Abstraction and Multi-Encodings in SAT

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Abstraction: Idea & Motivation

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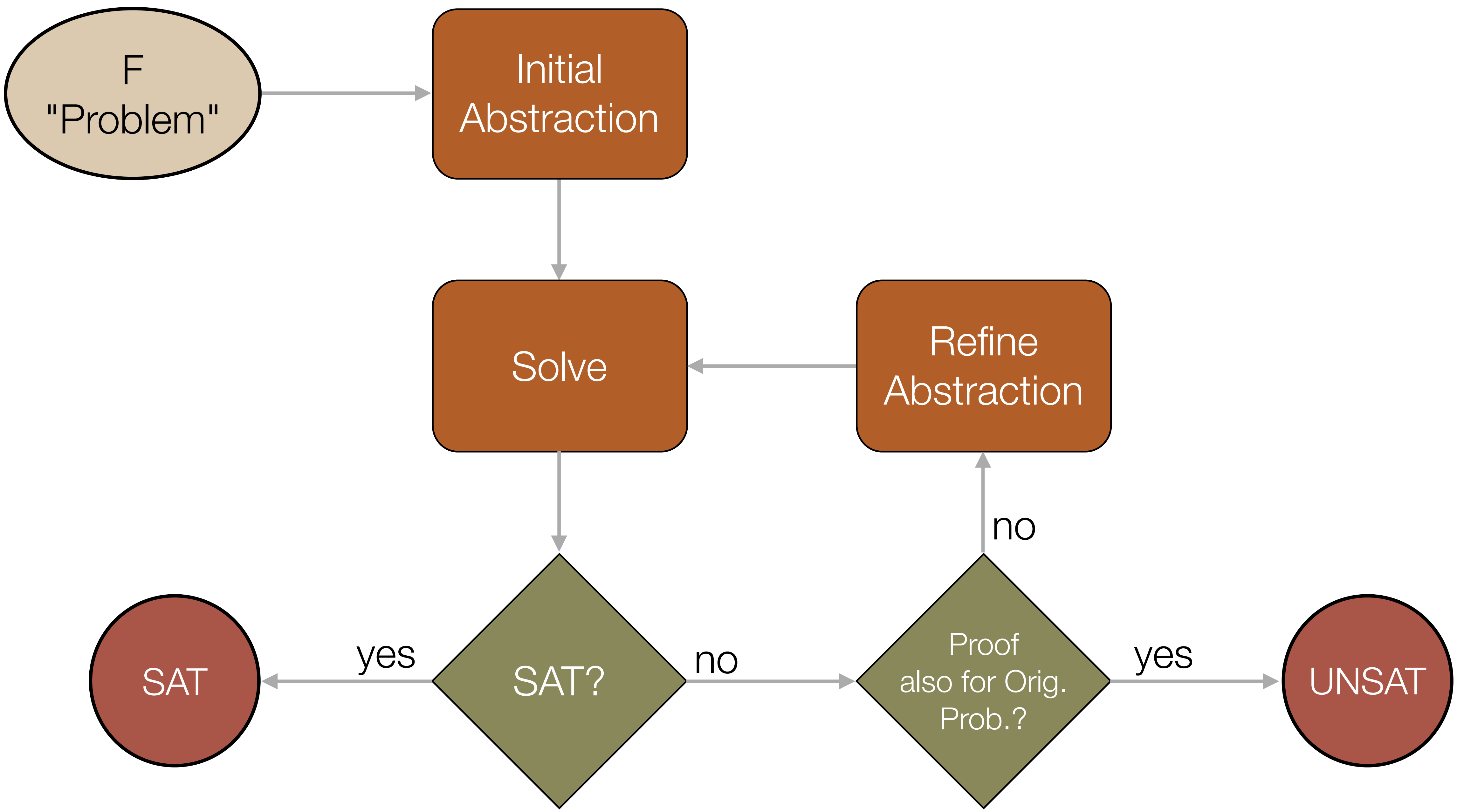
- **Question: Does S ⇒ P hold?**
  - **S**: System
  - **P**: Property
- **Applications:**
  - Model Checking, Software Verification, ...
- **Observations:**
  - Large state space (large formulas)
  - Only small part of **S** might be needed to show **P**
- **Abstraction:**
  - Replace **S** (or **P**) by approximate formalization **S'** (or **P'**)

  ➜ Hope that **S' ⇒ P** is easier to prove than **S ⇒ P**
- **In what follows:** "F SAT/UNSAT" instead of "S ⇒ P valid"

