

Global/Local Conjectures in Representation Theory of Finite Groups (14w5024)

March 16 – March 21, 2014

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday

*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday

*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday

Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)

***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in the lecture theater in the TransCanada Pipelines Pavilion (TCPL). An LCD projector, a laptop, a document camera, and blackboards are available for presentations.

SCHEDULE

Sunday

16:00 Check-in begins (Front Desk - Professional Development Centre - open 24 hours)

17:30–19:30 Buffet Dinner, Sally Borden Building

20:00 Informal gathering in 2nd floor lounge, Corbett Hall (if desired)

Beverages and a small assortment of snacks are available on a cash honor system.

Monday

7:00–8:45 *Breakfast*

8:45–9:00 Introduction and Welcome by BIRS Station Manager, TCPL

9:00–9:50 **M. Cabanes**, Inductive McKay conditions for finite simple groups of Lie type A

10:00–10:30 *Coffee Break, TCPL*

10:30–11:10 **S. Koshitani**, The inductive Alperin-McKay and blockwise Alperin weight conditions for cyclic defect blocks

11:30–13:00 *Lunch*

13:00–14:00 *Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall*

14:00 *Group Photo; meet in foyer of TCPL (photograph will be taken outdoors so a jacket might be required).*

14:10–15:00 **M. Linckelmann**, A characterisation of nilpotent blocks

15:00–15:30 *Coffee Break, TCPL*

15:30–16:00 **C. Vallejo**, McKay natural correspondences of characters

16:20–16:50 **J.-B. Gramain**, Perfect isometries and Murnaghan-Nakayama Rules

17:30–19:30 *Dinner*

Tuesday

- 7:00–9:00 *Breakfast*
9:00–9:50 **D. Craven**, Broué’s conjecture: Brauer trees and beyond
10:00–10:30 *Coffee Break, TCPL*
10:30–11:00 **J. Müller**, The Abelian Defect Group Conjecture for (certain) sporadic simple groups
11:20–11:50 **B. Sambale**, The Alperin-McKay Conjecture for a special class of defect groups
12:00–13:30 *Lunch*
14:20–15:00 **B. Srinivasan**, The BMM global-local bijection for $GL(n, q)$
15:00–15:30 *Coffee Break, TCPL*
15:30–16:00 **M. Geline**, Vertices, sources, and the Brauer-Feit bound on character heights
16:20–17:00 **R. Boltje**, p -permutation equivalences between blocks
17:30–19:30 *Dinner*

Wednesday

- 7:00–9:00 *Breakfast*
9:00–9:50 **O. Dudas**, Decomposition numbers via Deligne-Lusztig theory
10:00–10:30 *Coffee Break, TCPL*
10:30–11:10 **G. Hiss**, Harish-Chandra series in unitary groups and crystal graphs
12:00–13:30 *Lunch*
Free Afternoon
17:30–19:30 *Dinner*

Thursday

- 7:00–9:00 *Breakfast*
9:00–9:50 **B. Elias**, Soergel bimodules and the p -canonical basis
10:00–10:30 *Coffee Break, TCPL*
10:30–11:00 **A. Schaeffer Fry**, Self-normalizing Sylow subgroups and Galois automorphisms
11:20–11:50 **J. Taylor**, Character Sheaves and GGGRs
12:00–13:30 *Lunch*
14:20–15:00 **A. Turull**, Generalizations and reductions for Jordan’s theorem
15:00–15:30 *Coffee Break, TCPL*
15:30–16:00 **O. Brunat**, On characters of positive height in blocks of finite quasi-simple groups
16:20–17:00 Question and Discussion Session
17:30–19:30 *Dinner*

Friday

- 7:00–9:00 *Breakfast*
9:00–9:40 **A. Evseev**, Wreath products and graded RoCK blocks of symmetric groups
10:00–10:30 *Coffee Break, TCPL*
10:30–11:10 **M. Fayers**, Representations of the alternating group that remain irreducible in characteristic p
11:30–13:30 *Lunch*
Checkout by **12 noon.**

** 5-day workshop participants are welcome to use BIRS facilities (BIRS Coffee Lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout by 12 noon. **

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ABSTRACTS

(in alphabetic order by speaker surname)

Speaker: **Robert Boltje** (UC Santa Cruz)

Title: *p*-permutation equivalences between blocks

Abstract: This is joint work with Philipp Perepelitsky. Let F be an algebraically closed field of characteristic $p > 0$. For blocks A and B of group algebras FG and FH , respectively, we define a *p*-permutation equivalence to be an element $\gamma \in T^\Delta(A, B)$ satisfying $\gamma \cdot_H \gamma^\circ = [A] \in T^\Delta(A, A)$ and $\gamma^\circ \cdot_G \gamma = [B] \in T^\Delta(B, B)$. Here, $T^\Delta(A, B)$ denotes the free abelian group on isomorphism classes of indecomposable *p*-permutation (A, B) -bimodules with twisted diagonal vertices, $\gamma^\circ \in T^\Delta(B, A)$ denotes the F -dual of γ and \cdot_H is induced by \otimes_{FH} . This notion of equivalence is motivated by Broué's abelian defect group conjecture and splendid Rickard equivalences.

