ABSTRACTS

Algebraic design theory with Hadamard matrices:
applications, current trends and future directions
BIRS Workshop 14w2199
Banff, Alberta
July 11 – 13, 2014

Permutation Coverings

Charlie Colbourn
Arizona State University, Tempe, Arizona

A permutation π covers a t-subsequence $(x_1, \ldots, x_t)$ when these t elements appear in the specified order in π. A *genus* of t-subsequences is simply a set of t-subsequences; a permutation covers the genus when it covers at least one of the subsequences in the genus. A number of permutation covering problems of the following type have been addressed: *Given a set of genera, what is the minimum number of permutations so that every genus is covered by at least one of the permutations?*

We mention two examples. If every t-subsequence on v symbols forms a genus by itself, a permutation covering is a sequence covering array. When permutations from such an array are used to test a sequential system of v events, every ordering of every t-subset of the events is tried – so if an error results from a small number of events being executed in a bad ordering, we should detect it. This is the basic idea in event sequence testing. A second example looks quite different. Let $G = (V,E)$ be a simple graph. Whenever $\{w,x\}$ and $\{y,z\}$ are disjoint edges of G, form a genus containing the eight 4-subsequences in which either w and x precede y and z, or y and z precede w and x. The minimum number of permutations needed to cover all such genera has been called the *separation dimension* of G, and shown to the the same as the minimum dimension k for which k can be represented as the intersection graph of axis-parallel boxes in $\mathbb{R}^k$. There are also connections to directed designs and to insertion-deletion codes.

In this talk we describe a general framework for permutation covering problems of this type. We outline a simple conditional expectation algorithm for constructing them, and report on some computational results.

Update on Hadamard's maximal determinant problem

William Orrick

Indiana University, Bloomington, Indiana

Hadamard's maximal determinant asks for the largest determinant of a matrix of given size with elements ± 1. I will review recent progress in the field, including constructions, proofs of maximality, and bounds.

Chromatic index of Steiner triple systems and Latin squares

Ian Wanless

Monash University, Clayton, Victoria, Australia

Steiner triple systems and Latin squares can both be written as sets of triples. It then makes sense to try to colour the triples so that triples that intersect get different colours. The chromatic index is the minimum number of colours that makes this possible. The chromatic index captures important combinatorial information, such as whether a latin square has a mate, or whether an STS has a resolution. I will survey results in the area, including some recent progress in constructing examples with larger chromatic index than normal.

On some special classes of Hadamard matrices

Vladimir Tonchev

Michigan Technological University, Houghton, Michigan

This talk surveys some open problems and results concerning special types of Hadamard matrices such as Bush type matrices, Hadamard difference sets, generalized Hadamard matrices over groups, and related combinatorial designs and codes.

Parity of transversals in latin squares

Darcy Best
Monash University, Clayton, Victoria, Australia

A transversal of a latin square of order n is a set of n entries which has exactly one representative from each row, column and symbol. A long standing conjecture of Ryser states that the number of transversals in a latin square is congruent to the order of the latin square modulo 2. In 1990, Balasubramanian confirmed this conjecture to be true in the even orders. In this talk, we extend this proof to show that in squares of order 2 mod 4, the number of transversals is necessarily a multiple of 4. Moreover, the set of transversals may be further broken down into smaller classes which also contain a special property modulo 2.

The Smith and Critical Groups of a Graph

Qing Xiang
University of Delaware, Newark, Delaware

Let G be a finite graph and A its adjacency matrix. The Laplacian matrix of G is defined by $L := D - A$, where D is the diagonal matrix of degrees. Associated with G are two abelian groups. The first is the Smith group $S(G)$ and the second the critical group $K(G)$. We will talk about these groups, with emphasis on the critical group. In particular, we will discuss the recent computations of the Smith and critical group of the Paley graph (in joint work with David Chandler and Peter Sin) using representation theory and number theory.

