A Framework For Problem Standardization and Algorithm Comparison in Multibody Dynamics

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This work is in collaboration with

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Outline

- Benchmark Problems for Multibody Dynamics (BPMD)
- Different models and solvers in our framework
- Results for comparison of different algorithms with problems from simulators and physical experiments
- Conclusions & Future work

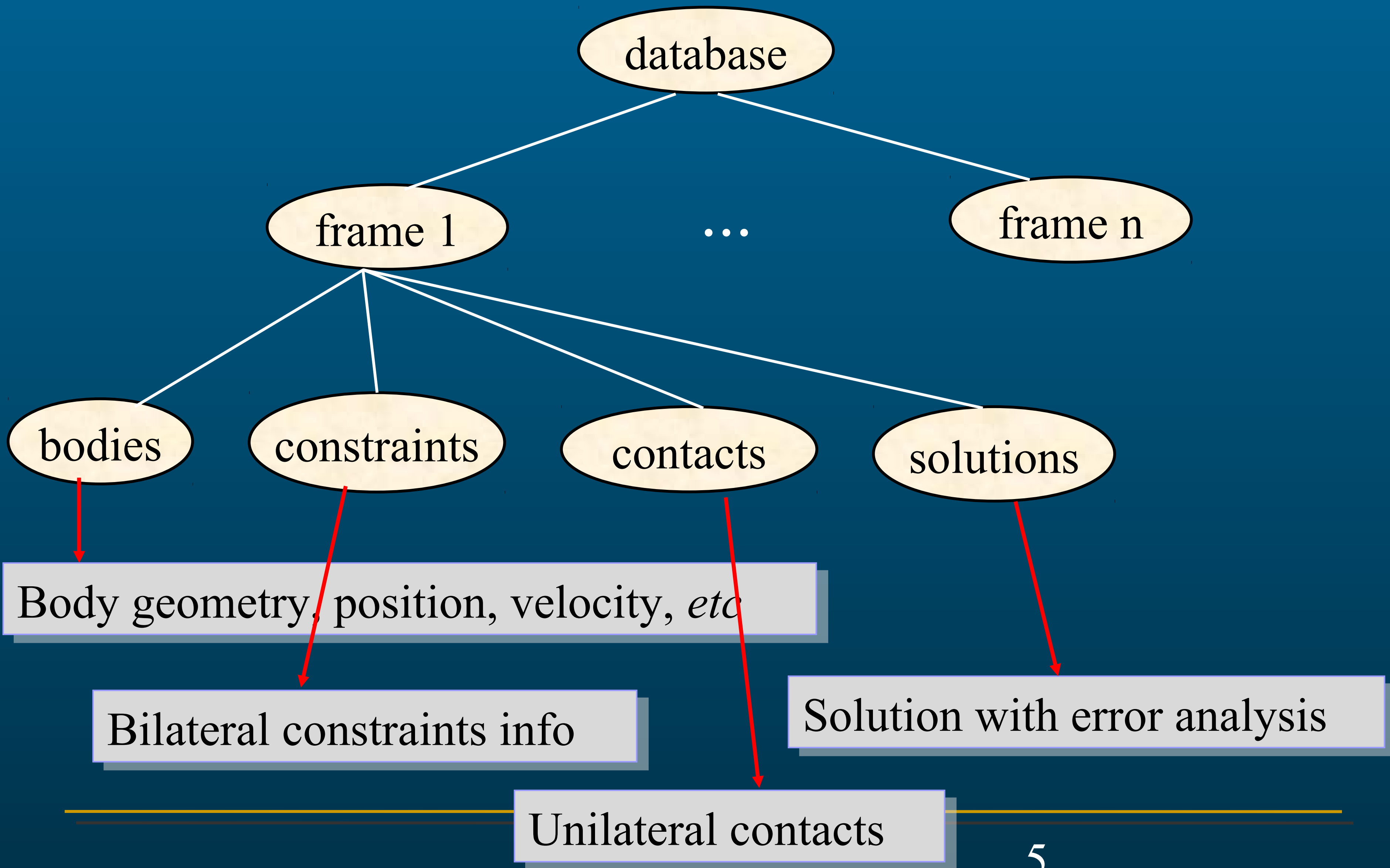
Why Compare Simulation Algorithms

- The Complementarity Problem (CP) takes the form of $0 \le x \perp f(x) \ge 0$
- The accuracy and efficiency in solving the CPs play an important role in simulation.
- In order to fairly compare different algorithms, we have to collect a set of benchmark testing problems.

Why Compare Simulation Algorithms

Different Physics engines → Different models Different solvers Different test problems → Which algorithm is best ?

