Entropy Methods, PDEs, Functional Inequalities, and Applications

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in the TransCanada Pipelines Pavilion (TCPL). LCD projector and blackboards are available for presentations. Ceiling-mounted video cameras are installed in the main lecture room of 201, TCPL. To enable both half workshops to use the recording facilities during the week, one group will be assigned to Room 201 during the mornings and to Room 202 in the afternoons. The Station Manager will confirm these details with you prior to your workshop.

SCHEDULE

Sunday

16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
17:30–19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall Beverages and a small assortment of snacks are available on a cash honor system.

Monday

	Breakfast
8:45–9:00	Introduction and Welcome by BIRS Station Manager, TCPL
9:00–9:40	K.-T. Sturm: "The curvature-dimension condition with finite N: consequences and transformations"
9:45–10:10	B. Nazaret: "Entropy method in some weighted fast diffusion equations and related functional inequalities"
	Coffee Break, TCPL
10:40–11:20	I. Gentil: "Solution of a class of reaction-diffusion systems via logarithmic Sobolev inequality"
11:25–11:50	G. Jankowiak: "Sobolev and Hardy-Littlewood-Sobolev inequalities"
	Lunch
13:00–14:00	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:00–14:40	C. Schmeiser: "Convergence to equilibrium in a model for the distribution of filament ends along the leading edge of a lamellipodium"
14:45–15:10	M.-T. Wolfram: "On numerical simulations of nonlinear convection-aggregation equations by evolving diffeomorphisms"
	Coffee Break, TCPL
15:40–16:05	A. Blanchet: "The parabolic-parabolic Keller-Segel system as a gradient flow"
16:10–16:35	P. Laurençot: "A thin film approximation of the Muskat problem"
16:40–17:05	C. Léonard: "An entropic approach to Navier-Stokes equation"
	Dinner

Tuesday

	Breakfast
9:00–9:25	M. Liero: "Dissipation distances for reaction-diffusion equations - the Hellinger–Kantorovich distance"
9:30–10:10	G. Savaré: "Entropy-transport optimization problems and the Hellinger-Kantorovich distance"
	Coffee Break, TCPL
10:40–11:20	J. Maas: "A gradient flow approach to chemical master equations"
11:25–11:50	U. Stefanelli: "Nonlinear evolution as convex minimization"
11:55	Group Photo; meet in the lobby of TCPL (photo will be taken outside)
	Lunch
14:00–14:40	J.A. Carrillo: "Minimizing interaction energies"
14:45–15:10	T. Laurent: "Dimensionality of local minimizers of the interaction energy"
	Coffee Break, TCPL
15:40–16:05	L. Wu: "Nonlocal interaction equations in heterogeneous environments with boundaries"
16:10–16:35	R. Stanczy: "Entropy methods for systems of gravitating particles"
16:40–17:05	K. Craig: "A blob method for the aggregation equation"
	Dinner

Wednesday

	Breakfast
9:00–9:40	P. Degond: "From kinetic to macroscopic models through local Nash equilibria"
9:45–10:10	A. Bertozzi: "Contagion shocks in one dimension"
	Coffee Break, TCPL
10:40–11:20	K. Fellner: "Towards global existence and optimal equilibration rates for reaction-diffusion-systems"
	Lunch
	Free Afternoon
	Dinner

Thursday

	Breakfast
9:15–9:55	A. Arnold: "Entropy method for hypocoercive Fokker-Planck equations with linear drift"
10:00–10:25	L. Desvillettes: "The interplay between entropy and duality methods for parabolic systems"
	Coffee Break, TCPL
10:55–11:20	J.A. Cañizo: "Entropy-entropy dissipation inequality for the linear Boltzmann equation"
11:25–11:50	I. Kim: "Approximating oblique boundary problems with Fokker-Planck type equations"
	Lunch
14:00–14:40	A. Stevens: "Haptotaxis and trail following: qualitative behavior of a drift-diffusion model coupled to an ODE"
14:45–15:10	Y. Yao: "Finite time singularity of a vortex patch model in the half plane"
	Coffee Break, TCPL
15:40–16:05	L. Chen: "Critical exponents for degenerate Keller-Segel system"
16:10–16:35	M. Chugunova: "On the Benilov-Vynnycky blow-up problem"
16:40–17:05	Y. Huang: "Long time asymptotics of a porous medium equation with fractional pressure"
	Dinner

