Quantum Curves and Quantum Knot Invariants (14w5073)
Arriving Sunday, June 15 and departing Friday June 20, 2014

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in the TransCanada Pipelines Pavilion (TCPL). LCD projector and blackboards are available for presentations. Ceiling-mounted video cameras are installed in the main lecture room of 201, TCPL.

SCHEDULE

Sunday

16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
17:30–19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall Beverages and a small assortment of snacks are available on a cash honor system.

Monday

7:00–8:45	Breakfast
8:45–9:00	Introduction and Welcome by BIRS Station Manager, TCPL
9:00–10:00	Paul Norbury: Keynote Lecture I: A quantum curve - What is it? Why is it important? A mathematical introduction to quantum curves
10:00–10:30	Coffee Break, TCPL
10:30–11:30	Piotr Sułkowski: Keynote Lecture II: An introduction to quantum curves from a physicist's point of view
11:30–13:30	Lunch
13:00–13:45	(For those who wish to see the campus) Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
13:45–14:45	Stavros Garoufalidis: The Slope Conjecture for 2-fusion knots
14:45–15:15	Coffee Break, TCPL
15:15–16:15	Marko Stošić:
16:30–17:30	Gaëtan Borot: $SU(N)$ Chern-Simons in Seifert spaces at large N: a matrix model analysis
17:30–19:30	Dinner

Tuesday

7:00–9:00	Breakfast
9:00–10:00	Victor Ginzburg: Quantization of line bundles on Lagrangian subvarieties
10:00	Group Photo; meet in the lobby of TCPL (photo will be taken outside)
10:00–10:30	Coffee Break, TCPL
10:30–11:30	Lotte Hollands:
11:30–13:30	Lunch
13:30–14:30	Pavel Etingof: Cherednik algebras and torus knots
14:30–15:00	Coffee Break, TCPL
15:00–16:00	Anne Schilling: Crystal operators and flag Gromov-Witten invariants
16:30–17:30	Alexei Oblomkov: $SL(n)$-knot homology vs. HOMFLY homology via Cherednik algebras
17:30–19:30	Dinner
20:00–	Informal seminar: Conjectures on knot homology of algebraic knots, organized by Alexei Oblomkov

Wednesday

7:00–9:00	Breakfast
9:00–10:00	Tudor Dimofte: $3d$-$3d$ Correspondence and Knot Homology
10:00–10:30	Coffee Break, TCPL
10:30–11:30	Olivia Dumitrescu: Topological recursion for Hitchin fibrations of rank 2 and quantum curves
11:30–13:30	Lunch
	Free Afternoon
17:30–19:30	Dinner

Thursday

7:00–9:00	Breakfast
9:00–10:00	Katrin Wendland: An instance of mirror symmetry on $K3$
10:00–10:30	Coffee Break, TCPL
10:30–11:30	Satoshi Nawata: J-functions, integrable systems and the AGT relation
11:30–13:30	Lunch
13:30–14:30	Lenny Ng: Knot contact homology and the augmentation polynomial
14:30–15:00	Coffee Break, TCPL
15:00–16:00	Chiu-Chu Melissa Liu: Eynard-Orantin topological recursion and equivariant Gromov-Witten invariants of the projective line
16:30–17:30	Jun Murakami: Logarithmic invariant of knots

Friday

7:00–9:00	Breakfast
9:00–10:00	Hiroyuki Fuji: Super-A-polynomial
10:00–10:30	Coffee Break, TCPL
10:30–11:30	John Harnad: $2D$ Toda τ-functions as combinatorial generating functions
11:30–13:30	Lunch
Checkout by 12 noon.	

** 5-day workshop participants are welcome to use BIRS facilities (BIRS 2nd floor lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

Quantum Curves and Quantum Knot Invariants (14w5073)
June 15 – June 20, 2014

ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **Gaëtan Borot** (Max-Planck-Institut für Mathematik, Bonn)
Title: *$SU(N)$ Chern-Simons in Seifert spaces at large N: a matrix model analysis*
Abstract: The partition function of $SU(N)$ Chern-Simons theory on $M =$ Seifert space can be written as a matrix model, and the colored HOMFLY of basic knots (those going along generators of $\pi_1(M)$) as the correlators of this model. From there follows that the large N expansions satisfy the topological recursion. The task is then to compute the initial data, i.e. the spectral curve. For this purpose, we use a machinery based on pseudoreflection groups and root systems. We obtain many information on the spectral curves, which in turn have consequences for analyticity properties of (the large rank expansion of) quantum knot invariants.

This is a joint work with Bertrand Eynard.