We show that a *p*-permutation equivalence $\gamma \in T^\Delta(A, B)$ preserves important invariants of the blocks A and B : It induces an isomorphism between defect groups and fusion systems; it induces *p*-permutation equivalences on the local levels and an isotopy between A and B ; it induces Morita equivalences between the Brauer correspondents of A and B ; it preserves the 2-cohomology classes on the inertia groups and also the Külshammer-Puig classes for corresponding centric subgroups of the defect groups.

We determine restrictive properties of the shape of a *p*-permutation equivalence, show that the group of *p*-permutation auto-equivalences of A is finite, and determine its structure in very simple cases.

Speaker: **Olivier Brunat** (Univ. Paris 7)

Title: *On characters of positive height in blocks of finite quasi-simple groups*

Abstract: This talk is concerned with a conjecture of Eaton and Moretó saying that, whenever the defect group of a block is non-abelian, the smallest non-zero height in the block and in its defect group are the same. We check this conjecture for all principal 2-blocks of finite quasi-simple groups, for all blocks of groups of Lie type when the prime is the defining characteristic and for all covering groups of symmetric and alternating groups. This is joint work with Gunter Malle.

Speaker: **Marc Cabanes** (Univ. Paris 7)

Title: *Inductive McKay conditions for finite simple groups of Lie type A*

Abstract: Isaacs-Malle-Navarro (2007) have shown that McKay's conjecture on character degrees of finite groups reduces to a so-called inductive condition to be satisfied by all finite simple groups. The purpose of the talk is to give the methods used in the verification of the inductive McKay condition for simple groups of type A (projective special linear or unitary groups over finite fields). Building on this, we also show how one can verify the more involved conditions on the inductive Alperin-McKay conditions. Thereby we give a criterion that is strongly adapted to those groups. In the end I point out the difficulties in generalizing this result to other blocks. This is joint work with B. Späth.

Speaker: **David Craven** (Univ. Birmingham)

Title: *Broué's conjecture: Brauer trees and beyond*

Abstract: with the completion of the classification of Brauer trees for unipotent blocks of groups of Lie type, we now move to what happens in the non-cyclic defect group case. In this talk I will discuss the techniques needed to solve the Brauer tree case, which of these techniques can extend, and how the derived category of a unipotent ℓ -block of $G(q)$ is independent not only of q , but also of ℓ .

Speaker: **Olivier Dudas** (Univ. Paris 7)

Title: *Decomposition numbers via Deligne-Lusztig theory*

Abstract: Joint work with Gunter Malle. I will report on new techniques to compute decomposition numbers for finite groups of Lie type in non-defining characteristic. These techniques are based on modular analogues of Lusztig’s results on the cohomology of Deligne-Lusztig varieties, some of which are still conjectural. Potential applications of these techniques include

- a definition of families for representation in positive characteristic
- unitriangular shape of the decomposition matrix.
- bounds on the number of (Morita classes of) unipotent blocks depending only on the rank of the underlying algebraic group.

Speaker: **Ben Elias** (MIT)

Title: *Soergel bimodules and the p -canonical basis*

Abstract: Fix a semisimple algebraic group G . In 1990, Soergel introduced a monoidal category SBim of bimodules over a polynomial ring, which encapsulates category \mathcal{O} for the complex semisimple Lie algebra attached to G . In 2000, he showed that this same monoidal category, except defined over a field k of finite characteristic, encapsulates the rational representation theory of the algebraic group $G(k)$ “around the Steinberg weight.” Both the characteristic zero and characteristic p versions have Grothendieck group isomorphic to the Hecke algebra H of the Weyl group, and the symbol of an object, when expressed in the standard basis of H , encodes certain multiplicities. In characteristic zero, Geordie Williamson and I have recently (re)proven the Soergel conjecture, which states that the symbols of indecomposable bimodules yield Kazhdan-Lusztig’s canonical basis of H . In characteristic p , the symbols of indecomposable bimodules yield a basis called the p -canonical basis; its computation is an interesting and difficult open problem.

In recent work with Geordie Williamson, extending earlier work of Libedinsky and Elias-Khovanov, we have a presentation of (a monoidal subcategory of) SBim by generators and relations. This allows for a direct algebraic approach to the computation of the p -canonical basis, and approach which has already led to Williamson’s recent disproof of the Lusztig conjecture. Our goal in this talk is to give a gentle introduction to Soergel bimodules, explain what they have to do with modular representation theory, and describe this direct approach.

