

# Mathematical Finance: Arbitrage and Portfolio Optimization

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## 1 Overview of the Field

Mathematics of financial markets is one of the most active and exciting areas of contemporary applied mathematics. It provides the mathematical community with a rich source of challenges, which, in turn, enrich the society's understanding of the financial system and help both regulators and the financial industry make better and more informed choices. Its proximity to practice, in addition to its purely mathematical appeal, makes this field especially attractive to young mathematicians. Moreover, it provides for wider employment opportunities both within the academic world and outside of it.

One of the most challenging areas within mathematical finance, namely its foundations, aims to understand the basic structure shared by all financial markets. It uses probabilistic tools, together with a variety of methods from stochastic, functional and convex analysis and partial differential equations, and draws from a host of other mathematical disciplines to accomplish its goals. Prior advances in the foundational issues have not only made a huge impact on the financial practice, but have also inspired a number of breakthroughs in related areas of mathematics traditionally regarded as theoretical.

## 2 Scientific Progress Made

**Arbitrage theory.** A number of researchers presented results in arbitrage theory. With first conceptual results going back to the work of Harrison, Kreps and Pliska (see [HK79], [HP81] and [Kre81]), a major (mathematical) breakthrough was accomplished by Delbaen and Schachermayer, with their work culminating in the papers [DS94] and [DS98]. The intensity of the research output in the arbitrage theory only increased since, with new contributions being made even today. Indeed, our meeting featured presentations of Beatrice Acciaio, Sara Biagini, Christa Cucchiero, Marco Frittelli, Monique Jeanblanc, Martin Schweizer and Josef Teichmann all of which discussed important new contributions to this rich theory.

**Optimal investment.** The roots of the optimal-investment (utility maximization) theory can be traced to the seminal work [Mer71] of Robert Merton, or, one may argue, even to Daniel Bernoulli (see [Ber54]). The theory has been adopted by the financial-mathematics community through the work of Karatzas, Kramkov, Lehoczky, Schachermayer, Shreve, Xu (see, e.g., [KLSX91] and [KS99]) and many others, during the 1990s. The intensity of mathematical research has not waned ever since; it is still strong today, as evidenced by a number of interesting talks by Peter Bank, Luciano Campi, Christoph Czichowsky, Paolo Guasoni, Vicky Henderson, Tomoyuki Ichiba, Jan Kallsen, Johannes Muhle Karbe, Steve Shreve, Ronnie Sircar and Peter Tankov, we had the opportunity to hear at the meeting.

**Equilibrium Theory and Related Fields.** The topic of competitive equilibria - traditionally housed in economics - has found its way to the mainstream of mathematical research in quantitative finance thanks to the versatility and mathematical richness of the stochastic models it supports. With the early research by Duffie, Huang, Karatzas, Lehoczky, Shreve (see [DH85] or [KLS90]) setting the stage, new developments (including the treatment of “incomplete markets”) have been appearing regularly to this day. Recent connections with the theory of systems of parabolic PDE and backward SDE have been particularly visible in an array of talks on the subject (and some related fields), by Michalis Anthropelos, Umut Çetin, Jakša Cvitanić, Christoph Frei, Ioannis Karatzas, Kasper Larsen, Sergio Pulido and Hao Xing.

**Other Topics.** In addition to the three “classical” domains of research outlined above, financial mathematics casts a broad net over a variety of related mathematical fields and problem areas. One, recent and extremely active area of research is so-called “robust finance” where the dependence on the choice of the underlying probability measure (more precisely, its equivalence class under mutual absolute continuity) is studied and universal (robust) results with respect to this dependence formulated. The talks of Mathias Beiglboeck, Alex Cox, Marco Frittelli, Marcel Nutz and Jan Obloj provide excellent illustration of the scope and depth of this exiting subject.

On the other hand, problems stemming from finance have been providing a steady inflow of interesting problems and inspiration to the areas of optimal stochastic control and games for several decades now. Two talks, one by Bruno Bouchard and the other by Mihai Sîrbu, delivered at this meeting, provide par-excellence examples of such a synergy.

Martin Larson and Johannes Ruf revisited the problem of identifying when a local martingale is a martingale through Novikov’s condition, providing a novel treatment that enhances intuition and contributes to the field of stochastic analysis.

