Modern Developments in M-Theory

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1   Overview of the Field

M-theory originates from string theory, a proposed quantum description of fundamental particles. String theory, in turn, generalises quantum field theory – the most successful description of fundamental physics to date – and reduces to it at low energies. At higher energies, typically those of relevance to quantum phenomena in gravitational systems, string theory departs quite radically from quantum field theory. It provides a specific mechanism to solve the problem of inconsistency at high energies that plagues field-theoretic formulations of gravity.

String theory is consistent only in 10 space-time dimensions but can be easily reduced to a 4-dimensional theory by compactifying it on a suitable manifold. Its formalism is endowed with a deep mathematical beauty that connects to many important results and questions in pure mathematics. Not only does string theory benefit from known theorems in mathematics but mathematics itself has benefited greatly from unexpected connections and unexplored problems suggested by string theory.

When string theory is strongly coupled, its standard description breaks down. In this limit an alternate description becomes available, involving one higher dimension of space. The resulting 11-dimensional theory is called M-theory. One can reduce this theory to string theory by compactifying it on a small circle, however its power and beauty are most evident directly in 11 dimensions. Its low-energy limit is a very special classical field theory: 11-dimensional supergravity. It is notable that supersymmetry is impossible to attain above 11 dimensions, and that under certain minimal assumptions this supergravity is the unique supersymmetric field theory in 11 dimensions.

Just as string theory contains extended excitations: strings and "branes", M-theory too contains extended branes. The mathematics of these branes is a particularly challenging question in view of their strongly coupled nature, inherited directly from the bulk M-theory in which they propagate.

In addition to their potential utility in providing unified theories of fundamental particles and interactions, as well as in stimulating new lines of mathematical research, string and M-theory have come to play a striking new role in the last couple of decades. When they are formulated on (at least asymptotically) anti-de Sitter spacetime, they are dual (i.e. equivalent, though expressed in different variables) to conformal quantum field theories without gravity. In this duality finds that in the quantum field theory, phenomena that would be hard to compute due to strong quantum effects are replaced by a classical, weakly curved gravity description in string or M-theory. This AdS/CFT duality is therefore considered a potentially useful tool to understand a range of strongly coupled systems including quark-gluon plasma, condensed matter systems and fluids.

While there are many analogies between string theory and M-theory (the presence of extended objects, the possibility of compactification to lower dimensions, the reduction to conventional field theory at low

energies, the existence of dualities with conformal field theories and the connection to beautiful mathematical structures) it is almost always the case that M-theory is more difficult to study, largely because of its lack of a small coupling constant in which to perturb. As a consequence, progress in M-theory has tended to proceed at a limited pace. However from time to time, dramatic new results have emerged. This accounts for the special appeal of M-theory to a subset of the string theory community and provides a motivation to hold a focused meeting about it.

2 Recent Developments and Open Problems

Since 2007-2008 there have been striking developments in the construction of the previously unknown field theory on multiple membranes in M-theory. Families of field theories of Chern-Simons type were shown by Bagger and Lambert and independently by Gustavsson to posses maximal superconformal symmetries which made them strong candidates for field theories on multiple parallel membranes. This theory relied on a mathematical structure called "3-algebras". Further analysis of this BLG theory by other authors revealed that in fact it can be reduced to ordinary Chern-Simons-type theory albeit in a particular combination that preserves parity. It was argued that BLG theory describes multiple membranes on orbifold spaces where the rank k of the orbifold group is given by the level of the Chern-Simons field theory. At the particular value $k = 1$ the orbifold is absent, and then the theory should describe membranes in flat space. It was also shown that via a novel version of the Higgs mechanism, these theories on the Higgs branch acquire Yang-Mills gauge fields with a fully dynamical gauge field, enabling their comparison with D2-brane theories to which M2-branes reduce in certain limits. Eventually it was realised that BLG theory cannot describe more than two M2-branes and this led to a fresh search for a theory possessing all or most of its relevant properties but allowing for an arbitrary parameter N labeling the number of branes. In a notable breakthrough, Aharony, Bergman, Jafferis and Maldacena, obtained such a theory by reducing the required manifest symmetries. This theory was explicitly proved, via a brane construction, to describe multiple M2-branes. These new developments have illuminated aspects of M-theory and also provided new insights into the dynamics of 2+1 dimensional quantum field theories and their integrability.

Among the open problems involving M-theory membranes, one can list a better understanding of the strongly coupled limit (absence of the orbifold), the construction of compactifications in which these branes arise, as well as verification of the integrability of the field theory and its role within the AdS/CFT context.