Friday

	Breakfast
9:00–9:40	F. Filbet: "On discrete functional inequalities for some finite volume schemes"
9:45–10:10	A. Tudorascu: "One-dimensional pressureless gas systems with/without viscosity"
10:15–10:55	J. Zimmer: "Scale-bridging for entropic flows in the presence of energy or noise"
	Coffee Break and Unstructured Discussions, TCPL
	Lunch
	Departure
Checkout by 12 noon.	

** 5-day workshop participants are welcome to use BIRS facilities (BIRS 2nd floor lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

ABSTRACTS
(in alphabetic order by speaker surname)

Anton Arnold (TU Wien)
Entropy method for hypocoercive Fokker-Planck equations with linear drift
In the last 15 years the entropy method became an invaluable tool for analyzing the large-time behavior (in particular the convergence to a steady state) for wide classes of PDEs: starting from linear Fokker-Planck equations, to various dissipative kinetic models, and up to several quasi-linear equations. For Fokker-Planck equations, the essence of the method is to first derive a differential inequality between the first and second time derivative of the relative entropy, and then between the entropy dissipation and the entropy. For degenerate parabolic equations, the entropy dissipation may vanish for states other than the equilibrium. Hence, the standard entropy method does not carry over. For hypocoercive Fokker-Planck equations, we first establish a condition that is equivalent to the existence of a unique normalised steady state. By introducing an auxiliary functional (of entropy dissipation type) we prove the exponential decay of the solution towards the steady state in relative entropy. Originally, this requires the initial condition to lie in a weighted H^1-space. But exploiting the parabolic/hypocoercive regularization, this requirement can be reduced to a finite relative entropy. Finally, we show that the obtained rate is indeed sharp (both for the logarithmic and quadratic entropy).
References:

1. J. Erb, A. Arnold. Sharp entropy decay for hypocoercive Fokker-Planck equations with linear drift, preprint 2014.

2. A. Arnold, E. Carlen, Q. Ju. Large-time behavior of non-symmetric Fokker-Planck type equations Communications in Stochastic Analysis 2, Nr. 1 (2008) 153-175.

3. A. Arnold, P. Markowich, G. Toscani, A. Unterreiter. On logarithmic Sobolev inequalities and the rate of convergence to equilibrium for Fokker-Planck type equations. Comm. PDE 26/1-2 (2001) 43-100.

Andrea Bertozzi (UCLA)
Contagion Shocks in One Dimension
We consider an agent-based model of emotional contagion coupled with motion in one dimension that has recently been studied in the computer science community. The model involves movement with a speed proportional to a *fear* variable that undergoes a temporal consensus averaging based on distance to other agents. We study the effect of Riemann initial data for this problem, leading to shock dynamics that are studied both within the agent-based model as well as in a continuum limit. We examine the behavior of the model under distinguished limits as the characteristic contagion interaction distance and the interaction timescale both approach zero. The limiting behavior is related to a classical model for pressureless gas dynamics with *sticky* particles. In comparison, we observe a threshold for the interaction distance vs. interaction timescale that produce qualitatively different behavior for the system - in one case particle paths do not cross and there is a natural Eulerian limit involving nonlocal interactions and in the other case particle paths can cross and one may consider only a kinetic model in the continuum limit. This is joint work with Jesus Rosado, Martin Short, and Li Wang.

