

# Spin Glasses and Related Topics

## July 20–25, 2014

### MEALS

\*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday

\*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday

\*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday

Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)

**\*Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

### MEETING ROOMS

All lectures will be held in the lecture theater in the TransCanada Pipelines Pavilion (TCPL). An LCD projector, a laptop, a document camera, and blackboards are available for presentations.

### SCHEDULE NOTES

(see talk schedule below)

#### Sunday:

**16:00** Check-in begins (Front Desk - Professional Development Centre - open 24 hours)

**17:30–19:30** Buffet dinner, Sally Borden Building

**20:00** Informal gathering in 2nd floor lounge, Corbett Hall

Beverages and a small assortment of snacks are available on a cash honor system.

#### Monday:

**8:45–9:00** Introduction and welcome by BIRS Station Manager, TCPL

Wednesday: Free afternoon, group photo (time TBD)

#### Friday:

**9:00–11:00** Discussion Sessions

**Checkout by 12 noon.**

\*\* Workshop participants are welcome to use BIRS facilities (BIRS Coffee Lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. \*\*

	Monday	Tuesday	Wednesday	Thursday	Friday
7:00-9:00	Breakfast	Breakfast	Breakfast	Breakfast	Breakfast
8:45-9:00	Introduction				
9:00-9:30	<i>Stein</i>	<i>Dembo</i>	<i>Franz</i>	<i>Gamarnik</i>	Discussion
9:35-10:05	<i>Gayrard</i>	<i>Parisi</i>	<i>Kirkpatrick</i>	<i>Sun</i>	
10:05-10:30	Coffee Break	Coffee Break	Coffee Break	Coffee Break	Coffee Break
10:30-11:00	<i>Sly</i>	<i>Arguin</i>	<i>Chen</i>	<i>Fyodorov</i>	Discussion
11:05-11:35	<i>Jagannath</i>	<i>Svejda</i>	<i>Genovese</i>	<i>Zindy</i>	
11:35-13:30	Lunch	Lunch	Lunch	Lunch	Lunch
13:30-14:00	<i>Schweinsberg</i>	<i>Giardina</i>	Free Afternoon	<i>Read</i>	Checkout by noon
14:05-14:35	<i>Starr</i>	<i>Berestycki</i>		<i>Mingione</i>	
14:35-15:00	Coffee Break	Coffee Break		Coffee Break	
15:00-15:30	<i>Bovier</i>	<i>Newman</i>	Free Afternoon	<i>Belius</i>	
15:35-16:05	<i>Auffinger</i>	<i>Tantari</i>		<i>Barra</i>	
17:30-19:30	Dinner	Dinner	Dinner	Dinner	

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### ABSTRACTS

(in alphabetic order by speaker surname)

Speaker: **Arguin, Louis-Pierre** (University of Montreal)

Title: *Fluctuation bounds for interface free energies in spin glasses*

Abstract: One way to understand the structure of the Gibbs states of disordered systems is to get good bounds on the fluctuations of the free energy difference between two states. This approach has led to the proof of the absence of phase transition in the 2D Random Field Ising Model (RFIM) by Aizenman and Wehr. We will explain a method to obtain lower bounds for the variance of the free energy difference of the Edwards-Anderson (EA) spin glass model on  $\mathbb{Z}^d$  between certain incongruent states (if they exist...). Unlike the RFIM, there is no dominance of the (+) and (-) states in the EA model. One interesting point of the method is to overcome this lack of monotonicity. The lower bound is also used to rule out particular structures of the Gibbs states in  $d = 2$ . This is joint work with C. Newman, D. Stein and J. Wehr.

Speaker: **Auffinger, Antonio** (University of Chicago)

Title: *Free energy and complexity of bipartite spherical spin glasses*

Abstract: In this talk, we overview some predictions and some recent rigorous results on bipartite spin systems. We investigate both free energy and complexity of a class of bipartite spherical spin glass models. We first provide a formula for the limiting free energy based on the Crisanti-Sommers representation of the mixed spherical model. Second, we show that the mean number of local minima at low levels of energy is exponentially large in the size of the system and we derive a bound on the location of the ground state energy. Based on a joint-work with Wei-Kuo Chen.