A talk on the efficient estimation of the distribution of perpetuities was presented by Scott Robertson, with immediate applications in Actuarial science and Insurance.

Finally, let us mention the talk by Hans Föllmer, which outlined a whole new domain of research at the interface of financial mathematics (risk-measure theory) and statistical mechanics, with envisioned applications in the study of systemic financial risk. Such models are expected to take a leading role in the mathematical support of large-scale financial regulation and the avoidance of and the mitigation of the effects of future catastrophic system-wide financial events.

### 3 Presentation Highlights

#### Beatrice Acciaio

*“Arbitrage of the first kind and filtration enlargements in semimartingale financial models”*

**ABSTRACT.** Given a financial market where no arbitrage profits are possible, and an agent with additional information with respect to it, I investigate whether the extra information can generate arbitrage profits. I will first justify why the right concept of arbitrage to consider here is the so-called Arbitrage of the First Kind (or, equivalently, Unbounded Profit with Bounded Risk). Then I will illustrate a simple and general condition ensuring that no arbitrage is available to the informed agent either. The preservation of No-Arbitrage under additional information is shown for a general semimartingale model both when this information is disclosed progressively in time and when it is fully added at the initial time (which correspond to the initial and to the progressive enlargement of filtration, respectively). In addition, I will provide a characterization of such a stability in a robust context, that is, for all possible semimartingale models. (This is joint work with C. Fontana and C. Kardaras.)

## Michalis Anthropelos

*“Equilibrium in risk sharing games”*

ABSTRACT. The paper studies the equilibrium sharing of investment risks among agents whose random endowments are private information. Given that the sharing rules are the ones that optimally allocate the submitted endowments, we propose a Nash game where agents’ strategic sets consist of the endowments are going to submit for sharing. First, it is proved that the best response problem admits a unique solution, which shall be called best endowment response and differs from the true agent’s risk exposure. The Nash equilibrium risk sharing admits a finite dimensional characterization and it is proved to be unique in the case of two agents. The game benefits the low risk averse agents since their expected utilities are higher at the Nash risk sharing equilibrium than the optimal risk sharing one. (This is joint work with C. Kardaras.)

## Peter Bank

*“Optimal investment with price impact”*

ABSTRACT. We present a price impact model where bid and ask prices are specified by a system of coupled diffusions driven both by a common noise and the buy and sell orders of an investor. The coupling causes the spread to decline after a transaction. The resulting nonlinear wealth dynamics are shown to be separable in a profit and loss part given, as in a linear model, by a stochastic integral and a separate cost term which only depends on the large investor’s transactions and the resulting spread. Absence of arbitrage is easily derived from the corresponding liquid model. For the illiquid optimal investment problem we obtain existence of optimal investment policies for utility from terminal wealth. In the special case of arithmetic Brownian motion and exponential utility, the singular optimal control problem turns out to have a deterministic solution which can be computed explicitly. (This is joint work with Moritz Voss)

## Mathias Beiglboeck

*“An optimal principle from mass transport and applications to model-independence.”*

ABSTRACT. Model-independent pricing has grown into an independent field in Mathematical Finance during the last 15 years. A driving inspiration in this area has been the fruitful connection to the Skorokhod embedding problem. We discuss a more recent approach to model-independent pricing, based on a link to Monge-Kantorovich optimal transport. Based on a similar technique in optimal transport we derive a “variational principle” that is applicable to model-independent pricing. This transport-viewpoint also sheds new light on Skorokhod’s classical problem.

## Sara Biagini

*“Robust Fundamental Theorem for Continuous Processes”*

ABSTRACT. We study a continuous-time financial market with continuous price processes under model uncertainty; more precisely, under a family  $\mathcal{P}$  of possible physical measures. A robust notion  $\text{NA}_1(\mathcal{P})$  of no-arbitrage of the first kind is introduced; it postulates that a nonnegative, nonvanishing claim cannot be

superhedged for free by using simple trading strategies. Our first main result is a version of the fundamental theorem of asset pricing:  $\text{NA}_1(\mathcal{P})$  holds if and only if every  $P \in \mathcal{P}$  admits a measure  $Q$  that is an equivalent martingale measure before a certain lifetime. The second main result provides the existence of optimal superhedging strategies for general contingent claims and a representation of the superhedging price in terms of martingale measures.

