Rigorously Verified Computing for Infinite Dimensional Nonlinear Dynamics

September 21-26, 2014

Organizers: J.-P. Lessard (Université Laval), K. Mischaikow (Rutgers University), S. Rump (Hamburg University of Technology), J.B. van den Berg (VU University Amsterdam), JF Williams (Simon Fraser University)

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in the lecture theater in the TransCanada Pipelines Pavilion (TCPL). An LCD projector, a laptop, a document camera, and blackboards are available for presentations.

SCHEDULE

Sunday

16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
17:30–19:30	Buffet Dinner, Sally Borden Building

Monday

7:00–8:45	Breakfast
8:45–9:00	Introduction and Welcome by BIRS Station Manager, TCPL
9:00–9:45	**Lecture 1 : J.-P. Lessard**
9:45–10:15	Discussion (identifying future lines of research and important problems to work on)
10:15–10:45	Coffee Break, TCPL
10:45–12:00	Break up into groups
12:00–13:30	Lunch There is an optional guided tour of the Banff Centre from 13:00 to 14:00. The meeting is in the 2nd floor lounge, Corbett Hall.
14:00–14:10	Group Photo; meet in foyer of TCPL (photograph will be taken outdoors so a jacket might be required).
14:10–15:00	**Student presentations**
15:00–15:30	Coffee Break, TCPL
15:30–16:15	**Lecture 2 : S. Rump**
16:15–16:45	Discussion (identifying future lines of research and important problems to work on)
16:45–18:00	Break up into groups
18:00–19:30	Dinner

Tuesday

7:00–9:00	Breakfast
9:00–9:45	**Lecture 3 : J.D. Mireles James**
9:45–10:15	Discussion (identifying future lines of research and important problems to work on)
10:15–10:45	Coffee Break, TCPL
10:45–12:00	Break up into groups
12:00–13:30	Lunch
13:30–14:15	**Lecture 4 : P. Zgliczynski**
14:15–15:00	Discussion (identifying future lines of research and important problems to work on)
15:00–15:30	Coffee Break, TCPL
15:30–18:00	Break up into groups
18:00–19:30	Dinner

Wednesday

7:00–9:00	Breakfast
9:00–9:45	**Lecture 5 : M. Plum**
9:45–10:15	Discussion (identifying future lines of research and important problems to work on)
10:15–10:45	Coffee Break, TCPL
10:45–12:00	Break up into groups
12:00–13:30	Lunch
	Free Afternoon
17:30–19:30	Dinner

Thursday

7:00–9:00	Breakfast
9:00–9:45	**Lecture 6 : C. Reinhardt (9:00–9:20) and J. Cyranka (9:25–9:45)**
9:45–10:15	Discussion (identifying future lines of research and important problems to work on)
10:15–10:45	Coffee Break, TCPL
10:45–12:00	Break up into groups
12:00–13:30	Lunch
13:30–14:15	**Lecture 7 : A. Neumaier**
14:15–15:00	Discussion (identifying future lines of research and important problems to work on)
15:00–15:30	Coffee Break, TCPL
15:30–18:00	Break up into groups
18:00–19:30	Dinner

Friday

7:00–9:00	Breakfast
9:00–10:15	Discussion
10:15–10:45	Coffee Break, TCPL
10:45–12:00	Discussion
12:00–13:30	Lunch
Checkout by 12 noon.	

** 5-day workshop participants are welcome to use BIRS facilities (BIRS Coffee Lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

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ABSTRACTS

Speaker: **Jacek Cyranka** (University of Warsaw)
Title: *Some results on a 2D convection-diffusion PDE*
Abstract: First, I will present some interesting conjectures, supported by numerical calculations, regarding a 2D convection-diffusion PDE $P(u) \cdot \nabla u - \Delta u = \lambda F(x)$, where P is the Helmholz projector. We are especially interested in analyzing the structure of solutions for large external forces. We approach this problem by variety of techniques: rigorous numerics , linear algebra , and functional analysis . As this is work in progress, I will report on current state of the research.