Speaker: **Barra, Adriano** (University of Rome La Sapienza)

Title: *Cavity fields, stochastic stability and mechanical analogies in mean field spin glasses*

Abstract: Using the Curie-Weiss model as a starting test-case, I will show at first how to analyze it through the smooth cavity technique, then I will show that all the results that can be obtained within this route are exactly those that can be obtained with stochastic stability technique (for gauge-invariant systems only). One step forward I will discuss a mechanical analogy that allows to solve for the model free energy via an Hamilton-Jacobi equation and analyze its phase transition in terms of shock waves and its self-averaging in terms of Noether currents. The whole route will be then quickly applied to the Van der Waals model and with more details to the Sherrington-Kirkpatrick model.

Speaker: **Belius, David** (University of Montreal)

Title: *The subleading order of two dimensional cover times*

Abstract: The cover time is the time it takes for a Markov process to visit the whole state space. This time can be studied by considering the extrema of a correlated random field, namely the field of occupation time (is the minimum occupation time zero or positive?), placing it in a similar category to many spin glass problems. In this talk I will present a recent result on the cover time of the two dimensional torus by Brownian motion, where the field of occupation times turns out to be log-correlated. We use techniques inspired by those used to study Branching Brownian Motion (BBM) to establish the existence of a subleading correction to the cover time, which corresponds to the well-known 3/2-correction for BBM (the leading order was established by Dembo, Peres, Rosen and Zeitouni [Ann. of Math., 160 (2004)]). This involves a multiscale analysis of the field of occupation times, and the identification of an approximate hierarchical structure. Joint work with Nicola Kistler.

Speaker: **Berestycki, Nathanael** (University of Cambridge)

Title: *Liouville Brownian motion*

Abstract: I will introduce and discuss a canonical notion of Brownian motion in the random geometry of Liouville quantum gravity, called Liouville Brownian motion. I will discuss some of its basic properties, for instance related to the time spent in the thick points of the Gaussian Free Field. Time permitting I will also discuss the construction of this process in the supercritical phase of Liouville quantum gravity and show a certain duality with the subcritical phase.

Speaker: **Bovier, Anton** (Rheinische Friedrich-Wilhelms-Universität Bonn)

Title: *Extremal processes of Gaussian processes indexed by trees*

Abstract: Gaussian processes indexed by trees form an interesting class of correlated random fields where the structure of extremal processes can be studied. One popular example is Branching Brownian motion, which has received a lot of attention over the last decades, non the least because of its connection to the KPP equation. In this talk I review the construction of the extremal process of BBM (with Arguin and Kistler) and present some more recent results on “variable speed” BBM, obtained with Lisa Hartung.

Speaker: **Chen, Wei-Kuo** (University of Chicago)

Title: *On properties of the Parisi measure*

Abstract: This talk is concerned about the functional ordered parameter, the Parisi measure, in the mixed  $p$ -spin models. We will first prove its uniqueness by establishing the strict convexity of the Parisi functional. We will then discuss some conjectured properties of this measure and present partial results along this direction. This talk is based on a joint work with Antonio Auffinger.

Speaker: **Dembo, Amir** (Stanford University)

Title: *Spin glasses on locally tree like graphs*

Abstract: Based on joint work with Gerschenfeld and Montanari.

Speaker: **Franz, Silvio** (University of Paris-Sud 11)

Title: *Glassy critical points and the Random Field Ising Model*

Abstract: Research in recent years has emphasized the importance of fluctuations in understanding glassy phenomena in supercooled liquids. The present comprehension of long lived dynamical heterogeneities in these systems compares the growth of their typical size to the appearance of long range correlations at second order phase transition points. Unfortunately, in supercooled liquids, the theoretical study of these correlations beyond the mean field is just at an embryonic level. One of the difficulties lies in the fact that -with good physical reasons- the critical point corresponds to an unstable field theory. It turns out that one can cure the instability introducing appropriate physical constraints. In that case true critical points appear and they can be analyzed theoretically through the replica method. I will present the field theory of these critical point and show that the universality class of the Random Field Ising Model appears.

Speaker: **Fyodorov, Yan** (Queen Mary University of London)

Title: *High-dimensional random landscapes and random matrices*

Abstract: I am going to discuss a picture of the “topology trivialization transition” (in the sense of an abrupt reduction of the number of stationary points and minima of the underlying energy landscape) which takes place at zero temperature in  $p$ -spin spherical model of spin glasses with increasing random magnetic field, as well as in related high-dimensional models not restricted to the sphere. In particular, I will emphasize the role of the “edge scaling” and the Tracy-Widom distribution of the largest eigenvalues of random matrices for providing some universal features of the above transition. For the simplest case  $p = 2$ , I will discuss the large deviation function for the minimal energy extracted via a variant of the replica formalism. If time allows I will also discuss how random matrix methods can be used to get insights into topology of random real algebraic varieties. The talk is based on the paper arXiv:1307.2379 as well as on joint works with Pierre Le Doussal [arXiv:1304.0024], Celine Nadal [arXiv:1207.6790], Antonio Lerario and Erik Lundberg [arXiv:1404.5349].