Speaker: **Anton Evseev** (Univ. Birmingham)

Title: *Wreath products and graded RoCK blocks of symmetric groups*

Abstract. An important step in the proof of the Broué abelian defect group conjecture for symmetric groups in characteristic p is a theorem of Chuang and Kessar establishing a Morita equivalence between the wreath product of the principal block of the symmetric group \mathfrak{S}_p with \mathfrak{S}_w and a certain special block of weight w of a symmetric group, called RoCK block (or Rouquier block), for $w < p$. In an effort to generalise this result to the case $w \geq p$ (i.e. to blocks of non-abelian defect), Turner conjectured that in this general case the same wreath product is Morita equivalent to a certain idempotent truncation of the RoCK block. The talk will outline a proof of Turner’s conjecture, which also is a new (and more explicit) proof of the Chuang–Kessar theorem. The proof is based on an isomorphism discovered by Brundan and Kleshchev between the group algebra of a symmetric group and a certain cyclotomic Khovanov–Lauda–Rouquier (KLR) algebra. It involves computations with KLR generators and uses the grading of KLR algebras. In fact, the Morita equivalence in Turner’s conjecture is shown to be that of graded algebras.

Speaker: **Matthew Feyers** (Queen Mary University London)

Title: *Representations of the alternating group that remain irreducible in characteristic p*

Abstract: Given any finite group G and any prime p , it is interesting to ask which ordinary irreducible representations of G remain irreducible in characteristic p . This question was solved for the symmetric groups several years ago, by the speaker and others. Here we address the case of the alternating group when p is odd. We’ll translate the question to one about representations of the symmetric group, and then explain how to answer it.

Speaker: **Michael Geline** (Northern Illinois University)

Title: *Vertices, sources, and the Brauer Feit bound on character heights*

Abstract: Brauer and Feit proved that the irreducible characters in a block of defect $d \geq 2$ have height at most $d-2$. Their argument used Brauer's "subsections." It turns out the same result can be obtained using vertices and sources. Unfortunately, though, the argument only seems to apply to irreducible characters whose values generate a tamely ramified extension of the p -adics.

Speaker: **Jean-Baptiste Gramain** (University of Aberdeen)

Title: *Perfect isometries and Murnaghan-Nakayama Rules*

Abstract: Broué's perfect isometries are the shadow at the level of characters of derived equivalences between blocks of finite groups. In 1988, Michel Enguehard proved that two blocks of symmetric groups with the same weight are perfectly isometric. In this talk, we generalize this result to many families of groups, as alternating groups, double covers of symmetric and alternating groups, Weyl groups of types B and D , and certain wreath products. This is joint work with Olivier Brunat.

Speaker: **Gerhard Hiss** (RWTH Aachen)

Title: *Harish-Chandra series in unitary groups and crystal graphs*

Abstract: Harish-Chandra series of Brauer characters in groups of Lie type have been introduced more than 20 years ago. By work of Geck, the unipotent Brauer characters of the finite unitary groups $\mathrm{GU}_n(q)$ are labelled by partitions of n . The question of describing the division of the unipotent Brauer characters of the unitary groups into Harish-Chandra series is still open.

We present a series of conjectures relating this division with crystal graphs of certain integrable highest weight modules of the quantum group corresponding to a suitable affine Lie algebra of type A , and give some evidence for these conjectures.

Speaker: **Shigeo Koshitani** (Chiba University)

Title: *The inductive Alperin-McKay and blockwise Alperin weight conditions for cyclic defect blocks*

Abstract: We will be discussing the inductive Alperin-McKay condition and blockwise Alperin weight condition, for blocks with cyclic defect groups. The work [Isaacs-Malle-Navarro, Inv.Math. 2007] convinces us that similar reduction procedure should work for the other cases and even for the other conjectures. Actually a similar treatment is given in an interesting paper [Navarro-Tiep, Inv.Math. 2011] which treats with the non-blockwise version of the Alperin weight conjecture. Späth gets blockwise reduction conditions for the two conjectures. We have been able to prove that at least for any block B with cyclic defect groups, B satisfies the reduction conditions due to Späth. This is joint work with Britta Späth.

Speaker: **Markus Linckelmann** (City University London)

Title: *A characterisation of nilpotent blocks*

Abstract: This is joint work with R. Kessar and G. Navarro. The starting point is a result by Isaacs, showing that a finite group G is p -nilpotent if and only if the highest power of p dividing the sum of the squares of the characters of degree prime to p is equal to the index of the p -focal subgroup in a Sylow p -subgroup of G .

We show that this statement admits a blockwise version. The proof involves the hyperfocal subalgebra of a block. We state a variation of Brauer's height zero conjecture for hyperfocal subalgebras.