Adrien Blanchet (U Toulouse)
The parabolic-parabolic Keller-Segel system as a gradient flow

We construct weak, global in time solutions to the parabolic-parabolic Keller-Segel model in dimension two assuming that the total mass is below the critical value. Our construction is based on the fact that the Keller-Segel model can be seen as a gradient flow in a suitable functional space. This allows us to employ a hybrid variational principle which is a generalisation of the minimising implicit scheme in the Monge-Kantorovich-Wasserstein metric developed by Jordan, Kinderlehrer and Otto. The main difficulty is to prove some regularity at the level of the discrete scheme. For this purpose we use an extension of the method developed by Matthes, McCann and Savaré. It is a joint work with J. Carrillo, D. Kinderleherer, M. Kowalczyk, P. Laurençot and S. Lisini.

José A. Cañizo (U Birmingham)
Entropy-entropy dissipation inequality for the linear Boltzmann equation
I will present a recent result, obtained in collaboration with B. Lods (Torino) and M. Bisi (Parma), on an entropy production inequality for the linear Boltzmann equation. This inequality is linked to analogs of the Blachman-Stam inequality for the Boltzmann operator proved by Villani (1998) and Matthes & Toscani (2012). When taking the grazing collisions limit, the inequality becomes a logarithmic Sobolev inequality.

José A. Carrillo (Imperial College)
Minimizing Interaction Energies
I will start by reviewing some recent results on qualitative properties of local minimizers of the interaction energie to motivate the main topic of my talk: to discuss global minimizers. We will show the existence of compactly supported global miminizers under quite mild assumptions on the potential in the complementary set of classical H-stability in statistical mechanics. A strong connection with the classical obstacle problem appears very useful when the singularity is strong enough at zero. An approach from discrete to continuum is also quite nice under convexity assumptions on the potential. This is based on three preprints/works in preparation one together with F. Patacchini, J.A. Cañizo, another one with M. Delgadino and A. Mellet, and finally with M. Chipot and Y. Huang.

Li Chen (U Mannheim)
Critical exponents for degenerate Keller-Segel system
Keller-Segel (KS) systems are important models in the study of chemotaxis in mathematical biology. They describe the collective motion of cells or the evolution of the density of bacteria. Two totally different mechanisms (diffusion and nonlocal aggregation) are reflected in the KS-system. In 2-d, the system contains a critical mass above which the solution blows up in finite time. We will explain the generalization of the critical mass from 2-d to multi-dimension in nonlinear diffusion case. The talk is based on joint works with Jian-Guo Liu and Jinhuan Wang.

Marina Chugunova (Claremont GU)
On the Benilov-Vynnycky Blow-up Problem
We study an initial-boundary value problem for a fourth-order parabolic partial differential equation with an unknown velocity. The equation originated from a local asymptotic analysis of behavior of the contact line velocity in a two-dimensional Couette flow model. In the case of a $180°$ contact angle between liquid and a moving plate Benilov and Vynnycky conjectured that the speed of the contact line blows up to infinity in finite time. We present numerical simulations and qualitative analysis of the model. We show that depending on the initial data and parameter values different regimes can be observed. Joint work with Chiu-Yen Kao and Sarun Seepun

Katy Craig (Rutgers)
A blob method for the aggregation equation
The aggregation equation describes the motion of particles (or, more generally, densities) according to the minimization of a nonlinear interaction energy. Often, the interaction between particles is chosen to scale according to a power law potential, leading to aggregation or repulsion, depending on the sign of

the potential. In the case of the Newtonian potential, the aggregation equation shares many important features with the vorticity formulation of the Euler equations. In this talk, I will present joint work with Andrea Bertozzi on a new numerical method for the aggregation equation, inspired by vortex blob methods for the Euler equations. I will present quantitative results on the convergence of the method along with many numerical examples exploring its qualitative behavior.

Pierre Degond (Imperial College)
From kinetic to macroscopic models through local Nash equilibria
We propose a mean field kinetic model for systems of rational agents interacting in a game theoretical framework. This model is inspired from non-cooperative anonymous games with a continuum of players and Mean-Field Games. The large time behavior of the system is given by a macroscopic closure with a Nash equilibrium serving as the local thermodynamic equilibrium. Applications of the presented theory to social and economical models will be given.

