

Complex Monge-Ampère equations on Compact Kähler manifolds

BIRS, 06-11.04.2014

Sunday 6th April 2014

- 16:00-: Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
- 17:30-19:30: Buffet Dinner, Sally Borden Building
- 20:00-: Informal gathering in 2nd floor lounge, Corbett Hall. Beverages and a small assortment of snacks are available on a cash honor system.

Monday 7th April 2014

- 9:00-9:10: Welcome of participants
- 9:10-10:00: V.Tosatti (Northwestern University, USA)
Collapsing of Ricci-flat Calabi-Yau manifolds.
- Coffee break [10:00-10:30]
- 10:30-11:20: J.Song (Rutgers University, USA)
Riemannian geometry of Kähler-Einstein currents.
- Lunch [11:30-13:15]
- 13:15-14:00: Guided Tour of The Banff Centre

- 14:00-14:10: Group Photo
- 14:10-15:00: X.Zhu (Beijing University, China)
Compactness of almost Kaehler-Ricci solitons.
- Coffee break [15:00-15:30]
- 15:30-16:20: D.Witt-Nyström (Cambridge, UK)
Homogeneous Monge-Ampère Equations and Canonical Tubular Neighbourhoods in Kähler Geometry.
- 16:30-17:20: A.Zeriahi (Toulouse University, France)
Weak solutions to degenerate complex Monge-Ampère flows.
- Dinner [17:30-19:30]

Tuesday 8th April 2014

- 9:10-10:00: F.Campana (Nancy University, France)
Classification of compact Kähler non-projective threefolds.
- Coffee break [10:00-10:30]
- 10:30-11:20: T.Collins (Columbia University, USA)
The boundary of the Kähler cone.
- Lunch [11:30-13:30]
- 14:10-15:00: S.Dinew (Jagiellonian University, Poland)
Local regularity of the complex Monge-Ampère equation
- Coffee break [15:00-15:30]
- 15:30-16:20: C.Li (Stony Brook University, USA)
On critical exponents for some complex Monge-Ampère equations.
- 16:30-17:20: S.T.Paul (University of Wisconsin, USA)
Introduction to (Semi)stable Pairs and Applications.
- Dinner [17:30-19:30]

Wednesday 9th April 2014

- 9:00-9:50: G.Szekelyhidi (Notre Dame University, USA)
The J-flow and stability.
- 10:00-10:50: H.Guenancia (ENS Paris, France)
Kähler-Einstein metrics on stable varieties.
- 11:00-11:50: J.Ross (Cambridge, UK)
Partial Bergman Kernels.
- Lunch [11:50-13:30]
- Free afternoon; Dinner [17:30-19:30]

Thursday 10th April 2014

- 9:10-10:00: M.Haskins (Imperial College, London, UK)
Asymptotically cylindrical Calabi-Yau manifolds.
- Coffee break [10:00-10:30]
- 10:30-11:20: M.Paun (KIAS, Korea)
Metrics with conic singularities and applications.
- Lunch [11:30-13:30]
- 14:10-15:00: H.Auvray (ENS Cachan, France)
Complete Kähler metrics and Monge-Ampère equations on the complement of a divisor
- Coffee break [15:00-15:30]
- 15:30-16:20: H.C.Lu (CTH, Sweden)
Complex Monge-Ampère equations on quasi-projective varieties.
- 16:30-17:20: X.Wang (Rutgers University, USA)
Connecting toric manifolds by conical Kähler-Einstein metrics.
- Dinner [17:30-19:30]

Friday 11th April 2014

- 9:10-10:00: B.Weinkove (Northwestern University, USA)
Monge-Ampère equations, $(n-1, n-1)$ forms & Hermitian metrics.
- Coffee break [10:00-10:30]
- 10:30-11:20: C.Arezzo (ICTP, Italy)
Desingularizations of Kcsc orbifolds
- Lunch [11:30-13:30]
- **Check out by 12 noon.**

** Workshop participants are welcome to use BIRS facilities (BIRS Coffee Lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

Abstracts

1. Claudio AREZZO (ICTP, Italy)

Desingularizations of Kcsc orbifolds

In a joint work with L.Mazziere and R. Lena, we have completed the picture started by Rolin-Singer-Pacard and myself about blowing up and desingularizing orbifolds with isolated quotient singularities. The missing case, orbifolds with holomorphic vector fields, turns out to have a different answer depending on which groups create the singularities. In particular crepant and non-crepant resolutions give rise to completely different balancing conditions.