Speaker: **Tudor Dimofte** (Institute for Advanced Study)
Title: *3d-3d Correspondence and Knot Homology*
Abstract: The compactification of K $M5$ branes on a 3-manifold M should produce a 3d supersymmetric quantum field theory $T[M]$ that is a topological invariant of M. Among many other properties, the vacua of this theory must match flat $SL(K, \mathbb{C})$ connections on M. I will review work with Gaiotto and Gukov that constructs a subsector of $T[M]$ (with vacua matching irreducible flat connections), using cluster algebra methods. Then I will explain how knot homology helps us understand how to construct a complete $T[M]$ (including all flat connections), and, conversely, how partition functions of the full $T[M]$ reproduce Poincare polynomials of colored $SL(2)$ knot homology.

Speaker: **Olivia Dumitrescu** (Leibniz Universität Hannover)
Title: *Topological recursion for Hitchin fibrations of rank 2 and quantum curves*
Abstract: In this talk I will discuss the Eynard-Orantin theory to the family of spectral curves of Hitchin fibrations over a smooth base curve C of genus at least 2. We study rank 2 holomorphic Higgs field with traceless spectral data and we apply the Eynard-Orantin recursion to quantize its spectral curve.

We generalize this construction to meromorphic Higgs fields and spectral curve in the compactified cotangent bundle. We present as simple examples of our theory, some well-known classical equations (Catalan function, Airy function) as spectral curves of meromorphic Higgs bundles over the rational curve.

The talk is based on my joint work with Motohico Mulase.

Speaker: **Pavel Etingof** (Massachusets Institute of Technology)
Title: *Cherednik algebras and torus knots*
Abstract: The Cherednik algebra $B(c, n)$, generated by symmetric polynomials and the quantum Calogero-Moser Hamiltonian, appears in many areas of mathematics. It depends on two parameters - the coupling constant c and number of variables n. I will talk about representations of this algebra, and in particular about a mysterious isomorphism between the representations of $B(m/n, n)$ and $B(n/m, m)$ of minimal functional dimension. This symmetry between m and n is made manifest by the fact that the characters of these representations can be expressed in terms of the colored HOMFLY polynomial of the torus knot $T(m/d, n/d)$, where $d = GCD(m, n)$. The talk is based on my joint work with E. Gorsky and I. Losev.

Speaker: **Hiroyuki Fuji** (Tsinghua University)
Title: *Super-A-polynomial*

Abstract: In this talk, I will discuss about the q-difference structure and asymptotic limit of the refined quantum knot invariant. Via categorification, a refinedment of the quantum knot invariant can be found. In the work of S. Gukov and M. Stošić, the triply-graded homology which categorifies the colored HOMFLY polynomial is proposed, and its Poincare polynomial is called as colored superpolynomial. Based on some manifest expressions of this refined knot invariant, we will disccuss about an analogue of the volume conjecture proposed by R. Kashaev, H. Murakami, J. Murakami, and S. Gukov, and AJ (quantum volume) conjecture proposed by S. Garoufalidis and S. Gukov. This talk is based on works in collaboration with H. Awata, S. Gukov, P. Sułkowski, and M. Stošić.

Speaker: **Stavros Garoufalidis** (Georgia Institute of Technology)
Title: *The Slope Conjecture for 2-fusion knots*
Abstract: The Slope Conjecture relates a quantum knot invariant, (the degree of the colored Jones polynomial of a knot) with a classical one (boundary slopes of incompressible surfaces in the knot complement). We will discuss how to produce slopes of incompressible surfaces in the knot complement via an incompressibility criterion (joint with Nathan Dunfield) and how to compute the degree of the colored Jones polynomial using quadratic integer programming (joint with Roland van der Veen). The combination of these approaches prove the Slope Conjecture for all 2-fusion knots.

Speaker: **Victor Ginzburg** (University of Chicago)
Title: *Quantization of line bundles on Lagrangian subvarieties*
Abstract: We apply the technique of formal geometry to give a necessary and sufficient condition for a line bundle supported on a smooth Lagrangian subvariety to deform to a sheaf of modules over a fixed deformation quantization of the structure sheaf of an algebraic symplectic variety.

Speaker: **John Harnad** (CRM, Université de Montreal, and Concordia University)
Title: *2D Toda τ-functions as combinatorial generating functions*
Abstract:

Speaker: **Lotte Hollands** (University of Oxford)
Title: *TBA*
Abstract:

Speaker: **Chiu-Chu Melissa Liu** (Columbia University)
Title: *Eynard-Orantin topological recursion and equivariant Gromov-Witten invariants of the projective line*
Abstract: I will describe an equivariant version of the Norbury-Scott conjecture based on joint work with Bohan Fang and Zhengyu Zong.

Speaker: **Jun Murakami** (Waseda University)
Title: *Logarithmic invariant of knots*
Abstract: I will explain the logarithmic invariant of knots in a 3-sphere, which is constructed with K. Nagatomo by using the centers in the Jacobson radical of the restricted quantum group. Then I will extend the logarithmic invariant to knots in a 3-manifold by combining with the Hennings invariant, which comes from the right integral of the restricted quantum group. The logarithmic invariant is a version of Kashaevs invariant of knots, whose certain limit is conjectured to the hyperbolic volume of the knot complement, and I will also explain such relation between the logarithmic invariant and the hyperbolic volume of the complement by some examples.