Speaker: **Jürgen Müller** (Friedrich-Schiller-Universität Jena)

Title: *The Abelian Defect Group Conjecture for (certain) sporadic simple groups*

Abstract: Let G be a finite group, let A be a prime block of G having an abelian defect group P , let N be the normaliser in G of P , and let B be the Brauer correspondent of A . Then the Abelian Defect Group Conjecture (in various forms due to M. Broué and J. Rickard) says that the bounded derived categories of the module categories of A and B are equivalent as triangulated categories. Although being in the focus of intensive study since two decades now, and considerable progress has been made, a general proof seems to be out of sight.

A possible strategy is a reduction to the simple (and closely related) groups. Hence it seems worthwhile to tackle these groups first. In view of this, in this talk we show how a combination of theoretical strategies and techniques from computational representation theory can be pursued to prove the Abelian Defect Group Conjecture for some of the sporadic simple groups. This is joint work with Shigeo Koshitani (Chiba) and Felix Noeske (Aachen).

Speaker: **Benjamin Sambale** (Friedrich-Schiller-Universität Jena)

Title: *The Alperin–McKay Conjecture for a special class of defect groups*

Abstract: After summarizing the results on blocks with metacyclic defect groups, I present a new result which states that the Alperin–McKay Conjecture holds for blocks with metacyclic, minimal non-abelian defect groups. These are precisely the metacyclic groups whose derived subgroup have order p . In the special case $p = 3$, I also verify Alperin’s Weight Conjecture for these defect groups. The proofs do *not* rely on the classification of the finite simple groups.

Speaker: **Amanda Schaeffer Fry** (Michigan State University)

Title: *Self-normalizing Sylow subgroups and Galois automorphisms*

Abstract: Let G be a finite group, p a prime, and P a Sylow p -subgroup of G . A recent refinement, due to G. Navarro, of the McKay conjecture suggests that there should exist a bijection between irreducible characters of p' -degree of G and irreducible characters of p' -degree of $N_G(P)$ which commutes with certain Galois automorphisms. This Galois automorphism refinement of the McKay conjecture has several interesting consequences. I will discuss my progress regarding one of these consequences, namely a way to read off from the character table of G whether a Sylow 2-subgroup of G is self-normalizing. In particular, I will discuss the reduction of the problem to simple groups.

Speaker: **Bhama Srinivasan** (University of Illinois at Chicago)

Title: *The BMM global-local bijection for $GL(n, q)$*

Abstract: Broué, Malle and Michel gave a bijection between characters in a block of a general linear or classical group on one hand, and characters of a corresponding complex reflection group on the other hand. In the first part of the talk we give a description of that connection, as arising from two equivalent representations of a Heisenberg algebra acting on a Fock space. In the second part of the talk we make some remarks on the ℓ -decomposition numbers of $GL(n, q)$ for large ℓ , again using the Fock space.

Speaker: **Jonathan Taylor** (TU Kaiserslautern)

Title: *Character Sheaves and GGGRs*

Abstract: In 1985 Lusztig introduced an interesting class of perverse sheaves on a connected reductive algebraic group G known as character sheaves. If $F : G \rightarrow G$ is a Frobenius endomorphism then each F -stable character sheaf admits a class function of the finite group G^F . Determining the values of these class functions is of central importance to the determination of the ordinary character table of the finite group G^F . In this talk we present a recent result which gives the restriction of each such class function, associated to a character sheaf, to the set of unipotent elements. We will then describe how this fits into a larger goal of understanding Kawanaka’s generalised Gelfand-Graev representations and the decomposition numbers of G^F in non-defining characteristic.

Speaker: **Alexandre Turull** (University of Florida)

Title: *Generalizations and reductions for Jordan’s theorem*

Abstract: A celebrated theorem of C. Jordan states the following. For any d there exists a constant C such that any finite subgroup G of $GL(d, \mathbb{C})$ has an abelian subgroup $A \leq G$ of index at most C , and A can be generated by at most d elements. While the theorem is about finite linear groups, a number of analogues of it have been proposed, where the new statements concern finite subgroups of other important infinite groups. In this talk, we discuss some of these conjectures and results. Our focus is joint work with Ignasi Mundet i Riera where these conjectures are reduced to simpler questions. These joint results are used by Mundet to prove a celebrated conjecture of Étienne Ghys for manifolds without odd cohomology.

Speaker: **Carolina Vallejo** (Universidad de Valencia)

Title: *McKay natural correspondences of characters*

Abstract: If $p > 2$, G is a finite group, $P \in \text{Syl}_p(G)$ and $N_G(P) = PC_G(P)$, then we prove that there is a natural correspondence between the irreducible characters of G of degree not divisible by p in the principal block and those of $N_G(P)$. (Joint work with G. Navarro and P. H. Tiep).