## Bruno Bouchard

*“Stochastic target games via regularized viscosity solutions: application to super-hedging under coefficients’ uncertainty”*

ABSTRACT. We study a class of stochastic target games where one player tries to find a strategy such that the state process almost-surely reaches a given target, no matter which action is chosen by the opponent. Our main result is a geometric dynamic programming principle which allows us to characterize the value function as the viscosity solution of a non-linear partial differential equation. Because abstract measurable selection arguments cannot be used in this context, the main obstacle is the construction of measurable almost-optimal strategies. We propose a novel approach where smooth supersolutions are used to define almost-optimal strategies of Markovian type, similarly as in verification arguments for classical solutions of Hamilton-Jacobi-Bellman equations. The smooth supersolutions are constructed by an extension of Krylov’s method of shaken coefficients. We apply our results to a problem of option pricing under model uncertainty with different interest rates for borrowing and lending.

## Luciano Campi

*“Utility indifference pricing for non-smooth payoffs in a model with non-tradable assets”*

ABSTRACT. We consider the problem of exponential utility indifference valuation under the simplified framework where traded and nontraded assets are uncorrelated but where the claim to be priced possibly depends on both. Traded asset prices follow a multivariate Black and Scholes model, while nontraded asset prices evolve as generalized Ornstein-Uhlenbeck processes. We provide a BSDE characterization of the utility indifference price (UIP) for a large class of non-smooth, possibly unbounded, payoffs depending simultaneously on both classes of assets. Focusing then on European claims and using the Gaussian structure of the model allows us to employ some BSDE techniques (in particular, a Malliavin-type representation theorem due to Ma and Zhang (2002)) to prove the regularity of  $Z$  and to characterize the UIP for possibly discontinuous European payoffs as a viscosity solution of a suitable PDE with continuous space derivatives. The optimal hedging strategy is also identified essentially as the delta hedging strategy corresponding to the UIP. Since there are no closed-form formulas in general, we also obtain asymptotic expansions for prices and hedging strategies when the risk aversion parameter is small.

## Umut Çetin

*“Equilibrium with risk averse market makers and related inverse problems”*

ABSTRACT. We analyse the existence of a continuous-time Nash equilibrium in a financial market with risk averse market makers and an informed trader with a private information. When the insiders signal is static, the optimal strategies of the agents turn out to be solutions of a forward-backward system of partial and

stochastic differential equations. If the private signal of the insider varies in time, existence of equilibrium depends on the solution of a linear inverse problem, which is equivalent to a backward parabolic PDE with an initial condition.

### Alex Cox

*“An optimal stopping approach to the  $n$ -marginal Root problem, and applications to variance options”*

ABSTRACT. Recently, the problem of finding robust bounds on option prices which incorporate information from vanilla options has generated renewed interest in solutions to the classical Skorokhod Embedding Problem (SEP). It is natural to consider generalisations of the problem where the prices of the vanilla options are known both at the maturity of the option, and also at intermediate times. In this talk, we consider a generalisation of Root’s solution to the SEP where we look for an ordered sequence of stopping times, each of which embeds a given distribution. In particular, we are able to identify these stopping times as the exit times from certain domains, and we are able to characterise these domains naturally as the stopping regions of a suitable multiple stopping problem. Moreover, we are able to show optimality for these stopping times, and hence derive sub-hedging strategies which enforce the price bounds in any suitable model. (Joint work with Jan Obloj and Nizar Touzi.)

### Christa Cuchiero

*“A convergence result for the emery topology and a variant of the proof of the fundamental theorem of asset pricing”*

ABSTRACT. We work in the general setting of admissible portfolio wealth processes as introduced by Y. Kabanov. We show that in this setting the (NUPBR) condition (No unbounded profit with bounded risk) implies the so called P-UT property, a boundedness property in the Emery topology which has been introduced by C. Stricker. Combining this with results from Memin-Slominski and a maximality assumption leads to convergence in the Emery topology and thus to a short variant of the proof of the fundamental theorem of asset pricing initially proved by Delbaen and Schachermayer. (The talk is based on joint work with Josef Teichmann.)

