Banff International Research Station

for Mathematical Innovation and Discovery

Geometric Tomography and Harmonic Analysis
March 9 - March 14, 2014

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in the lecture theater in the TransCanada Pipelines Pavilion (TCPL). An LCD projector, a laptop, a document camera, and blackboards are available for presentations.

SCHEDULE

Sunday

16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
17:30–19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall (if desired) Beverages and a small assortment of snacks are available on a cash honor system.

Monday

7:00 - 8:45	Breakfast
8:45 - 9:00	Introduction and Welcome by BIRS Station Manager, TCPL
9:00 - 9:30	Wolfgang Weil, *Area Measures and Fourier Operators*
9:35 - 10:05	Mathieu Meyer, *Affine points and duality*
10:05 - 10:40	Coffee Break
10:40 -	Vitali Milman, *Characterizing summations of convex sets*
- 11:45	Liran Rotem, *On isotropicity with respect to a measure*
11:45 - 13:00	Lunch
13:00 - 14:00	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:00	Group Photo; meet in foyer of TCPL (photograph will be taken outdoors so a jacket might be required).
14:15 - 14:45	Franz Schuster, *Cosine and Radon Transforms in the Theory of Minkowski Valuations*
14:45 - 15:20	Coffee Break
15:20 - 15:50	Florian Besau, *Binary Operations in Spherical Convex Geometry*
15:55 - 16:25	Susanna Spektor, *Quantitative version of a Silverstein's result*
17:30 - 19:30	Dinner

Tuesday

7:00 - 9:00	Breakfast
9:00 - 9:30	Eric Grinberg, *The Torus Transform on Symmetric Spaces of Compact Type*
9:35 - 10:05	Yves Martinez-Maure, *Can hedgehogs be useful for Geometric Tomography?*
10:05 - 10:40	Coffee Break
10:40 - 11:10	Matthieu Fradelizi, *Functional versions of L_p-affine surface area and entropy inequalities*
11:15 - 11:45	Joseph Lehec, *Bounding the norm of a log-concave vector via thin-shell estimates*
11:45 - 13:30	Lunch
13:30 - 14:00	Ronen Eldan, *A Two-Sided Estimate for the Gaussian Noise Stability Deficit*
14:05 - 14:35	Yoav Kallus, *The Ball as a Pessimal Shape for Packing*
14:35 - 15:10	Coffee Break
15:10 - 15:40	Alexander Litvak, *Approximating the covariance matrix with heavy tailed columns and RIP*
15:45 - 16:15	Galyna Livshyts, *Maximal surface area of a convex set in $\mathbb{R}^n$ with respect to log concave rotation invariant measures*
17:30 - 19:30	Dinner

Wednesday

7:00 - 9:00	Breakfast
9:00 - 9:30	Mark Rudelson, *Small ball probabilities for linear images of high dimensional distributions*
9:35 - 10:05	Peter Pivovarov, *Volume of the polar of random sets and shadow systems*
10:05 - 10:40	Coffee Break
10:40 - 11:10	Luis Rademacher, *TBA*
11:15 - 11:45	Konstantin Tikhomirov, *On the distance of polytopes with few vertices to the Euclidean ball*
11:45 - 13:30	Lunch
	Free Afternoon
17:30 - 19:30	Dinner

Thursday

7:00 - 9:00	Breakfast
9:00 - 9:30	Elisabeth Werner, *Equality characterization and stability for entropy inequalities*
9:35 - 10:05	Pierre Youssef, *Almost orthogonal contact points*
10:05 - 10:40	Coffee Break
10:40 - 11:10	Carsten Schütt, *Order statistics*
11:15 - 11:45	Astrid Berg, *Log-concavity properties of Minkowski valuations*
11:45 - 13:30	Lunch
13:30 - 14:00	Christos Saroglou, *Remarks on the conjectured log-Brunn-Minkowski inequality*
14:05 - 14:35	Petros Valettas, *Neighborhoods on the Grasmannian of marginals with bounded isotropic constant*
14:40 - 15:10	Patrick Spencer, *A note on intersection bodies and Lorentz balls in dimensions greater than 4*
15:10 - 15:30	Coffee Break
15:30 - 17:30	Informal Discussions
17:30 - 19:30	Dinner

Friday

7:00 - 9:00	Breakfast
9:00 - 11:30	Informal Discussions
10:00 - 11:00	Coffee Break
11:30 - 13:30	Lunch
	Checkout by 12 noon.