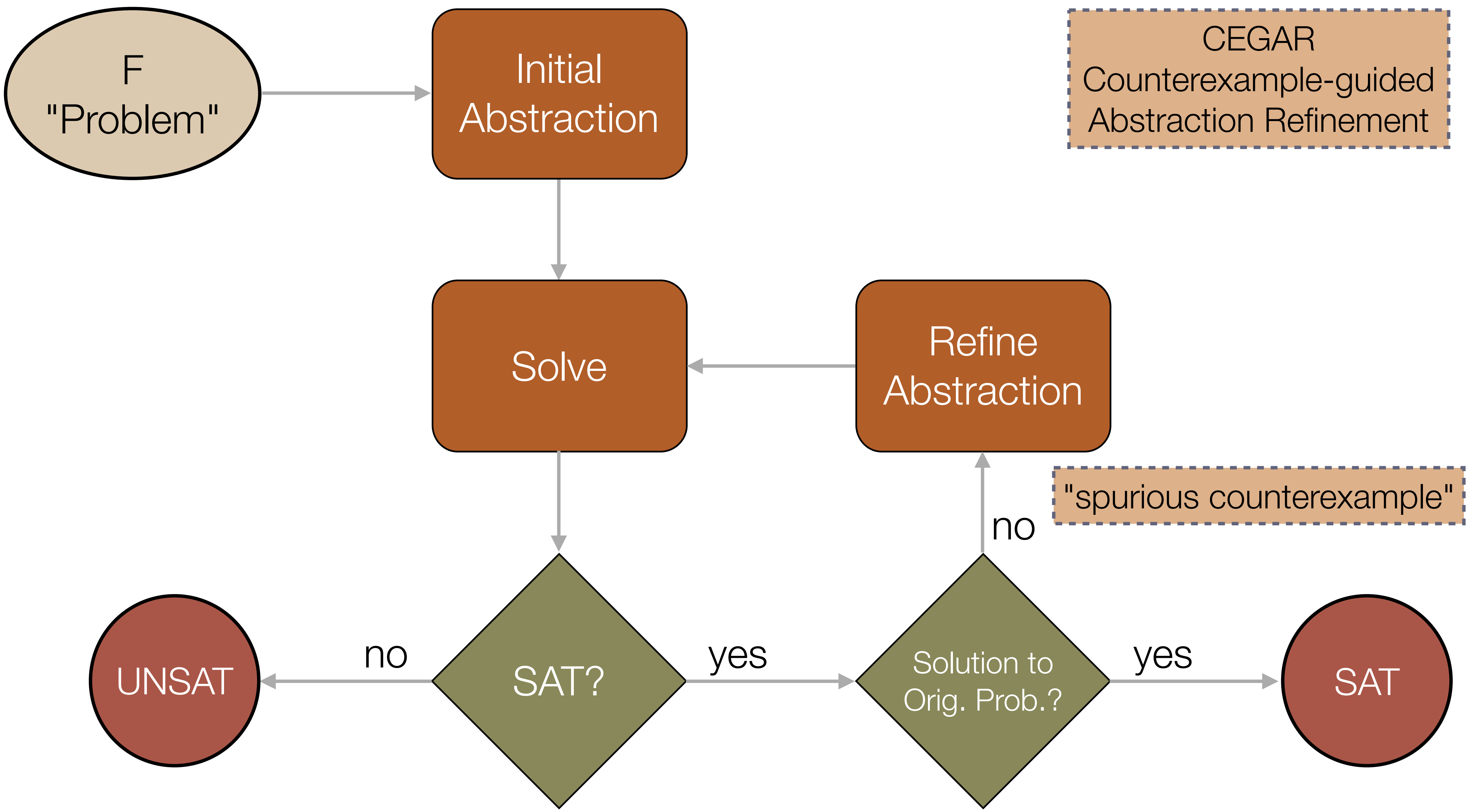
Abstraction

- **Two variants of abstraction:**
  - Under-approximation: abstraction has **fewer** solutions
  - Over-approximation: abstraction has **more** solutions
- **Definitions:**
  - F' is an *under-approximation* of F if: $\alpha \models F'$ **implies** $\alpha \models F$
  - F' is an *over-approximation* of F if: $\alpha \models F$ **implies** $\alpha \models F'$
- **Simple properties:**
  - For all F: F' = `false` is an under-approximation of F
  - For all F: F' = `true` is an over-approximation of F
- **Abstraction** is an extremely successful technique in model checking and (bit-blasting) SMT solvers
- **Refinement:** "better" approximation
- **Refinement loop:** gradually compute improved approximations

Under-Approximation



Over-Approximation



CEGAR: What do we need?