BPMD database



BPMD Standard Table

Fields	Names	Math Notation	Sizes	Physical meaning
bodies	ids	$\boldsymbol{i}$	$n_b \times 1$	unique body id
	masses	M	$n_b \times 1$	masses
	forces	$\boldsymbol{\lambda} = \begin{bmatrix} f \\ \boldsymbol{\tau} \end{bmatrix}$	$6n_b \times 1$	external forces
	inertia	$\boldsymbol{I}$	$3n_b \times 3$	inertia
	positions	$\boldsymbol{u} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$	$3n_b \times 1$	body positions
	quaternions	$Q = \begin{bmatrix} q_{rel} \\ q_{img} \end{bmatrix}$	$4n_b \times 1$	body orientations
	velocities	$\boldsymbol{\nu} = \begin{bmatrix} v \\ w \end{bmatrix}$	$6n_b \times 1$	generalized velocities

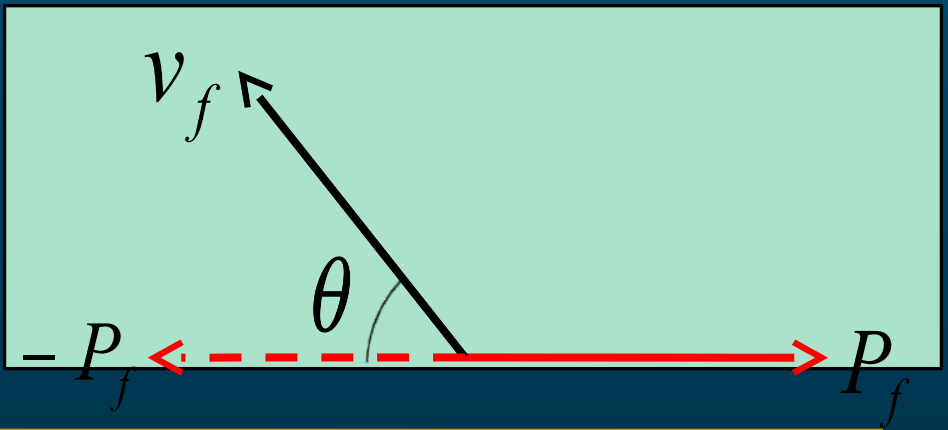
BPMD Standard Table (cont'd)

Fields	Names	Math Notation	Sizes	Physical meaning
bilateral constraints (n_j) (joints)	violations	ϕ	$n_j \times 1$	joint violations
	jacobians	J	$(n_j \times n_{type}) \times 3$	Jacobians
	pairs	$pair$	$n_j \times 2$	body pair ids
	bounds	$bound$	$n_j \times 2$	joint limit
	rows	row	$n_{type} \times 3$	Jacobian rows
unilateral constraints (n_c) (contacts)	gap	ψ	$n_c \times 1$	gap distance
	pairs	$pair$	$n_c \times 2$	body pair ids
	mu	μ	$n_c \times 1$	friction coefficient
	normals	$\hat{n}$	$n_c \times 3$	contact normals
	points	$\hat{p}$	$n_c \times 3$	contact points

BPMD Standard Table (cont'd)

Fields	Names	Math Notation	Sizes	Physical meaning
solutions	iterations	$iter$	$frames \times 1$	solver iterations
	total_error	$totalErr$	$iter \times 1$	total errors
	normal_error	$normErr$	$iter \times 1$	normal errors
	friction_error	$fricErr$	$iter \times 1$	friction errors
	stick	$stick$	$iter \times 1$	state of contact

$$fricErr = \sum_{i=1}^{n_c} \left\| P_{fi} \right\| \cdot \left\| v_{fi} \right\| \cdot \left\| \theta_i \right\|$$



BPMD Framework

- Modeling Coordinates
  - Maximal Coordinate (Redundant Coordinate)
  - Minimal Coordinate (Reduced Coordinate)
- Formulation Models
  - mNCP
  - LCP
  - mLCP
- Algorithms that solve CPs

Reformulation Functions

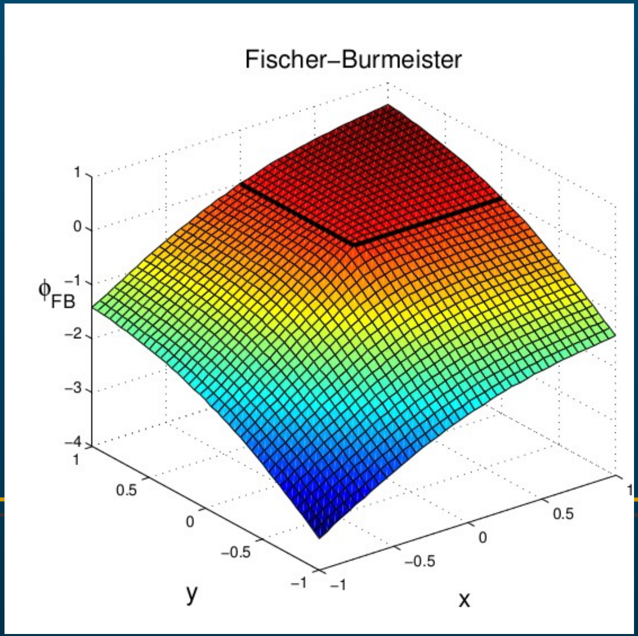
CP: $0 \le x \perp f(x) \ge 0$ Let: $y = f(x)$