** 5-day workshop participants are welcome to use BIRS facilities (BIRS Coffee Lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

Geometric Tomography and Harmonic Analysis
March 9 - March 14, 2014

ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **Astrid Berg** (Vienna University of Technology)
Title: *Log-concavity properties of Minkowski valuations*
Abstract: In this talk, new log-concavity properties of rigid motion compatible Minkowski valuations of arbitrary degree are presented. All the Brunn-Minkowski inequalities for Minkowski valuations established before turn out to be special cases of our new results. From new monotonicity properties of these Minkowski valuations, we are able to deduce a complete characterization of equality cases without any of the smoothness assumptions required before. (Joint work with Lukas Parapatits, Franz E. Schuster and Manuel Weberndorfer).

Speaker: **Florian Besau** (Vienna University of Technology)
Title: *Binary Operations in Spherical Convex Geometry*
Abstract: We establish characterizations of binary operations between convex bodies on the Euclidean unit sphere. Our main result shows that the convex hull is essentially the only non-trivial projection covariant operation between pairs of convex bodies contained in open hemispheres. Moreover, we prove that any continuous and projection covariant binary operation between all proper spherical convex bodies must be trivial. (Joint work with Franz E. Schuster).

Speaker: **Ronen Eldan** (Microsoft Research)
Title: *A Two-Sided Estimate for the Gaussian Noise Stability Deficit*
Abstract: The Gaussian Noise Stability of a set A in Euclidean space is the probability that for a Gaussian vector X conditioned to be in A, a small Gaussian perturbation of X will also be in A. Borel's celebrated Isoperimetric inequality states that a half-space maximizes noise stability among sets with the same Gaussian measure. We will present a novel short proof of this inequality, based on stochastic calculus. Moreover, we prove an almost tight, two-sided, dimension-free robustness estimate for this inequality: We show that the deficit between the noise stability of a set A and an equally probable half-space H can be controlled by a function of the distance between the corresponding centroids. As a consequence, we prove a conjecture by Mossel and Neeman, who used the total-variation distance.

Speaker: **Matthieu Fradelizi** (Université Paris-Est Marne-la-Vallée)
Title: *Functional versions of L_p-affine surface area and entropy inequalities*
Abstract: We introduce a functional form of the L_p-affine surface area for convex bodies for log-concave and s-concave functions. We show that this generalizes the original L_p-affine surface area for convex bodies, we prove duality relations and affine isoperimetric inequalities for log concave and s-concave functions. This leads to an inverse log-Sobolev inequality for log-concave and s-concave densities. In collaboration with Umut Caglar, Olivier Guédon, Joseph Lehec, Carsten Schütt and Elisabeth Werner.

Speaker: **Eric Grinberg** (University of Massachusetts, Boston)
Title: *The Torus Transform on Symmetric Spaces of Compact Type*
Abstract: TBA

Speaker: **Yoav Kallus** (Princeton University)
Title: *The Ball as a Pessimal Shape for Packing*
Abstract: The ball is conjectured to be the worst shape out of all 3-dimensional centrally-symmetric convex shapes in terms of its lattice packing density. I will show that it is at least a local minimum, by relating the problem to tomographic data about the shape to be packed, recasting the problem as a tomographic problem. In other dimensions where the densest packing of balls is known (2–8, 24) the ball is not even a local minimum.