Laurent Desvillettes (ENS Cachan)
The interplay between entropy and duality methods for parabolic systems
We present on various examples of complex parabolic systems how it is possible to combine entropy methods and duality estimates in order to obtain results of existence or weak stability. The sets of PDEs which are considered come out of chemistry and biology and include degnerate reaction-diffusion systems, coagulation-fragmentation systems and cross-diffusion systems.

Marco Di Francesco (U Bath)
Entropy solutions to scalar conservation laws via "follow-the-leader" particle systems
We provide a rigorous derivation of the unique entropy solution to the Lighthill-Whitham-Richards model for traffic flow from the follow-the-leader system as a many particle limit. More precisely, we prove that the empirical measure obtained from the approximating particle system converges in the 1-Wasserstein distance to the unique entropy solution of the LWR equation in the sense of Kružkov. The proof relies on discrete BV estimates, which allow in particular to prove the $L^\infty \to$ BV regularizing effect in the limit. This is a joint work with Massimiliano D. Rosini (Warsaw).

Klemens Fellner (U Graz)
Towards global existence and optimal equilibration rates for reaction-diffusion-systems
Systems of nonlinear reaction-diffusion equations, in particular when considering reversible kinetics, pose still many open problems. Examples are the lack of global smooth solutions in physical space and convergence to equilibrium with explicit/optimal rate. This talk presents recents advances on the large-time-theory of reaction-diffusion-systems by means of entropy- and duality methods and discusses ideas towards proving optimal convergence rates as well as applying entropy methods for reaction networks, which do not satisfy detailed balance.

Francis Filbet (Lyon I)
On discrete functional inequalities for some finite volume schemes
We prove several discrete Gagliardo-Nirenberg-Sobolev and Poincaré-Sobolev inequalities for some approximations with arbitrary boundary values on finite volume meshes. The keypoint of our approach is to use the continuous embedding of the space BV(Ω) into $L^{N/(N-1)}(\Omega)$ for a Lipschitz domain $\Omega \subset \mathbb{R}^N$, with $N \geq 2$. Finally, we give several applications to discrete duality finite volume (DDFV) schemes which are used for the approximation of nonlinear and non isotropic elliptic and parabolic problems.

Ivan Gentil (Lyon I)
Solution of a class of reaction-diffusion systems via logarithmic Sobolev inequality
We study global existence, uniqueness and positivity of weak solutions of a class of reaction-diffusion systems of chemical kinetics type, under the assumptions of logarithmic Sobolev inequality and appropriate exponential integrability of the initial data. This is a joint work with P. Fougères and B. Zegarlinski.

Yanghong Huang (Imperial College)
Long time asymptotics of a porous medium equation with fractional pressure
The entropy method is used to show the long time asymptotics and exponential convergence of many evolution equations. In this talk, the same method are examined for the porous medium equations with fractional pressure in different approaches, including the Bakry-Emery method and transport inequalities, showing the exponential convergence (in similarity variables) to steady states only in one dimension. This is a joint work with J. A. Carrillo, M. C. Santos and J. L. Vazquez.

Gaspard Jankowiak (Paris IX)
Sobolev and Hardy-Littlewood-Sobolev inequalities
This talk will be dedicated to a improvement of the Sobolev inequality with an additional term involving its dual counterpart, the Hardy-Littlewood-Sobolev inequality. We will focus on the optimal constant in this improved inequality, and on several ways to obtain estimates using completion of the square methods and spectral bounds given by the linearization around the Aubin-Talenti profiles. More estimates can be obtained using a non linear diffusion flow which is closely related to the above inequalities. We will also give an overview of how this result extends to other settings. This is a joint work with Jean Dolbeault, Maria J. Esteban and Van Hoang Nguyen.