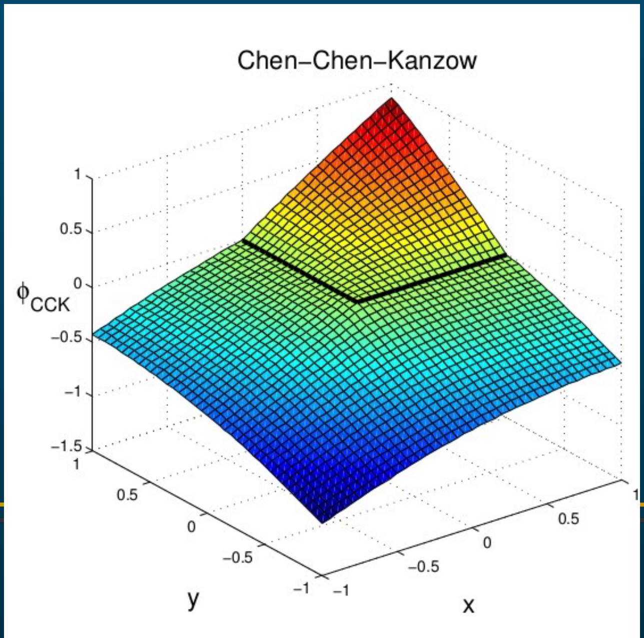
- Fischer-Burmeister function (FB)

$$\phi_{FBi}(x_i, y_i) = x_i + y_i - \sqrt{x_i^{\,2} + y_i^{\,2}} = 0$$

- Chen-Chen-Kanzow function (CCK)

$$\phi_{CCKi}(x_i, y_i, \lambda) = \lambda \phi_{FBi}(x_i, y_i) + (1-\lambda) x_i^+ y_i^+ = 0$$



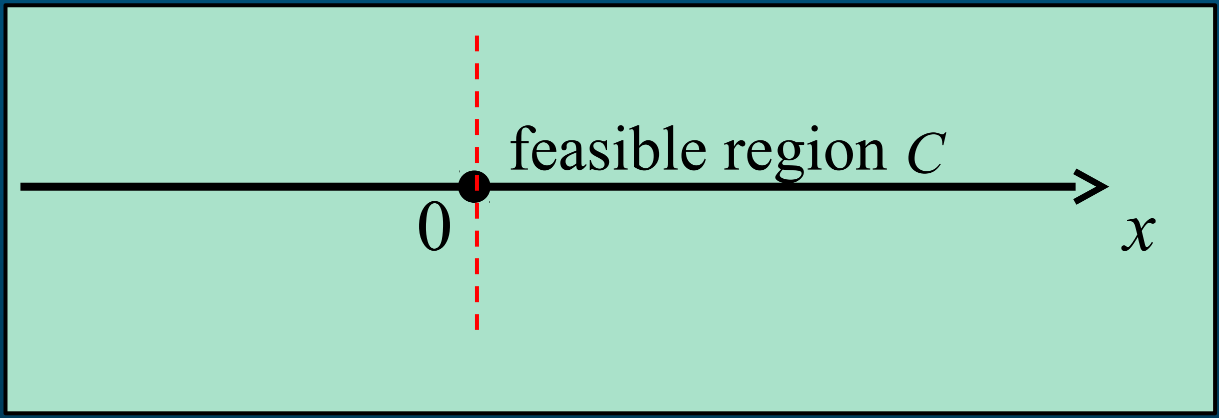


Reformulation Functions (cont'd)

CP: $0 \le x \perp f(x) \ge 0$ Let: $y = f(x)$

- The prox function

$$\phi_{prox}(x, y, r) = x - prox_C(x - ry) = 0$$



- Minimum-map function

$$\phi_{\min}(x, y) = \min(x, y) = 0$$

Available Methods & Solvers

Formulation	Available Solvers
LCP	Lemke
	Projected Gauss Seidel
	Nonsmooth Newton (FB)
	Nonsmooth Newton (CCK)
	Nonsmooth Newton (min)
mLCP	PATH
	Fixed-point iteration (prox)
mNCP	Fixed-point iteration (prox)
	Nonsmooth Newton (FB)
	Nonsmooth Newton (CCK)
	Implicit
Convex model	Solver for convex problems