- **Initial abstraction**
  - Should be close to problem, but easy to solve
- **Checking abstract counterexample**
- **Abstraction refinement**
  - Better approximation
  - Based on counterexample
- **Example: Check, whether F (in CNF) is satisfiable**
  - **Initial abstraction:** 2-clauses of F
  - **Checking abstract counterexample:** all clauses of F satisfied?
  - **Refinement:** add all clauses not satisfied by counterexample

CEGAR: Example

$F = \{ \{ a, b, \neg c \}, \{ a, c \}, \{ \neg a, b \}, \{ \neg b, \neg d \}, \{ \neg a, b, \neg c, \neg d \}, \{ \neg b, d \} \}$

1. Initial approximation: (2-clauses of F)

   $F_0^{over} = \{ \{ a, c \}, \{ \neg a, b \}, \{ \neg b, \neg d \}, \{ \neg b, d \} \}$

2. Solve: SAT, $\alpha = \{ \neg a, \neg b, c, \neg d \}$
3. Check α for F: $\alpha \not\models F$
4. Refine:

   $F_1^{over} = \{ \{ a, c \}, \{ \neg a, b \}, \{ \neg b, \neg d \}, \{ \neg b, d \}, \{ a, b, \neg c \} \}$

5. Solve: UNSAT

➜ Clause $\{ \neg a, b, \neg c, \neg d \}$ not needed

➜ CEGAR improves "locality"

Abstraction in SAT

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- **Abstraction not widely employed in SAT solving**
  - Exceptions: bounded model checking, SMT solvers
- **Why?**

	Total	MiniSAT	MiniSAT + Abstraction	MiniSAT + 10 Abstr. Rounds
SAT-Race 2010	**100**	**68**	**48**	**67**

- **Reasons:**
  - Abstraction-refinement approach in experiment too simple
  - Not enough high-level information for effective abstraction-refinement

Where Abstraction in SAT Works

- SMT solver, theory of arrays
- McCarthy (read-over-write) axiom:

$$\texttt{read}(\texttt{write}(a,i,x),j) = \begin{cases} x & \text{falls } i = j \\ \texttt{read}(a,i) & \text{sonst} \end{cases}$$

- Approximation: do not encode functional consistency for `read`
- **Example:** array logic formula F:

  $\texttt{read}(a, i) = x \ \wedge\ \texttt{read}(a, j) = y \ \wedge\ i = j \ \wedge\ x \neq y$

  - SAT encoding: (with 1-bit bit vectors)

    $\{ \{ \neg r_{a,i}, x \}, \{ r_{a,i}, \neg x \}, \{ \neg r_{a,j}, y \}, \{ r_{a,j}, \neg y \}, \{ \neg i, j \}, \{ i, \neg j \}\ \{ x, y \}, \{ \neg x, \neg y \},$

    $\{ \neg i, \neg j, \neg r_{a,i}, r_{a,j} \}, \{ \neg i, \neg j, r_{a,i}, \neg r_{a,j} \}, \{ i, j, \neg r_{a,i}, r_{a,j} \}, \{ i, j, r_{a,i}, \neg r_{a,j} \} \}$

  - Initial abstraction:

    $F_0^{over} = \{ \{ \neg r_{a,i}, x \}, \{ r_{a,i}, \neg x \}, \{ \neg r_{a,i}, y \}, \{ r_{a,i}, \neg y \}, \{ \neg i, j \}, \{ i, \neg j \},$
    $\{ x, y \}, \{ \neg x, \neg y \} \}$

Where Abstraction in SAT Works (II)

- Modulo-Operation in bit-blasting SMT solver

  $z = x \mathtt{\ mod\ } y$ (`bvurem`, truncated division)

  - Encoded precisely using a circuit (e.g. for 32-bit signed integers)
- **Possible abstractions:** replace $z = x \mathtt{\ mod\ } y$ by
  - $0 \leq z < y$
  - $z = x >> k$, if $y = 2^k$, where k is constant
  - $x < y => z = x$
  - $y \leq x < 2y \;\Rightarrow\; z = x - y$