Considerable work has also gone into the construction of the M5-brane field theory, but with comparatively less success. In this case, apart from the strongly coupled nature of the field theory, there is the additional issue of a self-dual tensor field. Even at the abelian level such a field does not admit a covariant Lagrangian. Attempts to formulate M5-brane field theories have attempted to circumvent this problem in various ways, for example by using non-self-dual tensor fields and imposing self-duality as an auxiliary condition at a later stage, and/or by relaxing the requirement of manifest Lorentz invariance. Interesting relations have been proposed between M5-branes and the 3-algebra structures that describe M2-branes. A striking conjecture due to Douglas and to Lambert, Papageorgakis and Schmidt-Sommerfeld identifies M5-brane field theory as the strongly coupled limit of supersymmetric Yang-Mills theory in 5 dimensions, locating the extra degrees of freedom that (among other things) render the theory superconformal in 6d within the soliton sector of the 5d theory.

While M2 and M5 branes have been the primary focus of study in recent years, there have been attempts to study M-theory directly in the bulk and these have had some partial success. Attempts to define M-theory nonperturbatively were made over a decade ago using matrices. This led to several related descriptions, the primary one being that of a matrix quantum mechanics based on the D0-brane of type IIA string theory, and others corresponding to 2d matrix string theories or 0d random matrix models. These approaches were later subsumed into the AdS/CFT correspondence but they have their own appeal. One would like to understand degree of validity of such descriptions as well as the symmetries and dynamics of M-theory that follow from them.

Higher-derivative corrections to the low-energy effective action of M-theory have an intricate and beautiful mathematical structure. Much work in recent years has been devoted to the explicit computation of these corrections. Open problems in this area involve not merely computing higher-order corrections, but rather gaining an insight into large classes of computable corrections using profound mathematical tools.

The construction of superconformal quantum field theories in dimension 4 and lower by wrapping M5-branes on Riemann surfaces and other manifolds, dates from the early days of M-theory. The recent work of Gaiotto and others has brought about dramatic progress, illuminating the dualities and other dynamical aspects of a wide variety of SCFT's and providing a powerful way to construct new ones. This subject is growing, with many different perspectives coming from various directions like type IIA string theory and F-theory.

A relatively recent approach to M-theory involves concepts like generalised geometry and doubled field theory, in which the U-duality symmetries of M-theory are manifest. Finding the symmetries, backgrounds and the geometric governing principle behind doubled field theory are all key open problems.

3 Presentation Highlights

3.1 M2-branes

In the presentation "Exact results and non-perturbative effects in M-theory", Marcos Marino computed the partition function of M2-branes (ABJM theory) on a three-sphere by localization in terms of a matrix integral, resulting in a derivation of the $N^{\frac{3}{2}}$ behavior for a theory of M2-branes. This partition function was found to contain more information including non-perturbative, exponentially small effects at large N. These effects correspond to non-perturbative corrections due to worldsheet and membrane instantons in string theory/M-theory. The resulting expansion highlights the important role of membrane instantons. The corrections due to them can be studied in the Fermi gas approach to the partition function, and are encoded in a system of integral equations of the TBA type. A semi-classical expansion of this system in the ABJM coupling k was studied, corresponding to the strong coupling expansion of the type IIA string. Using these semi-classical results, he verified conjectures for the form of the one-instanton correction at finite k proposed by Hatsuda, Moriyama and Okuyama (HMO), based on a conjectural cancellation of divergences between worldsheet instantons and membrane instantons. Analytic expressions in k were proposed for the full two-membrane instanton correction and for higher-order non-perturbative terms. These pass many consistency checks and provide further evidence for the HMO mechanism. On the mathematical side, the above non-perturbative effects turn out to be connected to topological string theory and its refinements, and to the quantization of algebraic curves.

Stefano Kovacs presented new results on monopole operators in M2-brane theory. Using the AdS/CFT correspondence he presented a proposal for studying the ABJM duality in a genuinely M-theoretic regime. By focussing on a large angular momentum sector, the field theory was studied in a regime in which the gravitational background is eleven-dimensional and the physical states correspond to M2-brane excitations. On the gravity side this sector is well approximated by the pp-wave matrix model, which is weakly coupled. The spectra computed on the two sides were found to agree at leading order, thus verifying the validity of the ABJM duality beyond the previously considered type IIA limit and also providing a new test of the matrix model approach to M-theory.