### Jakša Cvitanić

*“Moral Hazard in Dynamic Risk Management”*

ABSTRACT. We consider a contracting problem in which a principal hires an agent to manage a risky project. When the agent chooses volatility components of the output process and the principal observes the output continuously, the principal can compute the quadratic variation of the output, but not the individual components. This leads to moral hazard with respect to the risk choices of the agent. Using a recent theory of singular changes of measures for Ito processes, we formulate a principal-agent problem in this context, and solve it in the case of CARA preferences. In that case, the optimal contract is linear in these factors: the contractible sources of risk, including the output, the quadratic variation of the output and the cross-variations between the output and the contractible risk sources. Thus, path-dependent contracts naturally arise when there is moral hazard with respect to risk management. We also provide comparative statics via numerical examples, showing that the optimal contract is sensitive to the values of risk premia and the initial values of

the risk exposures. (Joint with N. Touzi and D. Possamai)

### **Christoph Czichowsky**

*“Strong Supermartingales and Portfolio Optimisation under Transaction Costs”*

ABSTRACT. In this talk, I will sketch how to develop a duality theory for portfolio optimisation under transaction costs and some of the new phenomena arising in this context. This talk is based on joint work with Walter Schachermayer.

### **Hans Föllmer**

*“Spatial risk measures: local specification and phase transition”*

ABSTRACT. In the spatial setting of a large network, the local specification of convex risk measures can be seen as a non-linear extension of the local specification of equilibrium states in Statistical Mechanics. We discuss the corresponding aggregation problem of passing from local to global risk measures and the appearance of phase transitions. This will involve a non-linear extension of backwards martingale convergence, arguments from preference theory, and a spatial version of Dynkin’s construction of entrance boundaries for Markov processes.

### **Christoph Frei**

*“Finding local equilibria by splitting multidimensional BSDEs”*

ABSTRACT. We consider a model of a financial market where investors take not only their own absolute performance, but also the relative performance compared to their peers into account. The goal is to find equilibria where every investor has an individually optimal strategy. This problem is related to the study of multidimensional backward stochastic differential equations (BSDEs). We introduce a new notion of local solution by splitting multidimensional BSDEs over time. This allows us to show that the BSDE from our financial problem is locally but not globally solvable. From this, we deduce that there exist local but no global equilibria in our model of a financial market. By considering the relative performance, investors may ruin each other so that equilibria exist only over a short time.

### **Marco Frittelli**

*“Robust Arbitrage under Uncertainty”*

ABSTRACT. In a model independent financial market, we introduce a topological notion of Robust Arbitrage, without fixing an a-priori set of reference probability measures. This notion relies only on the market structure and can be dually represented in terms of weakly open sets of probability measures. We then show that the absence of Robust Arbitrages with respect to an opportune filtration enlargement, guarantees the existence of full support martingale measures.

## Paolo Guasoni

*“The Limits of Leverage”*

ABSTRACT. When trading incurs proportional costs, leverage can scale an asset’s return only up to a maximum multiple, which is sensitive to the asset’s volatility and liquidity. In a continuous-time model with one safe and one risky asset with constant investment opportunities and proportional transaction costs, we find the efficient portfolios that maximize long term expected returns for given average volatility. As leverage and volatility increase, rising rebalancing costs imply a declining Sharpe ratio. Beyond a critical level, even the expected return declines. For funds that seek to replicate multiples of index returns, such as leveraged ETFs, our efficient portfolios optimally trade off alpha against tracking error. (Joint work with Eberhard Mayerhofer)

## Vicky Henderson

*“The Value of Being Lucky: Option Backdating and Non-diversifiable risk”*

ABSTRACT. The practice of executives influencing their option compensation by setting a grant date retrospectively is known as backdating. Since these options are usually granted at-the-money, selecting an advantageous grant date will be valuable to the executive. There is substantial evidence that backdating took place in the US, particularly prior to the tightening of SEC reporting requirements. In this talk, we develop and solve a utility-indifference model to quantify the value of the opportunity to backdate options. We show that the magnitude of ex ante gains from backdating is significant. Our model can be used to explain why backdating was more prevalent at firms with highly volatile stock prices. Explanations hinge on the executive’s inability to perfectly hedge and desire to exercise options early. Joint work with Jia Sun (China Credit Ratings) and Elizabeth Whalley (Warwick Business School)

## Tomoyuki Ichiba

*“Some Aspects of Universal Portfolios”*

ABSTRACT. We discuss Cover’s universal portfolios in the context of Stochastic Portfolio Theory. By enlarging the class of portfolio generating functions, we see universal portfolios are generated by functions, given excess growth rates of constant rebalanced portfolios. These generating functions and resulting universal portfolios can be represented as integrations with respect to tilted version of maximal entropy measure. In this way we may answer one of the open questions posed by Fernholz & Karatzas (2009). With analyses of concentration of measures we evaluate performance of universal portfolios. Finally, we discuss universal portfolios under large equity market models.