Inwon Kim (UCLA)
Approximating oblique boundary problems with Fokker-Planck type equations
We consider solutions of quasi-linear parabolic PDEs with zero oblique boundary data in a bounded domain in $\mathbb{R}^n$. Our main result states that the solutions can be approximated by solutions of Fokker-Planck type PDEs in the whole space with a penalizing drift term which also converges to zero outside the original domain. The convergence is uniform and error estimates are obtained.

Philippe Laurençot (U Toulouse)
A thin film approximation of the Muskat problem
Existence of nonnegative weak solutions is shown for thin film approximations of the Muskat problem which include either gravity forces or capillary forces or both. The model describes the space-time evolution of the heights of the two fluid layers and is a fully coupled system of two second or fourth order degenerate parabolic equations in $\mathbb{R}^d$ which can be viewed as a gradient flow for the 2-Wasserstein distance. In the absence of capillarity, a classification of self-similar solutions is given.

Thomas Laurent (Loyola Marymount)
Dimensionality of Local Minimizers of the Interaction Energy
We consider local minimizers (in the topology of transport distances) of the interaction energy associated to a repulsive-attractive potential. We show how the dimensionality of the support of local minimizers is related to the repulsive strength of the potential at the origin. This is a joint work with D. Balague, J. A. Carrillo, and G. Raoul.

Christian Léonard (Paris X)
An entropic approach to Navier-Stokes equations
In 1966, Arnold proposed to look at Euler's equation for perfect fluids as describing the geodesic flow of volume preserving diffeomorphisms. In the same spirit, in 1989 Brenier designed a least action principle based on optimal quadratic transport that allows for getting rid of the high regularity assumptions that underly Arnold's approach. Replacing deterministic geodesics on the state space by sample paths of Brownian bridges and optimal transport by minimal entropy, we obtain a least action principle for the Navier-Stokes equation, very much in the spirit of Brenier's representation of the Euler equation. This is a joint work with M. Arnaudon, A.-B. Cruzeiro and J.-C. Zambrini.

Matthias Liero (WIAS Berlin)
Dissipation distances for reaction-diffusion equations - the Hellinger–Kantorovich distance

It was recently shown by Mielke that a wide class of reaction-diffusion systems can be formulated in a natural way via gradient structures for the relative entropy or free energy. The metric gradient of the driving functional is determined via a state-dependent Onsager operator containing a diffusion part of Wasserstein type and an additional reaction term. With the Onsager operator we can associate a dissipation distance in the sense of Benamou–Brenier by infimizing the total dissipation over all connecting curves. The question of attainment of this infimum, which is the same as the existence of geodesic curves, is an open question in most cases. In this talk we present first results for a simple scalar reaction-diffusion equation. The associated Onsager operator gives rise to the Hellinger–Kantorovich distance, which has to be understood as the inf-convolution of the Kantorovich–Wasserstein and the Hellinger distance. It is described as a function of any two nonnegative measures in such a way that the competition between transport and reaction is clearly displayed. We present a characterization of the distance in terms of a minimization of coupling measures and a cost function describing the transport part. The crucial point is the identification of the amount of transport versus the amount of reaction. Finally, we provide the explicit construction of Hellinger–Kantorovich geodesic curves between measures that highlight the competing effects. (Joint work with Alexander Mielke and Giuseppe Savaré.)

Jan Maas (U Bonn)
A gradient flow approach to chemical master equations
It was recently shown (M. 2011, Mielke 2011) that forward equations of reversible Markov chains can be interpreted as gradient flow equations for the entropy with respect to a suitably modified transportation metric. In this talk we apply this gradient flow structure to chemical master equations satisfying detailed balance. In the thermodynamic limit we obtain a gradient flow structure for the Liouville equation induced by Mielke's gradient flow structure for reaction rate equations. We also analyse geodesic convexity properties of the relative entropy functional. This is joint work with Alexander Mielke.