Speaker: **Gamarnik, David** (Massachusetts Institute of Technology)

Title: *Limits of local algorithms for randomly generated constraint satisfaction problems*

Abstract: The talk is devoted to the problem of constructing algorithms for solving randomly generated constraint satisfaction problems, such random K-SAT problem. We establish a fundamental barrier on the power of local algorithms to solve such problems, despite the conjectures put forward in the past. In particular, we refute a conjecture regarding the power of so-called i.i.d factors to find nearly largest independent sets in random regular graphs. Similarly, we show that a broad class of local algorithms, including the so-called Belief Propagation and Survey Propagation algorithms, cannot find satisfying assignments in random NAE-K-SAT problem above a certain asymptotic threshold, below which even simple algorithms succeed with high probability. Our negative results exploit fascinating geometry of feasible solutions of random constraint satisfaction problems, which was first predicted by physicists heuristically and now confirmed by rigorous methods. According to this picture, the solution space exhibits a clustering property whereby the feasible solutions tend to cluster according to the underlying Hamming distance. We show that success of local algorithms would imply violation of such a clustering property thus leading to a contradiction. Joint work with Madhu Sudan (Microsoft Research).

Speaker: **Gayraud, Véronique** (Aix-Marseille Université and CNRS)

Title: *Aging in mean-field spin-glasses*

Abstract: I will present recent results on the activated aging dynamics of mean-field spin glasses (REM,  $p$ -spin SK model, GREM-like trap model).

Speaker: **Genovese, Giuseppe** (University of Zurich)

Title: *SK-Spherical spin glass approximation for the Hopfield model with Gaussian patterns*

Abstract: By means of a mapping into an appropriate bipartite spin glass, it is possible to decompose the free energy of the Hopfield model with Gaussian patterns into the free energies of a SK model and a spherical one. We will discuss some results in this direction obtained in collaboration with A. Barra, F. Guerra and D. Tantari.

Speaker: **Giardina, Cristian** (University of Modena and Reggio Emilia)

Title: *Central limit theorems for Ising model on random graphs*

Abstract: For various classes of random graphs with  $N$  vertices, we prove that the total spin of the Ising model defined on them satisfies a central limit theorem, provided it is centered by the total magnetization and rescaled by the square root of  $N$ . We consider both quenched and annealed measures in the one-phase region. We conjecture that when the vertex degrees do not fluctuate, the variance of the limiting Gaussian law is the same in the two settings. We substantiate this claim with the analysis of some configuration models. Joint work with C. Giberti, R. van der Hofstad, M.L. Prioriello.

Speaker: **Jagannath, Aukosh** (New York University)

Title: *Approximate ultrametricity for random measures and applications to spin glasses*

Abstract: We introduce a notion called “Approximate Ultrametricity” which encapsulates the phenomenology of a sequence of random probability measures having supports that behave like ultrametric spaces insofar as they decompose into nested balls. We provide a sufficient condition for a sequence of random probability measures on the unit ball of a separable Hilbert space to admit such a decomposition. We also characterize the laws of the measures of the sets in this regime by showing that they converge in law to the weights of a Ruelle Probability Cascade. These results apply to a large class of classical models in mean field spin glasses. In particular, they show that for this class of models, the Gibbs measure admits an approximate decomposition into “pure states” and their “combinations” at large but finite volume, as predicted in the physics literature. We further illustrate the notion of approximate ultrametricity by proving two important conjectures related to mixed  $p$ -spin glasses.

Speaker: **Kirkpatrick, Kay** (University of Illinois at Urbana-Champaign)

Title: *Non-normal asymptotics of the mean-field Heisenberg model*

Abstract: I will discuss spin models of magnets and superconductors, with spins in the circle (XY model) and in the sphere (Heisenberg model)—with interesting phase transitions. I will discuss work with Elizabeth Meckes on the mean-field Heisenberg model and its non-normal behavior at the phase transition. There is much that is still unclear about these models; I'll mention work in progress with Tayyab Nawaz and Leslie Ann Ross.