## Monique Jeanblanc

*“Arbitrages in progressive enlargement of filtrations”*

ABSTRACT. We study a financial market in which some assets, with prices adapted w.r.t. a reference filtration  $\mathbb{F}$  are traded. One then assumes that an agent has some extra information, and may use strategies adapted to

a larger filtration  $\mathbb{G}$ . This extra information is modeled by the knowledge of some random time  $\tau$ , when this time occurs. We restrict our study to progressive enlargement setting, and we pay a particular attention to honest times. Our goal is to detect if the knowledge of  $\tau$  allows for some arbitrage (classical arbitrages and arbitrages of the first kind), i.e., if using  $\mathbb{G}$ -adapted strategies, one can make profit.

The results presented here are based on two joint papers with Aksamit, Choulli, Deng, in which the authors study No Unbounded Profit with Bounded Risk (NUPBR) in a general filtration  $\mathbb{F}$  and the case of classical arbitrages in the case of honest times, density framework and immersion setting. We shall also study the information drift and the growth optimal portfolio resulting from that model. (Based on joint work with A. Aksamit, T. Choulli, J. Deng, T. Schmidt.)

### **Jan Kallsen**

*“On portfolio optimization and indifference pricing with small transaction costs: rigorous proofs based on duality”*

ABSTRACT. Portfolio optimization problems with frictions as e.g. transaction costs are hard to solve explicitly. In the limit of small friction, solutions are often of much simpler structure. In the last twenty years, considerable progress has been made both in order to derive formal asymptotics as well as rigorous proofs. However, the latter usually rely on rather strong regularity conditions, which are hard to verify in concrete models. Some effort is still needed to make the results really applicable in practice. This talk is about a step in this direction. More specifically, we discuss portfolio optimization for exponential utility under small proportional transaction costs. As an example, we reconsider the Whalley-Willmott results of utility-based pricing and hedging in the Black-Scholes model. We relax the conditions required by Bichuch who gave a rigorous proof for smooth payoffs under sufficiently small risk aversion.

### **Ioannis Karatzas**

*“The Inflationary Bias of Real Uncertainty and a Harmonic Fisher-Equation”*

ABSTRACT. We argue that real uncertainty by itself causes long-run nominal inflation. We start with an infinite-horizon, cash-in-advance economy with a representative agent, and with real uncertainty modelled by independent, identically distributed endowments. Suppose the central bank fixes the nominal rate of interest. We show that the equilibrium long-run rate of inflation is strictly higher, on almost every path of endowment realizations, than it would be if these endowments were constant. Indeed, we present an explicit formula for the long-run rate of inflation, based on the famous Fisher equation. This posits that, in the absence of real uncertainty, the rate of inflation should depend on the monetary rate of interest and on the time-preference of the agents in the economy, and on nothing else. The long-run Fisher equation for our stochastic economy turns out to be similar, but with the rate of inflation replaced by the harmonic mean of the growth rate of money.

We also show that these features are also present, albeit with less explicit results, when one considers an economy with fiat money and a continuum of agents, one non-durable commodity, countably many time periods, and a central bank. (This is joint work with John Geanakoplos, Martin Shubik and William D. Sudderth.)

## **Kasper Larsen**

*“Taylor approximation of incomplete Radner equilibrium models”*

ABSTRACT. In the setting of exponential investors and uncertainty governed by Brownian motions we first prove the existence of an incomplete equilibrium for a general class of models. We then introduce a tractable class of exponential-quadratic models and prove that the corresponding incomplete equilibrium is characterized by a coupled set of Riccati equations. Finally, we prove that these exponential-quadratic models can be used to approximate the incomplete models we studied in the first part. (Joint work with Jin Hyuk Choi.)