2. Hugues AUVRAY (ENS Cachan, France)

Complete Kähler metrics and Monge-Ampère equations on the complement of a divisor

Let D be a simple normal crossing divisor in a compact Kähler manifold X . One can always produce, relatively to any smooth Kähler class on X , Kähler metrics on $X \setminus D$ with cusp singularities along D ; the space of such metrics can in turn be endowed with a Mabuchi metric, as in the compact case. The aim of this talk is, first, to describe how one can extend to this non-compact case the resolution of a homogeneous Monge-Ampère equation on the product of $(X \setminus D)$ with a complex annulus with boundary, and therefore draw weak geodesics in the space of metrics. Secondly, we address the problem of solving Monge-Ampère equations on $(X \setminus D)$ with mildly singular right-hand side; restricting to the case when $K_X[D]$ is ample, we will explain how this leads to uniqueness statements for constant scalar curvature Kähler metrics with cusp singularities.

3. Frédéric CAMPANA (Nancy University, France)

Classification of compact Kähler non-projective threefolds.

A compact Kähler n -fold ($n \geq 2$) X is said to be ‘simple’ if it is not covered by subvarieties of any dimension d , $n > d > 0$. Usual fibrations (Albanese and Algebraic reductions) permit to reduce the structure of any such X to those which are either projective, Tori, or ‘simple’.

When $n = 2$, simple Kähler surfaces are bimeromorphic to either K3-surfaces or Tori. Conjecturally, this should remain true in arbitrary dimension (in a suitable version).

The talk will explain how the solution of the ‘Abundance conjecture’ in dimension 3 for projective threefolds can be adapted to the Kähler situation to show that ‘simple’ compact Kähler threefolds are bimeromorphic to Torus quotients T/G , and to complete the bimeromorphic classification of compact Kähler threefolds.

4. Tristan COLLINS (Columbia University, USA)

The boundary of the Kähler cone.

We will discuss a geometric characterization of classes of positive volume on the boundary of the Kähler cone of a compact Kähler manifold. As an application, we will show that finite time singularities of the Kähler-Ricci flow always form along analytic subvarieties. This work is joint with Valentino Tosatti.

5. Slawomir DINEW (Jagiellonian University, Poland)

Local regularity of the complex Monge-Ampère equation

We shall review recent results on the local regularity of the complex Monge-Ampère equation. Specifically we shall investigate lower order regularity issues, in situations where standard tools are unavailable. We shall also report on the status of several open problems in the field.

6. Henri GUENANCIA (ENS Paris, France)

Kähler-Einstein metrics on stable varieties.

I will present a joint work Robert Berman where we prove the existence of Kähler-Einstein metrics on stable varieties (which are the higher dimensional analogue of stable curves). I will also try to discuss various applications of this result.

7. Mark HASKINS (Imperial College, London, UK)

Asymptotically cylindrical Calabi-Yau manifolds.

I will discuss complete Ricci-flat Kähler manifolds that are asymptotic to cylinders at infinity. The main result roughly states that after some natural modifications every such space can be written as a compact

projective algebraic variety minus an anticanonical divisor with trivial normal bundle. The converse statement holds as well. Joint work with Hans-Joachim Hein and Johannes Nordstrom.

8. Chi LI (Stony Brook University, USA)

On critical exponents for some complex Monge-Ampère equations.

I will explain how to determine the critical exponents of some complex Monge-Ampère equations arising from Kähler-Einstein problem, both in the global and local settings. Blow-up behaviors will also be discussed.

9. Hoang Chinh LU (Chalmers Techniska Högskola, Sweden)

Complex Monge-Ampère equations on quasi-projective varieties.

We study the Calabi conjecture on quasi-projective varieties. Let (X, ω) be a compact Kähler manifold and D a divisor on X . Let f be a positive and smooth density on $X \setminus D$. Under a mild condition on f we prove that the unique solution to the Monge-Ampère equation with density f is also smooth on $X \setminus D$. This is a joint work with Eleonora Di Nezza.

10. Sean T. PAUL (University of Wisconsin, USA)

Introduction to (Semi)stable Pairs and Applications.

*Let G be a complex reductive linear algebraic group. Let V be a finite dimensional rational complex representation of G . Given L , a G invariant proper (non-zero) linear subspace of $P(V)$, and a point u of $P(V)$ not lying on L we say that u is "semistable with respect to L " provided that the Zariski closure of the orbit Gu does not meet L . This idea plays an important role in many areas of mathematics and it is the aim of this talk to discuss some of these surprising connections. In particular, the speaker will discuss how this notion of stability (when suitably specialized) is *equivalent* to the existence of a Kähler Einstein metric on any Fano manifold with finite automorphism group via Hyperdiscriminants and Resultants of projective varieties.*

11. Mihai PAUN (KIAS, Korea)

Metrics with conic singularities and applications.