Bent Nilsson considered topologically gauged BLG: matter/Chern-Simons gauge theory with N = 8 superconformal symmetry coupled to conformal supergravity. His presentation highlighted the discovery of new features like SO(N) gauge groups for any N (instead of the SO(4) in BLG) and Higgsing to topologically massive supergravity and a number of possible "critical" backgrounds. Such results were first found in topologically gauged ABJM/ABJ theory with indications of a "sequential AdS/CFT".

3.2 M5-branes

Seok Kim addressed the problem of computing superconformal indices for M5-brane field theories. Starting with maximally supersymmetric Yang-Mills theory in 5d on $CP_2 \times S^1$, he was able to count the BPS local operators of the 6d theory. In a closely related approach, Kimyeong Lee explored the 6d (1,0) and (2,0) superconformal field theories on $R \times S^5/Z_k$ via supersymmetric 5d theories on $CP_2 \times R$. The index computed by these authors agreed with the large-N supergravity index on $AdS_7 \times S^4$ at low energies, and also yielded the negative 'Casimir energy' with an N^3 scaling which was previously calculated from a QFT on S^5.

Costis Papageorgakis examined soliton contributions to perturbative amplitudes in field theory, in the context of the proposed 5d-6d correspondence that conjectures M5-brane field theory to be the strongly coupled

limit of 5d Yang-Mills with the desired additional excitations coming from instanton-solitons. This was done by investigating the conditions under which the calculation of a matrix element in the quantum mechanics on the soliton moduli space leads to exponential suppression. In turn, this suggests that the instanton-solitons of 5d supersymmetric Yang-Mills theory will not be suppressed, supporting the proposed correspondence. The same correspondence was discussed by Neil Lambert in a Euclidean context, wherein he proposed that the 5d theory exhibits a hidden time-like dimension.

In a different approach to multiple M5-branes, Chong-Sun Chu put forward a non-abelian self-duality equation in six-dimensions. This led to a proposal for the equation of motion of the 3-form field strength. He discussed how the solutions of these equations have properties that match precisely with the physics of the M5-branes system obtained from other analysis, such as 11 dimensional supergravity and string theory. Dmitri Sorokin presented an alternate approach in which the six-dimensional worldvolume is subject to a covariant split into 3+3 directions by a triplet of auxiliary fields. He discussed the relation of this action to the original form of the M5-brane action and to a Nambu–Poisson 5–brane action based on the Bagger–Lambert–Gustavsson model with the gauge symmetry of volume preserving diffeomorphisms.

3.3   Mathematical structures in M-theory

In his presentation "Mathieu moonshine, umbral moonshine and M5-branes", Jeff Harvey related umbral moonshine to the Niemeier lattices: the 23 even unimodular positive-definite lattices of rank 24 with non-trivial root systems. A finite group was attached to each Niemeier lattice by considering a naturally defined quotient of the lattice automorphism group. For each conjugacy class of each of these groups, a vector-valued mock modular form was identified whose components coincide with mock theta functions of Ramanujan in many cases. This then led to the umbral moonshine conjecture, stating that an infinite-dimensional module is assigned to each of the Niemeier lattices in such a way that the associated graded trace functions are mock modular forms of a distinguished nature. These constructions and conjectures include the Mathieu moonshine conjecture that has been of particular interest in the mathematical physics community in recent times. This analysis also highlights a correspondence between genus zero groups and Niemeier lattices. As a part of this relation it was recognised that the Coxeter numbers of Niemeier root systems with a type A component are exactly those levels for which the corresponding classical modular curve has genus zero. In a related development he studied perturbative BPS states in the near-horizon background of two Neveu-Schwarz fivebranes in type IIA superstring theory compactified on $K3 \times S^1 \times R^1$. The brane world-volume wraps the $K3 \times S^1$ factor. He obtained a simple expression for the spacetime helicity supertrace in terms of the completion of the mock modular form $H^{(2)}(\tau)$. This form appears in studies of the decomposition of the elliptic genus of $K3$ surfaces into characters of the $N = 4$ superconformal algebra and manifests a moonshine connection to the Mathieu group M_{24}.

On the same topic, the presentation of Callum Quigley discussed the extension of the Mathieu moonshine conjectures to $N = 2$ string compactifications. In this context it was shown that dimensions of M_{24} representations appear in the new supersymmetric index of heterotic strings on $K3 \times T_2$ and the Gromov-Witten invariants of their type IIA duals. He reported on work in progress to relate these results to the elliptic genus of a wrapped M5-brane. The dimensions of M_{24} representations appear very naturally again in the type IIA context, but now as counting the degeneracies of D4−D2−D0 bound states (i.e. the Donaldson-Thomas invariants).