Hadamard matrices, PBDs and compressed sensing

Padraig Ó Catháin
University of Queensland, Brisbane, Queensland

In this talk we explore a construction for compressed sensing matrices which uses pairwise balanced designs and complex Hadamard matrices. We will describe how the structure of the CHMs influences the performance of the compressed sensing matrix, and derive a condition on linear combinations of rows of the CHMs which implies optimality for our construction. It is easily seen that the Fourier matrices of prime order are optimal, but we know of no other examples. We will also describe a new construction for pairwise balanced designs in which the number of blocks of each size is specified.

Some thoughts on permanents of Sylvester Hadamard matrices and Boolean functions

José Andrés Armario
Universidad de Sevilla, Seville, Spain

It is well-known that the evaluation of the permanent of an arbitrary $(-1,1)$-matrix is a formidable problem. Ryser's formula is one of the fastest known general algorithms for computing permanents. In this talk, we will show that Ryser's formula admits to be rewritten for the special case of Sylvester Hadamard matrices by using their cocyclic construction. A connection between this formula and the Walsh spectrum of Boolean functions will be explored and the effect of Boolean functions in m variables with small nonlinearity on the permanent of a Sylvester Hadamard matrix of order 2^m investigated.

A two-fold cover of strongly regular graphs with spreads and association schemes of class five

Sho Suda
Aichi University of Education, Kariya, Aichi, Japan

A spread of strongly regular graphs is a partition of the vertex set by Delsarte cliques. We consider imprimitive association schemes of class four which are two-fold covers of strongly regular graphs with spreads. It will be shown that a two-fold cover of a strongly regular graph with a spread provides a five class fission scheme of the imprimitive scheme of class four. Also we will give an example, which is obtained from mutually unbiased weighing matrices.

Circulant Generalized Weighing Matrices

Robert Craigen
University of Manitoba, Winnipeg, Manitoba

I will be speaking about some clever constructions for circulant and group-developed generalized weighing matrices in a paper I cowrote with Warwick de Launey before he died, but which still has not been published.

The weighing matrices of weight 9

Ferenc Szöllősi
Tohoku University, Sendai, Japan

We construct new examples of symmetric and skew-symmetric weighing matrices of weight 9 by means of numerical methods. In particular, we show that there exist a symmetric $W(14,9)$, $W(19,9)$ and $W(21,9)$, and there exists a skew $W(18,9)$. This settles all remaining open cases of weighing matrices of weigh 9.

This is joint work with Akihiro Munemasa.

Complex Hadamard matrices and almost Hadamard matrices

Karol Zyczkowski
Jagiellonian University, Kraków, Poland

Group invariant Hadamard matrices and Lander's conjecture

Bernhard Schmidt
Nanyang Technological University, Singapore

A $(\mathbf{v},\mathbf{k},\lambda,\mathbf{n})$**-difference set** in a finite group G of order v is a k-subset D of G such that every element $g \neq 1$ of G has exactly λ representations $g = d_1 d_2^{-1}$ with $d_1, d_2 \in D$. The positive integer $n = k - \lambda$ is called the **order** of the difference set. In his 1983 book *Symmetric Designs: An Algebraic Approach* E. S. Lander stated the following.

Conjecture: Let G be an abelian group of order v containing a difference set of order n. If p is a prime dividing v and n, then the Sylow p-subgroup of G cannot be cyclic.

Let G be a group of order v. We say that a $v \times v$ matrix $H = (h_{f,g})_{f,g \in G}$, indexed with the elements of G, is **G-invariant** if $h_{fk,gk} = h_{f,g}$ for all $f, g, k \in G$. It turns out that a G-invariant Hadamard matrix of order v is equivalent to a (v,k,λ,n)-difference set in G with $v = 4u^2$, $k = 2u^2 - u$, $\lambda = u^2 - u$, $n = u^2$ for some u.

Suppose G is any abelian group of order $4u^2$ which has a cyclic Sylow p-subgroup for some odd prime p. Then, by the above fact, any G-invariant Hadamard matrix would produce a counterexample to Lander's conjecture. We discuss an attempt to construct counterexamples to Lander's conjecture of this kind.