Error Metric

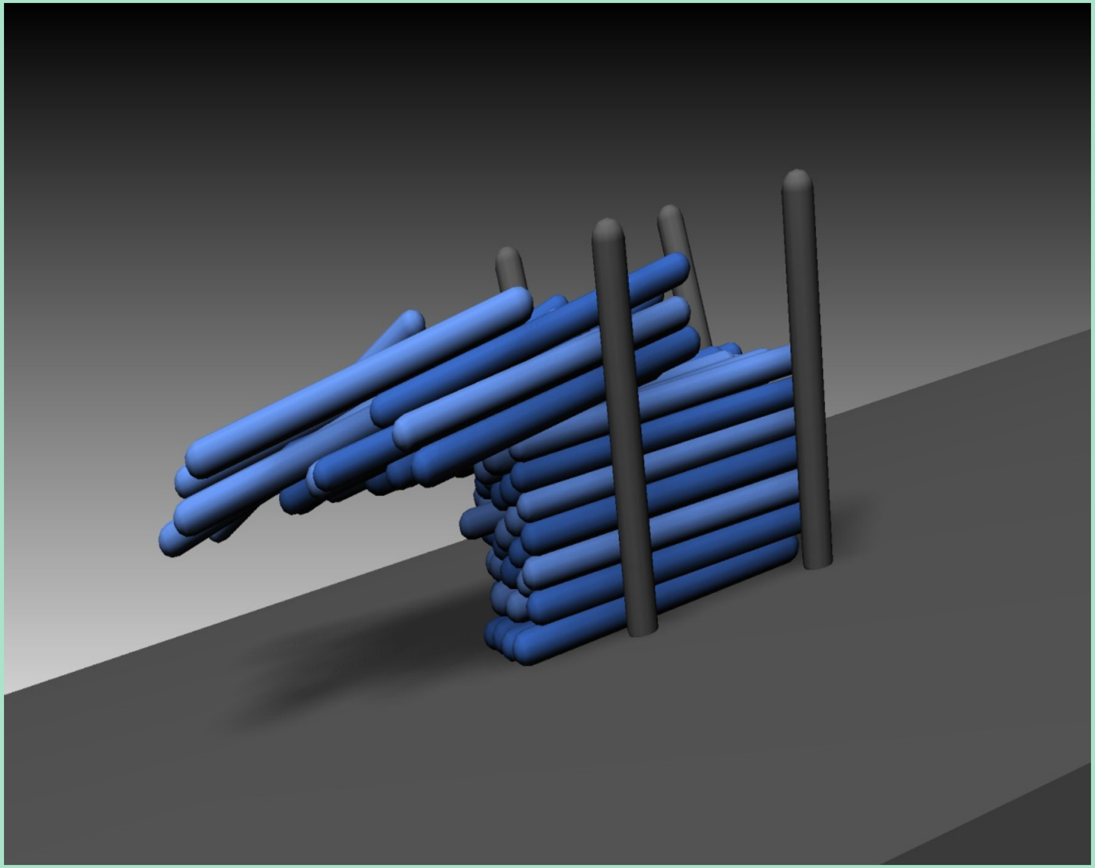
In order to compare the different models and solvers in an unbiased way, we have defined several standard error metric:

$$err_1 = \frac{1}{2}\phi_{FB}{}^T\phi_{FB}$$

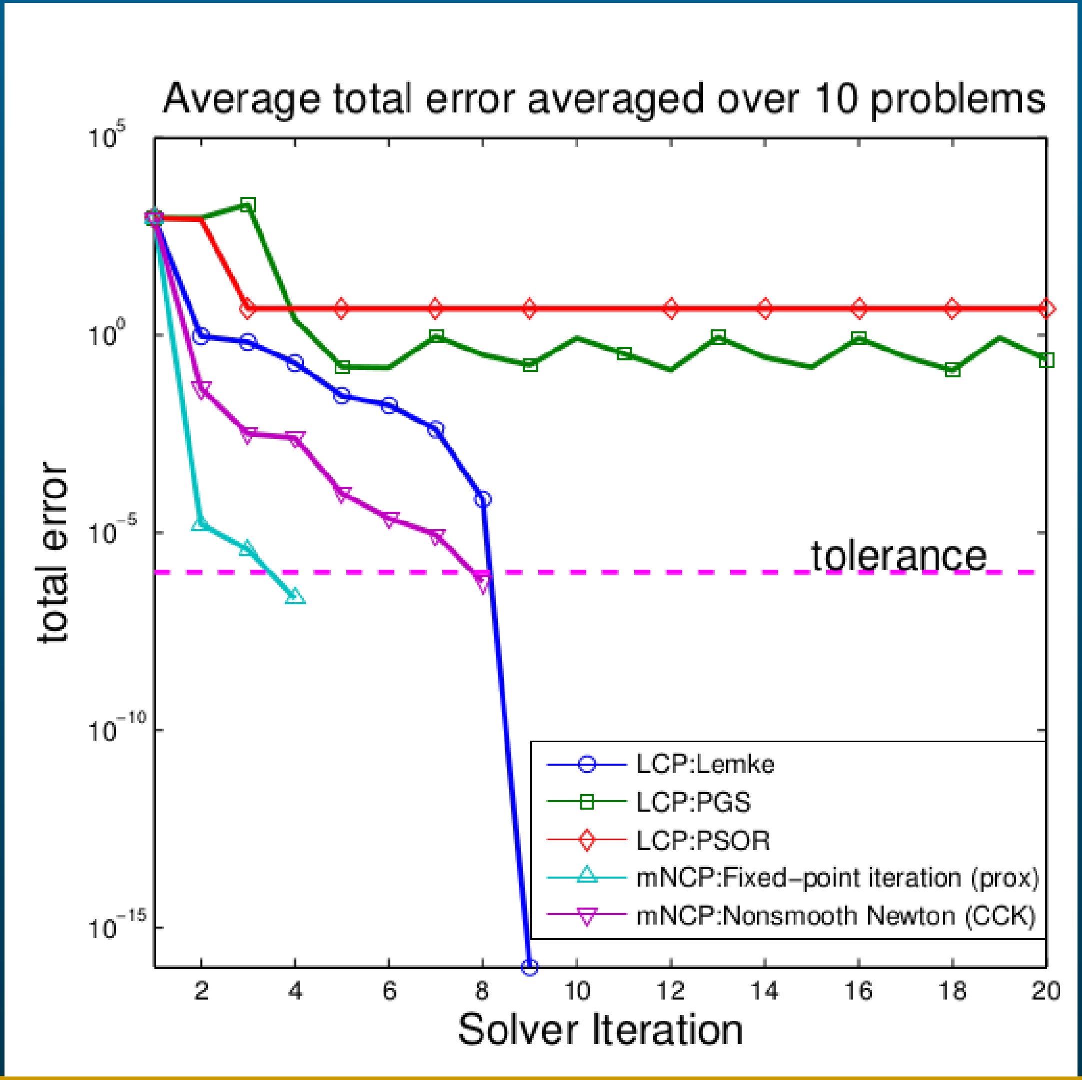
$$err_2 = \frac{1}{2}\phi_{CCK}{}^T\phi_{CCK}$$