Bruno Nazaret (Paris I)
Entropy method in some weighted fast diffusion equations and related functional inequalities
We investigate here the long time asymptotics in the weigthted fast diffusion equation $|x|^{-\gamma}\partial_t u = \Delta u^m$, with $m < 1$ and $\gamma \in (0, 2)$. In particular, we prove that the existence of radially symmetric minimizers for a 1-parameter family of Cafarelli-Kohn-Nirenberg inequalities implies that the entropy is bounded by $1/(2 - \gamma)$ times the Fisher information, wich gives the exponential decay of the entropy with the corresponding rate. The existence of such minimizers is proved for sufficiently small, by a pertubation argument using spectral estimates at the limit. This is a joint work with J. Dolbeault and M. Muratori.

Giuseppe Savaré (U Pavia)
Entropy-transport optimization problems and the Hellinger-Kantorovich distance
We will present a general class of variational problems involving entropy-transport minimization with respect to a couple of given finite measures with possibly unequal total mass. They can be regarded as a natural generalization of classical optimal transportation problems; with an appropriate choice of the entropy/cost functionals they provide a family of distances between measures that exhibit interesting geometric features. Such distances lie between the Hellinger and the Kantorovich-Wasserstein ones; their links with the entropy-transport minimization problems rely on convex duality in a surprising way. A suitable dynamic Benamou-Brenier characterization also show the role of these distances in dynamic processes involving creation or annihilation of masses. (Joint work with Matthias Liero and Alexander Mielke.)

Christian Schmeiser (U Wien)
Convergence to equilibrium in a model for the distribution of filament ends along the leading edge of a lamellipodium
The lamellipodium, a flat cell protrusion important for cell motility, is pushed forward by polymerizing actin filaments. The distribution of filament ends along the leading edge can be described by a system of nonlinear convection-reaction equations. Nontrivial stationary states of this system describe steadily

moving cells as observed in experiments. Existence results for steady states and long time convergence results, based on entropy-entropy dissipation methods, will be presented. (Joint work with Angelika Manhart.)

Robert Stanczy (U Wroclawski)
Entropy Methods for Systems of Gravitating Particles
We provide a few results on applications of the entropy methods to the systems of diffusing and gravitating particles, modeled by the generalized Smoluchowski-Poisson equation. We consider both the case of fixed temperature or the energy of the system. Some results concern the systems of particles obeying different statistics i.e. the Maxwell-Boltzmann, Fermi-Dirac, Bose-Einstein ensembles or the one modeling polytropic stars. A common feature of all the mentioned models is the pressure in the self-similar form. Parts of the results were obtained together with Piotr Biler and Jean Dolbeault.

Ulisse Stefanelli (U Wien)
Nonlinear evolution as convex minimization
The weighted inertia-energy-dissipation (WIDE) principle consists in a global-in-time variational approach to a large variety nonlinear evolution equations of either of dissipative or dispersive type. The problems that are have been presently tackled by this method include gradient flows, also in metric spaces, as well as doubly nonlinear and rate-independent evolutions. Triggered by a conjecture by De Giorgi, some first steps in the direction of hyperbolic situations have been made. The idea is to reformulate the differential systems as convex minimization problems in order to export the machinery of the Calculus of Variations (direct method, gamma-convergence, relaxation. . .) to evolutionary situations. I intend to present the basics of this methodology and comment on the available results.

Angela Stevens (U Münster)
Haptotaxis and Trail Following: Qualitative Behavior of a Drift-Diffusion Model Coupled to an ODE
Haptotaxis and trail following are common processes in cell movement in a variety of contexts. Movement can be along filaments but also according to non-diffusible signals. One model under consideration for such processes are Chemotaxis-like equations, but coupled to an ODE. Interestingly the pattern forming behavior of such systems is very different from the classical Keller-Segel model and also the mathematical techniques to tackle questions of global solutions and blow-up (localization of cells) differ from the ones used in the context of classical chemotaxis equations. Further, these systems can formally be set into context with reinforced self-attracting random walks and they reflect several rigorous results in this direction, at least formally.