Speaker: **Mingione, Emanuele** (University of Bologna)

Title: *Multi-species mean-field spin-glasses. Rigorous results.*

Abstract: In this talk we introduce a multi-species spin glass system where the density of each species is kept fixed at increasing volumes. The model reduces to the Sherrington-Kirkpatrick one for the single species case. Using Guerra's scheme and under a convexity condition on the interaction, we prove the existence of the thermodynamic limit, the replica symmetric and the replica symmetry breaking bounds. We show that the order parameter of the model is described by a multidimensional version of the Parisi Ansatz. Joint work with A. Barra, P. Contucci and D. Tantari.

Speaker: **Newman, Charles** (New York University)

Title: *Statistical Mechanics and the Riemann Hypothesis*

Abstract: In this talk we review a number of old results concerning certain statistical mechanics models and their possible connections to the Riemann Hypothesis. A standard reformulation of the Riemann Hypothesis (RH) is: The (two-sided) Laplace transform of a certain specific function  $\Psi$  on the real line is automatically an entire function on the complex plane; the RH is equivalent to this transform having only pure imaginary zeros. Also  $\Psi$  is a positive integrable function, so (modulo a multiplicative constant  $C$ ) is a probability density function. A (finite) Ising model is a specific type of probability measure  $\mathbb{P}$  on the points  $S = (S_1, \dots, S_N)$  with each  $S_j = +1$  or  $-1$ . The Lee-Yang theorem implies that for non-negative  $a_1, \dots, a_N$ , the Laplace transform of the induced probability distribution of  $a_1 S_1 + \dots + a_N S_N$  has only pure imaginary zeros. The big question here is whether it's possible to find a sequence of Ising models so that the limit as  $N$  tends to  $\infty$  of such distributions has density exactly  $C\Psi$ . We'll discuss some hints as to how one might try to do this.

Speaker: **Parisi, Giorgio** (University of Rome La Sapienza)

Title: *Fractal free energy landscapes in structural glasses*

Abstract: Glasses are amorphous solids whose constituent particles are caged by their neighbors and thus cannot flow. This sluggishness is often ascribed to the free energy landscape containing multiple minima (basins) separated by high barriers. I will show, using theory and numerical simulation, that the landscape is much rougher than is classically assumed. Deep in the glass, it undergoes a roughness transition to fractal basins, which brings about isostaticity and marginal stability on approaching jamming.

Critical exponents for the basin width, the weak force distribution and the spatial spread of quasi-contacts near jamming can be analytically determined. Their value is found to be compatible with numerical observations. This advance incorporates the jamming transition of granular materials into the framework of glass theory. Because temperature and pressure control what features of the landscape are experienced, glass mechanics and transport are expected to reflect the features of the topology I will discuss in this seminar.

Speaker: **Read, Nicholas** (Yale University)

Title: *The metastate interpretation of replica symmetry breaking in short-range Ising spin glasses*

Abstract: We show how the replica symmetry breaking (RSB) scheme of Parisi, applied to the Edwards-Anderson model of short-range Ising spin glasses, leads directly to a description in terms of a non-trivial metastate a la Newman and Stein (NS). In their “non-standard” picture, advanced as a “maximally mean-field-like” picture consistent with rigorous results, the metastate is a probability distribution on infinite-size Gibbs states, each of which has ultrametric structure; when the state is averaged over the metastate, the result is a mixed state with a decomposition into an uncountable number of pure states. This picture is

derived and extended here by interpreting a power-law decay of correlations found in replica calculations as the decay of correlations in the metastate-averaged state, yielding an effective fractal dimension, and implying that the logarithm of the number of pure states that can be distinguished using only correlations within a window of size  $W$  scales as  $W^{d-\zeta}$ , with  $\zeta = 4$  above six dimensions, the value found in replica calculations.

Speaker: **Schweinsberg, Jason** (University of California at San Diego)

Title: *The genealogy of a population undergoing selection*

Abstract: Consider a population of constant size  $N$  in which each individual dies at rate one and each individual experiences mutations at rate  $r$ . Mutations are assumed to be beneficial, so that the fitness of an individual with  $k$  mutations is  $1 + s(k - m)$ , where  $m$  is the mean number of mutations among individuals in the population. When an individual dies, a replacement is chosen at random from the population with probability proportional to the individual's fitness. We show that the genealogy of this population is given by the Bolthausen-Sznitman coalescent, confirming nonrigorous predictions of Neher and Hallatschek and of Desai, Walczak, and Fisher. The Bolthausen-Sznitman is the coalescent process that arises in connection with spin glass models, and was recently shown to describe the genealogy of branching Brownian motion with absorption.