## **Martin Larsson**

*“Novikov-type conditions for processes with jumps”*

ABSTRACT. We provide a novel proof for the sufficiency of Novikov-Kazamaki type conditions for the martingale property of nonnegative local martingales with jumps. The proof is based on explosion criteria for related processes under a possibly non-equivalent measure. (Joint work with Johannes Ruf.)

## **Johannes Muhle-Karbe**

*“Trading with small price impact”*

ABSTRACT. An investor trades a safe and several risky assets with linear price impact to maximize expected utility from terminal wealth. In the limit for small impact costs, we explicitly determine the optimal policy and welfare, in a general Markovian setting allowing for stochastic market, cost, and preference parameters. These results shed light on the general structure of the problem at hand, and also unveil close connections to optimal execution problems and to other market frictions such as proportional and fixed transaction costs. (Joint work with Ludovic Moreau and H. Mete Soner)

## **Marcel Nutz**

*“Nonlinear Lévy Processes and their Characteristics”*

ABSTRACT. We develop a general construction for nonlinear Lévy processes with given characteristics. More precisely, given a set  $\Theta$  of Lévy triplets, we construct a sublinear expectation on Skorohod space under which the canonical process has stationary independent increments and a nonlinear generator corresponding to the supremum of all generators of classical Lévy processes with triplets in  $\Theta$ . The nonlinear Lévy process yields a tractable model for Knightian uncertainty about the distribution of jumps for which expectations of Markovian functionals can be calculated by means of a partial integro-differential equation. (Joint work with Ariel Neufeld.)

## Jan Obłój

*“Robust hedging of barrier options with beliefs on implied Volatility”*

ABSTRACT. We develop an abstract robust modelling framework accommodating as inputs market priced of options and modelling beliefs formulated in terms of pathspace restrictions. This naturally allows us to talk about robust market models. As an example we consider pricing and hedging of barrier options with beliefs about future levels of implied volatilities. We construct local volatility models which satisfy such constraints and use them to combine static and robust hedging methods. We discuss asymptotic convergence when beliefs become stronger (model specific) or weaker (model independent). (Joint work with Sergey Nadtochiy)

## Sergio Pulido

*“Existence and uniqueness results for multi-dimensional quadratic BSDEs arising from a price impact model with exponential utility”*

ABSTRACT. In this work we study multi-dimensional systems of quadratic BSDEs arising from a price impact model where an influential investor trades illiquid assets with a representative market maker with exponential preferences. The impact of the strategy of the investor on the prices of the illiquid assets is derived endogenously through an equilibrium mechanism. We show that a relationship exists between this equilibrium mechanism and a multi-dimensional system of quadratic BSDEs. We also specify conditions on the parameters of the model that guarantee that the system of BSDEs has a unique solution, which corresponds to a family of unique equilibrium prices for the illiquid assets. The proof relies on estimates that exploit the structure of the equilibrium problem. Finally, we provide examples of parameters for which the corresponding system of BSDEs is not well-posed. (This is joint work with Dmitry Kramkov.)

## Scott Robertson

*“Continuous Time Perpetuities and the Time Reversal of Diffusions”*

ABSTRACT. In this talk we consider the problem of obtaining the distribution of a continuous time perpetuity, where the non-discounted cash flow rate is determined by an ergodic diffusion. Using results regarding the time reversal of diffusions, we identify the distribution of the perpetuity with the invariant measure associated to a certain (different) ergodic diffusion. This enables efficient estimation of the distribution via simulation and, in certain instances, an explicit formula for the distribution. Time permitting, we will talk about how Large Deviations Principles and results concerning Couplings of diffusions can be used to estimate rates of convergence, thus providing upper bounds for how long simulations must be run when obtaining the distribution.

## Johannes Ruf

*“Convergence of local supermartingales”*

ABSTRACT. We characterize the event of convergence of a local supermartingale. Conditions are given in terms of its predictable characteristics and jump measure. Furthermore, it is shown that  $L^1$ -boundedness of a

related process is necessary and sufficient for convergence. The notion of extended local integrability plays a key role. (Joint work with Martin Larsson.)