Speaker: **Joseph Lehec** (Université Paris-Dauphine)
Title: *Bounding the norm of a log-concave vector via thin-shell estimates*
Abstract: Chaining techniques show that if X is an isotropic log-concave random vector in R^n and G is a standard Gaussian vector then

$$E||X|| < Cn^{1/4}E||G||$$

for any norm $||*||$, where C is a universal constant. We show that under the thin-shell hypothesis $n^{1/4}$ can be replaced by $\log(n)$ in the inequality. As an application we recover (up to a logarithmic factor) the fact that a positive answer to the thin-shell conjecture implies a positive answer to the slicing problem. The proof is based on a stochastic construction due to Ronen Eldan and on a correlation inequality which we learnt from Bernard Maurey. This is a joint work with Ronen Eldan.

Speaker: **Alexander Litvak** (University of Alberta)
Title: *Approximating the covariance matrix with heavy tailed columns and RIP*
Abstract: Let A be a matrix whose columns $X_1, \dots, X_N$ are independent random vectors in $\mathbb{R}^n$. Assume that p-th moments of $\langle X_i, a\rangle$, $a \in S^{n-1}$, $i \le N$, are uniformly bounded. For $p > 4$ we prove that with high probability A has the Restricted Isometry Property (RIP) provided that Euclidean norms $|X_i|$ are concentrated around $\sqrt{n}$ and that the covariance matrix is well approximated by the empirical covariance matrix provided that $\max_i |X_i| \le C(nN)^{1/4}$. We also provide estimates for RIP when $\mathbb{E}\,\phi\left(|\langle X_i, a\rangle|\right) \le 1$ for $\phi(t) = (1/2)\exp(t^\alpha)$, with $\alpha \in (0, 2]$. (Joint work with O. Guédon, A. Pajor, N. Tomczak-Jaegermann).

Speaker: **Galyna Livshyts** (Kent State University)
Title: *Maximal surface area of a convex set in $\mathbb{R}^n$ with respect to log concave rotation invariant measures.*
Abstract: It was shown by K. Ball and F. Nazarov, that the maximal surface area of a convex set in $\mathbb{R}^n$ with respect to the Standard Gaussian measure is of order $n^{\frac{1}{4}}$ We establish the analogous result for all rotation invariant log concave probability measures. We show that the maximal surface area with respect to such measures is of order $\frac{\sqrt{n}}{\sqrt[4]{Var|X|}\sqrt{\mathbb{E}|X|}}$, where X is a random vector in $\mathbb{R}^n$ distributed with respect to the measure.

Speaker: **Yves Martinez-Maure** (Institut de Mathematiques de Jussieu)
Title: *Can hedgehogs be useful for Geometric Tomography?*
Abstract: Hedgehogs are geometrical objects that describe the Minkowski differences of arbitrary convex bodies in the Euclidean linear space $\mathbb{R}^{n+1}$. Many notions from the classical theory of convexity can be extended to hedgehogs and quite a number of classical results find their counterparts. Of course, a few adaptations are necessary. In particular, areas and volumes have to be replaced by their algebraic versions. In fact, many notions that are useful in Geometric Tomography are related to hedgehogs. For instance, stars bodies are related to hedgehogs by duality. I will try to expound the present state of our knowledge on the subject, insisting on notions and examples that can be useful for Geometric Tomography.

Speaker: **Mathieu Meyer** (Université Paris-Est Marne-la-Vallée)
Title: *Affine points and duality*
Abstract: Contained in the title.

Speaker: **Vitali Milman** (Tel Aviv University)
Title: *Characterizing summations of convex sets*
Abstract: We will discuss addition operations between convex sets. We show that under a very short list of natural assumptions, one has polynomiality of volume only for the Minkowski addition. We will also present two other characterization theorems. For the first theorem we define the induced homothety of an addition operation, and characterize additions by this homothety. The second theorem characterizes all additions which satisfy a short list of natural conditions. This is a joint work with Liran Rotem

Speaker: **Peter Pivovarov** (University of Missouri)
Title: *Volume of the polar of random sets and shadow systems*
Abstract: I will discuss inequalities for the volume of the polar of random sets, generated for instance by the convex hull of independent random vectors in Euclidean space. Extremizers are given by random vectors uniformly distributed in Euclidean balls. This provides a randomized extension of the Blaschke-Santal inequality which, in turn, can be derived by the law of large numbers. The method involves generalized shadow systems, their connection to Busemann type inequalities, and how they interact with functional rearrangement inequalities. (Joint work with D. Cordero-Erausquin, M. Fradelizi , G. Paouris).