Speaker: **Sly, Allan** (University of California, Berkeley)

Title: *Maximum independent sets in random  $d$ -regular graphs*

Abstract: We study the size of maximum independent sets in random  $d$ -regular graphs. We show that for  $d$  exceeding a constant  $d_0$ , there exist explicit constants  $A, C$  depending on  $d$  such that the maximum size has constant fluctuations around  $An - C \log n$  establishing the 1-RSB predictions. This is joint work with Jian Ding and Nike Sun.

Speaker: **Starr, Shannon** (University of Alabama, Birmingham)

Title: *About eigenvectors for random matrices*

Abstract: In analogy with the random overlap structure for spin glasses, it seems useful to know the distribution of the eigenvector inner-products for models of random matrices, especially non-Hermitian models such as Ginibre's ensemble. I will describe our attempt to determine these, and to find reported values in the literature.

Speaker: **Stein, Daniel** (New York University)

Title: *Predictability in nonequilibrium discrete spin dynamics*

Abstract: Consider a dynamical many-body system with a random initial state subsequently evolving through stochastic dynamics. What is the relative importance of the initial state ("nature") vs. the realization of the stochastic dynamics ("nurture") in predicting the final state? We discuss this question and present both old and new results for low-dimensional homogeneous and disordered Ising spin systems.

Speaker: **Sun, Nike** (Stanford University)

Title: *Potts and independent set models on  $d$ -regular graphs*

Abstract: We consider the ferromagnetic Potts on typical  $d$ -regular graphs, and the independent set model on typical bipartite  $d$ -regular graphs, with graph size tending to infinity. We show that the replica symmetric (Bethe) prediction applies for all parameter values in these two models. In this talk I will describe some of the proof techniques, which will give an indication of the contrast with the anti-ferromagnetic Potts model and the independent set model at high fugacity on non-bipartite graphs, where the Bethe prediction is known to fail. This is joint work with Amir Dembo, Andrea Montanari, and Allan Sly.

Speaker: **Svejda, Adela** (Technion)

Title: *Clock processes on infinite graphs and aging in Bouchaud's asymmetric trap model*

Abstract: Clock processes of random dynamics in random environments have recently been at the center of

attention in connection with the study of aging, which is a phenomenon that is believed to characterize the dynamical behavior of spin glasses. Based on a method for proving convergence of partial sum processes due to Durrett and Resnick, convergence criteria for clock processes were established for dynamics on finite graphs by Bovier and Gayraud. In this talk, we study dynamics that are defined on infinite graphs and present general convergence criteria for their clock processes. As an application we prove the existence of a normal aging regime in Bouchaud's asymmetric trap model on  $\mathbb{Z}^d$ , for all  $d \geq 2$ . Joint work with Véronique Gayraud.

Speaker: **Tantari, Daniele** (University of Rome La Sapienza)

Title: *Parallel retrieval in multitasking associative networks*

Abstract: Recently multitasking associative networks have been introduced in the statistical mechanics community to mimic the parallel processing capabilities of the immune system, the latter being thought of as a network of interacting B and T lymphocytes. In this talk, after a streamlined introduction to fundamentals of theoretical immunology, I will introduce these models (mainly bipartite spin glasses embedded on a finite connectivity topology) and study their phase diagrams, via replica trick and/or cavity techniques, in order to distinguish between a ferromagnetic region -where the system performs extensive parallel retrieval- and a spin glass one, where the amount of interferences among the patterns increases: I will prove that this clonal cross-talk diminishes the multitasking features of these networks. Further, I will show that a second order phase transition occurs when varying the level of load (number of memorized patterns), network's dilution and fast noise: from low to high load, from fully connected to finite connectivity regimes.

Speaker: **Zindy, Olivier** (Université Pierre et Marie Curie)

Title: *Poisson-Dirichlet statistics for the extremes of log-correlated Gaussian fields*

Abstract: Gaussian fields with logarithmically decaying correlations, such as branching Brownian motion and the 2D Gaussian free field, are conjectured to form a new universality class of extreme value statistics (notably in the work of Carpentier & Ledoussal and Fyodorov & Bouchaud). This class is the borderline case between the class of IID random variables, and models where correlations start to affect the statistics. In this talk, I will describe a general approach based on rigorous works in spin glass theory to describe features of the Gibbs measure of these Gaussian fields. I will focus on a model defined on the periodic interval  $[0; 1]$ . At low temperature, we show that the normalized covariance of two points sampled from the Gibbs measure is either 0 or 1. This is used to prove that the joint distribution of the Gibbs weights converges in a suitable sense to that of a Poisson-Dirichlet variable. This is joint work with Louis-Pierre Arguin.