Speaker: **Jean-Philippe Lessard** (Université Laval)
Title: *Rigorous computations for infinite dimensional problems: a functional analytic approach*
Abstract: Studying and proving existence of solutions of nonlinear dynamical systems using standard analytic techniques is a challenging problem. In particular, this problem is even more challenging for partial differential equations, variational problems or functional delay equations which are naturally defined on infinite dimensional function spaces. The goal of this talk is to present rigorous numerical technique relying on functional analytic methods to prove existence of steady states, time periodic solutions, traveling waves and connecting orbits for the above mentioned dynamical systems.We will spend some time identifying difficulties of the proposed approach as well as time to identify future directions of research.

Speaker: **Jason Mireles James** (Florida Atlantic University)
Title: *Fixed point approach to rigorous validated computation of connecting orbits in infinite dimensions*
Abstract: Many questions about the global behavior of a nonlinear system are addressed by studying connecting orbits between invariant sets. When the invariant set is normally hyperbolic then connecting orbits arise as intersections of stable/unstable manifolds. For finite dimensional dynamical systems there are a number of powerful tools which facilitate rigorous computation of these intersections. I will describe one approach which is based on solving a certain projected boundary value problem. Careful study of the local stable/unstable manifolds (and their derivatives) is a critical ingredient in this approach. I will discuss some work in progress which aims to extend the approach to infinite dimensions. The most delicate part of the argument is managing the infinite dimensional stable manifold with the help of the computer.

Speaker: **Arnold Neumaier** (Universität Wien)
Title: *To come*
Abstract: To come

Speaker: **Michael Plum** (Karlsruhe Institute of Technology)
Title: *Existence and multiplicity proofs for semilinear elliptic boundary value problems by computer assistance*
Abstract: Many boundary value problems for semilinear elliptic partial differential equations allow very stable numerical computations of approximate solutions, but are still lacking analytical existence proofs. In the talk, we propose a method which exploits the knowledge of a "good" numerical approximate solution, in order to provide a rigorous proof of existence of an exact solution close to the approximate one. This goal is achieved by a fixed-point argument which takes all numerical errors into account, and thus gives a mathematical proof which is not "worse" than any purely analytical one. The method is used to prove existence and multiplicity statements for some specific examples, including cases where purely analytical methods had not been successful.

Speaker: **Christian Reinhardt** (VU University Amsterdam)
Title: *Rigorous numerics for ODEs using Chebyshev series*
Abstract: In this talk we give an outline of how to use Chebyshev series to rigorously compute solutions of initial and boundary value problems induced by ordinary differential equations. The basic idea is to apply a spectral approach and reformulate the respective problem under consideration as a zero finding problem for a nonlinear operator defined on a space of suitably decaying Chebyshev coefficients. The validated solution of this problem is obtained by studying a fixed point operator associated with an approximate zero using computational fixed point methods. We explain how our methods can be combined with rigorous techniques for the computation of (un)stable manifolds to compute connecting orbits. We also indicate several extensions of the methods that are currently under investigation.

Speaker: **Siegfried Rump** (Hamburg University of Technology)
Title: *Computer-assisted proofs using floating-point arithmetic*
Abstract: We present methods for computer-assisted proofs, in particular using floating-point arithmetic. The goal is to produce verified results with the rigor of a mathematical proof. A number of so-called "verification methods" are discussed together with the necessary computational tools. We use INTLAB, the Matlab toolbox for reliable computing, allowing to write simple and easy-to-read code. Amongst the tools is affine arithmetic which, compared to interval arithmetic, produces sometimes better results.

Speaker: **Piotr Zgliczynski** (Jagiellonian University)
Title: *Geometric methods in the integration of evolutionary problems in infinite dimension*
Abstract: With an eye toward the computer assisted proofs (CAP) in the dynamics of infinite dimensional systems we will discuss some algorithms for the rigorous integration of evolutionary problems. When constructing such a procedure we need to keep in mind what kind of abstract theorems should be used in CAP and this implies some geometric postulates to be fullfilled by the proposed integration algorithms.

I will discuss the main ideas of the algorithms for the integration the following problems

- dissipative PDEs with periodic boundary conditions
- delay equations
- PDEs with indefinite tail with periodic boundary conditions
- dissipative PDEs in other bases

The talk will become more and more tantative as we will progress along the above list. I will try to highlight the basic ideas and the main difficulties.

Open problems and challenges will be also indicated.