mathcal{O}(G \rtimes_{\theta} P)$ is shown to be simple and in the case of finite cokernels, the core is a generalized Bunce-Deddens algebra in the sense of Orfanos. This implies, that it has many regularity properties. Using these results, we will indicate how to prove that $\mathcal{O}(G \rtimes_{\theta} P)$ is a unital UCT Kirchberg algebra.

Speaker: **Mak Trifkovic** (University of Victoria)

Title: *Bost-Connes systems and induction*

Abstract: We show that the Bost-Connes system of a number field K can be obtained from a Hecke algebra naturally associated with K by an induction from the group of totally positive principal ideals to the whole group of ideals. For an extension L/K of number fields we show that the Bost-Connes system for L embeds into a system obtained from the Bost-Connes system for K by an induction from the group of ideals in K to the group of ideals in L . This gives a correspondence from the Bost-Connes system for K to that for L , and thus defines a functor from number fields to C^* -dynamical systems with equivariant correspondences as morphisms. This is joint work with Marcelo Laca and Sergey Neshveyev.