We will survey a few recent results obtained in collaboration with F. Campana and H. Guenancia.

12. Julius ROSS (Cambridge, UK)

Partial Bergman Kernels.

I will discuss joint work with Michael Singer concerning asymptotics of the partial Bergman Kernel concerning sections of a positive hermitian line bundle that vanish along a submanifold, and relate this to David Witt-Nyström's talk on tubular neighbourhoods.

13. Jian SONG (Rutgers University, USA)

Riemannian geometry of Kähler-Einstein currents.

Let X be a projective Calabi-Yau variety with crepant singularities. There exists a unique Ricci flat Kähler current g constructed by Eyssidieux-Guedj-Zeriahi, smooth on X_{reg} with bounded local potentials in any polarization of X . We show that the metric completion of (X_{reg}, g) is homeomorphic to X as a projective variety with well-defined tangent cones at each singular point. We will also give some generalizations and applications.

14. Gabor SZEKELYHIDI (Notre-Dame University, USA)

The J-flow and stability.

The J-flow is a parabolic flow for Kähler metrics, introduced by Donaldson. I will discuss a new algebro-geometric stability condition, which is related conjecturally to convergence of the flow, and I will provide some evidence for this conjecture. I will present some examples due to Fang-Lai when the flow does not converge, and show how this is related to the stability condition. This is joint work with Mehdi Lejmi.

15. Valentino TOSATTI (Northwestern University, USA)

Collapsing of Ricci-flat Calabi-Yau manifolds.

I will discuss the problem of understanding how Ricci-flat Calabi-Yau manifolds collapse to lower-dimensional spaces, and how this is relevant to the Strominger-Yau-Zaslow picture of mirror symmetry. I will present some results in this direction which are joint work with Mark Gross and Yuguang Zhang.

16. Xiawei WANG (Rutgers University, USA)

Connecting toric manifolds by conical Kähler-Einstein metrics.

We give criteria for the existence of toric conical Kähler-Einstein metrics on any toric manifold in relation to the greatest Ricci lower bound. We also show that any two toric manifolds of the same dimension can be joined by a continuous path of toric manifolds with conical Kähler-Einstein metrics in the Gromov-Hausdorff topology. Based on joint work with V. Datar, B. Guo and J. Song.

17. Ben WEINKOVE (Northwestern University, USA)

Monge-Ampère equations, $(n-1, n-1)$ forms & Hermitian metrics.

A smooth function on C^n is called $(n-1)$ -plurisubharmonic in the sense of Harvey-Lawson if the sum of every $n-1$ eigenvalues of its complex Hessian is nonnegative. We show existence of smooth solutions to the associated Monge-Ampère equation on compact Kähler manifolds. As a consequence we obtain Calabi-Yau theorems for balanced and Gauduchon metrics on compact Kähler manifolds. I will also describe generalizations to Hermitian manifolds and some open problems. This is a joint work with Valentino Tosatti.

18. David WITT-NYSTRÖM (Cambridge, UK)

Homogeneous Monge-Ampère Equations and Canonical Tubular Neighbourhoods in Kähler Geometry.

I will talk about joint work with Julius Ross. We prove the existence of canonical tubular neighbourhoods around complex submanifolds of Kähler manifolds that are adapted to both the holomorphic and symplectic structure. This is done by solving the complex Homogeneous Monge-Ampère equation on the deformation to the normal cone of the submanifold. We use this to establish local regularity for global weak solutions, which in particular shows that the (weak) geodesic rays in the space of (weak) Kähler potentials associated to a given complex submanifold Y are regular near the orbit of Y . We also get an optimal regularity result for naturally defined plurisubharmonic envelopes and for the boundaries of their associated equilibrium sets.

19. Ahmed ZERIAHI (Institut Mathématique de Toulouse, France)

Weak solutions to degenerate complex Monge-Ampère flows.

Studying the (long-term) behavior of the Kähler-Ricci flow on mildly singular varieties, one is naturally lead to study weak solutions of degenerate parabolic complex Monge-Ampère equations. The purpose of this talk is to develop the first steps of a viscosity theory for degenerate complex Monge-Ampère flows on compact Kähler manifolds. This is joint work with P.Eyssidieux and V.Guedj.

20. Xiaohua ZHU (Beijing University, China)

Compactness of almost Kaehler-Ricci solitons.

We discuss the convergence of a sequence of almost Kaehler-Ricci solitons in Gromov-Hausdorff topology. We show that the limit is a normal Fano variety which admits a Kaehler-Ricci soliton on the smooth part of variety. Our result implies that a Fano manifold with the modified K-energy bounded below can be deformed to a normal Fano variety, which admits a Kaehler-Ricci soliton on the smooth part of variety.