Speaker: **Satoshi Nawata** (NIKHEF, Amsterdam)
Title: *J-functions, integrable systems and the AGT relation*
Abstract: The Givental's J-functions of some Fano varieties and GIT quotient spaces can be obtained from supersymmetric partition functions on S^2. In particular, the J-functions of (cotangent bundle of)

flag varieties become eigenfunctions of some integrable Hamiltonian. In addition, the J-functions can be realized as the equivariant partition functions of instanton moduli spaces, which provides the connection to the AGT relation in the presence of a surface operator.

Speaker: **Lenny Ng** (Duke University)
Title: *Knot contact homology and the augmentation polynomial*
Abstract: In this talk, I'll discuss the augmentation polynomial, a three-variable knot invariant derived from contact geometry. Recently the augmentation polynomial has been conjectured to coincide with the "Q-deformed A-polynomial" of Aganagic and Vafa, which in turn should be the classical limit of the recurrence relation for symmetric colored HOMFLY polynomials. I'll describe the motivation for the polynomial from contact homology, and report on joint work with Aganagic, Ekholm, and Vafa relating this story to topological strings.

Speaker: **Paul Norbury** (University of Melbourne)
Title: *Keynote Lecture I. A quantum curve - What is it? Why is it important? A mathematical introduction to quantum curves*
Abstract:

Speaker: **Alexei Oblomkov** (University of Massachusetts)
Title: *$SL(n)$-knot homology vs. HOMFLY homology via Cherednik algebras*
Abstract: Talk is based on the joint work with Gorsky and Rasmussen. As it is well-known the $SL(n)$ quantum knot invariant, which is a one variable polynomial of q, could be obtained from the HOMFLY knot invariant, which is a polynomial of a and q, by a simple substitution $a = q^n$. Analogous, though a bit more subtle, relation conjecturally holds on the level of homology: the conjecture due to Dunfeld-Gukov-Rasmussen asserts that there is a differential d_n on the HOMFLY homology of the knot such that its homology are $SL(n)$ homology. The finite dimensional representations of the Cherednik algebras provide a model for the HOMFLY homology of the torus knots. In my talk I will discuss a construction of d_n in terms of the Cherednik algebras. I will also discuss a conjectural formula for the homology of the Jones-Wentzel projectors.

Speaker: **Anne Schilling** (UC Davis)
Title: *Crystal operators and flag Gromov-Witten invariants*
Abstract: We apply ideas from crystal theory to affine Schubert calculus, flag Gromov-Witten invariants, positroid varieties, and Hall-Littlewood polynomials. By defining operators on certain decompositions of elements in the type-A affine Weyl group, we produce a crystal reflecting the internal structure of Specht modules associated to permutation diagrams (for which the representatives are stable Schubert polynomials, or Stanley symmetric functions). We show how this crystal framework can be applied to study the product of a Schur function with a k-Schur function. Consequently, we prove that a subclass of 3-point Gromov–Witten invariants of complete flag varieties for $\mathbb{C}^n$ enumerate the highest weight elements under these operators. Included in this class are the Schubert structure constants in the (quantum) product of a Schubert polynomial with a Schur function s_λ for all $|\lambda^c| < n$. Another by-product gives a highest weight formulation for fusion coefficients of the Verlinde algebra and our results apply to the Schubert decomposition of positroid varieties.

This is joint work with Jennifer Morse.

Speaker: **Marko Stošić** (Instituto Superior Técnico, Portugal)
Title: *TBA*
Abstract:

Speaker: **Piotr Sułkowski** (University of Warsaw and Caltech)
Title: *Keynote Lecture II. An introduction to quantum curves from a physicist's point of view*
Abstract:

Speaker: **Katrin Wendland** (Albert-Ludwigs-Universität Freiburg)
Title: *An instance of mirror symmetry on $K3$*
Abstract: The moduli space of superconformal field theories on $K3$ is well understood. It can be interpreted as a generalisation of the moduli space of Einstein metrics on $K3$. However, just as no smooth Einstein metrics on $K3$ are known explicitly, the explicit construction of CFTs associated to $K3$ in general remains an open problem. The only known constructions which allow to deal with families of CFTs give CFTs associated to $K3$ surfaces with orbifold singularities. We use a classical construction by Hiroshi Inose to explicitly construct a family of CFTs which are associated to a family of smooth algebraic $K3$ surfaces. Although these CFTs were known before, it is remarkable that they allow a description in terms of a family of smooth surfaces whose complex structure is deformed while all other geometric data remain fixed. We argue that these CFTs can be interpreted as mirror duals of certain orbifold conformal field theories on $K3$. As such, these models may come useful as examples, or as building blocks of examples, where one can test aspects of mirror symmetry.