$$err_3 = \frac{1}{2}\phi_{CCK}{}^T\phi_{CCK} + \sum_{i=1}^{n_c}\left\|P_{fi}\right\|\cdot\left\|\nu_{fi}\right\|\cdot\left\|\theta_i\right\|$$

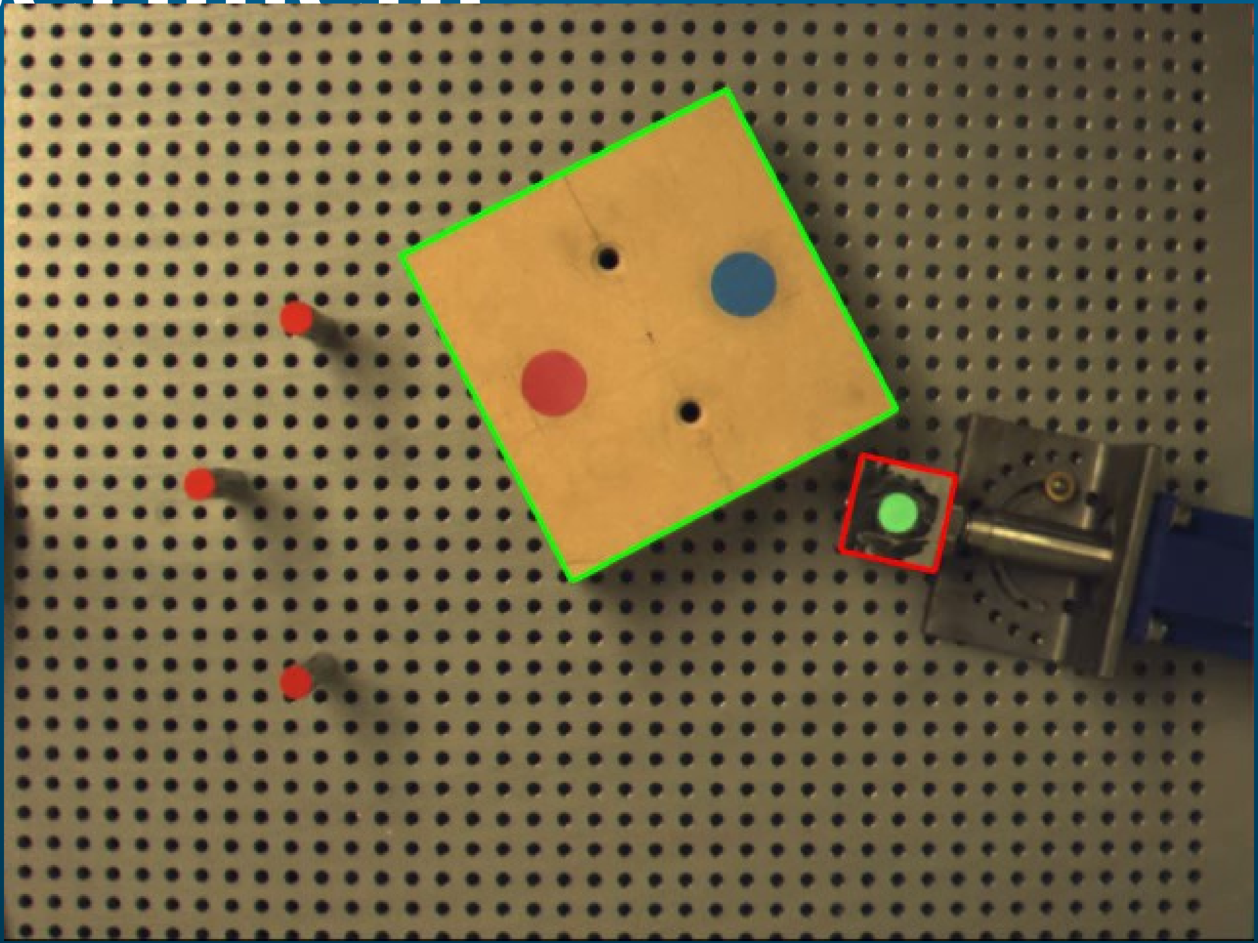
Logs Piling from Agx



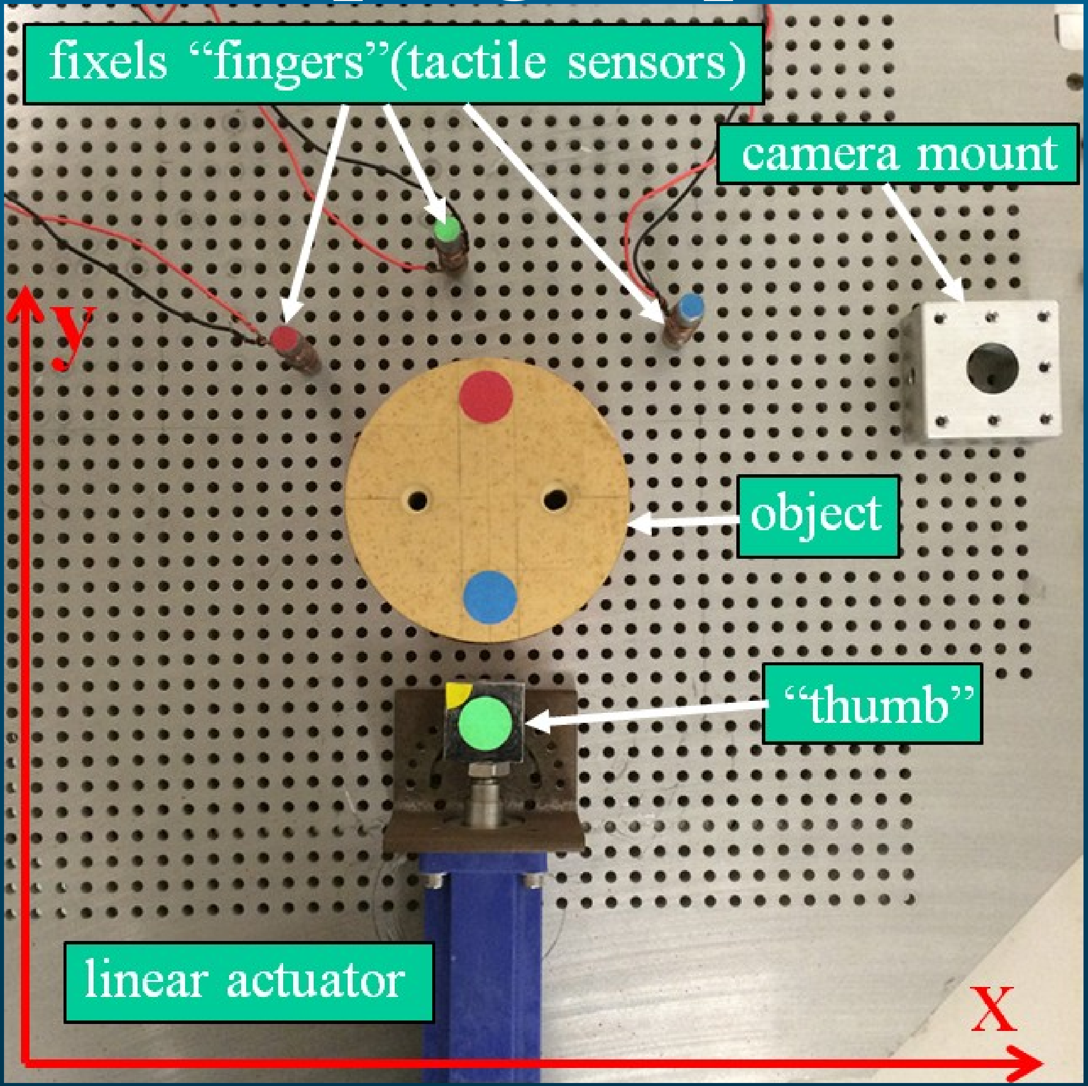
Results for Logs Piling



Planar Grasping Experiment



Planar Grasping Experiment



Initial experimental set up

Simulation Procedure

- Parameter Estimation: use the input from physical experiments to drive the simulation, uncertain model parameters such as friction coefficients are identified.
- Solver Comparison: with the estimated parameters, various solvers are evaluated to measure their suitability to predict the physical behavior, by comparing with experimental results