Speaker: **Luis Rademacher** (Ohio State University)
Title: *TBA*
Abstract: TBA

Speaker: **Liran Rotem** (Tel Aviv University)
Title: *On isotropicity with respect to a measure*
Abstract: In this talk we will define what it means for a body to be isotropic with respect to a measure. As it turns out, It is surprisingly non-trivial to check if a given body can be put in an isotropic position with respect to a given measure. We will explain this phenomenon, and will show that any body can be put in isotoropic position with respect to any rotation invariant measure. We will then discuss the properties of this position, and its relation to the well-known M-position. We will also give bounds on the corresponding isotropic constant. These results extend a theorem of Bobkov, where the measure involved is the Gaussian.

Speaker: **Mark Rudelson** (University of Michigan)
Title: *Small ball probabilities for linear images of high dimensional distributions*
Abstract: We study concentration properties of random vectors of the form AX, where X has independent coordinates and A is a given matrix. We show that the distribution of AX is well spread in space whenever the distributions of the coordinates of X are well spread on the line. This extends the bound obtained by Paouris for log-concave measures to random vectors with independent coordinates.

Joint work with Roman Vershynin.

Speaker: **Christos Saroglou** (Texas A&M University)
Title: *Remarks on the conjectured log-Brunn-Minkowski inequality*
Abstract: Böröczky, Lutwak, Yang and Zhang recently conjectured a certain strengthening of the Brunn-Minkowski inequality for symmetric convex bodies, the so-called log-Brunn-Minkowski inequality. We establish this inequality together with its equality cases for pairs of unconditional convex bodies with respect to the same orthonormal basis. Applications of this fact are discussed. Moreover, we prove that the log-Brunn-Minkowski inequality is equivalent to the (B)-Theorem for the uniform measure of the cube (this has been proven by Cordero-Erasquin, Fradelizi and Maurey for the gaussian measure instead).

Speaker: **Franz Schuster** (Vienna University of Technology)
Title: *Cosine and Radon Transforms in the Theory of Minkowski Valuations*
Abstract: In this talk, we discuss three different ways to represent an even smooth Minkowski valuation

which intertwines rigid motions, namely Crofton measures, Klain bodies, and generating functions. Moreover, we present explicit formulas, involving cosine and Radon transforms on Grassmannians, relating these different descriptions of such Minkowski valuations. (Joint work with Thomas Wannerer).

Speaker: **Susanna Spektor** (University of Alberta)
Title: *Quantitative version of a Silverstein's result*
Abstract: The talk is devoted to the quantitative version of a Silverstein's Theorem on the 4-th moment condition for convergence in probability of the norm of a random matrix. More precisely, we show that for a random matrix with i.i.d. entries, satisfying certain natural conditions, its norm cannot be small.

Speaker: **Patrick Spencer** (University of Missouri)
Title: *A note on intersection bodies and Lorentz balls in dimensions greater than 4.*
Abstract: We show that the unit ball of the n-dimensional Lorentz space $\ell^n_{w,q}$ is not an intersection body for $q > 2$ and $n > 4$.

Speaker: **Carsten Schütt** (University of Kiel)
Title: *Order statistics*
Abstract: The expression $\underset{1\le i\le n}{\text{k-min}}\, a_i$ denotes the k-smallest of all the numbers $a_1, \dots, a_n$.