Abstraction in SAT

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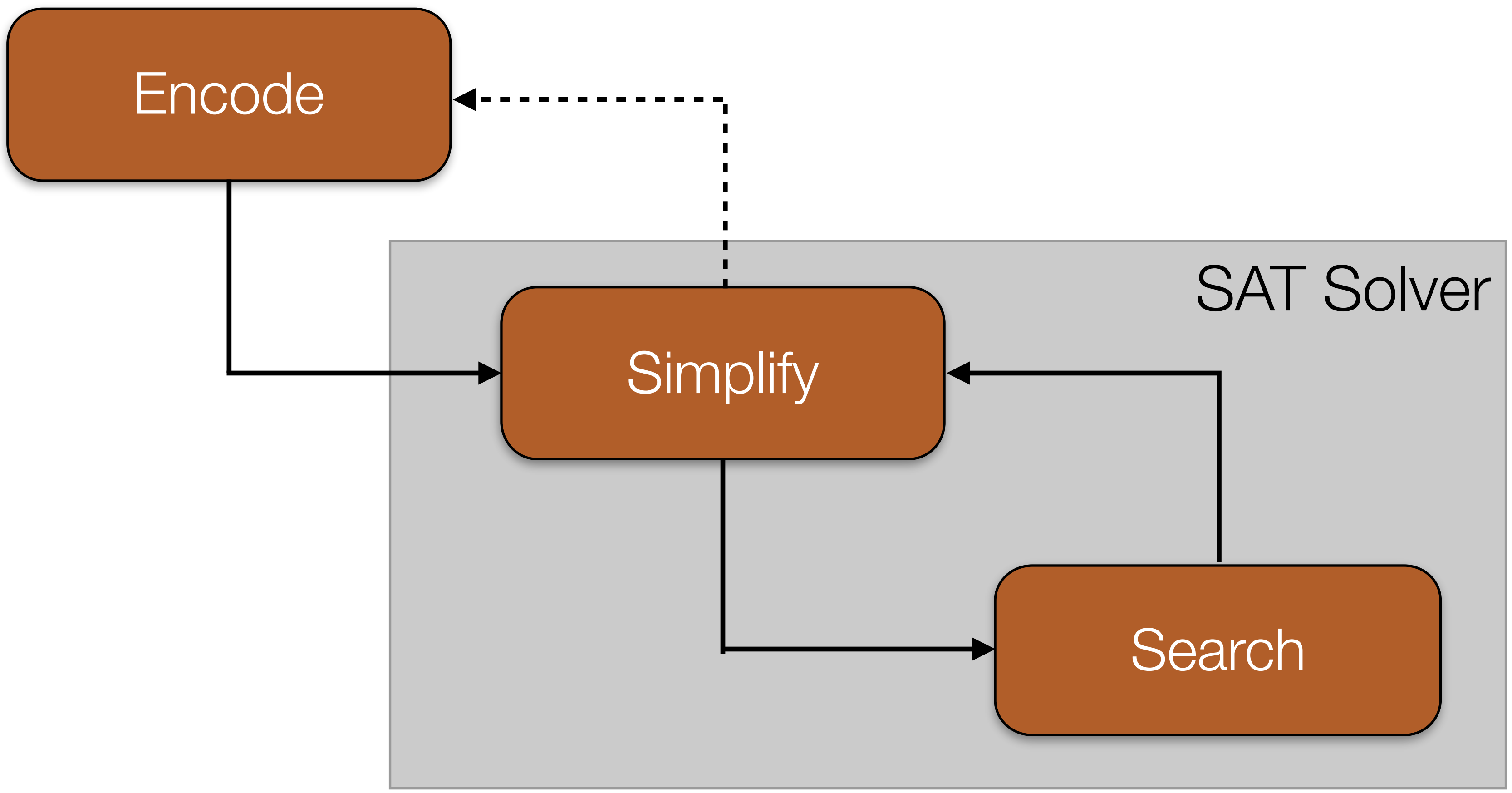
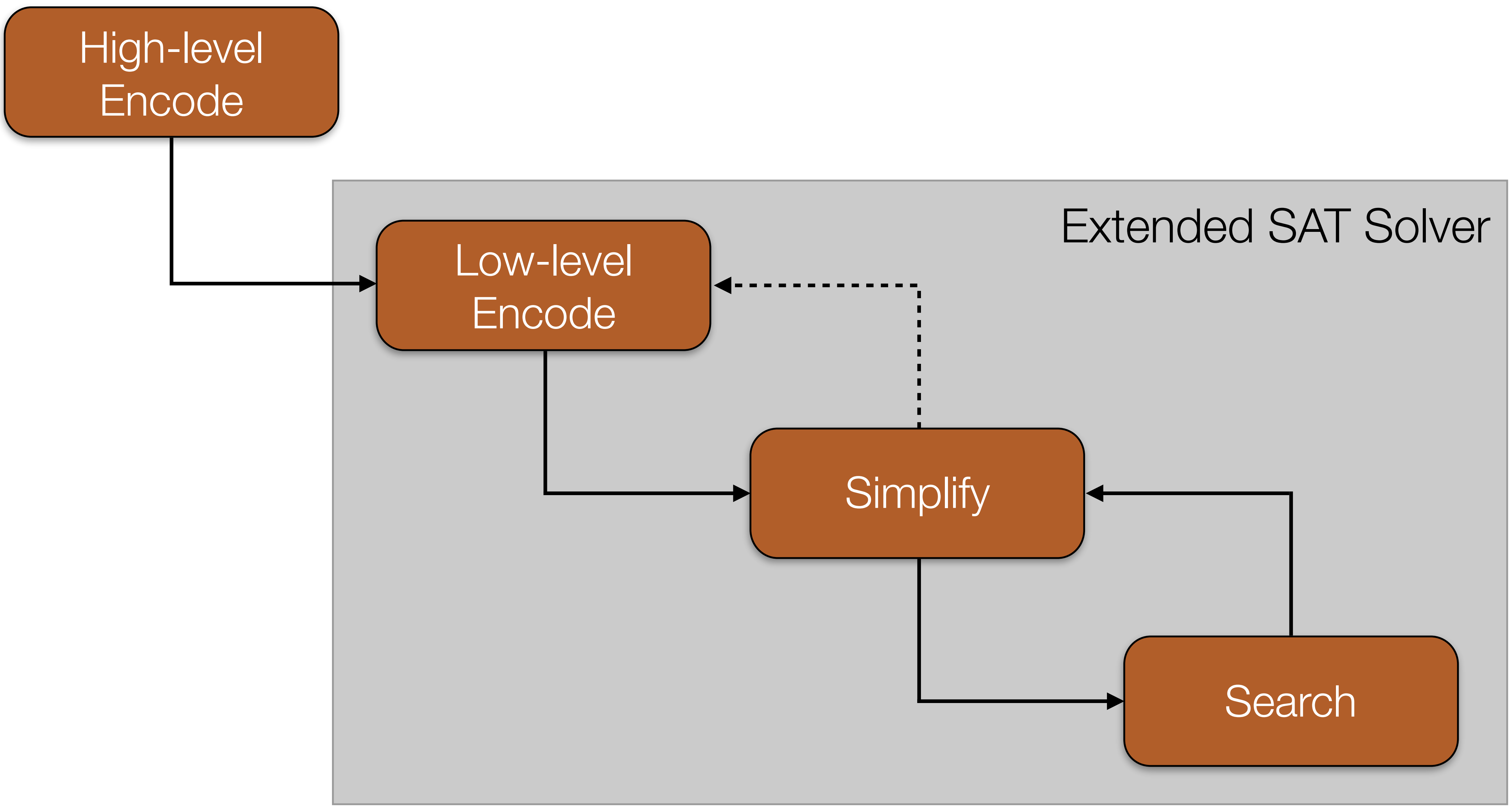
- **Abstraction seems to work, if high-level information can be used, e.g.**
  - Groups of clauses that encode the same high-level constraint
  - Semantics of group of clauses that can be abstracted
- **Extend SAT solvers?**
  - General abstraction-based SAT engine
  - Use high-level information about problem for abstractions
- **Possible ways to realize:**
  - Mark groups of clauses by "abstraction levels"
  - Provide different encodings ("**Multi-encoding**")
    - E.g., different ways to encode a cardinality constraint
  - High-level description language

Current SAT Solvers



[Adapted from Biere: Understanding Modern SAT Solvers]

Extended SAT Solvers



Summary

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- Abstraction techniques successful in certain SAT applications
- **Research direction:** make abstractions broadly applicable by
  - Passing high-level problem information to SAT solver
  - Using multiple encodings simultaneously
- **Does it also help in proof complexity research? Maybe**
  - Automatizability of proof procedures
  - Analyze SAT in conjunction with problem encoding
  - Shorter proofs by using multiple encodings simultaneously?