Parameter Estimation

The parameters are estimated by solving: $\min E$

$$E = \frac{1}{M}\sum_{j=1}^{M}\frac{1}{N_j}\left[\left(q_j^0-\bar{q}_j^0\right)^T\left(q_j^0-\bar{q}_j^0\right)\right]+\sum_{\ell=1}^{N_j}\left[\left(q_j^\ell-\bar{q}_j^\ell\right)^T\left(q_j^\ell-\bar{q}_j^\ell\right)\right]$$

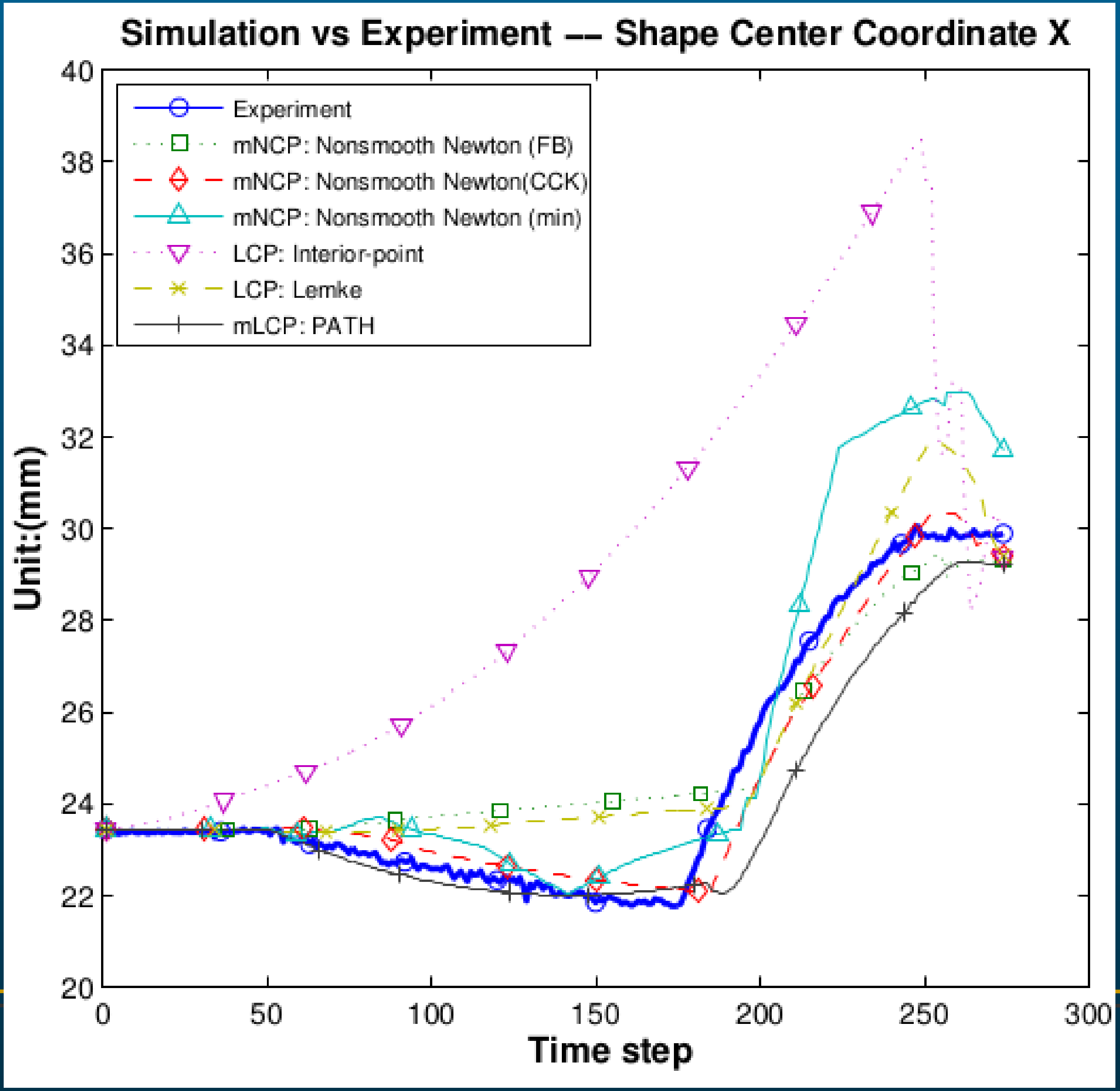
- q^ℓ is the object position in simulation at $\ell - th$ simulation step
- $\bar{q}^\ell$ is the object position measured in physical experiment
- M is the total number of experiments used for calibration
- N_j is the total number of frames in the $j-th$ experiment
- E is the error to be minimized