3.4   Superconformal field theory

The presentation by Leonardo Rastelli: "The superconformal bootstrap program", outlined the modern bootstrap program for four-dimensional theories with extended superconformal symmetry. He found "minibootstrap" equations for supersymmetric quantities, which can be solved analytically, and full-fledged bootstrap equations for non-protected quantities, which can be studied numerically. The entire program relies on general symmetry principles, with no need for "fields" or Lagrangians. The latter part of the talk focused on the numerical results of the $N = 4$ bootstrap and on their interpretation in $N = 4$ supersymmetric Yang-Mills theory. Consistency of the four-point function of the stress-energy tensor multiplet was used to impose significant upper bounds for the scaling dimensions of unprotected local operators as functions of the central

charge. At the threshold of exclusion, a particular operator spectrum was singled out by the bootstrap constraints. For large values of the central charge, this was compared and found to be compatible with that of supergravity in $AdS_5 \times S^5$. For finite central charge it was conjectured that the extremal spectrum is that of $N = 4$ SYM at an S-duality invariant value of the complexified gauge coupling.

3.5 Higher-spin gravity

In her presentation "Wilson lines in higher spin gravity", Alejandra Castro reviewed the interpretation of Wilson line operators in the context of higher spin gravity in 2+1 dimensional field theories and holography. She showed how a Wilson line encapsulates the thermodynamics of black holes and provides an elegant description of massive particles. This opens a new window of observables which can probe the true geometrical nature of higher spin gravity. Another presentation on higher spins, by Bengt Nilsson, reviewed the coupling of conformal supergravity to CFTs in three dimensions with eight supersymmetries. The so obtained SO(N) models have a new kind of scalar potential that gives rise to a number of possible background geometries. These are specified by a sequence of values for the parameters in the standard topologically massive gravity action that arises after the conformal symmetry breaking. Solutions corresponding to these values were identified, all of which are critical or special in some sense.

3.6 Supergravity solutions

Alessandro Tomasiello presented a seminar on all AdS_7 solutions of type- II supergravity. Generalising the important $AdS_7 \times S^4$ solutions (and their orbifolds) in M-theory, he described new AdS_7 supersymmetric solutions in type II supergravity with Romans mass (which does not lift to M-theory). His classification started from a pure spinor approach reminiscent of generalized complex geometry, this determined uniquely the form of the metric and fluxes, up to solving a system of ODE's. His presentation concluded with some work in progress about the CFT$_6$ duals to these solutions.

3.7 Higher-derivative corrections in M-theory

String scattering amplitudes, Feynman diagrams and M- theory and the relations between them were surveyed in the presentation of Michael Green. One class of recent computations involved the determination of explicit expressions for one-loop five-supergraviton scattering amplitudes in type II superstring theories using the pure spinor formalism. The type IIB amplitude was expressed in terms of a doubling of a ten-dimensional super Yang–Mills tree amplitude. A series of terms in the effective action of the schematic form $D^{2k}R^5$ for $0 \le k \le 5$ (where R is the Riemann curvature) were evaluated. Comparison with earlier analyses of the tree amplitudes and of the four-particle one-loop amplitude gave rise to an interesting extension of the action of SL(2,Z) S-duality on the moduli-dependent coefficients in the type IIB theory. The investigation also covered closed-string five-particle amplitudes that violate conservation of the U(1) R-symmetry charge and are forbidden in supergravity. Their low energy expansion was found to match the predictions of S-duality. For the six-point function it was shown that the analytic parts of the R^6 and $D^4 R^6$ interactions vanish in the ten-dimensional effective action, but not in lower dimensions.

Anirban Basu presented his results on constraining gravitational interactions in the M-theory effective action. Based on assumptions about the structure of supersymmetry, he was able to obtain the expressions for certain non-BPS operators in the effective action, part of the structure of which is fixed by superstring perturbation theory.

4 Outcome of the Meeting

The seminars and deliberations at this meeting were very successful in highlighting the outstanding open problems in the field and proposing new avenues to explore them. Discussions on topics like world-volume

formulations of M5-branes, localisation and matrix models for M2-branes, superconformal field theory in 4d, and the mathematics of Mathieu moonshine have led to significant developments during the following year. This meeting was followed up with the "Second Workshop on Developments in M-Theory" held in Gangwon-do, Korea during January 2015.