### **Martin Schweizer**

*“Some ideas on bubbly markets”*

ABSTRACT. We introduce a notion of bubbly markets. This is an economically motivated and formulated concept which captures the idea that a given financial markets contains “bubbles”. We give dual characterisations of bubbly markets in terms of numeraire and martingale measures, and we show in particular that in any “reasonable” bubbly market, discounted prices must be strict local martingales, for any numeraire and any martingale measure associated to that numeraire. This can be viewed as a “robust” notion of a market containing a bubble. We show by examples how different concepts in our setting are related, discuss related literature, and explain how existing work fits into our framework. If time permits, we also discuss the issue of valuation principles for financial products in markets with bubbles. (The talk is based on ongoing joint work with Martin Herdegen, ETH Zürich).

### **Steve Shreve**

*“Should banks escrow traders’ bonuses?”*

ABSTRACT. We consider the principle agent problem of a bank deciding whether it is advantageous to escrow traders’ bonuses. The escrow model is the following. Each year a trader manages a portfolio for a bank. At the end of the year, based on the trader’s performance, the bank pays a bonus into an escrow account. If he has not had a disastrous year, the trader can consume from the bonus paid the previous year. Within each year the trader has a continuous-time optimal control problem, and the problem across years is set up as infinite-horizon discounted dynamic programming. It is shown that is sometimes to the bank’s advantage to escrow the bonus, and sometimes it is not. (This is work in progress with Jing Wang.)

### **Mihai Sîrbu**

*“A new look at zero-sum stochastic differential games”*

ABSTRACT. We revisit the well studied problem of zero-sum stochastic differential games. We consider a symmetric formulation over (path-dependent) feed-back strategies, carefully restricted to produce strong solutions of the state equations. Using a stochastic modification of Perron’s method we show that the lower and upper values are solutions to the corresponding Hamilton-Jacobi-Bellman-Isaacs equations. If the Isaacs condition holds, the game has a value over such pure feed-back strategies. If the Isaacs condition does not hold, then generalized/mixed strategies need to be considered to obtain a value.

## Ronnie Sircar

*“Optimal Investment with Transaction Costs and Stochastic Volatility”*

ABSTRACT. Two major financial market frictions are transaction costs and uncertain volatility, and we analyze their joint impact on the problem of portfolio optimization. When volatility is constant, the transaction costs optimal investment problem has a long history, especially in the use of asymptotic approximations when the cost is small. Under stochastic volatility, but with no transaction costs, the Merton problem under general utility functions can also be analyzed with asymptotic methods. Here, we look at the long-run growth rate problem when both frictions are present, using separation of time scales approximations. This leads to perturbation analysis of an eigenvalue problem. We also describe some related infinite horizon problems where these methods can be effective.

## Peter Tankov

*“Optimal discrete-time hedging with directional views, or how to make some money while hedging your option”*

ABSTRACT. We consider the hedging error due to discrete-time rebalancing of a given hedging strategy, in the presence of drift of the underlying asset price. The problem is to choose the optimal rebalancing times. Under the mean-variance criterion, the choice of rebalancing times is reduced to a non-degenerate Linear-Quadratic optimal control. A modified version of the Sharpe ratio is also considered and we obtain explicit expressions for the optimal rebalancing times. These results are obtained under an asymptotic framework. (Joint with J. Cai, M. Fukasawa and M. Rosenbaum.)

## Josef Teichmann

*“FTAP for large financial markets”*

ABSTRACT. For a sequence of (small) markets defined on a single stochastic basis we can characterize the existence of an equivalent separating measure by a condition reminiscent to NFLVR. In particular no weak star closures are needed to state this condition. (Joint work with Christa Cuchiero and Irene Klein.)

## Hao Xing

*“Existence of close to Pareto optimal incomplete Radner equilibrium”*

ABSTRACT. We consider an equilibrium model between exponential investors whose random endowments cannot be spanned by the traded asset. We first characterize the set of endowments which induce Pareto optimal equilibrium. For endowments close to this set, we establish three existence results of equilibria which are not Pareto optimal. In a non-Markovian setting, the first existence result is established by analysing a system of coupling quadratic BSDEs, via the techniques introduced by Tevzadze (2008). Then the first result is improved by a BMO-norm estimate in the second result. In a Markovian setting, equilibrium is established using partial regularity results for system of parabolic PDEs with quadratic nonlinearity in gradient. (This is work in progress with Constantinos Kardaras and Gordan Žitković.)

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