Parameter Estimation

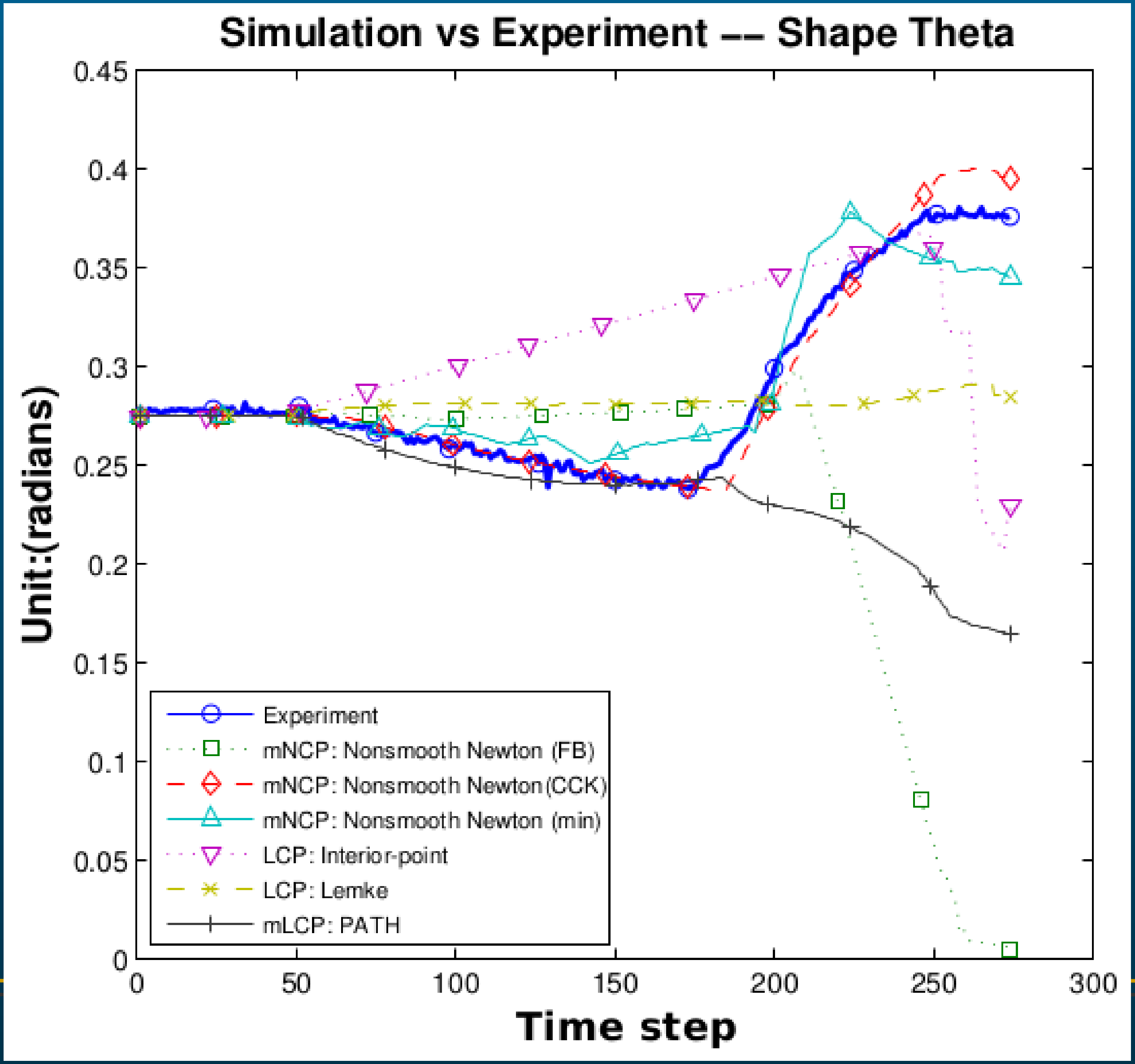
Solvers	μ_p	μ_f	μ_s	R_{tri}
mNCP: Nonsmooth Newton (FB)	0.1	0.8	0.1	25
mNCP: Nonsmooth Newton (CCK)	0.4	0.2	0.6	45
mNCP: Nonsmooth Newton (min)	0.6	0.3	0.1	45
LCP: Interior-point method	0.2	0.6	0.1	15
LCP: Lemke	0.1	0.1	0.3	65
mLCP: PATH	0.2	0.4	0.1	65

- μ_p is the friction coefficient between the object and pusher
- μ_f is the friction coefficient between the object and fingers
- μ_s is the friction coefficient between the object and surface
- R_{tri} is tripod radius, which allows for a simpler point-contact model of the contact between object and the surface

Results – position X



Orientation



Conclusions

- Nonsmooth Newton method with CCK function predicts the trajectory most close to experiment.

Future Work

- Improve the parameter estimation procedure
- Expand benchmark problem sets
- Include more formulations
- Include more solvers

Thank you !