Integer Sequences K–12

Gordon Hamilton (MathPickle), Neil Sloane (OEIS Foundation)

February 27–March 1, 2015

The On-Line Encyclopedia of Integer Sequences (OEIS) has many pedagogic gems that remain undiscovered by K–12 educators. These sequences need to be lifted out of obscurity and become a part of every child's experience of mathematics.

The primary objective of the Integer Sequences K–12 conference was to help unearth these gems by finding 13 integer sequences—one for each grade K–12. Here is the list we came up with:

Kindergarten

A034326 Hours struck by a clock.

$1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, \ldots$

Grade 1

A030227 Number of n-celled polyominoes with bilateral symmetry.

$1, 1, 2, 3, 6, 10, 20, 34, 70, 121, 250, 441, 912, 1630, 3375, \ldots$

Grade 2

A243205 The Nasty Mr. Sneeze: consider the $n \times n$ Go board as a graph; remove i nodes and let j be the number of nodes in the largest connected subgraph remaining; then $a(n) = minimum(i + j)$.

$1, 3, 5, 9, 12, 16, 20, 25, 29, 36?, 41?, 47?, \ldots$

Grade 3

A254873 Recamán [$\div, -, +, \times$]: starting at the seed number, 14, the sequence continues by dividing, subtracting, adding or multiplying by the step number, 2. Division gets precedence over subtraction which gets precedence over addition which gets precedence over multiplication. The new number must be a positive integer and not previously listed. The sequence terminates if this is impossible.

$14, 7, 5, 3, 1, 2, 4, 6, 8, 10, 12, 24, 22, 11, 9, 18, 16, 32, 30, 15, 13, 26, 28, 56, 54, 27, 25, 23, 21, 19, 17, 34, 36, 38, 40, 20.$ ⟨terminated⟩

Grade 4

A071983 Square chains: the number of permutations (reversals not counted as different) of the numbers 1 to n such that the sum of any two consecutive numbers is a square. Starting with $a(15) = 1$, the sequence is:

$1, 1, 1, 0, 0, 0, 0, 0, 3, 0, 10, 12, 35, 52, 19, 20, 349, \ldots$

OR (some of us want to try these in the classroom)

A253472 Numbers n such that $1, 2, ..., 2n$ can be partitioned into n pairs, where each pair adds up to a perfect square.

$4, 7, 8, 9, 12, 13, 14, 15, 16, \ldots$

Grade 5

A256174 Boomerang Fractions: starting with 1, on the first step add $1/n$, and on subsequent steps either add $1/n$ or take the reciprocal. $a(n)$ = fewest number of steps required to return to 1. (The sequence starts with $a(2)$.)

$4, 9, 7, 20, 6, 33, 13, 23, 16, 62?, 8, 75?, 18, 17, 25, \ldots$

Grade 6

A125508 Integral Fission: a prime factorization tree in which every pair of children is chosen so they are as equal as possible and the largest child goes on the right. $a(n)$ are the lowest numbers for which a new tree shape is encountered.

$2, 4, 8, 16, 20, 32, 40, 64, 72, 88, 128, 160, 176, 200, 220, 256, 272, 288, 320, 336, 360, 400, 420, 460, 480, 512, 540, 544, 640, 704, 864, 880, 920, \ldots$

Grade 7

A039834 Fibonacci numbers (A000045) extended to negative indices. $F(0) = 0$ in the sequence below.

$1, 1, 0, 1, -1, 2, -3, 5, -8, 13, -21, 34, -55, 89, -144, \ldots$

Grade 8

Similar to A226595 Lengths of maximal non touching increasing paths in $n \times n$ grids.

$0, 2, 4, 7, 9, 12, 15, 17, 20, \ldots$

$\ldots$ but actually

"Lengths of maximal non touching increasing paths in $n \times n$ grids starting at the upper left and ending at the lower right."

$0, 2, 4, 6, 9, 12?, 15?, 17?, 20, \ldots$

Grade 9

A069283 The number of ways that n can be written as the sum of at least two consecutive positive integers.

$0, 0, 0, 1, 0, 1, 1, 1, 0, 2, 1, 1, 1, 1, 1, 3, 0, 1, 2, 1, 1, 3, 1, 1, 1, 2, 1, 3, 1, 1, 3, 1, 0, 3, 1, 3, 2, 1, 1, 3, 1, 1, 3, 1, 1, 5, \ldots$

Grade 10

A225745 Smallest k such that n numbers can be picked in $\{1, \ldots, k\}$ with no four in arithmetic progression.

$1, 2, 3, 5, 6, 8, 9, 10, 13, 15, 17, 19, 21, 23, 25, 27, 28, 30, 33, 34, 37, 40, \ldots$

Grade 11

A000108 Catalan numbers: $C(n) = \text{binomial}(2n, n)/(n+1) = (2n)!/(n!(n+1)!)$.

$1, 1, 2, 5, 14, 42, 132, 429, 1430, \ldots$

Grade 12

A000127 Maximal number of regions obtained by joining n points around a circle by straight lines. Also number of regions in 4-space formed by $n - 1$ hyperplanes.

$1, 2, 4, 8, 16, 31, 57, 99, 163, 256, \ldots$

These are classroom ambassadors for Integer Sequences. The list took three weeks of emails after the conference before everyone was happy, but in the end we had consensus or near consensus and where differences of opinion remain, they are mild.

The secondary and more time intensive goal—soon to be started—is to develop and distribute teacher-friendly resources so that these integer sequences get the wide exposure they deserve.

Working documents around the Integer Sequences K-12 conference have been set up by Neil Sloane on OEIS: `https://oeis.org/w/index.php?title=Integer_Sequence_K-12_(Banff,_2015)`