Let $M = (M_1, \dots, M_n)$ be an n-tuple of increasing maps from $[0, \infty)$ onto itself. Then for all $x \in \mathbb{R}^n$ we define

$$\|x\|_M = \inf\left\{ \rho \,\middle|\, \sum_{i=1}^n M_i\left(\frac{x_i}{\rho}\right) \le 1 \right\}.$$

Theorem 0.1 *There is a universal constant C such that for all independent, positive and uniformly subregular sequences of random variables $\xi_1, \dots, \xi_n$ with continuous distribution functions $F_1, \dots, F_n$*

$$\mathbb{E}\left(\underset{1\le i\le n}{\text{k-min}}\, \xi_i\right) \sim \text{med}\left(\underset{1\le i\le n}{\text{k-min}}\, \xi_i\right) \sim \frac{1}{\|(1,\dots,1)\|_{\frac{F}{k-\frac{1}{2}}}}.$$

(Joint work with S. Kwapien).

Speaker: **Konstantin Tikhomirov** (University of Alberta)
Title: *On the distance of polytopes with few vertices to the Euclidean ball*
Abstract: Let n, N be natural numbers satisfying $n + 1 \le N \le 2n$, B_2^n be the unit Euclidean ball in $\mathbb{R}^n$ and $P \subset B_2^n$ be a convex n-dimensional polytope with N vertices and the origin in its interior. We prove that

$$\inf\{\lambda \ge 1 : B_2^n \subset \lambda P\} \ge cn/\sqrt{N-n},$$

where $c > 0$ is a universal constant. As an immediate corollary, for any covering of S^{n-1} by N spherical caps of geodesic radius ϕ, $\cos(\phi) \le C\sqrt{N-n}/n$ for an absolute constant $C > 0$. Both estimates are optimal up to the constant multiples c, C.

Speaker: **Petros Valettas** (Texas A&M University)
Title: *Neighborhoods on the Grasmannian of marginals with bounded isotropic constant.*
Abstract: We prove that for any isotropic, log-concave probability measure μ on $\mathbb{R}^n$ any k-dimensional ($k \le \sqrt{n}$) subspace of $\mathbb{R}^n$ and every $\epsilon > 0$ there exists k-dimensional subspace E such that $d(E, F) < \epsilon$ and the corresponding marginal of μ onto E has bounded isotropic in terms of ϵ. This is joint work with G. Paouris.

Speaker: **Wolfgang Weil** (Karlsruhe Institute of Technology)
Title: *Area Measures and Fourier Operators*

Abstract: In Goodey-Yaskin-Yaskina (2009) the use of certain spherical operators I_p which bare based on Fourier transforms in Convex Geometry was demonstrated. With the help of these operators, we give a representation of mean section bodies of a body K in terms of area measures of K. This leads to a new Crofton-type formula for area measures and also to a local version of the Principal Kinematic Formula. The talk is based on a forthcoming paper with Paul Goodey and on a joint project with Paul Goodey and Daniel Hug.

Speaker: **Elisabeth Werner** (Case Western Reserve University)
Title: *Equality characterization and stability for entropy inequalities*
Abstract: This talk is based on joint work with T. Yolcu.

We characterize the equality case in a recently established entropy inequality. To do so, we show that characterization of equality is equivalent to uniqueness of the solution of a certain Monge Ampere differential equation. We prove the uniqueness of the solutionusing methods from mass transport, due to Brenier and Gangbo-McCann.

We then give stability versions for this entropy inequality, as well as for a reverse log Sobolev inequality and for the L_p-affine isoperimetric inequalities for both, log concave functions and convex bodies. In the case of convex bodies such stability results have only been known in all dimensions for $p = 1$ and for $p > 1$ only for 0-symmetric bodies in the plane.

Speaker: **Pierre Youssef** (University of Alberta)
Title: *Almost orthogonal contact points*
Abstract: Let $K \subset \mathbb{R}^n$ be a convex body in John's position. The aim is to find a "large" number of contact points which is almost equivalent to an orthonormal basis of l_2. We prove this as a consequence of a more general result which asks about extracting a well conditioned block inside a given matrix.