Karl-Theodor Sturm (U Bonn)
The curvature-dimension condition with finite N: consequences and transformations
We give a new formulation of the curvature-dimension condition $CD(K, N)$ for metric measure spaces in terms of the so-called (K, N)-convexity of the Boltzmann entropy and we present some consequences of this new definition. Among them new Wasserstein contraction properties, gradient estimates and the full Bochner inequality (which finally leads to the whole zoo of Li-Yau estimates). Finiteness of the 'dimension' parameter N allows to combine spatial and time derivatives which leads to new transformation formulas under time changes (= changes of measure) and conformal changes of the metric.

Adrian Tudorascu (West Virgina U)
One-dimensional pressureless gas systems with/without viscosity
A general global existence result for one-dimensional pressure less Euler/Euler-Poisson systems with or without viscosity is obtained by employing the *sticky particles* model. We first construct entropy solutions for some appropriate scalar conservation laws, then we show that these solutions encode all the information necessary to obtain solutions for the pressureless systems. Another novel contribution is the stability and uniqueness of solutions, which is obtained via a contraction principle in the Wasserstein metric. This is based on joint work with T. Nguyen.

Marie-Therese Wolfram (U Wien)
On numerical simulations of nonlinear convection-aggregation equations by evolving diffeomorphisms
In this talk we present a numerical algorithm for solving nonlinear convection-aggregation equations, which is based on the Lagrangian coordinate representation of the original problem. These equations can be considered as gradient flows with respect to the quadratic transportation distance for a free energy functional. Here the evolution of the density is represented by evolving diffeomorphisms, mapping the unknown density to the constant distribution of mass on the domain. The proposed numerical algorithm is based on the variational formulation and automatically adapts the mesh to the shape of the mass distribution. Hence it allows to track the formulation of singularities in a natural manner. A feature that we illustrate with various numerical experiments.

Lijiang Wu (Carnegie Mellon)
Nonlocal interaction equations in heterogeneous environments with boundaries
We discuss biological aggregation in a heterogeneous environment and on nonconvex, nonsmooth domain. We model the heterogeneous environment as a Riemannian manifold with boundary and develop gradient flow approach in the space of probability measures on the manifold endowed with Riemannian 2-Wasserstein metric. We use the gradient flow structure to show the well-posedness of a class of nonlocal interaction equations. We discuss how heterogeneity of the environment leads to new dynamical phenomena. We also present a result on generalizing the well-posedness of weak measure solutions to a class of nonlocal interaction equations on nonconvex and nonsmooth domains. We use particle approximations, solve the discrete ODE systems and pass to the limit by stability property. The novelty here is that under mild regularity conditions on space(i.e. prox-regularity), we can show the well-posedness of the ODE systems and the stability properties with explicit dependence on the geometry of the space(prox-regular constant). The talk is based on collaborations with J. A. Carrillo and D. Slepcev.

Yao Yao (Wisconsin Madison)
Finite time singularity of a vortex patch model in the half plane In this talk, we consider a family of contour dynamics equations in the half plane with a parameter α, where $0 < \alpha < 1$. The $\alpha \to 1$ limit gives us the 2D Euler equation in the half plane, while the $\alpha \to 1$ limit gives us the inviscid quasi-geostrophic equation. We prove that for sufficiently small $\alpha > 0$, the solution may develop a singularity in finite time. This is a joint work with A. Kiselev, L. Ryzhik and A. Zlatos.

Johannes Zimmer (U Bath)
Scale-bridging for entropic flows in the presence of energy or noise
One often aims to describe systems out of equilibrium by the governing energy E and entropy S, as well as the corresponding evolution laws for E and S. How can we derive these ingredients of the macroscopic evolution from particle models? In recent years, a dynamic scale-bridging approach has been developed and applied to a number of problems; large deviation theory plays an important role. The talk will present some of these results, in particular discuss the Vlasov-Fokker-Planck equation as a system driven by energy and entropy. Time permitting, an approach of deriving stochastic equations mimicking the fluctuations in underlying mesoscopic models will be sketched as well, connecting fluctuating hydrodynamics and Wasserstein evolution. This is joint work with Hong Duong and Mark. A. Peletier respectively Rob Jack.